DOUBLY REINFORCED BEAMS

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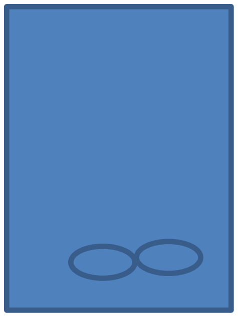
INTRODUCTION

- Concrete has very good compressive strength and almost negligible tensile strength. Hence, steel reinforcement is used on the tensile side of concrete. Thus, singly reinforced beams reinforced on the tensile face are good both in compression and tension.
- LIMITATIONS: MOR UP TO CERTAIN LIMIT-
- When B,D, Grade of Concrete and Grade of Steel is fixed.
- HOW TO INCREASE MOR
- *A) Increase the section-may not be possible in some cases*
- *B) Increase amount of Steel in tension zone A_{stbal} and steel in Compression zone*



THEORY

- MOR= (C or T)* Lever arm
- C is the compressive force of concrete in compression zone,(Limited to a Value) can not be increased more than $C=0.36*f_{ck}*b*x_{umax}$
- T is the tensile force due to steel in tension zone

$T=A_{st}*0.87f_y$

To increase MOR, we have to increase C and T,

And C=T

T can be increased by increasing amount of Ast,

To increase C, we have to add some other material in compressive zone to take compressive force along with concrete.

Let C_1 is the compressive force due to concrete and

C_2 is the compressive force of other material in compression zone,

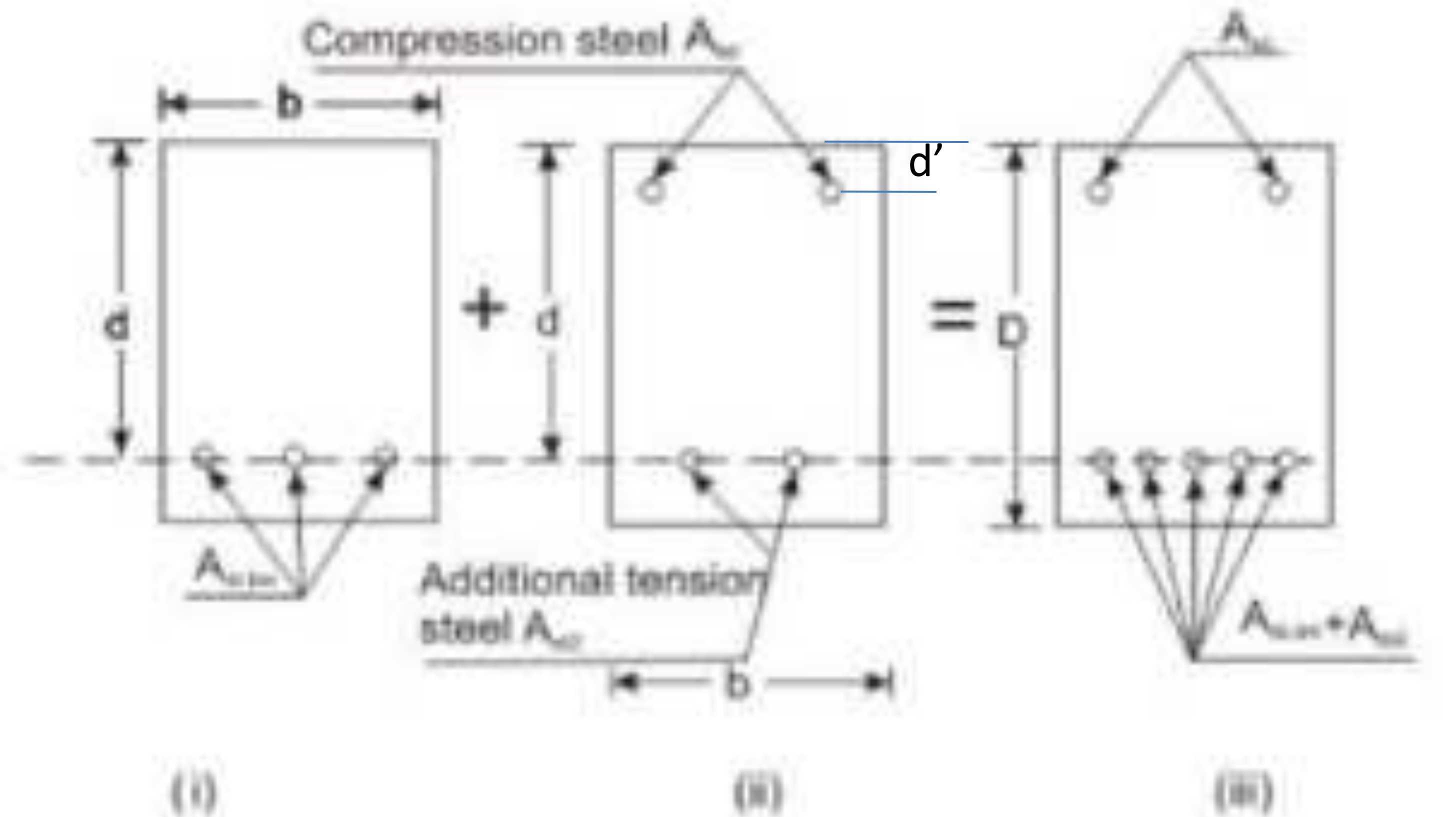
As steel is the material which is very strong in comp and tension, add steel in compression zonein compression zone,

Such reinforced concrete sections having steel reinforcement both on tensile and compressive faces are known as doubly reinforced section.

DOUBLY REINFORCED SECTION

- However, other than in doubly reinforced beams compression steel reinforcement is provided when:
- (i) some sections of a continuous beam with moving loads undergo change of sign of the bending moment which makes compression zone as tension zone or vice versa.
- (ii) the ductility requirement has to be followed.
- (iii) the reduction of long term deflection is needed.

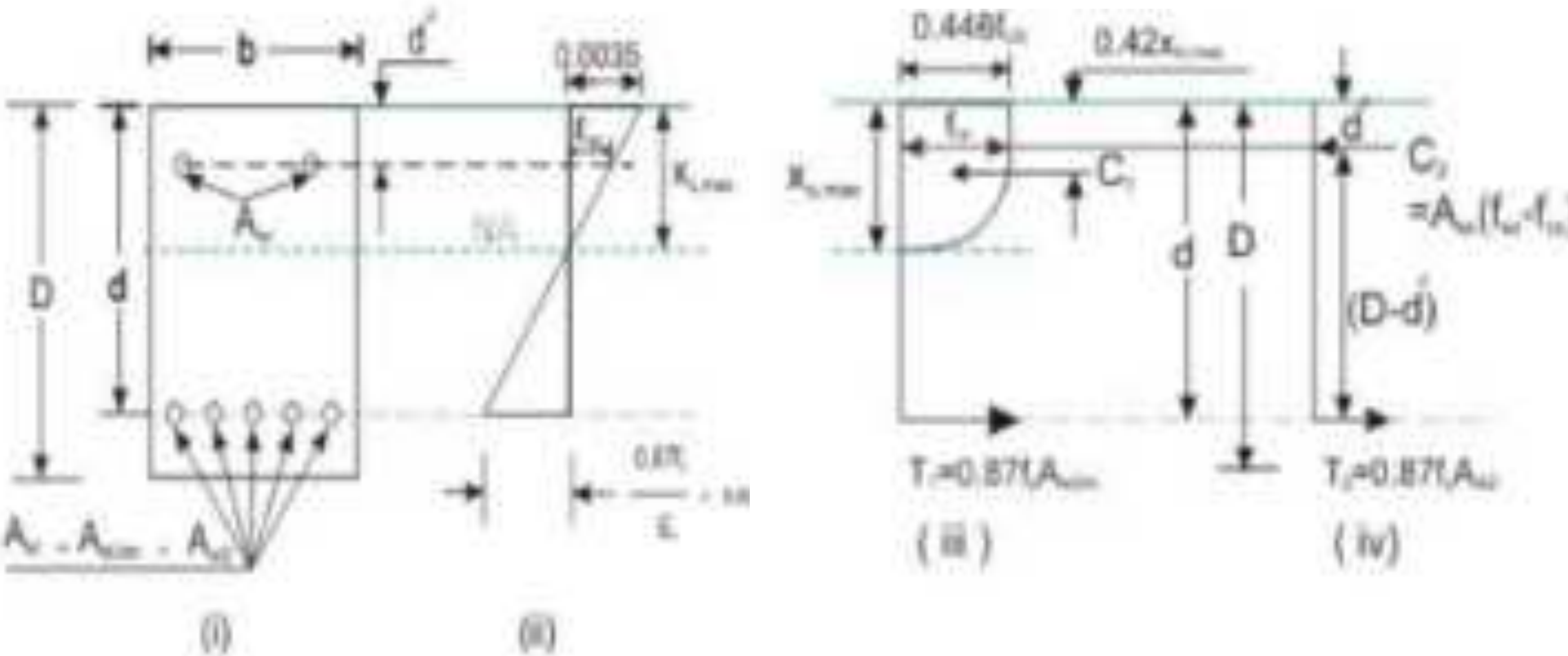
DOUBLY RC BEAMS



Fig(i) M_1=Mulimit=A_{stlim}*0.87f_y*(Lever arm=d-0.42x_{ulimit})
Fig(ii) M_2=A_{sc}*f_{sc}(Lever arm)=A_{st2}*.87f_y(d-d')
Total M=M_1+M_2

DERIVATIONS

- Assumptions
- (i) The assumptions of singly reinforced sections are also applicable here.
- (ii) Provision of compression steel ensures ductile failure .
- The stress-strain relationship of steel in compression is the same as that in tension. So, the yield stress of steel in compression is 0.87 f_y.



(I) Beam cross section

(ii) Strain diagram

(iii) Force diagram of beam of $M_{u,lim}$

(iv) Force diagram of beam of M_{u2}

DOUBLY RC BEAM

- Derivation $M_u = M_{limit} + M_{u2}$

$$M_{ulim} = 0.36 f_{ck}(x_{umax}/d) * [1 - 0.42 * (x_{umax}/d)]bd^2$$

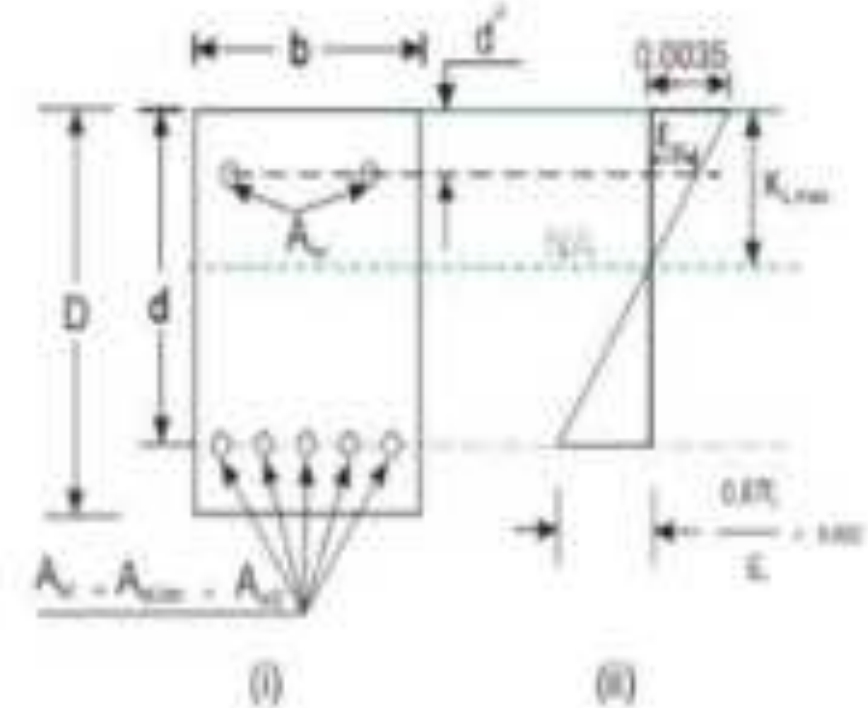
$$M_{ulimit} = T * z = 0.87 f_y * A_{st} * (d - 0.42 x_{ulim})$$

$$M_u = 0.87 f_y * A_{st} * d\left(1 - \frac{f_y * A_{st}}{f_{ck} * b * d}\right)$$

$$M_{u2} = A_{sc}(f_{sc} - f_{cc})(d - d')$$

$$M_{u2} = A_{st2}(0.87\, f_y)(d - d')$$

• Calculation of f_{sc} and f_{cc}



$$\frac{\epsilon_{sc}}{0.0035} = \frac{x_u - d'}{x_u}$$

- For fe250 stress strain diagram is linear, The strain at design yield stress that is ($0.87*fy=217.5$ N/mm^2)/Es=0.0010875
- *When ε_{sc} is less or equal to 0.0010875, $f_{sc} = \varepsilon_{sc} E_s$*
- *When ε_{sc} is greater than 0.0010875, f_{sc} = 217.5 N/mm^2*

DOUBLY RC BEAM

- For fe415 and fe500

Stress level	Fe 415		Fe 500	
	Strain ε_{sc}	Stress f_{sc} (N/mm^2)	Strain ε_{sc}	Stress f_{sc} (N/mm^2)
0.80 f_{yd}	0.00144	288.7	0.00174	347.8
0.85 f_{yd}	0.00163	306.7	0.00195	369.6
0.90 f_{yd}	0.00192	324.8	0.00226	391.3
0.95 f_{yd}	0.00241	342.8	0.00277	413.0
0.975 f_{yd}	0.00276	351.8	0.00312	423.9
1.0 f_{yd}	0.00380	360.9	0.00417	434.8

TYPES OF PROBLEMS

TWO TYPES OF PROBLEMS

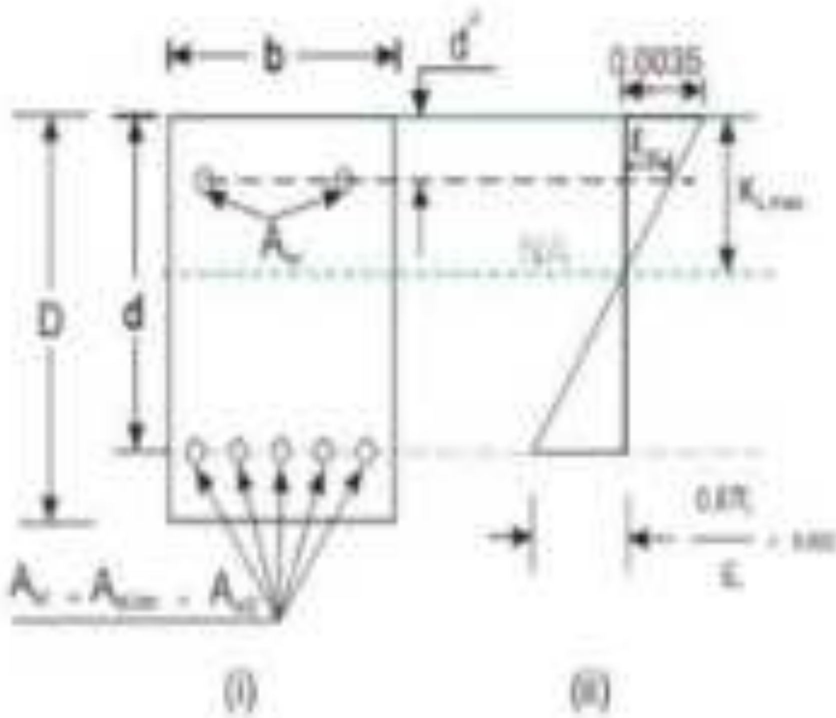
- 1. ANALYSIS TYPE:TO FIND OUT MOR
- 2. DESIGN TYPE: TO DESIGN A BEAM

MOR OF DOUBLY RC BEAMS

- PROBLEM STATEMENT: GIVEN
- CROSS SECTION
- A_{st}, A_{sc}
- GRADE OF CONCRETE, GRADE OF STEEL

DETERMINE MOR ?

MOR OF DOUBLY RC BEAMS



- METHOD-1: DIRECT METHOD
- STEP-1. CALCULATION OF x_u
- Assume some values of x_u
- Find ε_{sc}

$$\frac{\varepsilon_{sc}}{0.0035} = \frac{x_u - d'}{x_u}$$

- Find f_{sc}
- Find strain in tension steel

$$\frac{0.0035}{\varepsilon_{st}} = \frac{x_u}{d - x_u}$$

- Find stress in tension steel, genera
- Find $C=C_1+C_2$
- $C_1=0.36*f_{ck}*b*x_u$
- $C_2=A_{sc}*(f_{sc}-f_{cc})$
- Find $f_{cc}= 0.44*f_{ck}$
- Find $T=0.87f_y*A_{st}$

CALCULATION of f_{sc}

Stress level	Fe 415		Fe 500	
	Strain ε_{sc}	Stress f_{sc} (N/mm^2)	Strain ε_{sc}	Stress f_{sc} (N/mm^2)
0.80 f_{yd}	0.00144	288.7	0.00174	347.8
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1.0 f_{yd}	0.00380	360.9	0.00417	434.8

Linear interpolation may be done for intermediate values.

MOR OF DOUBLY RC BEAMS

- METHOD-1: DIRECT METHOD
- STEP-1. CALCULATION OF x_u
- If C=T, then Assumption is OK.
- Otherwise Assume New values of X_u
- STEP-2. CALCULATION OF MOR
- CALCULATIE MOR BY TAKING COMPRESSION INTO ACCOUNT

$$M = M_1 + M_2$$
$$= 0.36 * f_{ck} * b * x_u(d - 0.42x_u) + A_{sc} * (f_{sc} - f_{cc})(d - d')$$

- CALCULATIE MOR BY TAKING TENSION INTO ACCOUNT

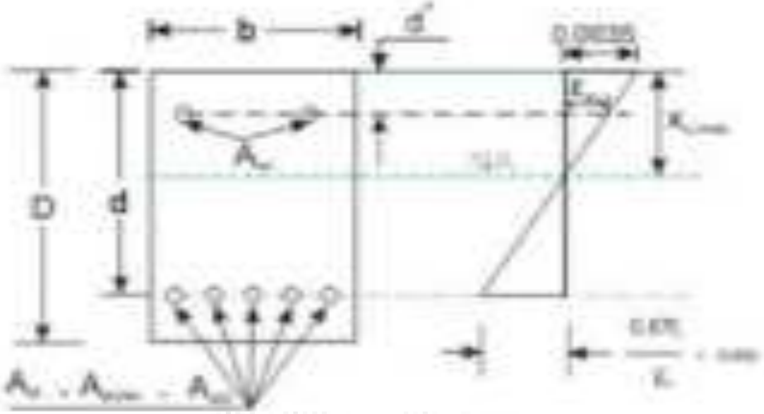
$$C_2 = [A_{sc}(f_{sc} - f_{cc})] = T_2 = [A_{st2}(0.87\ f_y)]$$

- Find A_{st2}, then Calculate A_{st1}=A_{st}-A_{st2}

$$M = M_1 + M_2 = A_{st1} * 0.87 * f_y(d - 0.42x_u) + A_{st2} * 0.87 f_y(d - d')$$

MOR OF DOUBLY RC BEAMS

- METHOD-2: USE OF SP 16
- STEP-1. CALCULATION OF x_u



$$C = C_1 + C_2 = 0.36 * f_{ck} * b * x_u + A_{sc} * (f_{sc} - f_{cc})$$

- Find f_{sc} from the following

f_y (N/mm^2)	d'/d				Strain at yield
	0.05	0.10	0.15	0.20	
250	217.4	217.4	217.4	217.4	0.0010869
415	355	353	342	329	0.0038043
500	412	412	395	370	0.0041739

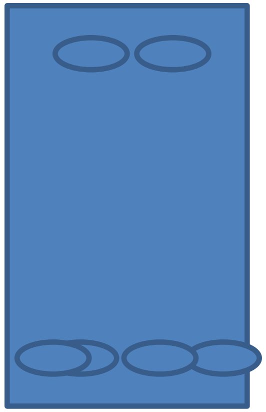
- For any intermediate value take the next higher value
- From the below formulae find x_u equating Total compressive force is equal to total Tensile force

$$[C = C_1 + C_2 = 0.36 * f_{ck} * b * x_u + A_{sc} * (f_{sc} - f_{cc})] = T = 0.87 * f_y * A_{st}$$

- STEP-2. CALCULATION OF MOR

PROBLEM-ANALYSIS TYPE

- Find out MOR of beam if 4,20 mm dia bars in tension side and 2, 16 mm dia bars on compression side, Beam size b=230 mm, d=460 mm, d'=40 mm, M20,Fe415.
- Solution:
- Given: b=230 mm, d=460 mm, d'=40 mm,
- A_{sc}=402 mm^2
- A_{st}= 1256 mm^2
- f_{ck}=20N/mm^2
- f_y=415 N/mm^2
- d'/d=40/460=0.087, Take next higher value 0.1, f_{sc}=353 MPa



$$[C = C_1 + C_2 = 0.36 * f_{ck} * b * x_u + A_{sc} * (f_{sc} - f_{cc})] = T$$
$$= 0.87 * f_y * A_{st}$$

PROBLEM-ANALYSIS TYPE

- Find x_u neglecting f_{cc}

$$[C = C_1 + C_2 = 0.36 * f_{ck} * b * x_u + A_{sc} * (f_{sc} - f_{cc})] = T = 0.87 * f_y * A_{st}$$

- x_u= 188.15 mm
- X_{umax}=0.48d =0.48*460=220.8 mm
- $X_u < x_{umax}$
- Find MOR

$$M = M_1 + M_2 = 0.36 * f_{ck} * b * x_u(d - 0.42x_u) + A_{sc} * (f_{sc} - f_{cc})(d - d')$$

- MOR=M_1+M_2
- M_1=118.70 kNm
- M_2=59.60 kNm
- M=MOR=178.3 kNm

PROBLEM-DESIGN TYPE

- GIVEN, LOAD, SECTION , TO FIND OUT A_{sc} AND A_{st}

$$[C = C_1 + C_2 = 0.36 * f_{ck} * b * x_u + A_{sc} * (f_{sc} - f_{cc})] = T = 0.87 * f_y * A_{st}$$

$$M = M_1 + M_2 = 0.36 * f_{ck} * b * x_u(d - 0.42x_u) + A_{sc} * (f_{sc} - f_{cc})(d - d')$$

PROBLEM-ANALYSIS TYPE

- Find out MOR of beam if 4,25 mm dia bars in tension side and 2, 20 mm dia bars on compression side, Beam size b=300 mm, d=450 mm, d'=50 mm, M20,Fe415.

