Principle of Rotating machines

Let the fluid enters the rotor at a radius r_1, then the rate of angular momentum at inlet is $\dot{m} V_{w_1} r_1$, where V_{w_1} is whirl component or tangential component of absolute velocity at Inlet. Similarly, if the fluid leaves the rotor at a radius r_2 then rate of angular momentum at exit is $\dot{m} V_{w_2} r_2$.

Rate of change of Angular momentum

$$= \dot{m} V_{w_2} r_2 - \dot{m} V_{w_1} r_1$$

But we know that

Rate of change of angular momentum = Torque

$$\Rightarrow \dot{m} V_{w_2} r_2 - \dot{m} V_{w_1} r_1 = T$$

$$\text{Power} = T \times w$$

$$P = (\dot{m} V_{w_2} r_2 - \dot{m} V_{w_1} r_1) w$$

$$P = \dot{m} [V_{w_2} r_2 w - V_{w_1} r_1 w]$$

$$\boxed{P = \dot{m} [V_{w_2} u_2 - V_{w_1} u_1]}$$ **

euler's equation for compressor and pump.

**

$$\boxed{P = \dot{m} [V_{w_1} u_1 - V_{w_2} u_2]}$$ For Turbine

When energy is transferred from fluid to rotor, then the device is turbine.

When energy is transferred from rotor to fluid then the device is compressor or pump.

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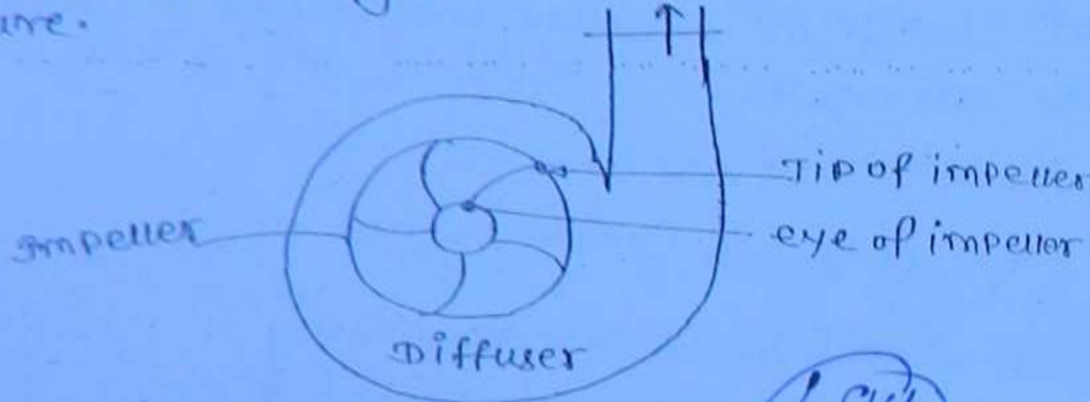
Centrifugal Compressor

Principle of operation of a centrifugal compressor ⇒

In centrifugal compressor, the fluid enters axially at the eye of impeller and leaves radially at the tip of impeller. When the fluid moves radially outwards, ue to centrifugal force the pressure increases. The mpeller also imparts kinetic energy to the fluid and ence the fluid leaves the impeller at high kinetic nergy and high pressure energy. The fluid after leaving he impeller enters diffuser, where K.E. of fluid is onverted into pressure energy. Thus the fluid leaves at high pressure.