PROBLEM-ANALYSIS TYPE

- Find out MOR of beam if 4,25 mm dia bars in tension side and 2, 20 mm dia bars on compression side, Beam size b=300 mm, d=450 mm, d'=50 mm, M20,Fe415.
- Solution:
- Given: b=300 mm, d=450 mm, d'=50 mm,
- A_{sc}=628 mm^2
- A_{st}= 1964 mm^2
- f_{ck}=20 N/mm^2
- f_y=415 N/mm^2
- d'/d=50/450=0.111, Take next higher value 0.15, f_{sc}=342 MPa

$$[C = C_1 + C_2 = 0.36 * f_{ck} * b * x_u + A_{sc} * (f_{sc} - f_{cc})] = T = 0.87 * f_y * A_{st}$$

PROBLEM-ANALYSIS TYPE

- Find x_u neglecting f_{cc}

$$[C = C_1 + C_2 = 0.36 * f_{ck} * b * x_u + A_{sc} * (f_{sc} - f_{cc})] = T$$
$$= 0.87 * f_y * A_{st}$$

- x_u= 228.90 mm
- X_{umax}=0.48d =0.48*450=216 mm
- $X_u > x_{umax}$ over reinforced, put $x_u = x_{umax}$
- Find MOR
- MOR=M_1+M_2
- M_1=167.62 kNm
- M_2=85.91 kNm
- M=MOR=253.53 kNm

$$M = M_1 + M_2$$
$$= 0.36 * f_{ck} * b * x_u(d - 0.42x_u) + A_{sc} * (f_{sc} - f_{cc})(d - d')$$

TYPE-II DESIGN PROBLEMS

- PROBLEM STATEMENT
- b, D, d, d', f_{ck}, f_y , Moment
- To find out A_{sc} and A_{st}
- SOLUTION
- STEP-1
- Find out MOR of a singly reinforced balance section.

$$MOR(Sin.Bal) = 0.36 f_{ck} bd\, x_{umax}(1 - 0.42\frac{x_{umax}}{d})$$

- STEP-2
- Calculation of A_{st1}
- a) If MOR(Sin.Bal) equal or Greater than Given Moment design the beam as singly RC Beam.
- b) If MOR(Sin.Bal) < Given Moment ,design the beam as Doubly RC balance Beam.
- M_1=MOR(Sin. Bal), Calulate A_{st1}= Steel required to make the section as singly Reinforced Balance section from the below equation.

$$M_{ultimit} = M_1 = T * z = 0.87 f_y * A_{st1} * (d - 0.42 x_{ulim})$$

TYPE-II DESIGN PROBLEMS

- STEP-3
- Calculation A_{st2} and A_{sc}
- a) Calculate M_2=M-M_1.
- b) Find out f_{sc} from d'/d or by any other method by finding ε_{sc} ,
- c) Calculation of A_{st2} and A_{sc} from these equations

$$M_2 = A_{st2}(0.87\, f_y)(d - d')$$

$$M_2 = A_{sc}(f_{sc} - f_{cc})(d - d')$$

- d) Calculation of total Ast and Asc

A_{st}=A_{st1}+A_{st2}

PROBLEM-ANALYSIS TYPE

- Find out reinforcement to resist a factored bending moment 200 kNm, Beam size b=230 mm, d=500 mm, d'=50 mm, M20,Fe415.

PROBLEM-ANALYSIS TYPE

- Find out reinforcement to resist a factored bending moment 200 kNm, Beam size b=230 mm, d=500 mm, d'=50 mm, M20,Fe415.
- Solution:
- Step-1
- x_{umax}=0.48*500=240 mm

$$MOR(Sin.Bal) = M_1 = 0.36 f_{ck} bd\, x_{umax}(1 - 0.42\frac{x_{umax}}{d})$$

$$M_1 = 0.36 * 20 * 230 * 500 * 240 * \left(1 - 0.42\frac{240}{500}\right)$$

$$= 158.65\ kNm$$

- Step-2
- Calculation of A_{st1}

$$M_{ulimit} = M_1 = T * z = 0.87 f_y * A_{st1} * (d - 0.42 x_{ulim})$$

$$M_1 = 158.65\ kNm =$$

$$= 0.87 * 415 * A_{st1} * (500 - 0.42 * 240)$$

- A_{st1}=1100.80 mm^2

- Step-3 Calculation of A_{st2} and A_{sc}
- $M_2 = M - M_1 = 200 - 158.65 = 41.35$ kNm

$$M_2 = A_{st2}(0.87\, f_y)(d - d')$$

$$M_2 = 41.35 * 10^6 = A_{st2} * (0.87 * 415)(500 - 50)$$

- $A_{st2} = 254.50\ mm^2$
- $A_{st} = A_{st1} + A_{st2} = 1100.80 + 254.50 = 1355.29\ mm^2$
- Provide 5, 20 mm dia bars as tension steel, $A_{st} = 1570\ mm^2$
- Calculation of A_{sc}
- d'/d=50/500=0.1 , fsc= 353 N/mm^2

$$M_2 = A_{sc}(f_{sc} - f_{cc})(d - d')$$

$$M_2 = 41.35 * 10^6 = A_{sc} * (353)(500 - 50)$$

- $A_{sc} = 260.3\ mm^2$
- Provide 2 nos of 16 mm dia, $A_{sc} = 402\ mm^2$

Flanged sections

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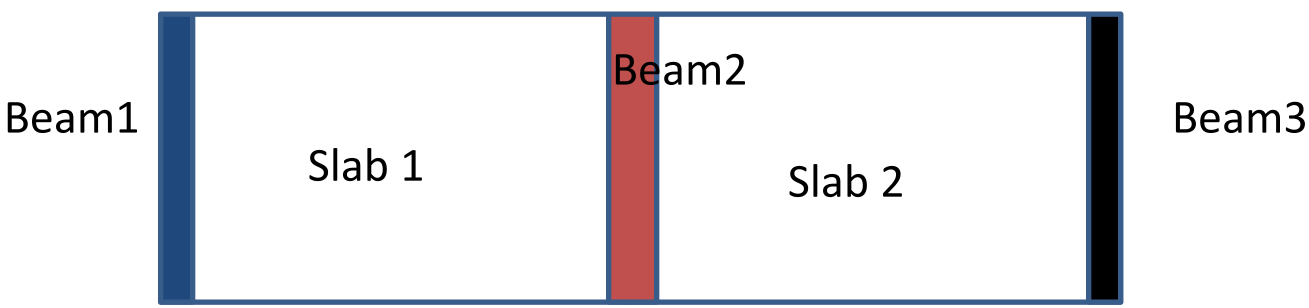
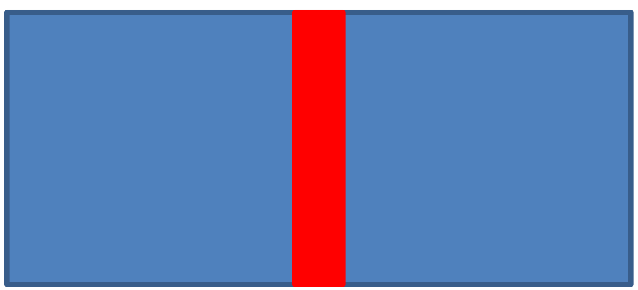
T AND L BEAMS

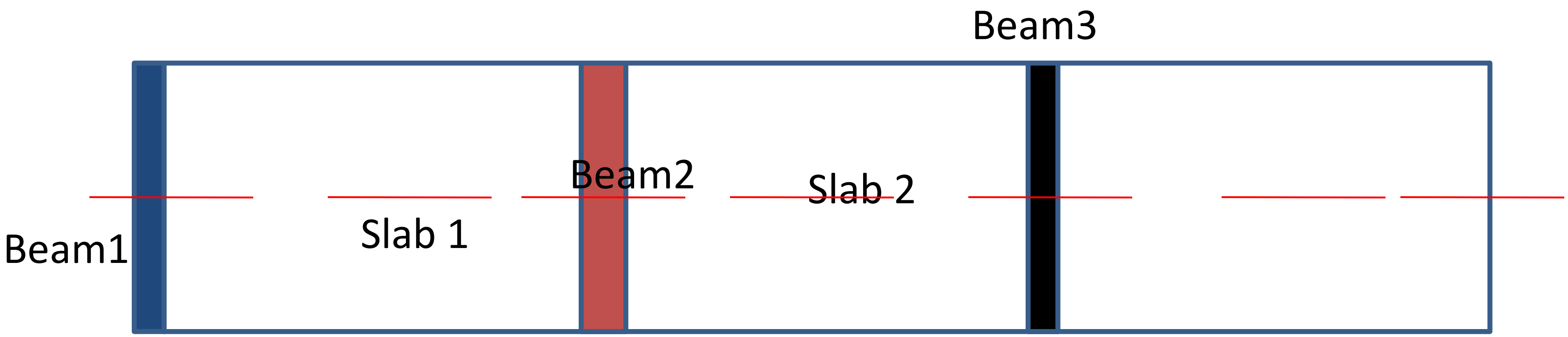
- In previous section we have studied rectangular beams.
- When slab and beam are cast simultaneously(monolithic), then some portion of the slab act as beam and bends along with beam in longitudinal direction. This slab portion is called the *flange* of the T- or L-beam. The beam portion below the flange is often termed the *web*, although, technically, the web is the full rectangular portion of the beam other than the overhanging parts of the flange. Indeed, in shear calculations, the web is interpreted in this manner.

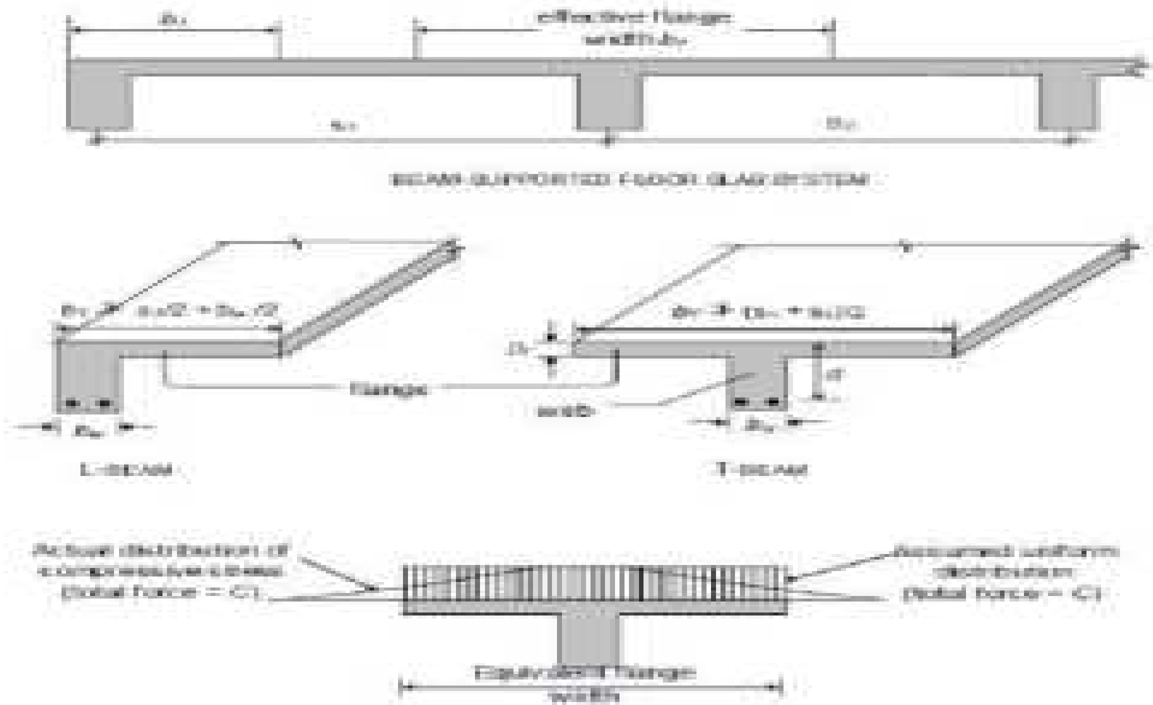


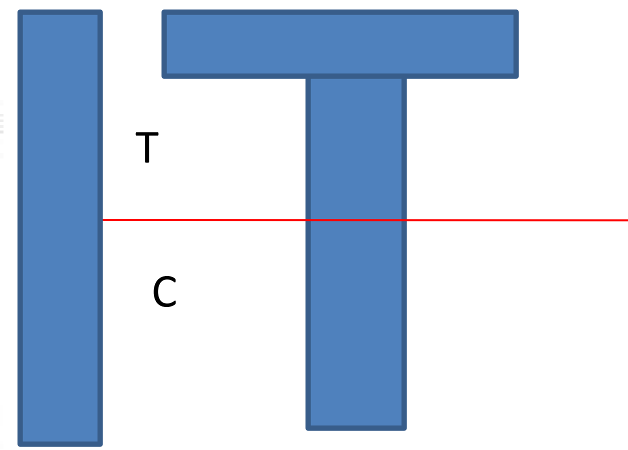



Portion of slab acting as beam

T AND L BEAMS

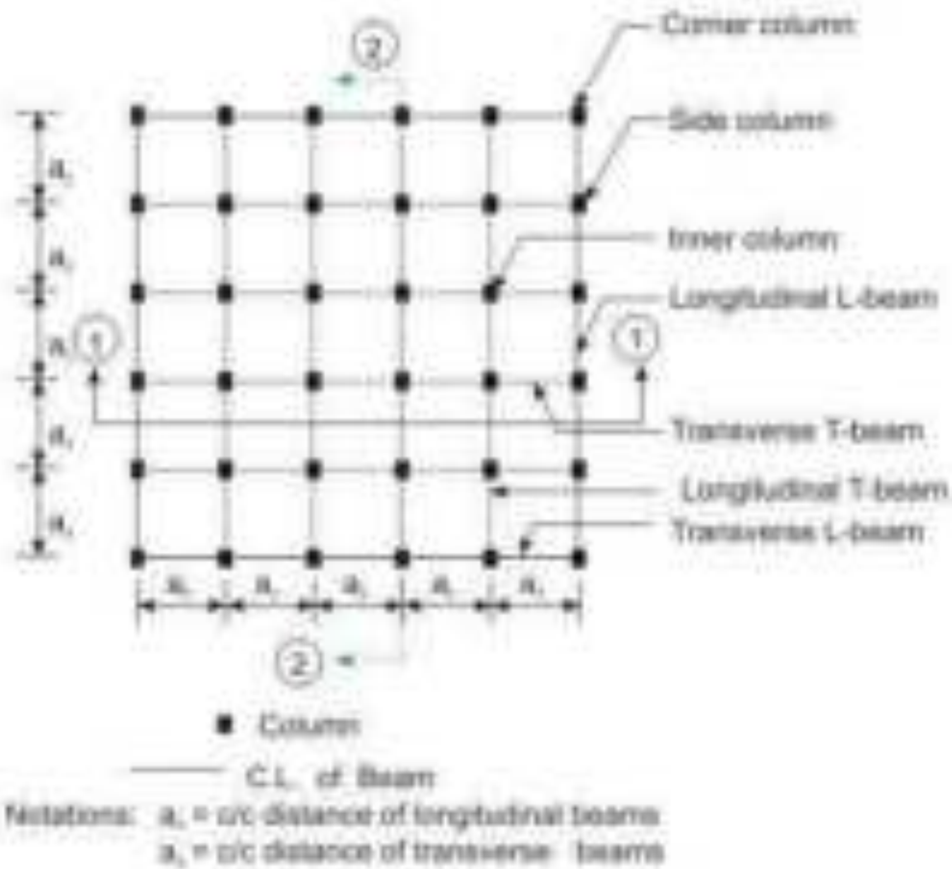

Beam3
Beam2
Slab 2
Beam1
Slab 1

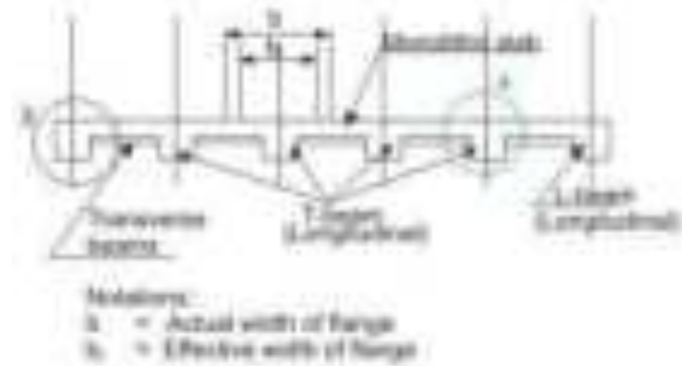

effective flange width
L-BEAM
T-BEAM
flange
web


T
C

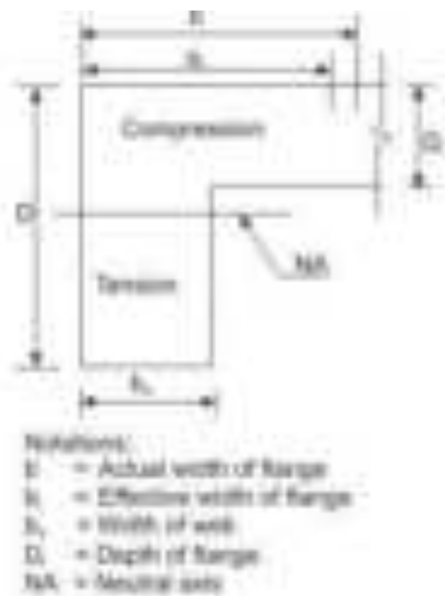
- Due to monolithic casting, beams and a part of the slab act together. Under the action of positive bending moment, i.e., between the supports of a continuous beam, the slab, up to a certain width greater than the width of the beam, forms the top part of the beam. Such beams having slab on top of the rectangular rib are designated as the flanged beams - either *T or L type depending on whether the slab is on both sides or on one side of the beam. As the flanged portion is in compression, these potion will resist more compression as area of flanged portion in compression is more.*

- *Over the supports of a continuous beam, the bending moment is negative and the slab, therefore, is in tension while a part of the rectangular beam (rib) is in compression. These beams T or L portion is of no use in taking the force as concrete takes no tension. So, these will be designed as simple rectangular beam.*
- *T or L beam is useful if flanged portion is under compression.*

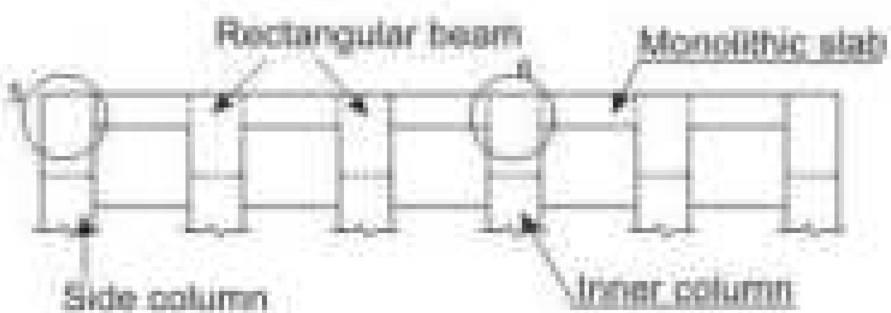




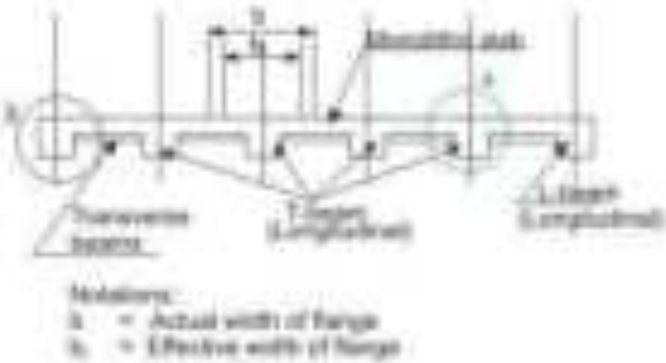
SECTION-1-1



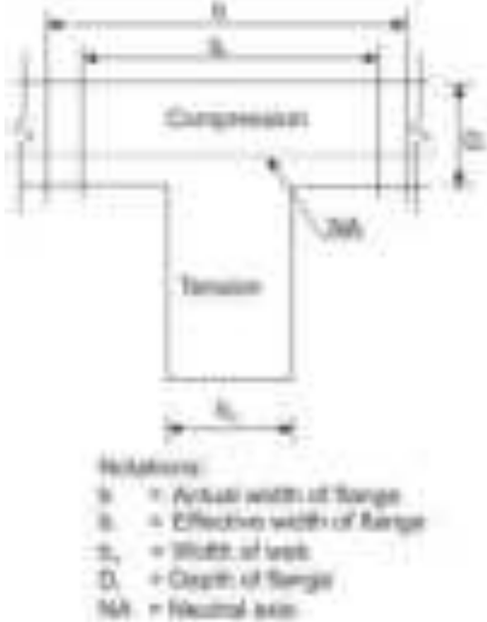
DETAILS AT 3 L BEAM



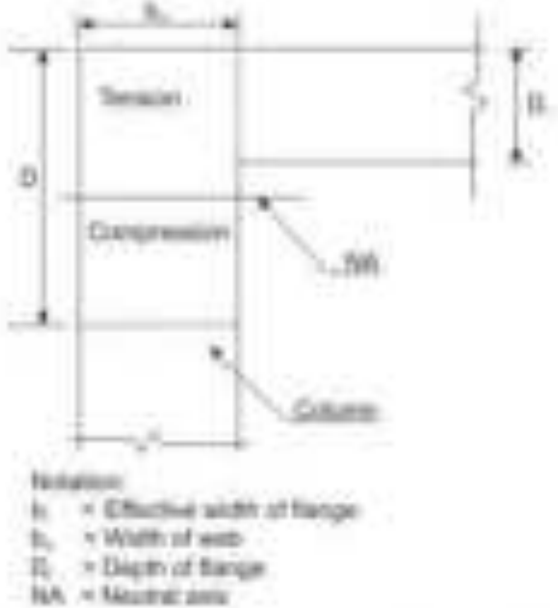
SECTION-2-2



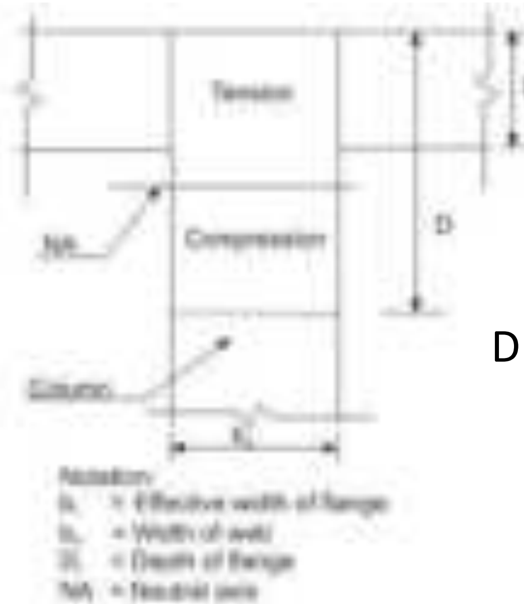
SECTION-1-1



DETAILS AT 4 T BEAM



DETAILS AT 5
RECTANGULAR BEAM



DETAILS AT 6 RECT. BEAM

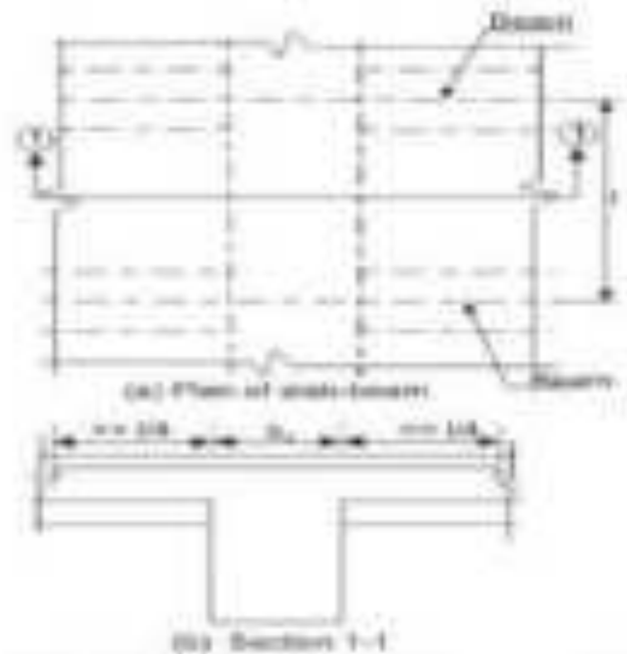
DETERNINATION OF EFFECTIVE WIDTH

- The total width of flanged portion can not be considered as effective width of the flanged section while calculating the total compressive force, because the total width may not be effective resisting the force.
- The actual width of the flange is the spacing of the beam, which is the same as the distance between the middle points of the adjacent spans of the slab, as shown in 3. However, in a flanged beam, a part of the width less than the actual width, is effective to be considered as a part of the beam. This width of the slab is designated as the effective width of the flange.
- IS CODE PROVISION FOR FLANGED BEAM

The following requirements (cl. 23.1.1 of IS 456) are to be satisfied to ensure the combined action of the part of the slab and the rib (rectangular part of the beam).

(a) The slab and the rectangular beam shall be cast integrally or they shall be effectively bonded in any other manner.

(b) Slabs must be provided with the transverse reinforcement of at least 60 per cent of the main reinforcement at the mid span of the slab if the main reinforcement of the slab is parallel to the transverse beam



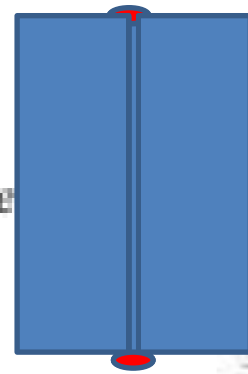
EFFECTIVE WIDTH CALCULATION

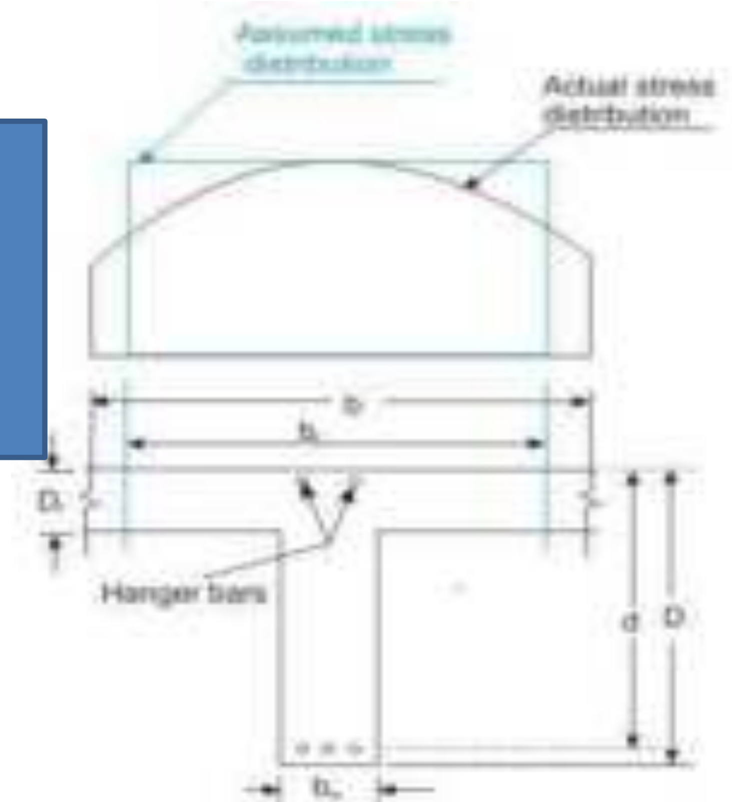
Clause 23.1.2 of IS 456 specifies the following effective widths of *T and L-beams:*

(a) For *T-beams, the lesser of*

$$i)\quad b_f = \frac{l_0}{6} + b_w + 6D_f$$

$$ii)\quad b_f = Actual\ width\ of\ flange$$





(b) For isolated *T-beams, the lesser of*

$$i)\quad b_f = \frac{l_0}{l_0/b + 4} + b_w$$

$$ii)\quad b_f = Actual\ width\ of\ flange$$

(c) For *L-beams, the lesser of*

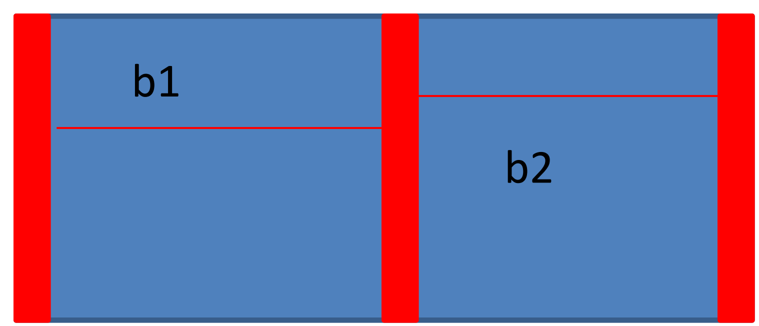
$$i)\quad b_f = \frac{l_0}{12} + b_w + 3D_f$$

$$ii)\quad b_f = Actual\ width\ of\ flange$$

(d) For isolated *L-beams, the lesser of*

$$i)\quad b_f = \frac{0.5 * l_0}{l_0/b + 4} + b_w$$

$$ii)\quad b_f = Actual\ width\ of\ flange$$



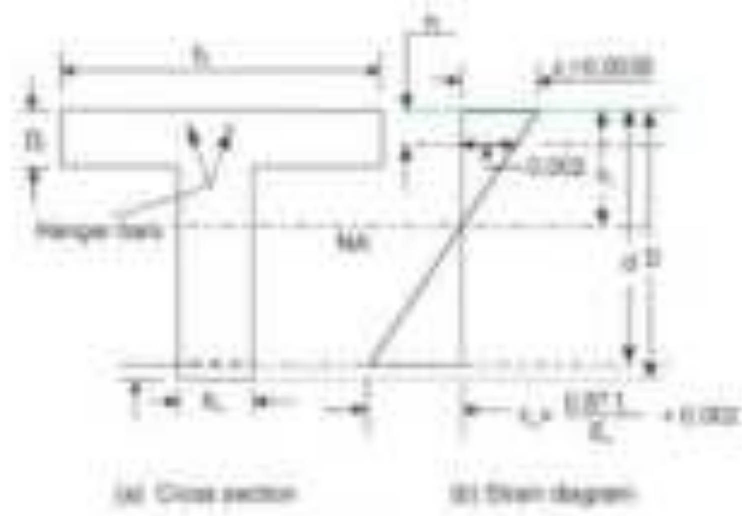
- where b_f = *effective width of the flange,*
- l_o = *distance between points of zero moments in the beam, which is the effective span for simply supported beams and 0.7 times the effective span for continuous beams and frames,*
- b_w = *breadth of the web,*
- D_f = *thickness of the flange,*
- and b = *actual width of the flange.*

ANALYSIS OF T AND L BEAM

ASSUMPTIONS:

As for singly RC section

Let us think about h, in which stress in concrete is $0.446f_{ck}$.



$$\frac{0.002}{0.0035} = \frac{x_u - h}{x_u} = 1 - \frac{h}{x_u}$$

$$\frac{h}{x_u} = \frac{3}{7} = 0.43$$

If $x_u = x_{umax}$, then

$h = 0.43 * x_{umax}$

$h = 0.43*53d = 0.227d$ foe Fe250

$h = 0.43*48d = 0.2064d$ foe Fe415

$h = 0.43*46d = 0.1978d$ foe Fe500

The value can be taken 0.2d

To find the relation between strain in steel and concrete

$$\frac{\epsilon_{st}}{\epsilon_c = 0.0035} = \frac{d - x_u}{x_u} \qquad \frac{\epsilon_{st}}{\epsilon_c} = \frac{d - x_u}{x_u} \qquad \frac{\epsilon_{st} + \epsilon_c}{\epsilon_c} = \frac{d}{x_u}$$

ANALYSIS

$$\frac{h}{x_u} = \frac{3}{7} = 0.43 \qquad \frac{\epsilon_{st} + \epsilon_c}{\epsilon_c} = \frac{d}{x_u}$$

$$\frac{\frac{h}{x_u}}{\frac{d}{x_u}} = \frac{0.43}{\frac{\epsilon_{st} + \epsilon_c}{\epsilon_c}} = \frac{h}{d} = \frac{0.43 * .0035}{0.0035 + 0.002 + 0.87 * f_y/E_s}$$

$$= \frac{0.0015}{0.0055 + 0.87 * f_y/E_s}$$

h/d will be 0.227, 205 and 0.195 for Fe250, Fe415 and Fe500 respectively.

From this derivation we found if h/d is approximately 0.2 or less then, that portion is having a stress 0.446 f_{ck}. If is more some portion is having stress 0.446fck and other portion stress is parabolic and less than 0.0446f_{ck}.

The maximum value of h may be D_f, at the bottom of the flange where the strain will be 0.002,

if D_f/d = 0.2. in this case, the position of the fiber of 0.002 strain will be in the web and the entire flange will be under a constant compressive stress of 0.446 f_{ck}.

On the other hand, if *D_f is > 0.2 d, the position of the fiber of 0.002 strain will be in the flange. In that case, a part of the slab will have the constant stress of 0.446 f_{ck} where the strain will be more than 0.002.Other portion of flange stress is less than 0.446f_{ck} and parabolic.*

ANALYSIS

Based on above analysis we have the following cases:

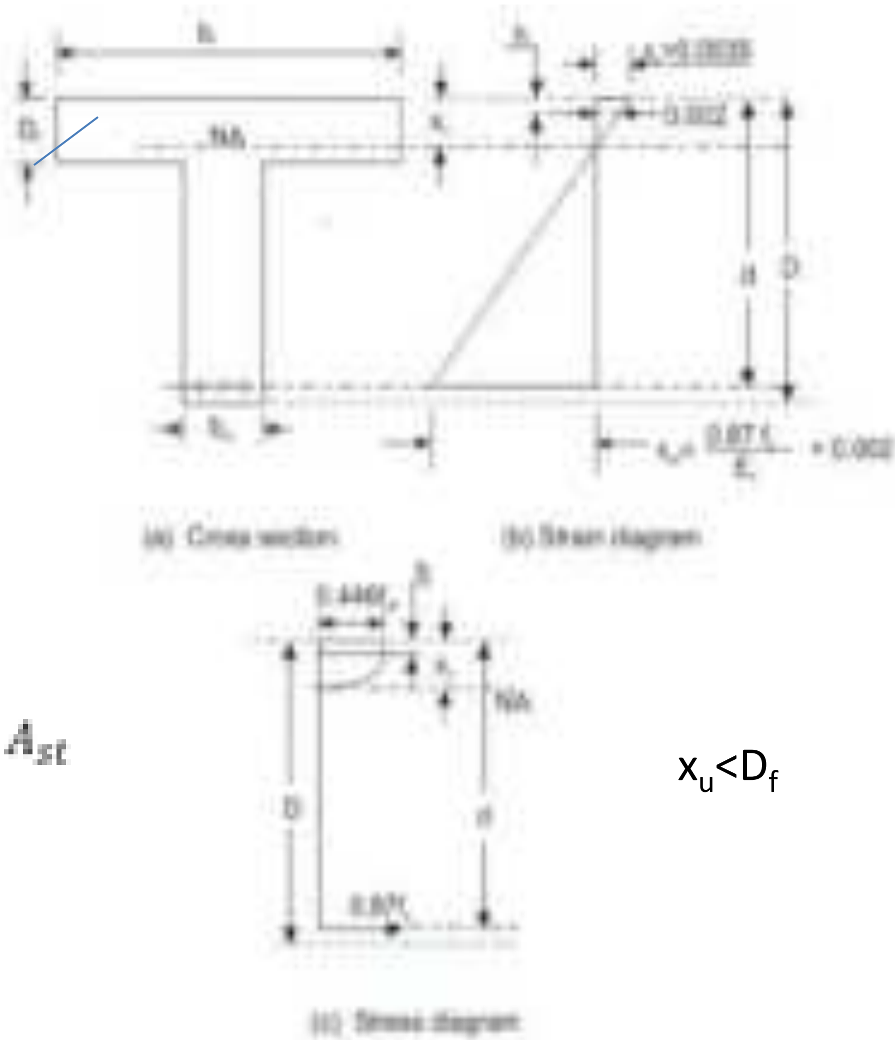
i) *When NA axis is within flange*

It will be analyzed as Concrete below the neutral axis is in tension and is ignored. The steel reinforcement takes the tensile force. Therefore, *T and L-beams are considered as rectangular beams of width b_f and effective depth d.*

Check whether the section is under reinforced, balance or over reinforced.

$$C = 0.36 * f_{ck} * b_f * x_u = T = 0.87 f_y * A_{st}$$

$$M_u = 0.36 * f_{ck} * b_f * x_u (d - 0.42 * x_u)$$

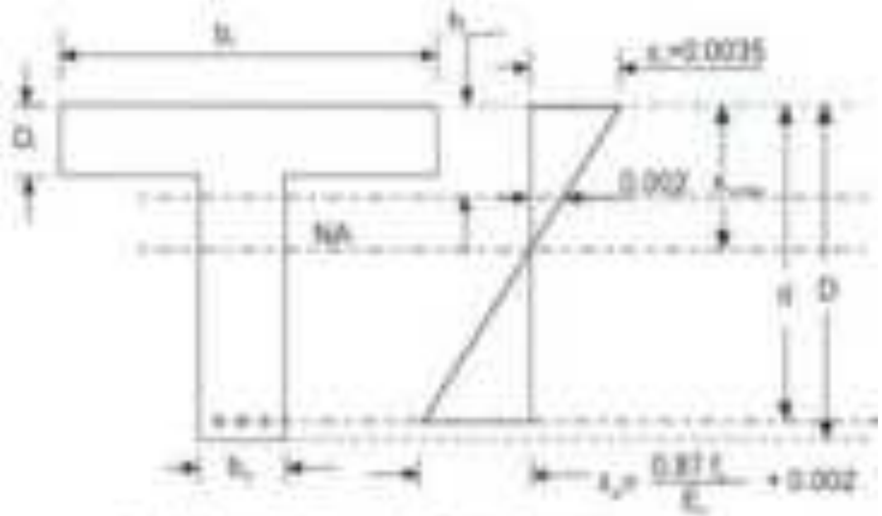


$x_u < D_f$

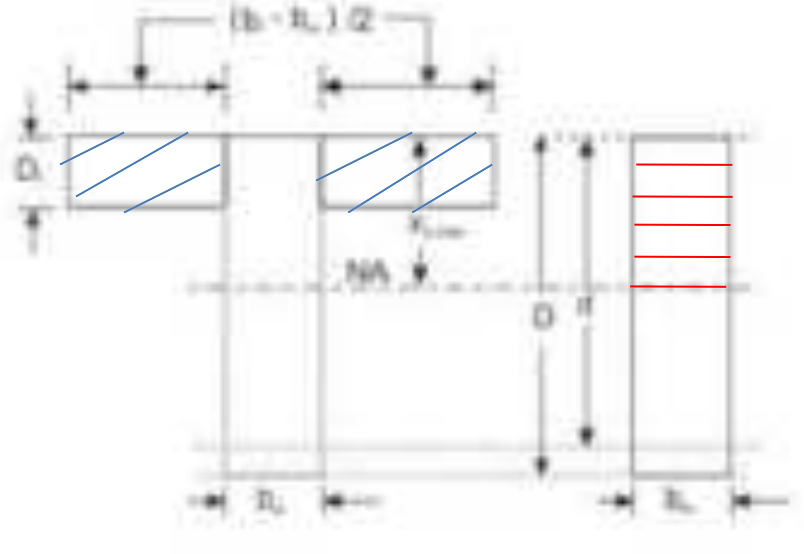
ANALYSIS

ii) NA axis is within web($x_u > D_f$) and a balance section

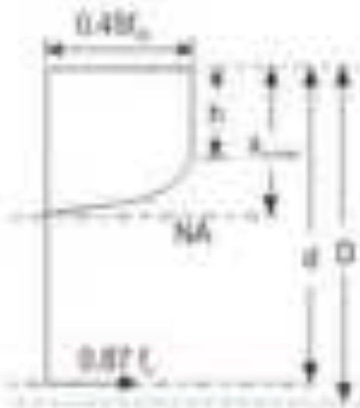
a) (h/d or D_f/d) is less than or equal to 0.2



(a) Cross section (b) Strain diagram



(d) Flange (e) Web



(c) Stress diagram

The entire flange portion is having constant stress 0.446fck.This can be analyzed taking two rectangular portions, flange portion as shown in d and total web as shown in e.

Flange is having width (b_f-b_w) and depth D_f having stress $0.446f_{.ck}$

Another rectangular from Flange and web having width (b_w) and depth xumax, having stress $0.446f_{.ck}$ to 0

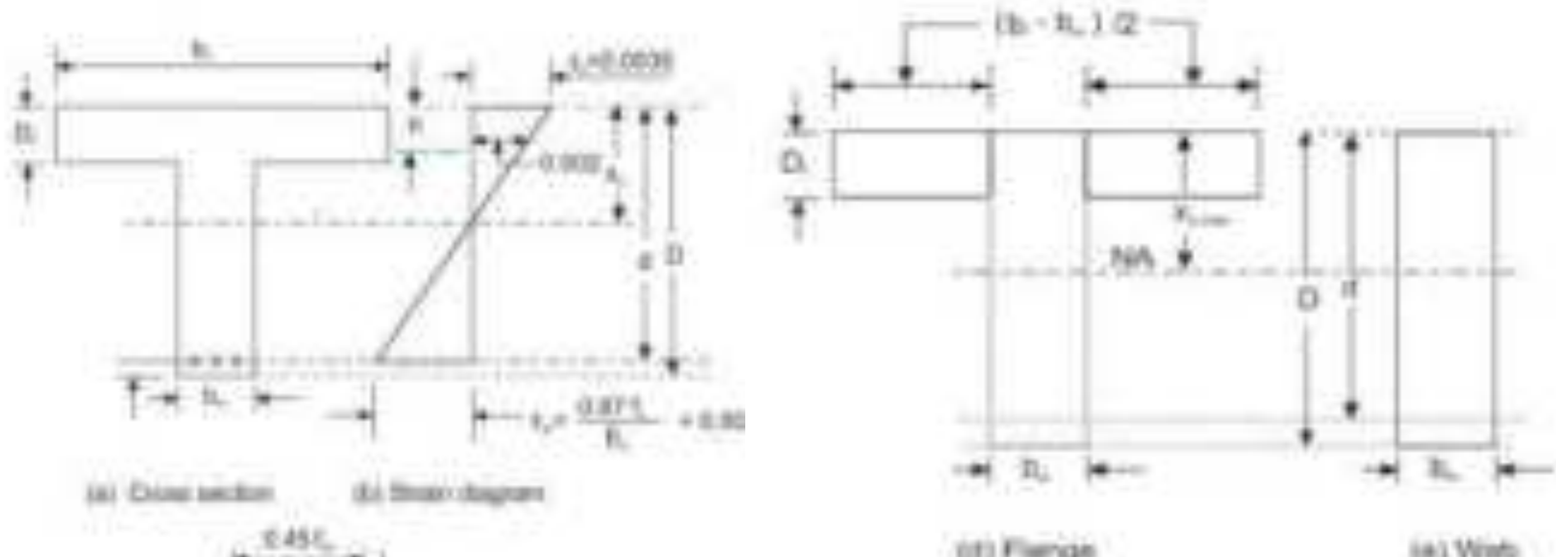
$$C = C_f + C_w$$
$$= 0.446 * f_{ck} * (b_f - b_w) * D_f + 0.36 * f_{ck} * b_w * x_{umax}$$
$$C = T = 0.87 f_y * A_{st}$$

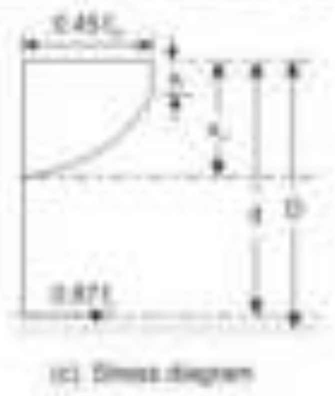
The lever arm of the rectangular beam (web part) is (*d - 0.42* x_{umax}*) and the same for the flanged part is (d - 0.5* D_f*).*
Moment of resistance will be

$$M = M_f + M_w$$
$$= 0.446 * f_{ck} * (b_f - b_w) * D_f * (d - \frac{D_f}{2}) + 0.36 * f_{ck} * b_w * x_{umax} * (d - 0.42 x_{umax})$$

ii) NA axis is within web($x_u > D_f$) and a balance section

b) (h/d or D_f/d) greater than 0.2





The entire flange portion is not having constant stress 0.446fck.This can be analyzed taking two rectangular portions, flange portion as shown in d and total web as shown in e.