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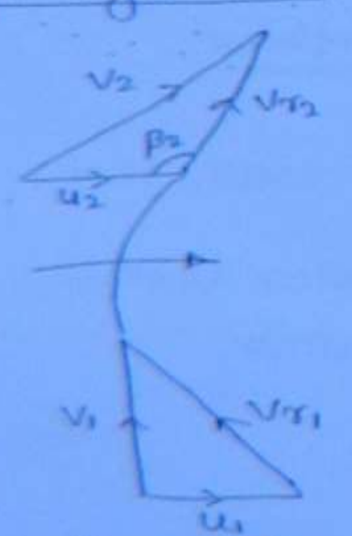
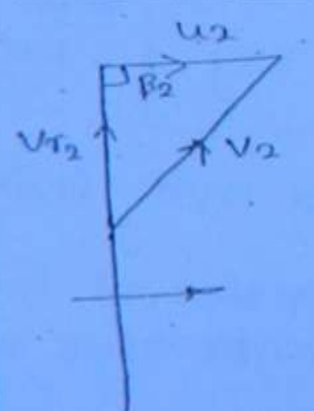
Blade Angle Triangles

orward Angle Blade



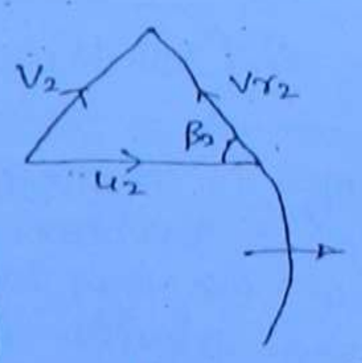
$\beta_2 > 90°$

Radial Blade



$\beta_2 = 90°$

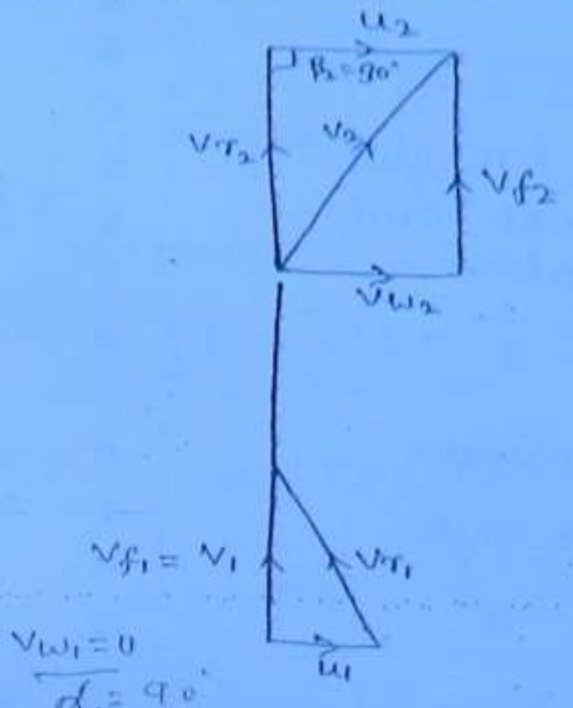
Backward angle blade



$\beta_2 < 90°$

If Nothing is mentioned then assume radial blade.

(#) Power Input for Radial Blades ⇒



$$P = \dot{m}\left[V_{w_2} u_2 - V_{w_1} u_1\right]$$

But $V_{w_1} = 0$

$$P = \dot{m}\left[V_{w_2} u_2\right]$$

But $V_{w_2} = u_2$

$$\therefore \boxed{P = \dot{m} u_2^2}$$

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(#) Slip in centrifugal compressor :⇒

Due to Inertia the fluid Particles are reluctant to move around the blade passage and because of those there is reduction in V_{w_2}.

$$\text{Slip} = u_2 - V_{w_2}$$

$$\text{Slip factor } \sigma = \frac{V_{w_2}}{u_2}$$

$$P = \dot{m}\left[V_{w_2} u_2 - V_{w_1} u_1\right]$$

for radial blade $V_{w_1} = 0$

$$P = \dot{m}\left[V_{w_2} u_2\right] \quad \text{but } V_{w_2} = \sigma u_2$$

$$\therefore \boxed{P = \dot{m} \sigma u_2^2}$$

(#) Power Input factor (ψ) ⇒

To account for frictional losses power input factor is introduced and it is the ratio of actual power input to the theortical power input

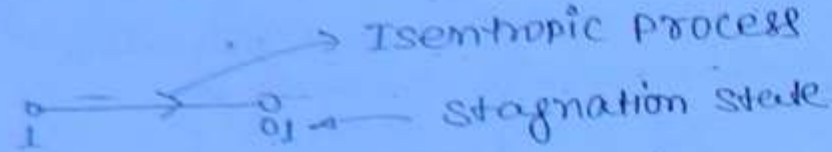
$$\boxed{P = \dot{m} \sigma \psi u_2^2} \qquad \psi = 1.035 \text{ to } 1.05$$

→ If ψ and σ are not given in the problem, then take them unity.