Flange is having width (b_f-b_w) and depth D_f having stress 0.446$f_{ck,}$ up to strain 0.002 and rest less than 0.446$f_{ck.}$

Another rectangular from Flange and web having width (b_w) and depth xumax, having stress 0.446$f_{.ck}$ to 0

Here for the flange portion some equivalent depth y_f may be assumed which will be having constant stress 0.446fck.

$$y_f = 0.15x_{umax} + 0.65D_f \text{ but in no case more than } D_f$$

$y_f=K_1*x_u+K_2*D_f$

When $D_f/X_u=3/7$, $y_f=D_f$

$$\frac{3}{7} * x_u = K_1x_u + K_2\left(\frac{3}{7}\right)x_u$$

When $D_f=x_u$, $y_f=0.8\ x_u$

$$0.8 * x_u = K_1x_u + K_2x_u$$

$K_1=0.15$ and $K_2= 0.65$

$$C = C_f + C_w$$

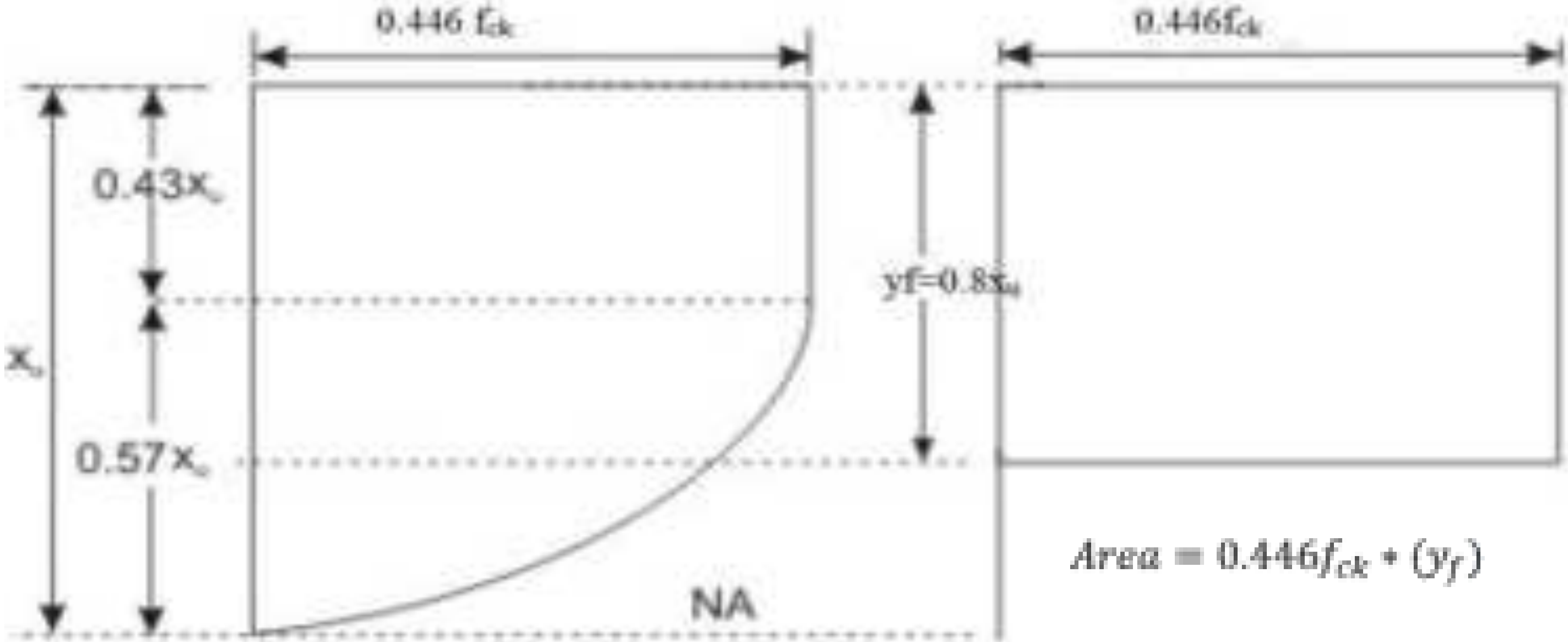
$$= 0.446 * f_{ck} * (b_f - b_w) * y_f + 0.36 * f_{ck} * b_w * x_{umax}$$

$$C = T = 0.87f_y * A_{st}$$

The lever arm of the rectangular beam (web part) is (*d - 0.42* $_{xumax}$*) and the same for the flanged part is (d - 0.5**y_f*). Moment of resistance will be*

$$M = M_f + M_w$$

$$= 0.446 * f_{ck} * (b_f - b_w) * y_f * (d - \frac{y_f}{2}) + 0.36 * f_{ck} * b_w * x_{umax} * (d - 0.42x_{umax})$$



$$Area = 0.446f_{ck} * (y_f)$$

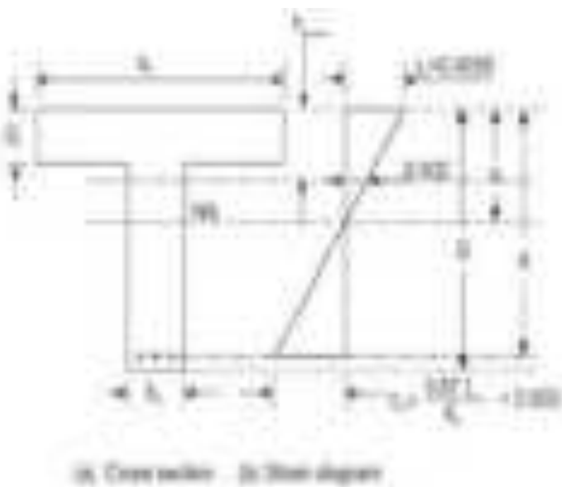
$$Area = 0.446f_{ck} * \left(\frac{3}{7} * x_u\right) + \left(\frac{2}{3}\right) 0.446f_{ck} * \left(\frac{4}{7} * x_u\right)$$

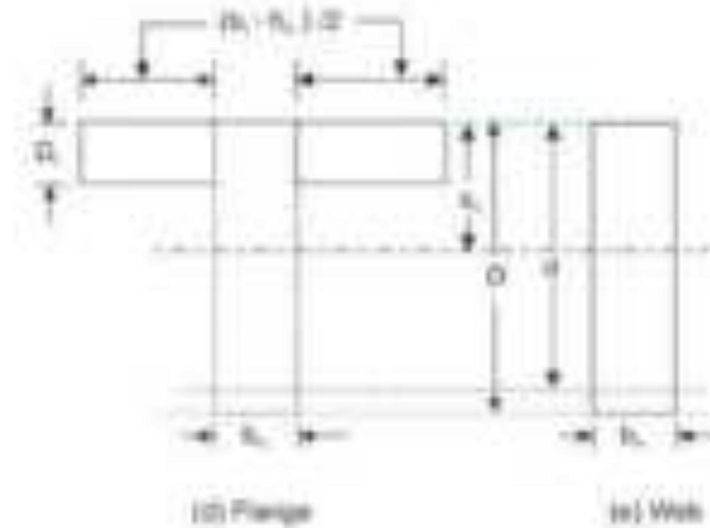
$$= 0.446f_{ck} * (0.8x_u)$$

ANALYSIS

iii) NA axis is within web($x_{umax} > x_u > D_f$) and under reinforced section

a) (D_f/X_u) is less than or equal to 0.43





The entire flange portion is having constant stress 0.446fck.This can be analyzed taking two rectangular portions, flange portion as shown in d and total web as shown in e.

Flange is having width (b_f-b_w) and depth D_f having stress 0.446$f_{.ck}$

Another rectangular from Flange and web having width (b_w) and depth x_u having stress 0.446$f_{.ck}$ to 0

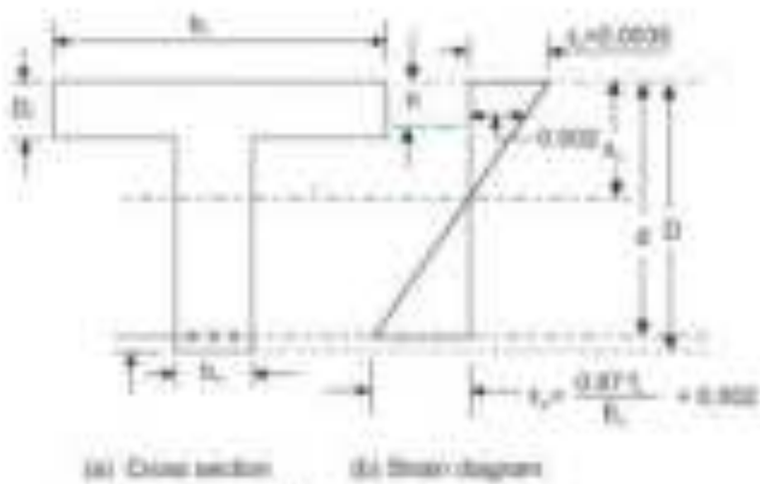
$$C = C_f + C_w$$
$$= 0.446 * f_{ck} * (b_f - b_w) * D_f + 0.36 * f_{ck} * b_w * x_u$$

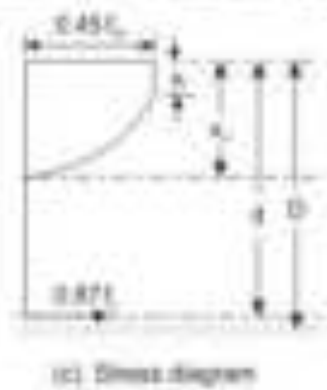
- The lever arm of the rectangular beam (web part) is (*d - 0.42* x_u*) and the same for the flanged part is (d - 0.5**D_f*).*

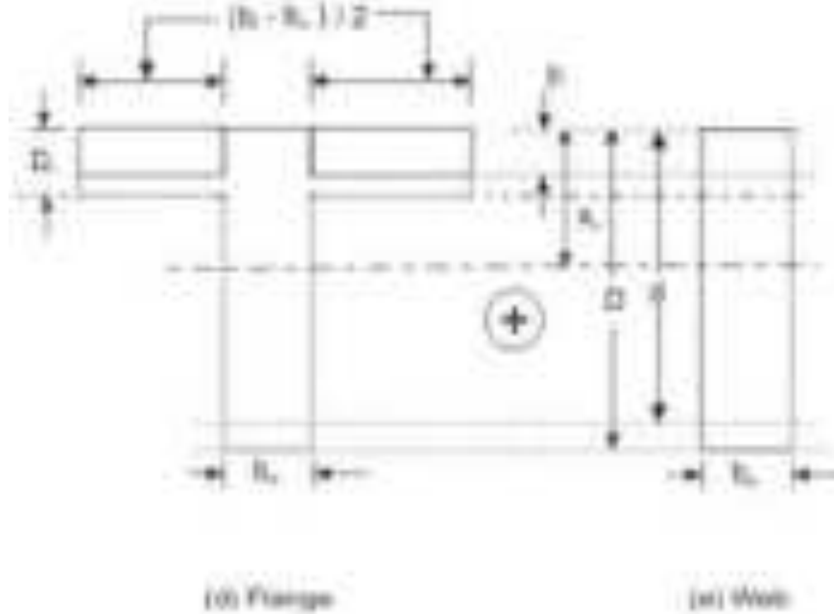
$$M = M_f + M_w$$
$$= 0.446 * f_{ck} * (b_f - b_w) * D_f * (d - \frac{D_f}{2}) + 0.36 * f_{ck} * b_w * x_u * (d - 0.42x_u)$$

iii) NA axis is within web($x_{umax} > x_u > D_f$) and under reinforced section

b) $(D_f/X_u) > 0.43$







The entire flange portion is not having constant stress 0.446fck.This can be analyzed taking two rectangular portions, flange portion as shown in d and total web as shown in e.

Flange is having width $(b_f\text{-}b_w)$ and depth D_f having stress $0.446f_{ck,}$ up to strain 0.002 and rest less than $0.446f_{ck.}$

Another rectangular from Flange and web having width (b_w) and depth xu, having stress $0.446f_{.ck}$ to 0

- Here for the flange portion some equivalent depth y_f may be assumed which will be having constant stress 0.446fck.

$$y_f = 0.15x_u + 0.65D_f \text{ but in no case more than } D_f$$

$$C = C_f + C_w$$
$$= 0.446 * f_{ck} * (b_f - b_w) * y_f + 0.36 * f_{ck} * b_w * x_u$$

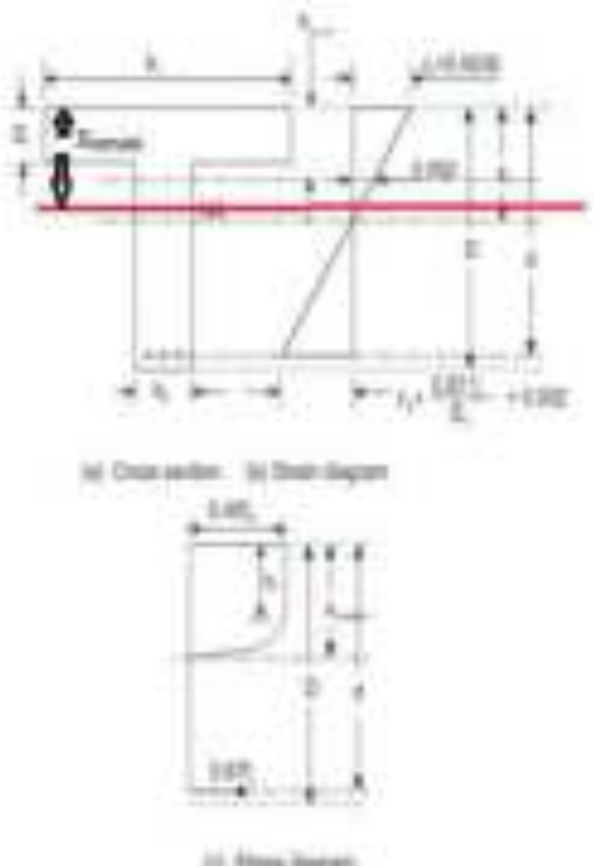
- The lever arm of the rectangular beam (web part) is (*d - 0.42* x_u*) and the same for the flanged part is (d - 0.5**y_f*).*
- *Moment of resistance will be*

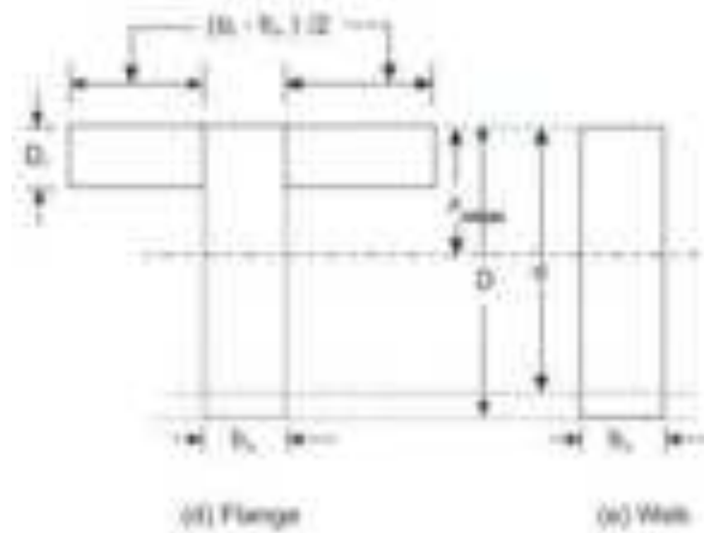
$$M = M_f + M_w$$
$$= 0.446 * f_{ck} * (b_f - b_w) * y_f * (d - \frac{y_f}{2}) + 0.36 * f_{ck}$$
$$* b_w * x_u * (d - 0.42x_u)$$

ANALYSIS

iv) NA axis is within web($x_u > x_{umax} > D_f$) and over reinforced section

a) (D_f/d) is less than0.2 or(D_f/X_u) less or equal to 0.43





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Another rectangular from Flange and web having width (b_w) and depth x_u having stress 0.446$f_{.ck}$ to 0

$$C = C_f + C_w$$
$$= 0.446 * f_{ck} * (b_f - b_w) * D_f + 0.36 * f_{ck} * b_w * x_{umax}$$

- The lever arm of the rectangular beam (web part) is (d - $0.42\ x_{umax}$) *and the same for the flanged part is* (d - $0.5*D_f$).
- *Moment of resistance will be*

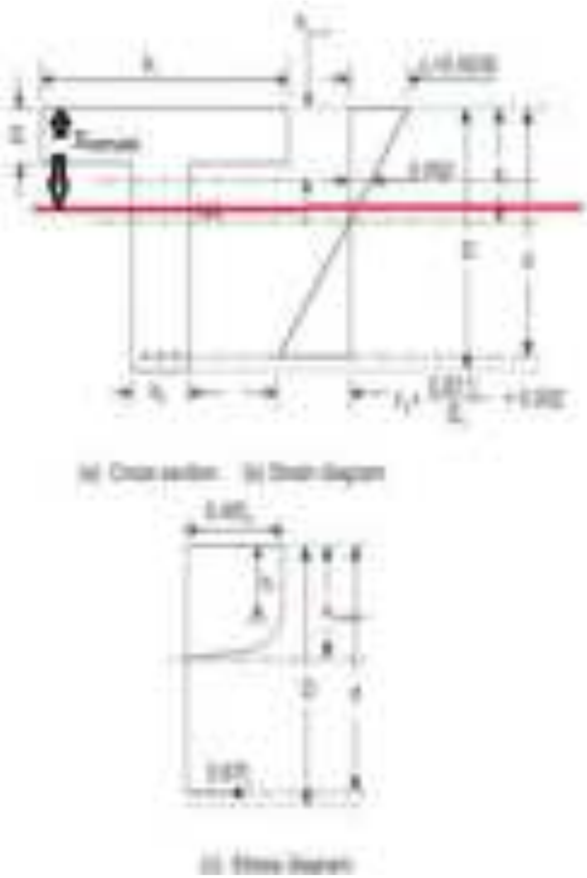
$$M = M_f + M_w$$
$$= 0.446 * f_{ck} * (b_f - b_w) * D_f * (d - \frac{D_f}{2}) + 0.36 * f_{ck} * b_w * x_{umax} * (d - 0.42x_{umax})$$

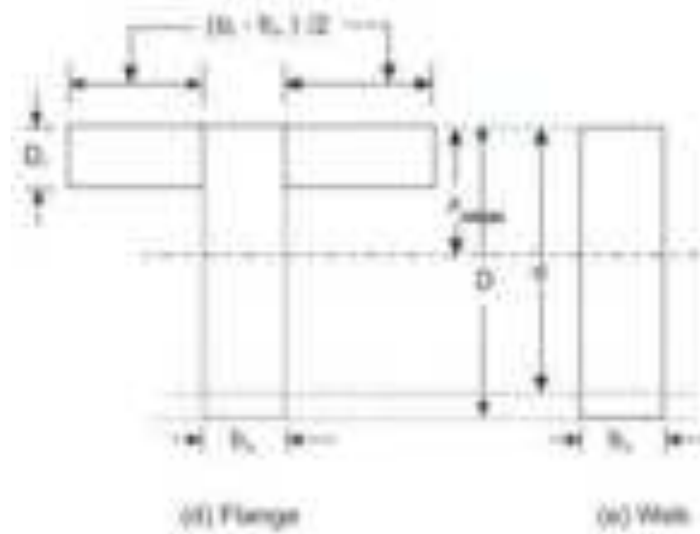
ANALYSIS

iv) NA axis is within web($x_u > x_{umax} > D_f$) and over reinforced section

b) (D_f/d) is greater than 0.2

or (D_f/x_u)> 0.43





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$$y_f = 0.15x_{umax} + 0.65D_f \text{ but in no case more than } D_f$$

$$C = C_f + C_w$$
$$= 0.446 * f_{ck} * (b_f - b_w) * y_f + 0.36 * f_{ck} * b_w * x_{umax}$$

- The lever arm of the rectangular beam (web part) is (*d - 0.42* x_{ulim}*) and the same for the flanged part is (d - 0.5**y_f*).*
- *Moment of resistance will be*

$$M = M_f + M_w$$
$$= 0.446 * f_{ck} * (b_f - b_w) * y_f * (d - \frac{y_f}{2}) + 0.36 * f_{ck} * b_w * x_{umax} * (d - 0.42x_{umax})$$

PROBLEM

Design the T beam if slab thickness is 150mm, slab beam casting monolithic, effective span 5 m. Slab is subjected to imposed load = 5kN/m^2, M20 and Fe415

Step1: Calculation of b_f

Df=150 mm

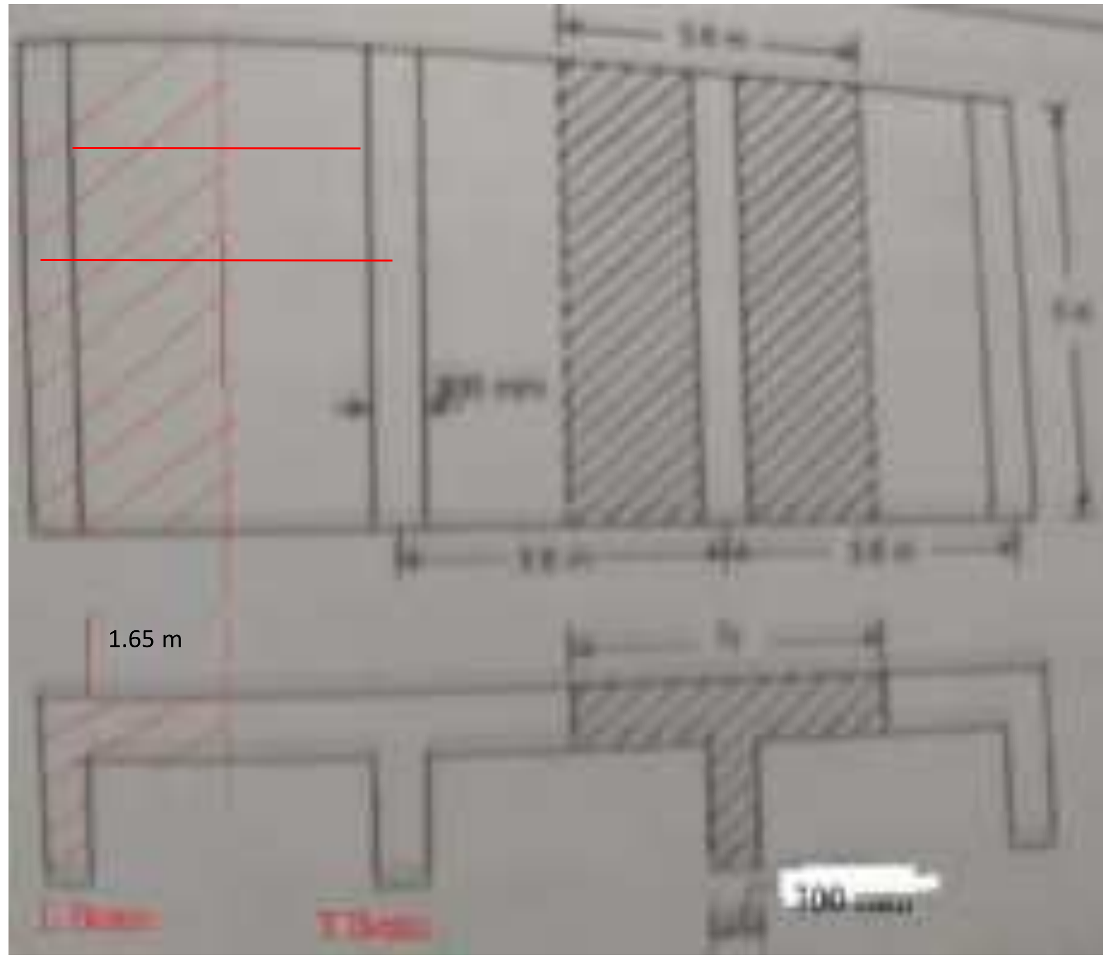
Actual width=1.65+0.3=1.95 m

lo=5 m

bw=300 mm

i) $b_f = \frac{l_0}{12} + b_w + 3D_f$

ii) b_f = Actual width of flange



- # Calculation of b_f (For L Beam)

$$b_f = \frac{l_0}{12} + b_w + 3D_f = \frac{5}{12} + .3 + 3 * .15 = 1.167 = 1.17\ m$$

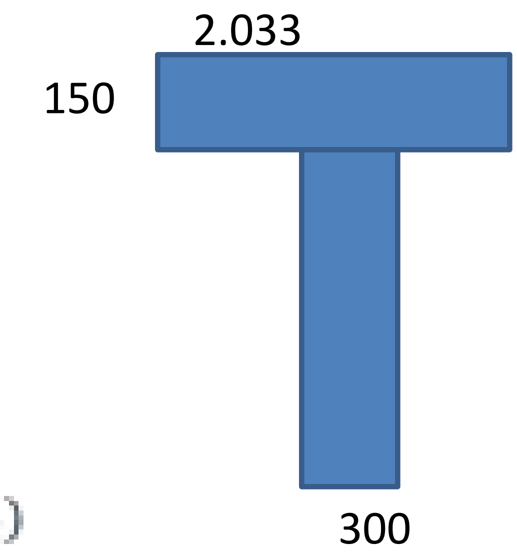
- b= actual width=1.95 m
- b_f=1.17 m
- Calculation of b_f (For T Beam)

$$b_f = \frac{l_0}{6} + b_w + 6D_f = \frac{5}{6} + .3 + 6 * .15 = 2.033\ m$$

- Actual width available= b=(3.60) >2.033 mt
- b_f=2.033 mt.
- Assuming the depth (D)as 1/12 of the length=5 m/12=400 mm
- Effective depth= 370 mm taking effective cover 30 mm
- Depth of web=400-150=250 mm
- LOAD CALCULATION:
- DL of Slab/m length of beam=1*3.6*.15*25=13.5 kN/m
- LL of Slab/m length of beam=1*3.6*5=18 kN/m
- DL of beam=1*0.25*0.3*25=1.875 kN/m
- Total load=w= 33.375 kN/m

DESIGN OF T BEAM

- Factored Load=w_u=1.5*33.375=50.0625 kN/m
- Factored moment=$M_u=w_u l^2/8$=50.0625*5^2/8=156.5 kNm
- Factored SF=$V_u=w_u l/2$=50.0625*5/2=125.15 kN
- Assuming NA passing through flange



$$C = 0.36 * f_{ck} * b_f * x_u = T = 0.87 * f_y * A_{st}$$

$$C = 0.36 * 20 * 2033.3 * x_u = T = 0.87 * 415 * A_{st}$$

$$x_u = 0.02466 * A_{st}$$

$$M = 0.87 * f_y * A_{st} * (d - 0.42 * x_u)$$

$$156.5 * 10^6 = 0.87 * f_y * A_{st} * (d - 0.42 * 0.0246 A_{st})$$

$$A_{st} = 1213\ mm^2 \qquad x_u = 30\ mm$$

- Let us provide 4, 20 mm dia bars= A_{st}=1256 mm^2
- A_{stmin}=0.85*bd/f_y=0.85*300*370/415= 228 mm^2
- A_{max}=0.04bD=0.04*300*400=4800 mm^2

- Check for Shear:
- Vu=125.15 kN

$$\tau_{vu} = \frac{V_u}{bd} = \frac{125.15 * 10^3}{300 * 370} = 1.127\ N/mm^2$$

- Calculation of τ_c
- $p_t = (A_{st}/bd) * 100 = 1.13\%$

$$\tau_c = 0.62 + \frac{0.67 - 0.62}{1.25 - 1.00} * (1.13 - 1.00) = 0.65\ N/mm^2$$

- $V_c = \tau_c * bd = 0.65 * 300 * 370 = 72.15$ kN
- V_{su}=Vu-Vc=125.15-72.15 kN= 53 kN
- Taking 2 legged 8 mm dia bars

$$s_v = \frac{0.87 * f_y * A_{sv} * d}{V_{su}} = \frac{0.87 * 415 * 100 * 370}{53 * 1000} = 252.05\ mm$$

Maximum spacing can not be greater than 0.75d=0.75*370=277.5 mm

It can not be more than 300 mm

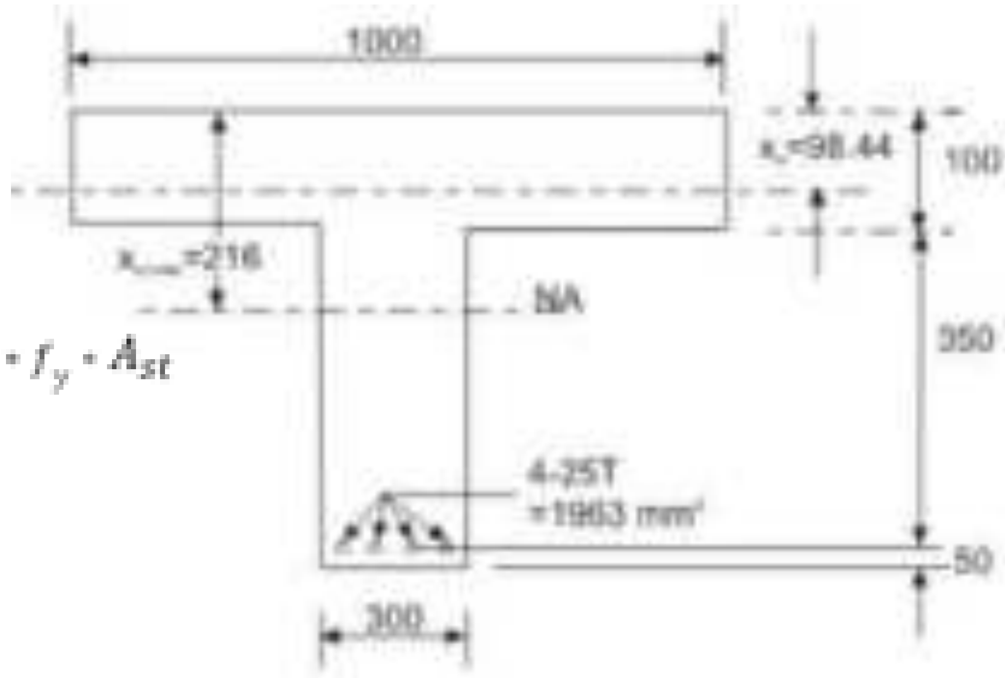
Provide 8mm dia. 2 legged stirrup 250 mm c/c.

- Check for deflection:
- L_{eff}/d_{eff}=5000/370=13.51
- Modification factor for tensile reinforcement:
- fs=.58*fy*(Ast required/Ast provided)=0.58*415*(1213/1256)=232.46
- Pt=1.13
- Modification factor= 1
- For compression NA
- For flanged section= bw/bf=300/2033=0.147, modification factor=0.8
- (L_{eff}/d_{eff}=5000/370=13.51) can not be greater than =20*1.0*.08=16
- It is ok.

PROBLEMS

- Problem-1:Determine the moment of resistance of the *T-beam of Fig shown. Given data: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm, d = 450 mm and A_{st} = 1963 mm^2 (4- 25 T). Use M 20 and Fe 415.*
- *Solution:*
- *Step:1- Determination X_u*

Assuming the neutral axis passing through flange portion



$$C = C_f = 0.36 * f_{ck} * b_f * x_u = T = 0.87 * f_y * A_{st}$$

$$x_u = 0.87 * f_y * A_{st}/(0.36 * f_{ck} * b_f)$$

$$x_u = 0.87 * 415 * \frac{1963}{0.36 * 20 * 1000} = 98.44 \text{ mm}$$

X_u< (D_f=100 mm) , Assumption is OK.

Neutral axis passes through flange

- Step:2-Calculation of x_{umax}
- x_{umax}=0.48*d=0.48*450=216 mm
- X_U< X_{UMAX}(So, under reinforced section)
- Step:2-Calculation of MOR
- Calculation of C or T

$$C = 0.36 * f_{ck} * b_f * x_u = 0.36 * 20 * 1000 * 98.44$$
$$= 708.77\ kN$$

$$T = 0.87 * f_y * A_{st} = 0.87 * 415 * 1963 = 708.77\ kN$$

- Calculation of MOR

$$M = M_f = 0.36 * f_{ck} * b_f * x_u * (d - 0.42x_u)$$

$$M = C * Lever\ arm$$
$$= 0.36 * 20 * 1000 * 98.44$$
$$* (450 - 0.42 * 98.44) = 289.64\ kN\text{m}$$

$$M = T * Lever\ arm$$
$$= 0.87 * 415 * 1963 * (450 - 0.42 * 98.44)$$
$$= 289.64\ kN\ \text{m}$$

PROBLEM

- Determine $A_{st,lim}$ *and* $M_{u,lim}$ *of the flanged beam of Fig.. Given data are:* b_f = *900 mm,* D_f = *90 mm,* b_w = *300 mm, cover = 50 mm and d = 450 mm.*

  Use M 20 and Fe 415.

SOLUTION:

STEP:1- Determination of X_{ulim}

d=360+90=450 mm

X_{ulim}*=0.48*d=0.48*450=216 mm*

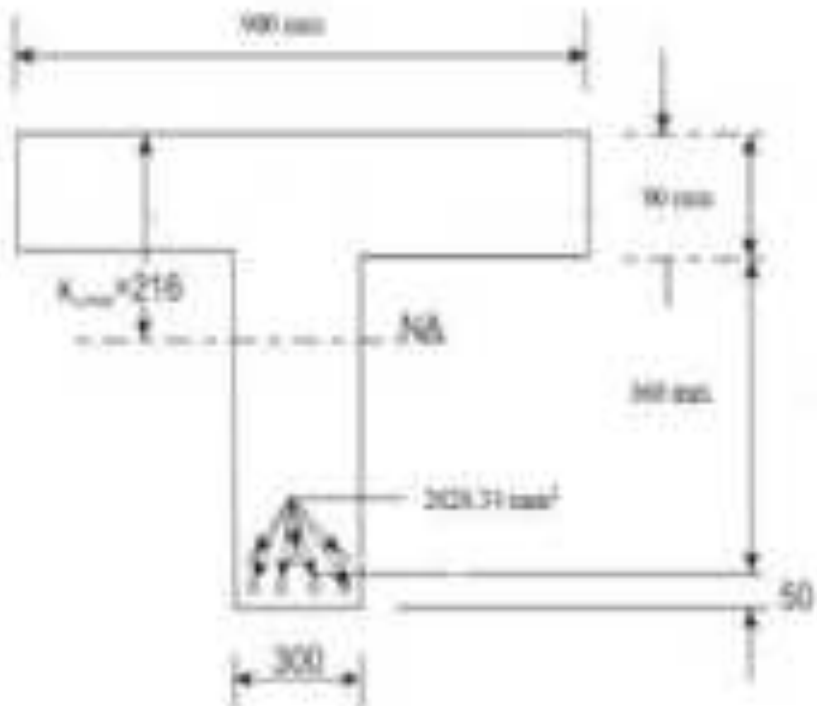
(X_{ulim}*=216 mm)> (*D_f*=90 mm)*

X_{ulim} *passes through web.*

Let us check (D_f*/d)=90/450=0.20*

It is balanced section, D_f*/d=0.2,*

it is in category (ii a)



STEP:2- Determination of MOR

Calculation of C and T

$$C = C_f + C_w = 0.446 * f_{ck} * (b_f - b_w) * D_f + 0.36 * f_{ck} * b_w * x_{ulim}$$

$$C = C_f + C_w$$
$$= 0.446 * 20 * (900 - 300) * 90 + 0.36 * 20 * 300 * 216$$
$$= 481680 + 466560 = 948.24\ kN$$

- Calculation A_{st}

$$T = 0.87 \cdot f_y \cdot A_{st} = C = 948240$$

- A_{st}=2626.33 mm^2
- STEP-III: Calculation MOR

$$M = M_f + M_w$$

$$= 0.446 * f_{ck} * (b_f - b_w) * D_f * (d - \frac{D_f}{2}) + 0.36 \cdot f_{ck} \cdot b_w * x_{ulim} * (d - 0.42x_{ulim})$$

$$M = M_f + M_w = 362.70\ kNm$$

- Mulim=0.87*fy*Ast*(d-0.42*xulim) DONOT USE THIS FORMULAE.
- Mulim=0.87*415*2626.34*(450-0.42*216)=3420.68

PROBLEM

- Determine the moment of resistance of the beam of when A_{st} = 4,825 mm^2. b_f = 1,000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Step 1: To determine x_u

Assuming NA axis passing through flange

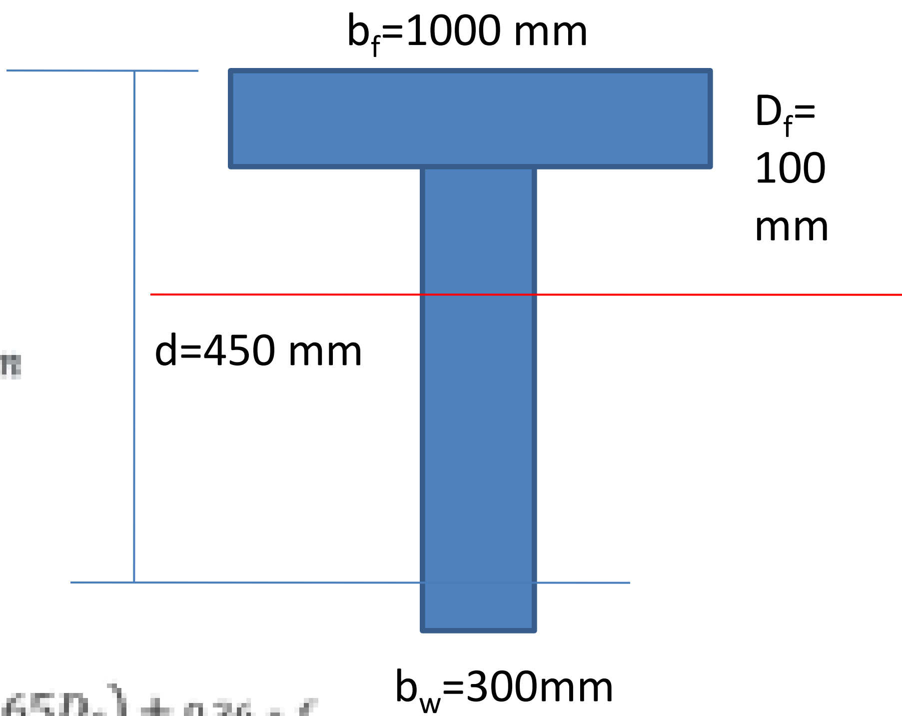
$$x_u = 0.87 * f_y * A_{st}/(0.36 * f_{ck} * b_f)$$

$$x_u = 0.87 * 415 * \frac{4825}{0.36 * 20 * 1000} = 241.95\ mm$$

NA axis is passing through web.

D_f/d=100/450=0.222 >0.2

There is requirement of calculation of yf



$$C = 0.446 * f_{ck} * (b_f - b_w) * (0.15x_u + 0.65D_f) + 0.36 * f_{ck} * b_w * x_u = T = 0.87 f_y * A_{st}$$

$$0.446 * 20 * (1000 - 300) * (0.15x_u + 0.65 * 100) + 0.36 * 20 * b_w * x_u = 0.87 * 415 * 4825$$

- Calculation of x_u, $x_u = 431.50\ mm$
- X_{ulim}=0.48*450=216 mm
- X_u> x_{ulim}, over reinforced, D_f/d>0.2
- y_f=0.15*x_u+0.65*D_f=0.36*216+0.65*100=97.4 mm < 100 mm

$$M = M_f + M_w$$
$$= 0.446 * f_{ck} * (b_f - b_w) * y_f * (d - \frac{y_f}{2}) + 0.36 * f_{ck} * b_w * x_{ulim} * (d - 0.42x_{ulim})$$

$$M = M_f + M_w$$
$$= 0.446 * 20 * (1000 - 300) * 97.4 * \left(450 - \frac{97.4}{2}\right) + 0.36 * 20 * 300 * 216 * (d - 0.42 * 216)$$
$$= 411.68\ kNm$$

ASSIGNMENT

Design the L beam if slab thickness is 150mm, slab beam casting monolithic, effective span 5 m. Slab is subjected to imposed load = 5kN/m^2, M20 and Fe415

Step1: Calculation of b_f

Df=150 mm

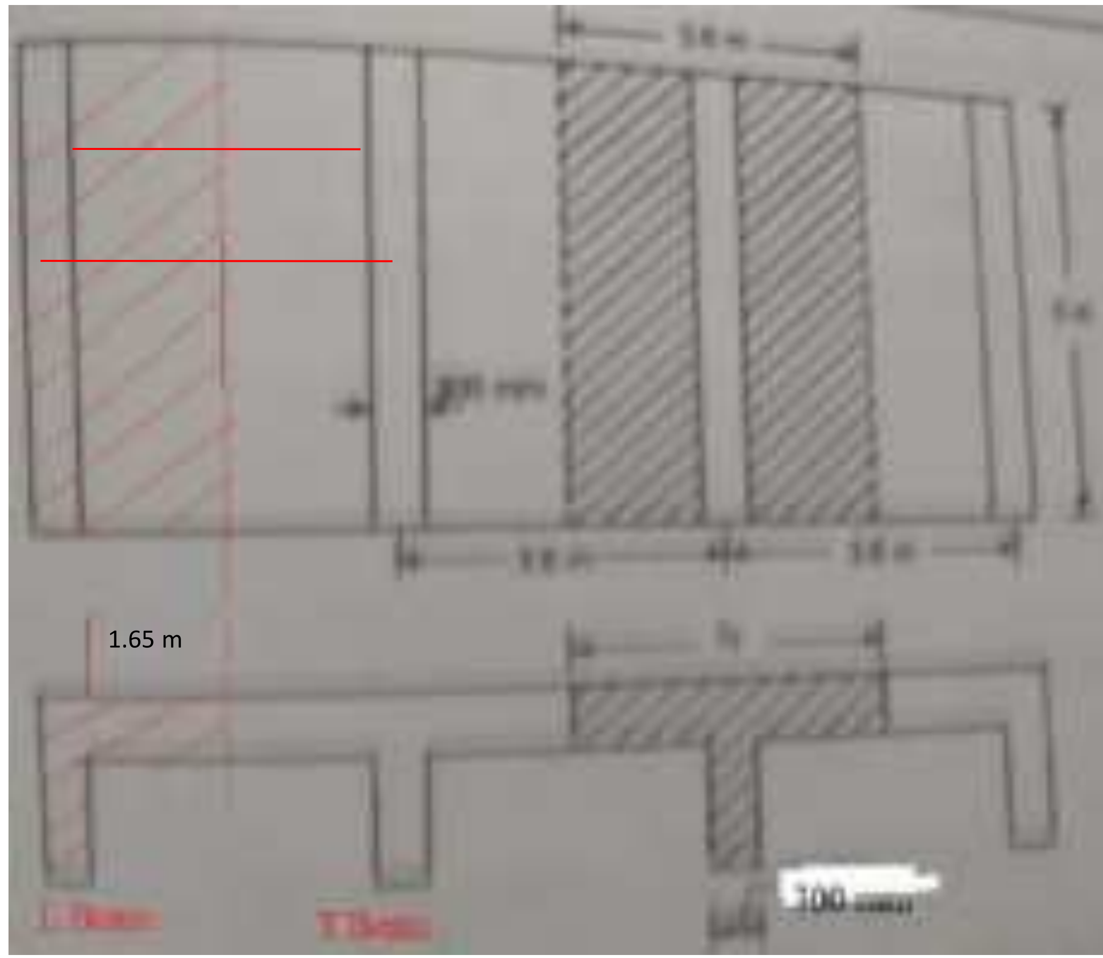
Actual width=1.65+0.3=1.95 m

lo=5 m

bw=300 mm

i) $b_f = \frac{l_0}{12} + b_w + 3D_f$

ii) $b_f = Actual\ width\ of\ flange$

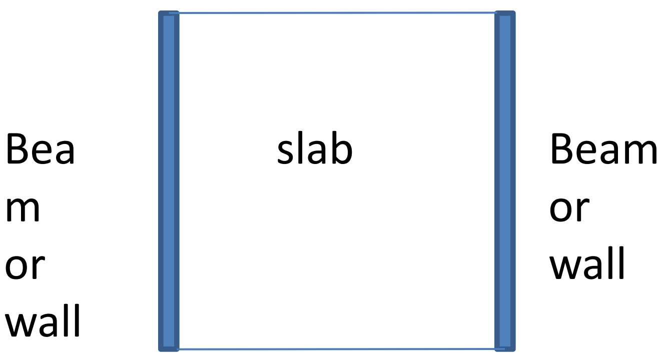


DESIGN OF SLABS

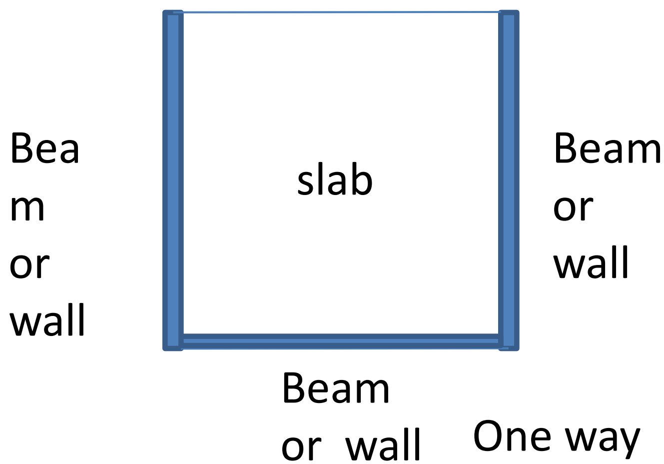
Dr.G.C.Behera

SLAB

- Slab is the covering to a structure. Generally slab is a horizontal structure. In some cases, these may be inclined also. The slab may be supported on four edges or it may be supported on three or two edges or one side as shown in figure. Depending upon the supporting conditions and distribution of load, slabs may be categorized as
- 1. One way slab
- 2. Two way slab.



One way slab

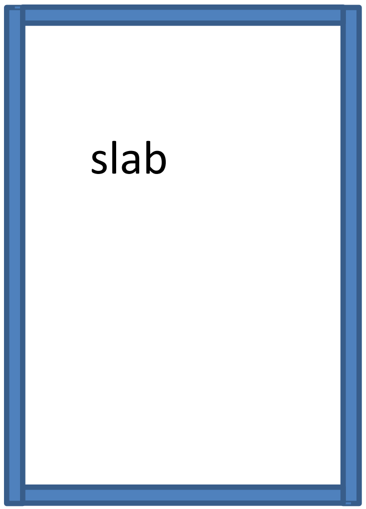


One way slab

SLAB

- 1. One way slab
- 2. Two way slab.

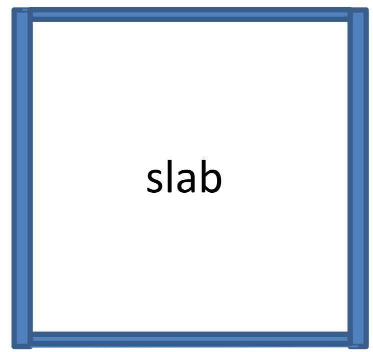
Beam or wall



Beam or wall

One way slab
If $L_y/L_x > 2$

Beam or wall



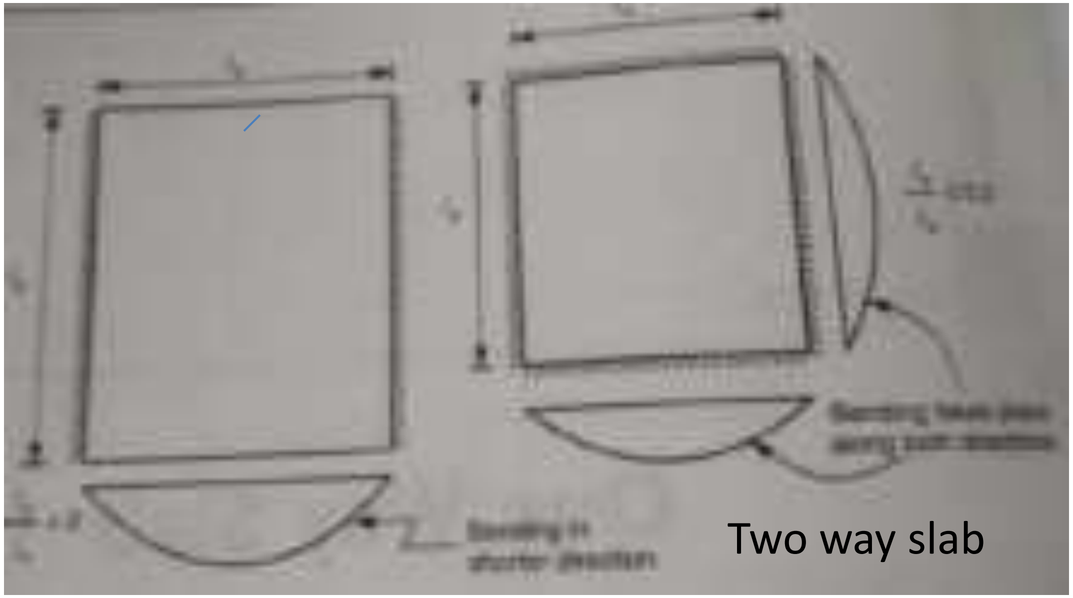
Beam or wall

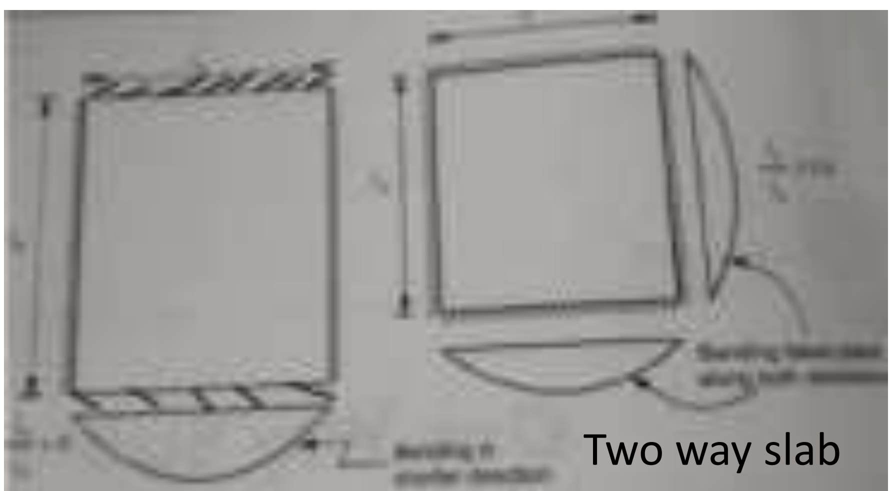
Beam or wall

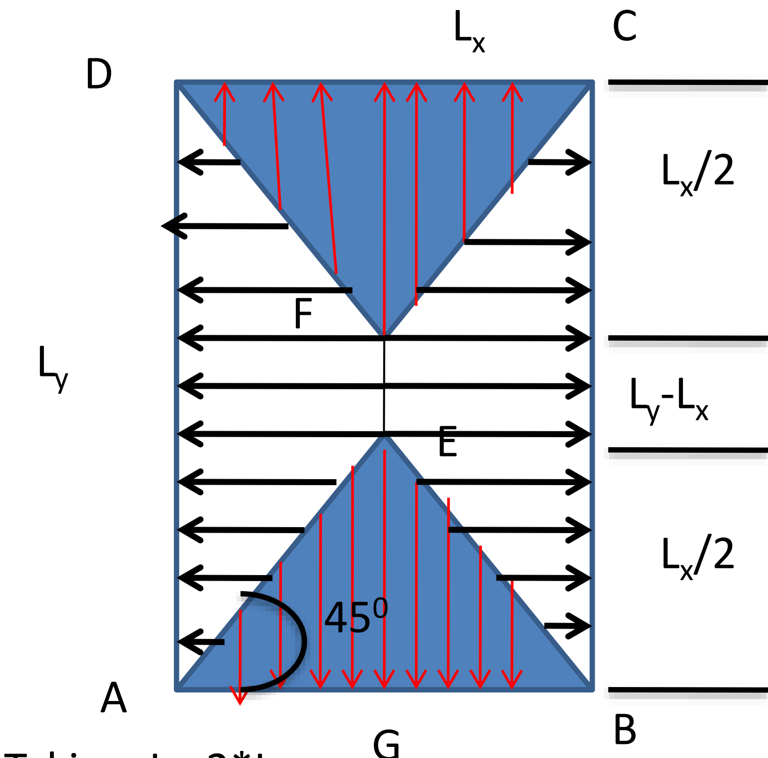
Two way slab
If L_y/L_x less or equal to 2

Where L_y and L_x are the effective length in longer and shorter direction of slab.