USE OF SFEE IN Centrifugal compressor ⇒

Stagnation State : When the fluid is brought to zero velocity isentropically then that state reached is known as stagnation state.

$$h_{01} = h_1 + \frac{V_1^2}{2}$$



$$h_1 + \frac{V_1^2}{2} + z_1 g + Q = h_2 + \frac{V_2^2}{2} + g z_2 + W$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} + W$$

$$h_{01} = h_{02} + W$$

$$W = h_{01} - h_{02}$$

$$W = -(h_{02} - h_{01})$$

$$W_{input} = h_{02} - h_{01}$$
$$= C_p (T_{02} - T_{01})$$

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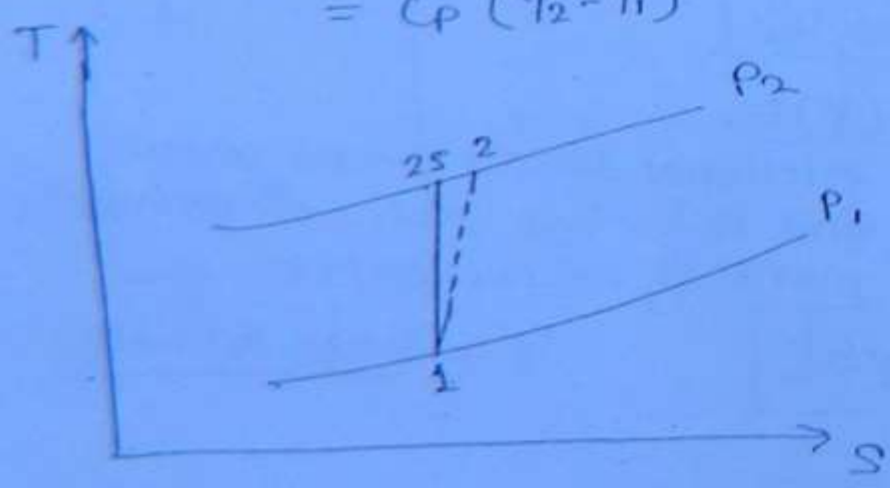
If kinetic energy is neglected then

$$W_{input} = h_2 - h_1$$
$$= C_p (T_2 - T_1)$$



$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}}$$

** We know:

$$\boxed{W = \sigma \psi u_2^2} \rightarrow \frac{m^2}{s^2} \rightarrow J/kg$$

$$W = C_p (T_2 - T_1) \rightarrow kJ/kg$$

$$\text{But } \frac{J}{kg} = \frac{Nm}{kg} = \frac{kg\,m \times m}{s^2\,kg} \Rightarrow \frac{m^2}{s^2}$$

Prewhirl in centrifugal compressor ⇒

To Reduce the mach No. at the inlet prewhirl is given. Generally this prewhirl is mentioned with respect to axial direction.

$\alpha \rightarrow$ prewhirl angle

Here $M = \frac{V_{r_1}}{C}$

for same u_1

** As α increases ↑ ⇒ V_{r_1} ↓

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PROB: ①

A centrifugal compressor has an impeller dia of 0.5 m at the tip and running at 7000 rpm. Assume zero whirl at inlet and $T_1 = 290K$, $\sigma = 1$, $\psi = 1$, and process of compression is isentropic. Determine the pressure ratio developed and specific work input. Take $C_p = 1.005$ KJ/kg-K

Soln.