LOAD DISTRIBUTION IN SLABS



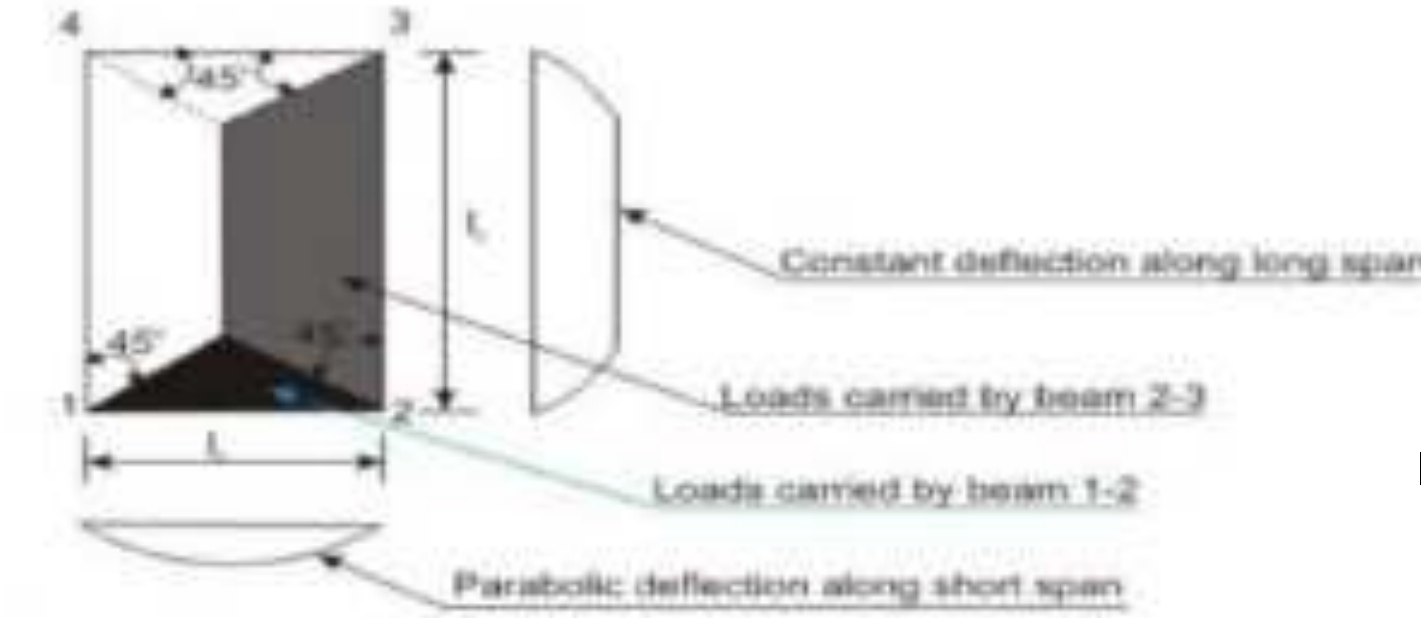




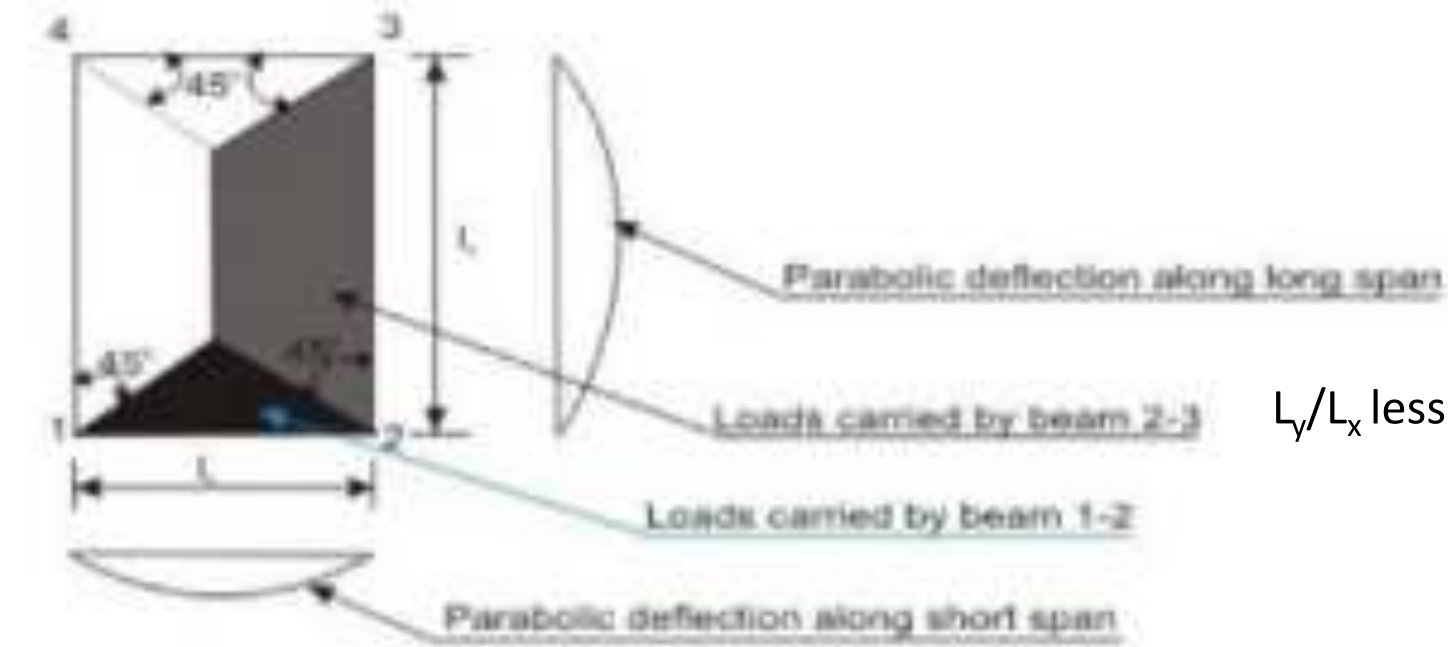
Taking $L_y=2*L_x$

Area of load on ABE and CDF along long span direction$=(1/2)*L_x*L_x/2=L_x^2/4$

Total Area of load on longer span=Area of ABE+Area of CDF$=2*L_x^2/4=L_x^2/2$



$L_y/L_x > 2$

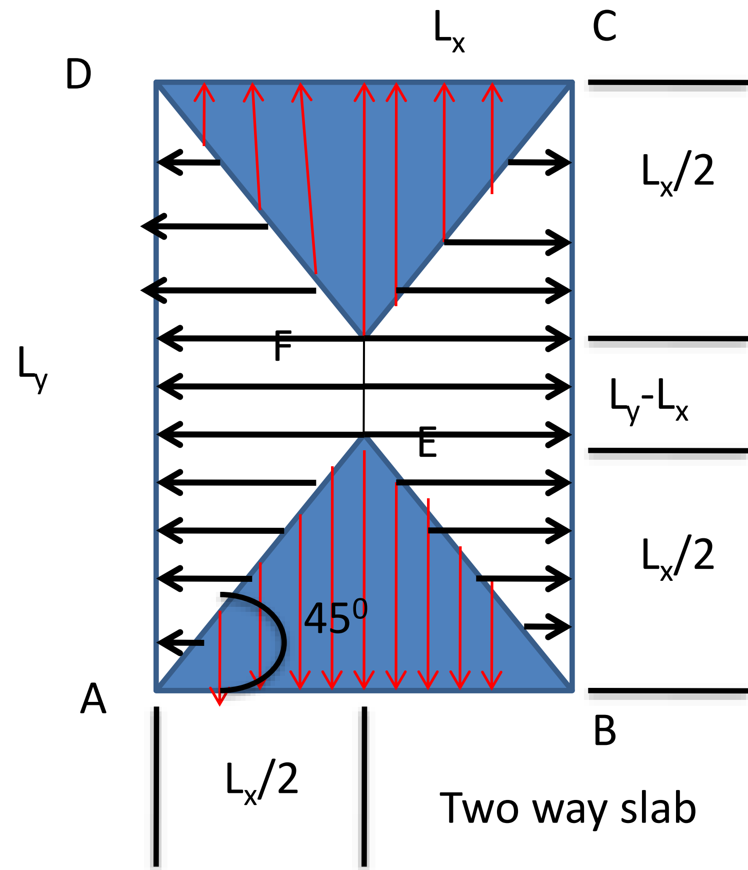


L_y/L_x less or equal to 2

LOAD DISTRIBUTION IN SLABS

Taking $L_y=2*L_x$
Area of load on ADEF and BEFC along short span direction=$[(L_y+L_y-L_x)/2]*L_x/2=3L_x^2/4$

Total Area of load on shorter span=Area of ADEF and BEFC $=2*3L_x^2/4=3L_x^2/2$
Load transferred along short span direction is 3 times more than that is transferred along long span direction.
If Ly> 2*Lx, then load on long direction will be very less. That is reason we are assuming load is transferred along short span direction. Bending will be in short span direction only known as one way slab
Otherwise load will be distributed in both directions and bending will be on both direction, it is known as two way slab.



Black shaded area will transferred load on long span direction, red portion area load will be transferred along short span direction.

DIFFERENCE IN ONE WAY AND TWO WAY SLAB

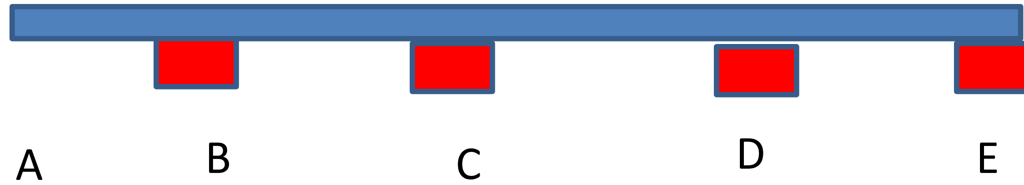
SN	ONE WAY SLAB	TWO WAY SLAB
1	$L_y/L_x>2$	L_y/L_x less or equal to 2
2	Bending takes place in one direction only.(Short span)	Bending takes place in both directions.
3	More bending moment. Depth required more.	Less bending moment. Depth required less.
4	Main steel is provided in shorter span direction. Distribution steel is provided along long span direction.	Main steel is provided in both directions.
5	Thickness more, more steel is required. Not economical	Thickness less, less steel is required. economical.

IS-456-2000 SLAB SPECIFICATION:

22.2 Effective Span

Unless otherwise specified, the effective span of a member shall be as follows:

a) *Simply Supported Beam or Slab*—The effective span of a member that is not built integrally with its supports shall be taken as clear span plus the effective depth of slab or beam or centre to centre of supports, whichever is less.



22.3 *Continuous Beam or Slab –*

B) In the case of continuous beam or slab,

if the width of the support is less than l/12 of the clear span, the effective span shall be as in 22.2(a). If the supports are wider than l/12 of the clear span or 600 mm whichever is less, the effective span shall be taken as under:

1) For end span with one end fixed and the other continuous or for intermediate spans,

the effective span shall be the clear span between supports;

2) For end span with one end free and the other continuous, the effective span shall be equal

to the clear span plus half the effective depth of the beam or slab or the clear span plus

half the width of the discontinuous support, whichever is less;

3) In the case of spans with roller or rocket bearings, the effective span shall always be the distance between the centres of bearings.

C)Cantilever-The effective length of a cantilever shall betaken as its length to the face of the

support plus half the effective depth except where it forms the end of a continuous beam

- where the length to the centre of support shall be taken.

D)Frames-In the analysis of a continuous frame,

- centre to centre distance shall be used.

DEFLECTION CONTROL

- FOR ONE WAY SLAB: As beams
- 23.2.1 The vertical deflection limits may generally be assumed to be satisfied provided that the span to depth ratios are not greater than the values obtained as below:
- Basic values of~span to effective depth ratios for spans up to 10 m:
- A) Cantilever 7
- B) Simply supported 20
- C) Continuous 26
- For spans above 10 m, the values in (a) may be multiplied by 10/span in metres, except for cantilever in which case deflection calculations should be made.
- Depending on the area and the stress of steel for tension reinforcement, the values in (a) or(b) shall be modified by multiplying with the modification factor obtained as per Fig. 4.
- Depending on the’ area of compression reinforcement, the value of span to depth ratiobe further modified by multiplying with the modification factor obtained as per Fig. 5.
- The provisions of 23.2 for beams apply to slabs also.
- Notes
- **For slabs spanning in two directions shorter of the two spans should be used for calculating the span to effective depth ratio.**
- **Simply supported =35**
- **Continuous=40**

REINFORCEMENT

Minimum reinforcement (cl.26.5.2.1 of IS 456)

- Both for one and two-way slabs, the amount of minimum reinforcement in either direction shall not be less than 0.15 and 0.12 per cents of the total cross-sectional area for mild steel (Fe 250) and high strength deformed bars (Fe 415 and Fe 500)/welded wire fabric, respectively.

Maximum diameter of reinforcing bars (cl.26.5.2.2)

- The maximum diameter of reinforcing bars of one and two-way slabs shall not exceed one-eighth of the total depth of the slab.

Maximum distance between bars (cl.26.3.3 of IS 456)

- The maximum horizontal distance between parallel main reinforcing bars shall be the lesser of (i) three times the effective depth, or (ii) 300 mm. However, the same for secondary/distribution bars for temperature, shrinkage etc. shall be the lesser of (i) five times the effective depth, or (ii) 450 mm.

Minimum distance between bars

- The minimum horizontal distance between parallel main reinforcing bars shall not be the lesser of
- (i) The diameter of bar (largest bar dia. If unequal dia. Are used), or
- (ii) 5 mm more than maximum size of the coarse aggregate
- The minimum vertical distance between two parallel main reinforcing bars shall be more than
- i) 15 mm
- Ii) 2/3 rd of maximum size of coarse aggregate
- Iii) Maximum size of the bar

SPECIFICATIONS

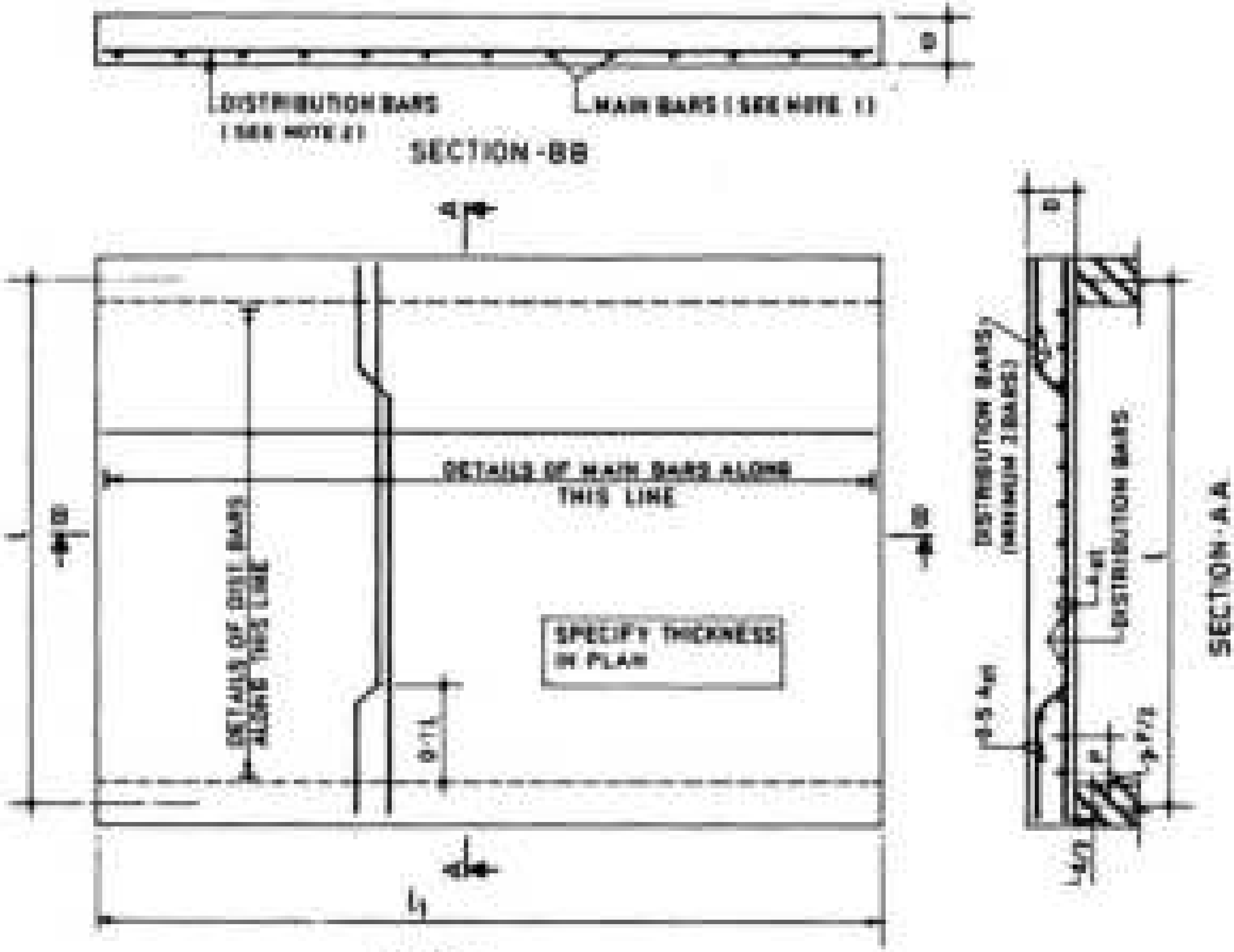
Nominal cover (cl.26.4 of IS 456)

- The nominal cover to be provided depends upon durability and fire resistance requirements. Table 16 and 16A of IS 456 provide the respective values. Appropriate value of the nominal cover is to be provided from these tables for the for particular requirement of the structure.
- **Design Shear Strength of Concrete in Slabs**
- Experimental tests confirmed that the shear strength of solid slabs up to a depth of 300 mm is comparatively more than those of depth greater than 300 mm. Accordingly, cl.40.2.1.1 of IS 456 stipulates the values of a factor *k to be multiplied with* τ_c *given in Table 19 of IS 456 for different overall depths of slab. Table 8.1 presents the values of k as a ready reference below:*

Overall depth of slab (mm)	300 or more	275	250	225	200	175	150 or less
k	1.00	1.05	1.10	1.15	1.20	1.25	1.30

DESIGN STEPS FOR ONE WAY SLAB

- STEP-1: Take width of slab as one meter.
- STEP-2:Assume the depth of slab from control of deflection. Take (leff/d) as 25 to 30 for simply supported and 10 for contriver.
- STEP-3: Find the load and BM. Calculate Factored BM and Factored SF.
- STEP4: Calculate the required depth from BM taking as balanced section. If calculated depth is less than assumed depth, design is OK. Otherwise redesign it.
- STEP-5: Find the main steel along short span direction and find distribution steel amount along long span direction and for others.
- STEP-6: Check for shear.
- STEP-7: Detailing.



PLAN

NOTE 1 — Diameter ≮ 8 mm for deformed bars, 10 mm for plain bars. Spacing ≯ 3d or 450 mm.

NOTE 2 — Diameter ≮ 6 mm. Spacing ≯ 5d or 450 mm.