$D_2 = 0.5$ m, $N = 7000$ rpm

$\alpha = 0$, $T_1 = 290K$, $\sigma = 1$, $\psi = 1$

$$u_2 = \frac{\pi D_2 N_2}{60} = 183.25 \text{ m/s}$$

$W = \sigma \psi u_2^2 = 1 \times 1 \times (183.25)^2$

$= 33584 \ J/kg$

$\boxed{W = 33.584 \ kJ/kg}$

Also $W = h_2 - h_1 = C_p (T_2 - T_1)$

$33.584 = 1.005 (T_2 - 290)$

$\boxed{T_2 = 323.4 \ K}$

$\left(\frac{P_2}{P_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{323.4}{290}\right)^{\frac{1.4}{1.4-1}}$

Pressure ratio = 1.465

PROB. ② Determine the isentropic efficiency of compressor which under test gave following result N = 11500 rpm, $T_1 = 21^\circ C = 273 + 21 = 294 K$, pressure ratio = 4, Impeller tip dia. $D_2 = 0.75 m$, slip factor $\sigma = 0.92$

m

$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.75 \times 11500}{60}$

(198)

$\boxed{u_2 = 451.6 \ m/s}$

$W = \sigma \psi u_2^2 = 0.92 \times 1 \times (451.6)^2$

$= 187630 \ J/kg$

$= 187.63 \ kJ/kg$

Also $W = C_p (T_2 - T_1)$

$187.63 = 1.005 (T_2 - 294)$

$\boxed{T_2 = 480.7 K}$

$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$\frac{T_{2s}}{294} = (4)^{\frac{1.4-1}{1.4}}$

$T_{2s} = 436.9 K$

Isentropic efficiency $\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$ $\overset{\shortparallel}{3}$

$$\eta_c = \frac{436.9 - 294}{480.7 - 294}$$

$$\boxed{\eta_c = 76.5\%}$$

I.E.S. 2009

PROB. (3) A centrifugal compressor running at N = 16000 rpm takes air at 17°C and compressor it to a pressure ratio of 4 with an isentropic efficiency of 82%. At the exit the blade are radially inclined and slip factor is 0.85. Guide vanes at inlet give air a prewhirl angle of 20° to the axial direction. The dia of impeller eye is 200 mm and absolute velocity ~~impeller eye is 200 mm~~ of air at inlet is 120 m/sec find impeller tip dia.

Soln $N = 16000$ rpm, $T_1 = 17°C = 273 + 17 = 290K$,

$\frac{P_2}{P_1} = 4$, $\eta_c = 82\%$, $\sigma = 0.85$, $\alpha = 20°$

$D_1 = 200 mm = 0.2 m$, $V_1 = 120$ m/sec.

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 16000}{60} = 167.55 \text{ m/s}$$

$$V_{w1} = V_1 \sin\alpha = 120 \times \sin 20°$$

$$V_{w1} = 41.04 \text{ m/s}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_{2s}}{290} = 1.486$$

$$\boxed{T_{2s} = 430.9 K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$0.82 = \frac{430.9 - 290}{T_2 - 290}$$

$$\boxed{T_2 = 461.8K}$$

$C_p (T_2 - T_1)$

$= 1.005 (461.8 - 290)$

$W = 172.688 \text{ KJ/kg}$

But also $W = (V_{w_2} u_2 - V_{w_1} u_1)$

But $V_{w_2} = \sigma u_2$

$10^3 \times 172.688 = [0.85 \times u_2^2 - 41.04 \times 167.55]$

$u_2^2 = 211252.6 \text{ m}^2/\text{s}^2$

$\boxed{u_2 = 459.6}$ m/s

But $u_2 = \frac{\pi D_2 N}{60}$

$459.6 = \frac{\pi \times D_2 \times 16000}{60}$

$D_2 = 0.549 \text{ m}$

O/P dia. $\boxed{D_2 = 549 \text{ mm}}$ Ans.

[illegible]

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POWER PLANT – [redacted]