FIG. 8.1 TYPICAL DETAILS OF A SLAB SPANNING IN ONE DIRECTION

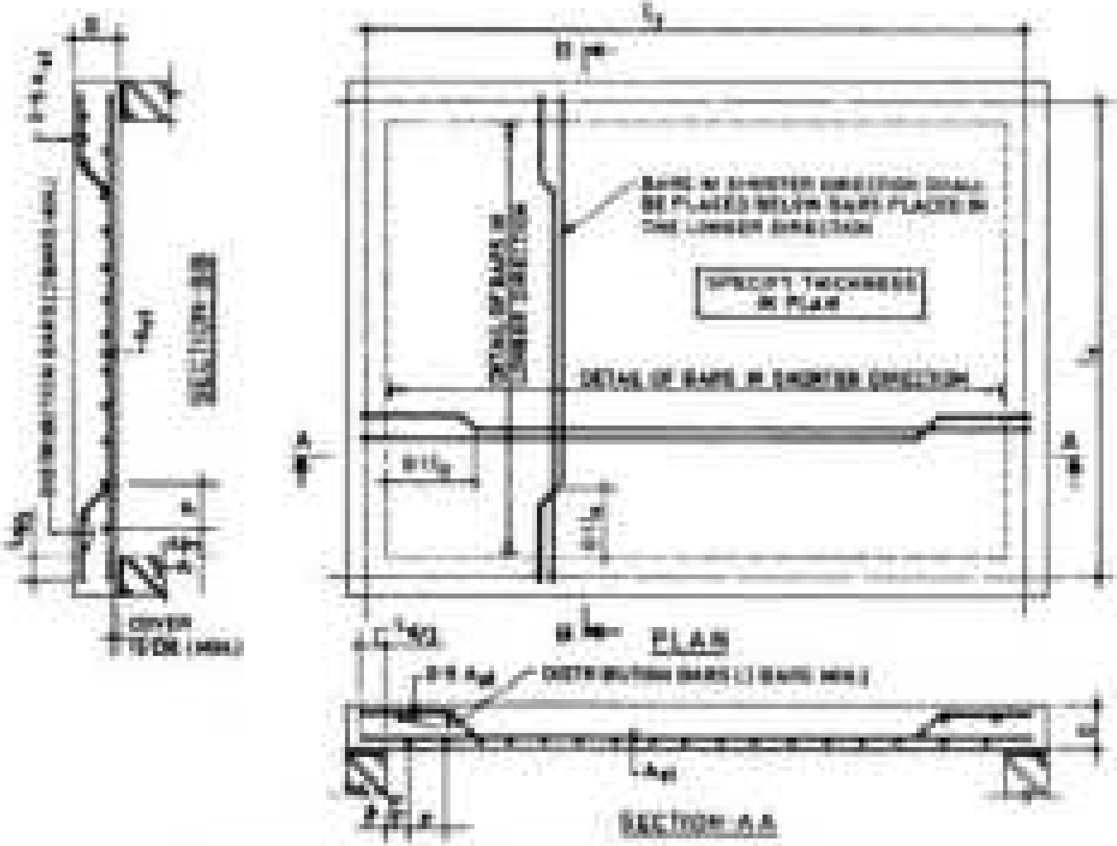
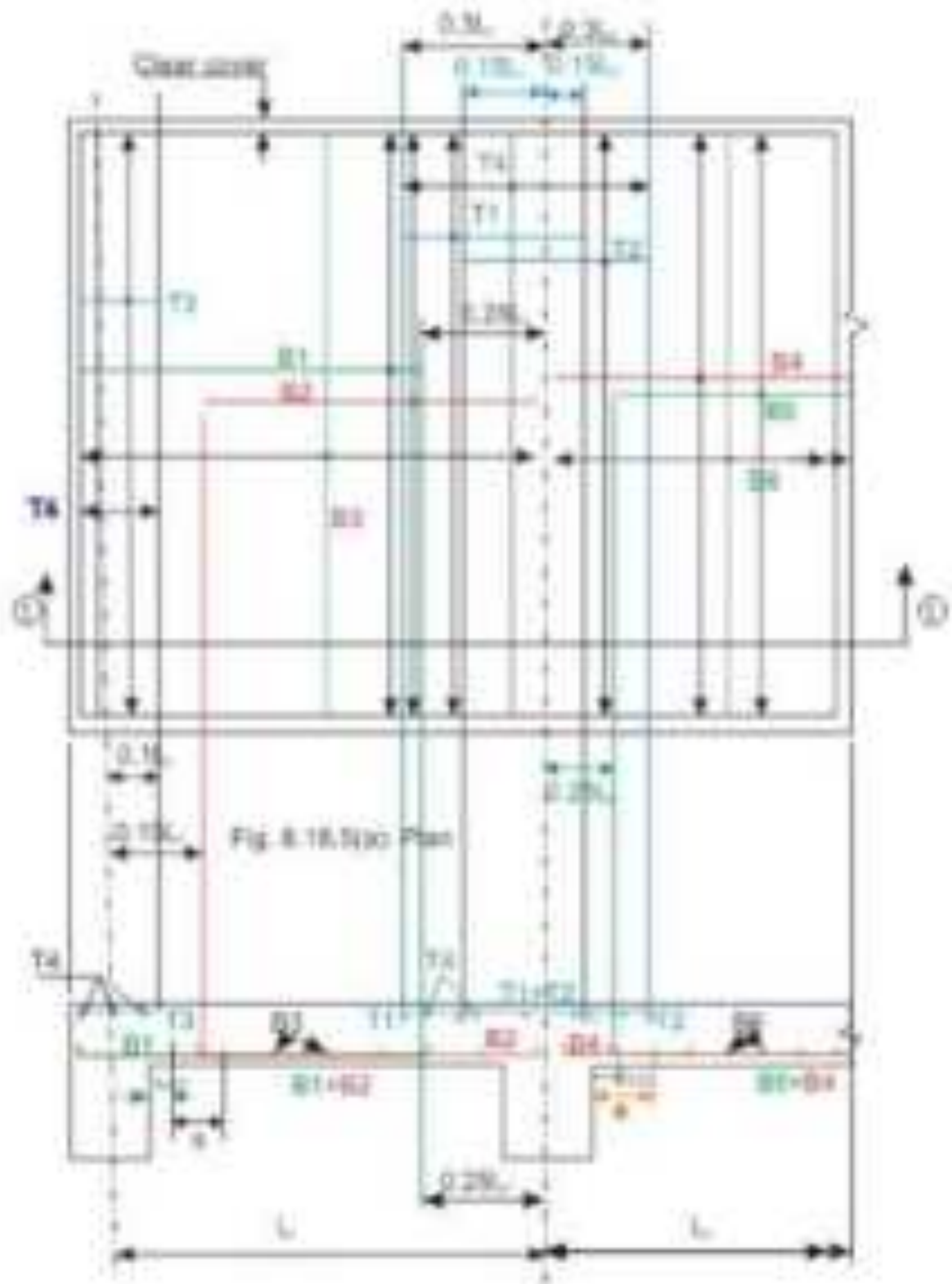


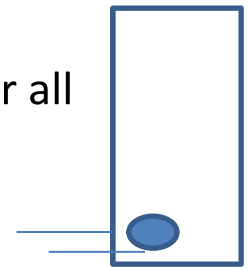
Fig. 5.2 Typical Details of a Slab Spanning in Two Directions



Detailing for one way continuous slab

DESIGN OF ONE WAY SLAB

- A simply supported slab of clear span 2.5 mt. of a corridor of a hospital building is supported on beams of width 230 mm. The slab is carrying live load of 5kN/m^2.Use M20and Fe415 grade concrete and steel respectively.



Assumption of Depth

Take L/d=25, l=2.5 m d=100 mm, over all depth =D=120 mm

Assuming effective cover 20 mm

Effective Length Calculation:

Centre to Centre =2.5+.23/2+0.23/2=2.73 m

$L_{cl}+d_{eff}$=2.5+0.1=2.6 m

L_{eff}=2.6 m

Calculation of Load:

Assuming width of slab as 1m

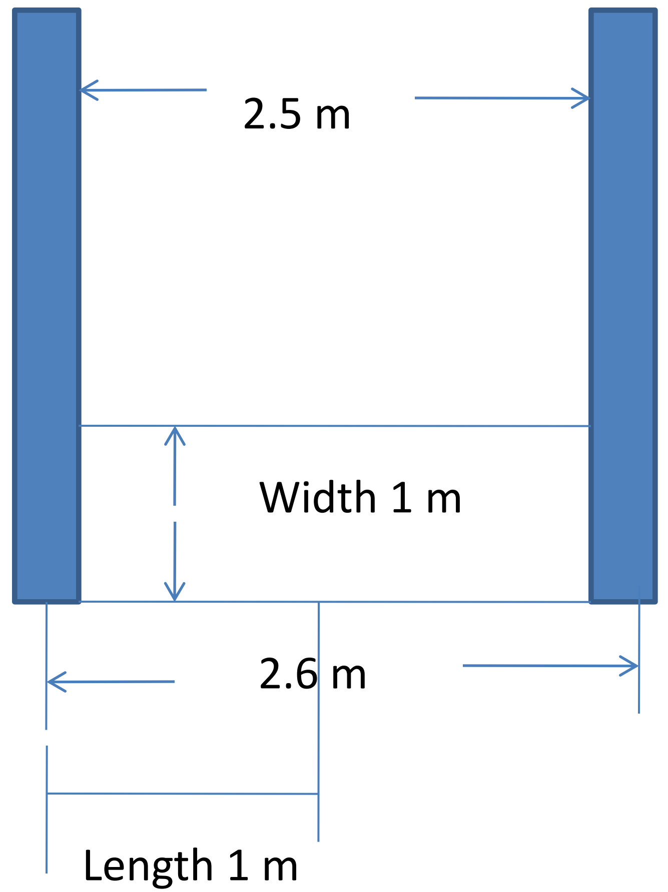
Self wt/m length= 1*1*.120*25=3.0 kN/m

LL= 1*1*5= 5 kN/m

Total w= 8 kN/m

W_u=1.5*8=12 kN/m

$M_u=w_u*l_{eff}^2/8$=12*2.6*2.6/8=10.14 kNm



DESIGN OF ONE WAY SLAB

- $V_u = w_u * l_{eef}/2 = 15.6$ kN
- **Required Effective Depth Calculation:**

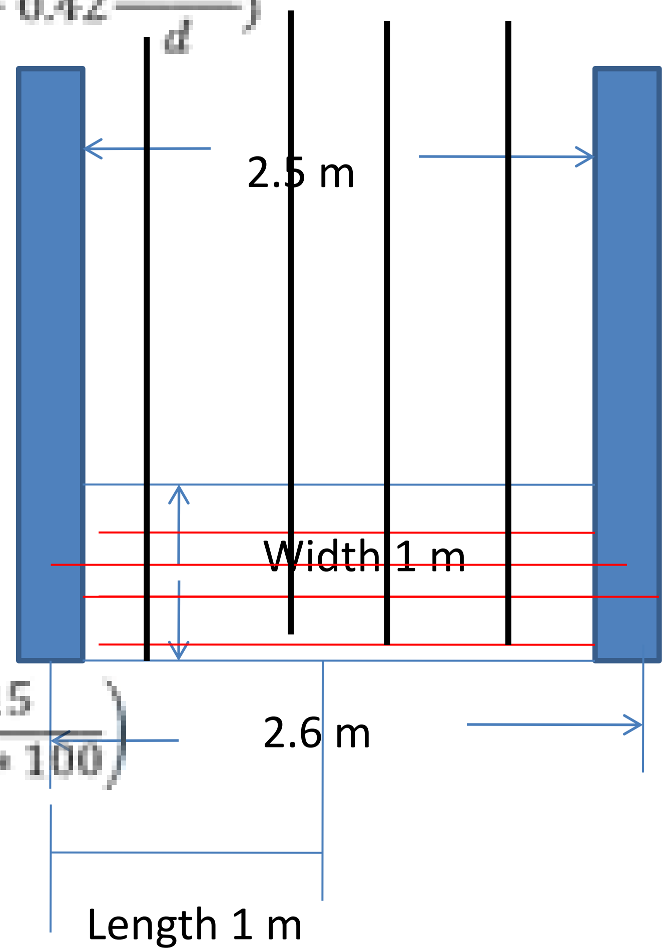
$$MOR = 10.14*10^6 = 0.36 f_{ck} bd\, x_{umax}(1 - 0.42\frac{x_{umax}}{d})$$

- b=1000 mm, $f_{ck}=20$, $x_{umax}=0.48d$
- d_{req}= 60.61 mm
- (d_{req}= 60.61 mm)< ($d_{assumed}$=100 mm)
- Calculation of Steel

$$M_u = 0.87 f_y * A_{st} * d\left(1 - \frac{A_{st} f_y}{f_{ck} bd}\right)$$

$$10.14*10^6 = 0.87*415*A_{st}*100\left(1 - \frac{A_{st}*415}{20*1000*100}\right)$$

$$A_{st} = 300\ mm^2$$



- Check for minimum steel:
- A_{st}=.12%*bD=0.12*1000*120/100=144 mm^2< 300 mm^2
- So, it is OK.
- Spacing of bars:
- Taking 8 mm diameter bars,

$$Spacing = \frac{Length}{no.of\ bars} = \frac{1000}{\frac{A_{st}}{50}} = 166.66\ mm$$

- Spacing can not be greater than 3d=3*100=300 mm
- Spacing can not be greater than 300 mm,
- Provide 8 mm dia bars 160 mm c/c. In short span direction.
- Distribution steel:
- In the long span direction provide distribution steel
- Area of distribution steel can not be less than 144 mm^2.
- Spacing of 8 mm dia bars

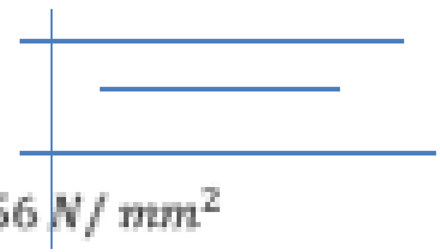
$$Spacing = \frac{Length}{no.of\ bars} = \frac{1000}{\frac{144}{50}} = 347.22\ mm$$

- For distribution steel spacing

$$Spacing = \frac{Length}{no.of\ bars} = \frac{1000}{\frac{144}{50}} = 347.22\ mm$$

- Spacing of distribution steel can not be greater than
- 5d=5*100=500 mm
- Can not be greater than 450 mm
- So, provide 8 mm dia bars 340 mm c/c.
- CHECK FOR SHEAR:
- Vu=15.6 kN

$$\tau_{vu} = \frac{V_u}{bd} = \frac{15600}{1000 * 100} = 0.156\ N/mm^2$$



- At the support, bending 50% of steel at a distance l/7 =371 mm from centre of support,
- A_{st}=(1000/320)*50=156.25 mm^2
- p_t=156.25*100/(1000*100)=0.156
- τ_c=0.28 N/mm^2
- k=1.3 for 120 mm depth

- $K*\tau_c$=1.3*0.28=0.364 N/mm^2
- shear resistance> nominal shear, so design is ok.
- Check for deflection:
- p_t=0.30 at centre
- f_s=.58*f_y*(Ast required/Ast provided)=0.58*415*(300/312.5)=231.07
- Modification factor=1.5
- Leff/deff=2.6/.1=26
- Maximum value=20*1.5=30
- 26<30, So it is ok.
- Check for Development Length:

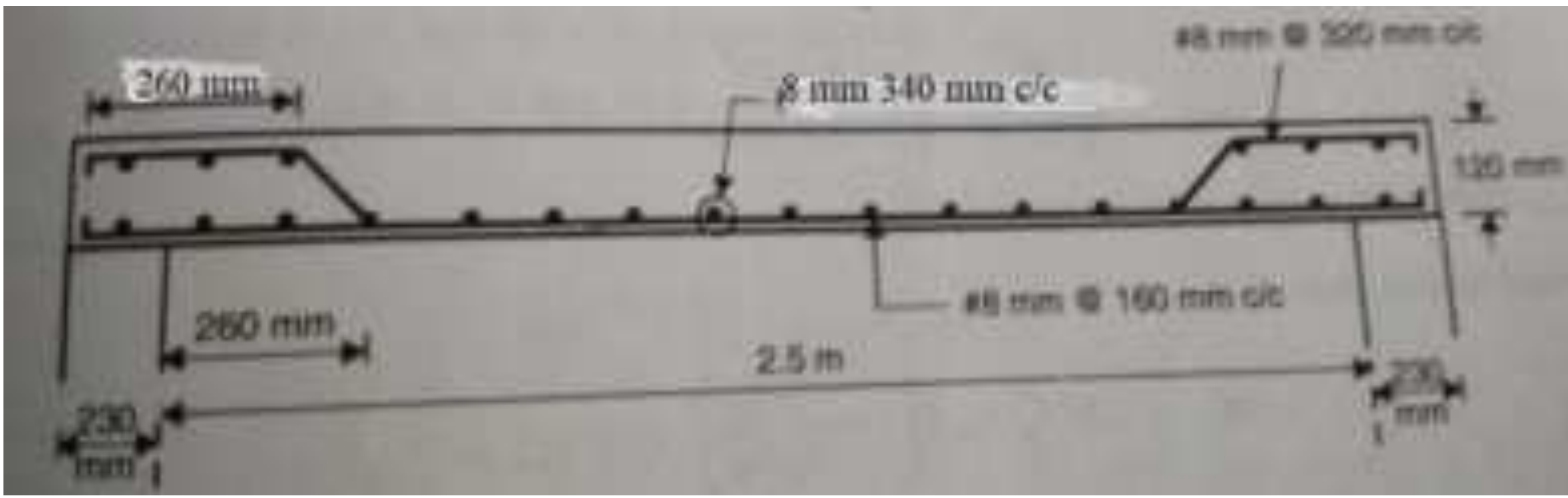
$$l_d = \frac{0.87 * f_y * \emptyset}{4 * \tau_{bd}} = 376.09\ mm$$

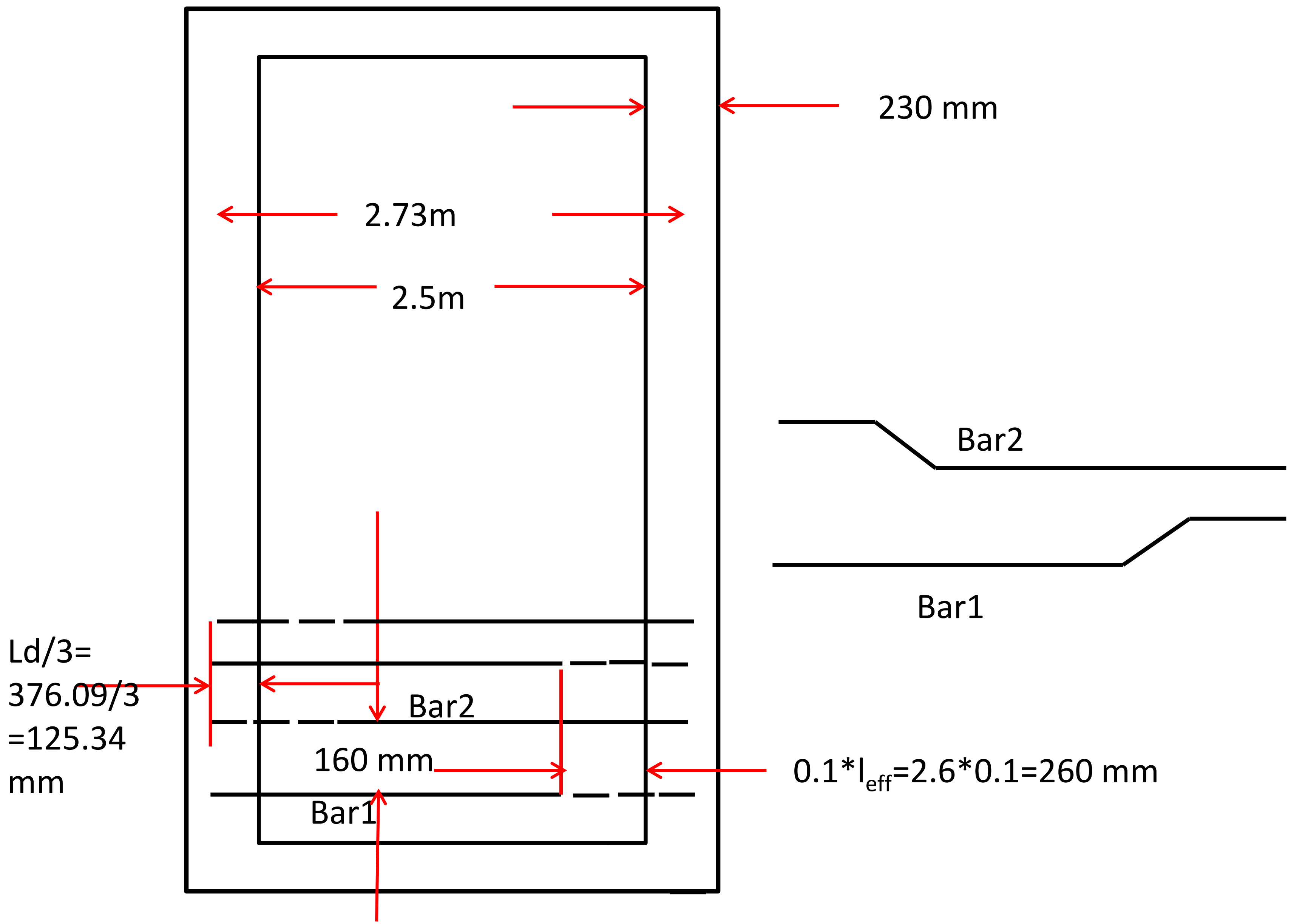
- M1= $$M_1 = T * z = 0.87 f_y * A_{st} * d\left(1 - \frac{A_{st} f_y}{f_{ck} bd}\right) = 54.50\ kNm$$
- Vu=15.6 kN, Using 90^0 bend, l_0=8*φ,
- l_0=8*φ=64 mm
- $$\frac{M_1}{V_u} + l_0 = \frac{54500059}{15600} + 8 * (\emptyset = 8) = 418.90$$

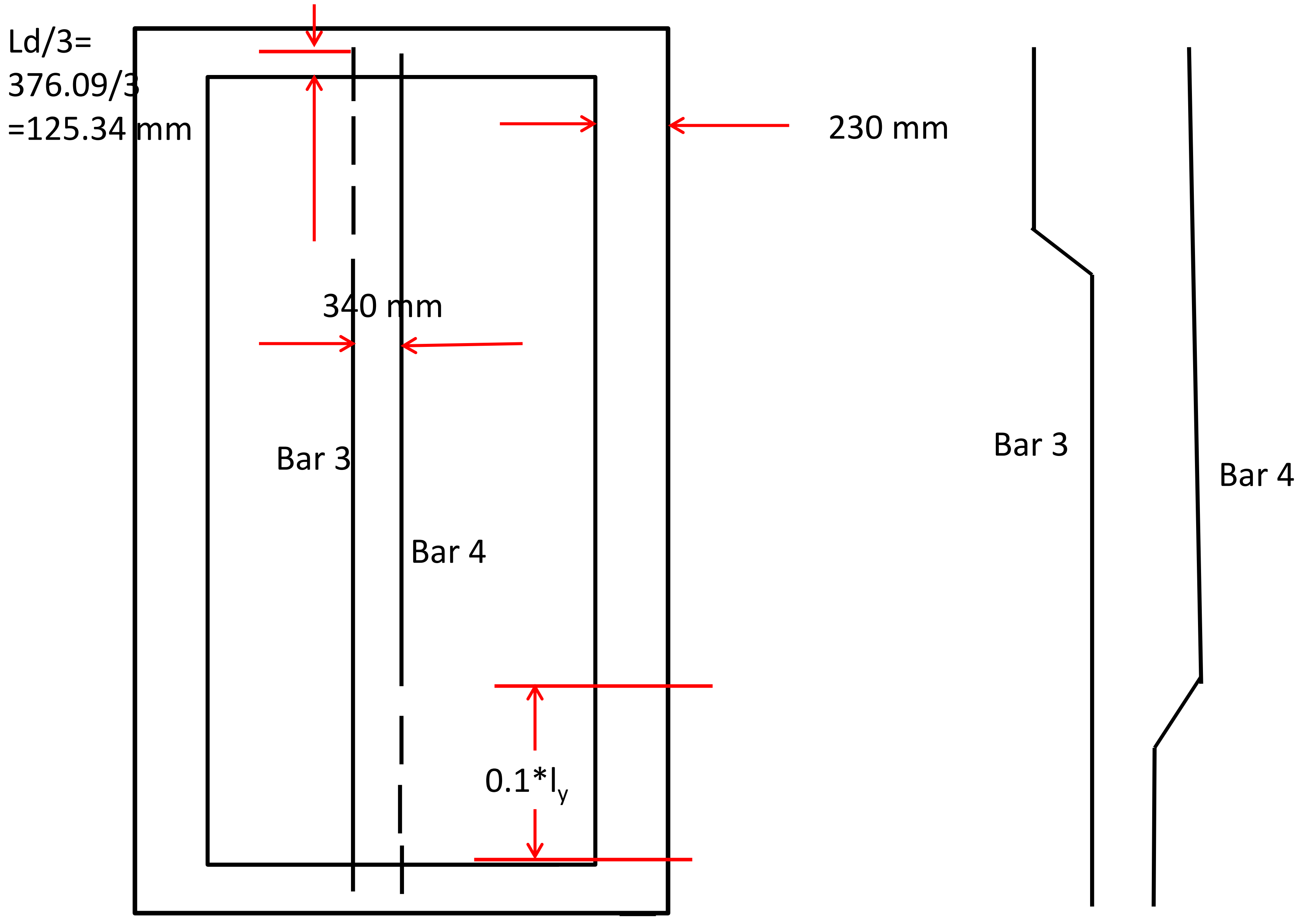
$$l_d = \frac{0.87 * f_y * \emptyset}{4 * \tau_{bd}}$$

$$= 376.09\ mm\ should\ not\ be\ greater\ than(\frac{M_1}{V_u} + l_0$$

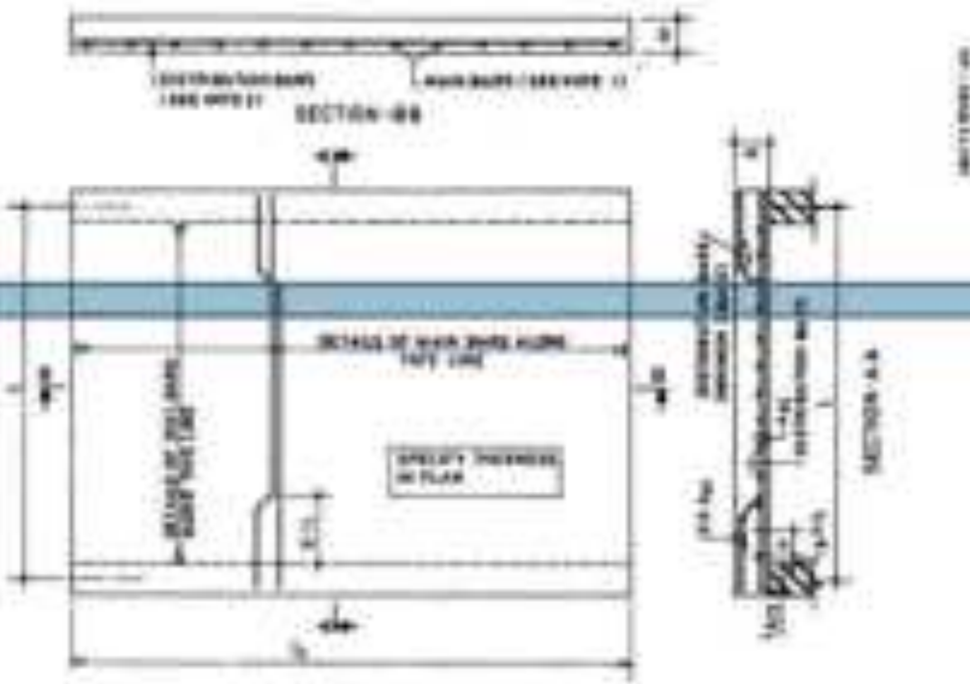
$$= \frac{54500059}{15600} + 8 * (\emptyset = 8) = 418.90mm)$$



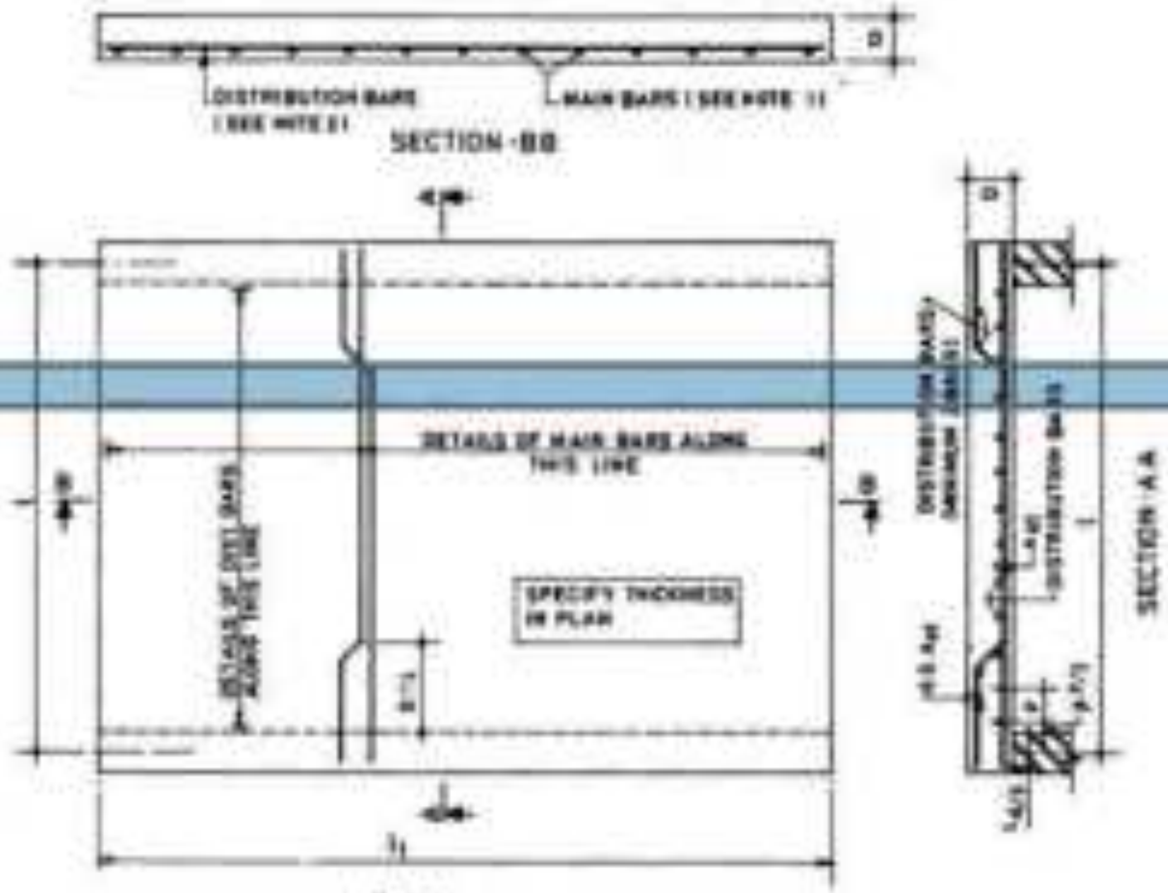

230 mm
2.73m
2.5m
Bar2
Bar1
Ld/3=
376.09/3
=125.34
mm
Bar2
160 mm
0.1*l_eff=2.6*0.1=260 mm
Bar1



Ld/3=
376.09/3
=125.34 mm
230 mm
340 mm
Bar 3
Bar 4
0.1*l_y
Bar 3
Bar 4



PLAN



DISTRIBUTION BARS
MAIN BARS
SECTION - BB
DETAILS OF MAIN BARS ALONG
THIS LINE
SPECIFY THICKNESS
IN PLAN
SECTION - A A
DISTRIBUTION BARS
PLAN

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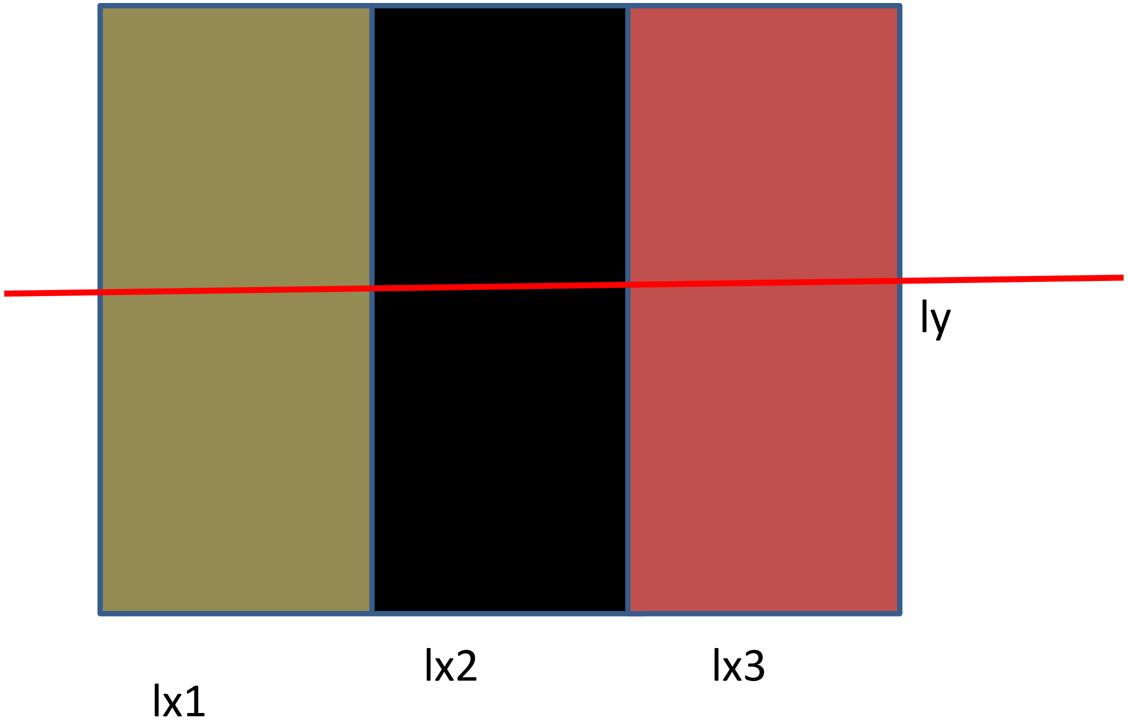
CONTINUOUS ONE WAY SLAB

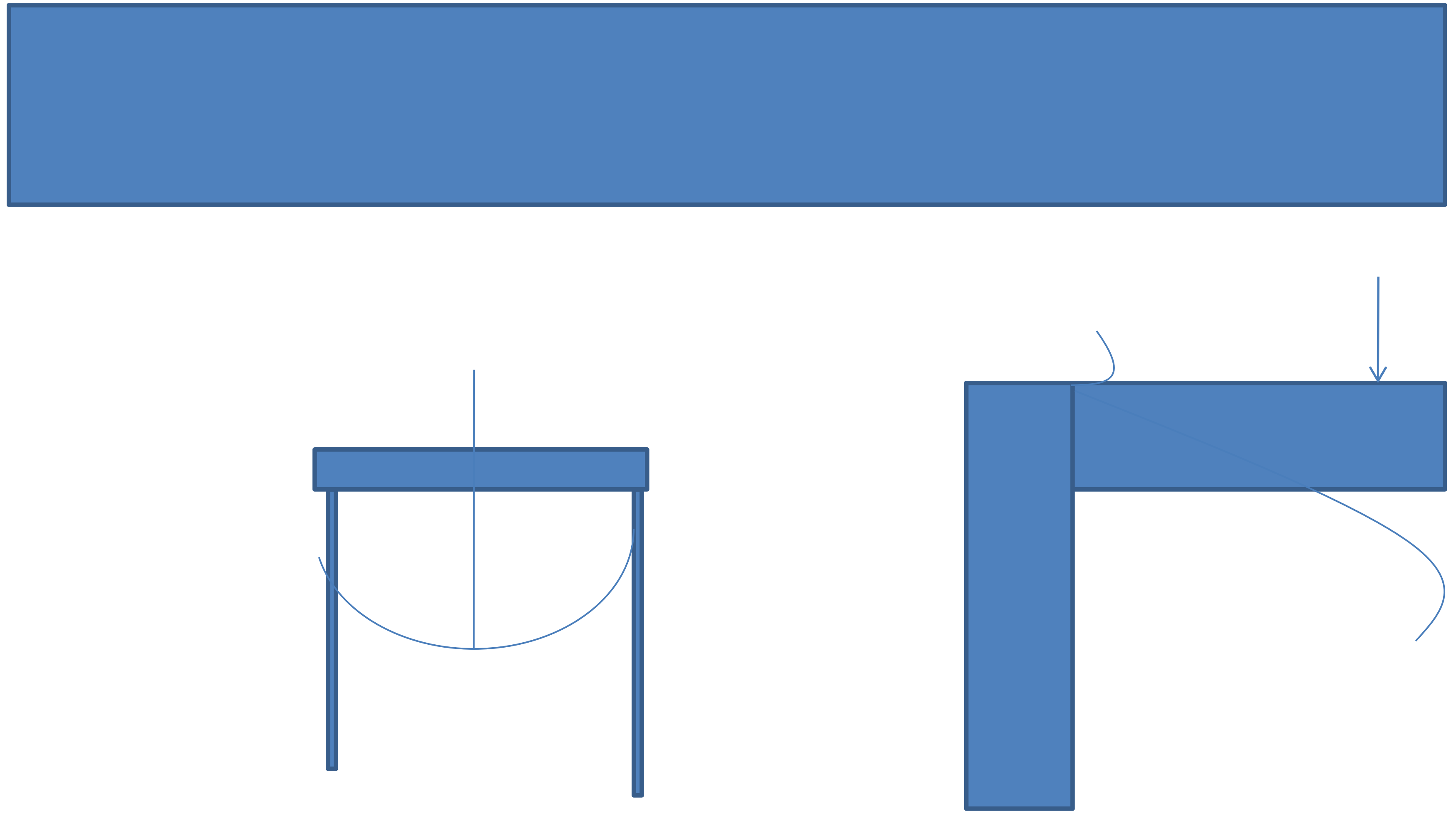
- Continuous one-way slab

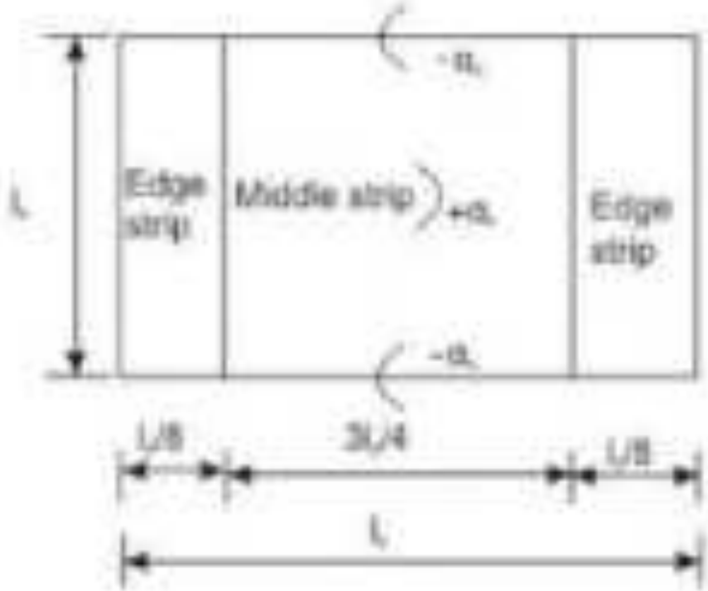


TWO -WAY SLAB(IS CODE METHOD)

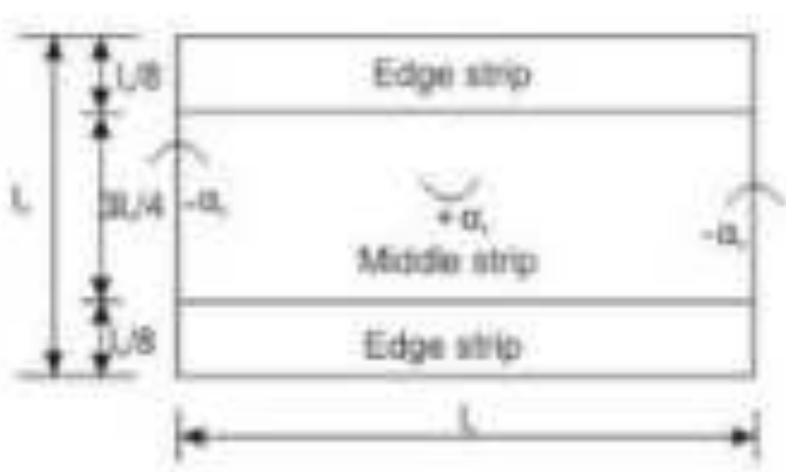
- IF L_y/L_x less than or equal to 2, two way slab.
- Two-way slabs subjected mostly to uniformly distributed loads resist them primarily by bending about both the axis. However, as in the one-way slab, the depth of the two-way slabs should also be checked for the shear stresses to avoid any reinforcement for shear. Moreover, these slabs should have sufficient depth for the control deflection. Thus, strength and deflection are the requirements of design of two-way slabs.
- Design for Shear:
- $V_u=w_u*L_x/2$

- BENDING MOMENT CALCULATION:
- Two-way slabs spanning in two directions at right angles and carrying uniformly distributed loads may be analysed using any acceptable theory. Pigeoud’s or Wester-guard’s theories are the suggested elastic methods and Johansen’s yield line theory is the most commonly used in the limit state of collapse method and suggested by IS 456 in the note of cl. 24.4. Alternatively, Annex D of IS 456 can be employed to determine the bending moments in the two directions for two types of slabs: (i) restrained slabs, and (ii) simply supported slabs. The two methods a
- **(i) Restrained slabs**
- Restrained slabs are those whose corners are prevented from lifting due to effects of torsional moments. These torsional moments, however, are not computed as the amounts of reinforcement are determined from the computed areas of steel due to positive bending moments depending upon the intensity of torsional moments of different corners. Thus, it is essential to determine the positive and negative bending moments in the two directions of restrained slabs depending on the various types of panels and the aspect ratio l_y/l_x.





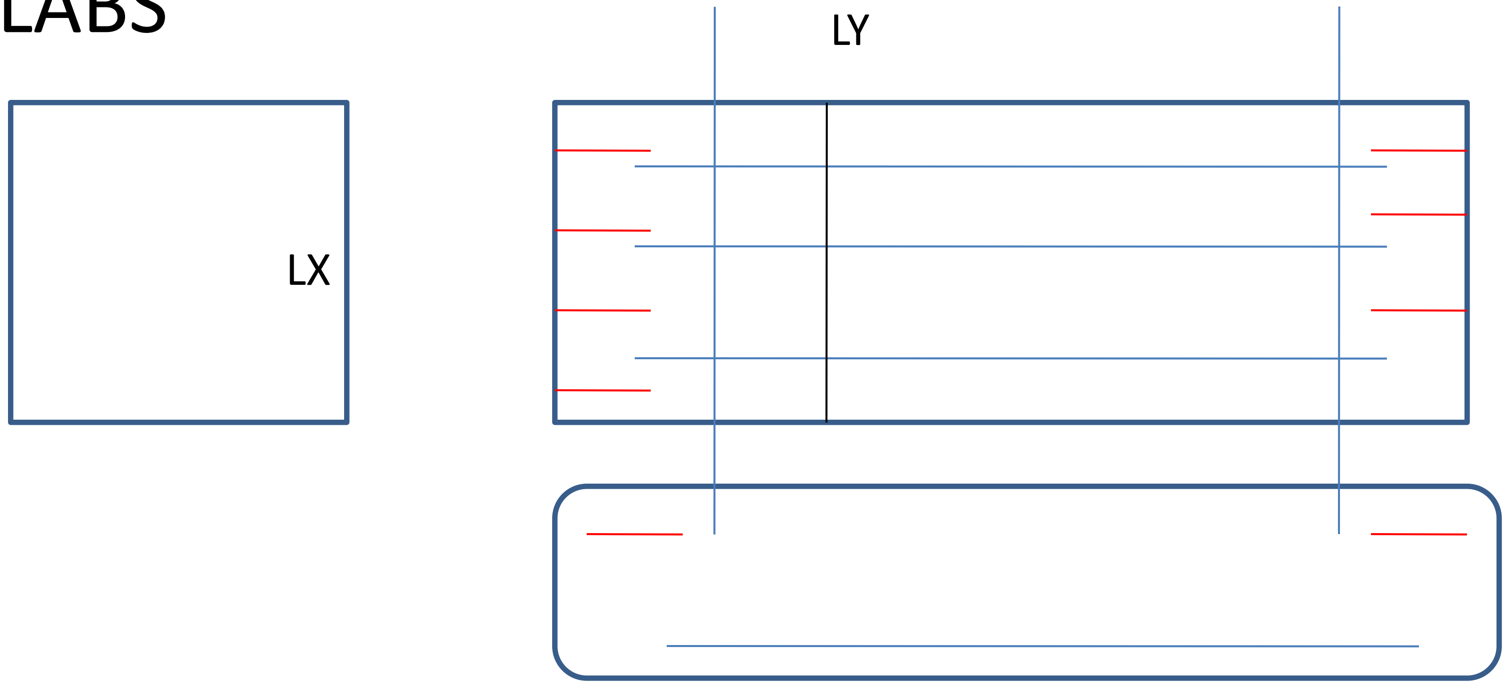
FOR L_X



FOR L_Y

Restrained slabs are considered as divided into two types of strips in each direction: (i) one middle strip of width equal to three-quarters of the respective length of span in either directions, and (ii) two edge strips, each of width equal to one-eighth of the respective length of span in either directions. Figures above present the two types of strips for spans l_x *and* l_y *separately.*

- SLABS



- The maximum positive and negative moments per unit width in a slab are determined from

$$M_x = \alpha_x * w * l_x^2$$

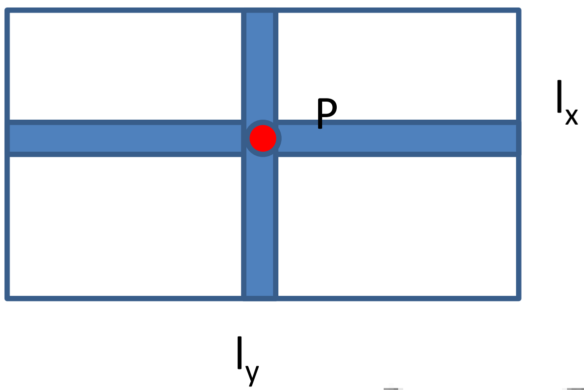
$$M_y = \alpha_y * w * l_x^2$$

Where α_x *and* α_y *and are coefficients given in Table 26 of IS 456, Annex D, cl. D-1.1. Total design load per unit area is w and lengths of shorter and longer spans are represented by* l_x *and* l_y*, respectively. The values of* α_x *and* α_y *, given in Table 26 of IS 456, are for nine types of panels having eight aspect ratios of* l_y/l_x *from one to two at an interval of 0.1. The above maximum bending moments are applicable only to the middle strips and no redistribution shall be made.*

Tension reinforcing bars for the positive and negative maximum moments are to be provided in the respective middle strips in each direction. Figure 8.19.2 shows the positive and negative coefficients α_x *snd* α_y .
The edge strips will have reinforcing bars parallel to that edge following the minimum amount as stipulated in IS 456.
The detailing of all the reinforcing bars for the respective moments and for the minimum amounts as well as torsional requirements will be discussed later).

- II) Simply supported Slabs:
- The maximum moments per unit width of simply supported slabs, not having adequate provision to resist torsion at corners and to prevent the corners from lifting, are determined from

$$M_x = \alpha_x * w * l_x^2 \qquad M_y = \alpha_y * w * l_x^2$$



Let us take w= load per unit length,
w_x and w_y is the load transferred along l_x and l_y direction respectively,
Deflection of point P

$$\frac{l_y}{l_x} = k \qquad \delta_{x\,at\,P} = 5 * w_x * l_x^4/384EI$$

$$\delta_{y\,at\,P} = 5 * w_y * l_y^4/384EI$$

$$\delta_{x\,at\,P} = \delta_{y\,at\,P} = 5 * w_x * \frac{l_x^4}{384EI} = 5 * w_y * l_y^4/384EI \qquad \frac{w_x}{w_y} = \frac{l_y^4}{l_x^4} = (\frac{l_y}{l_x})^4 = k^4$$

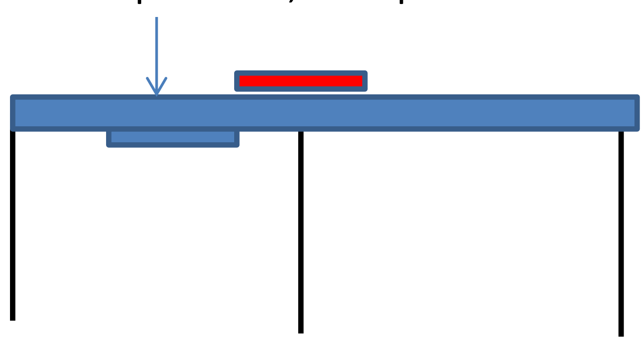
$$w_x + w_y = w \quad k^4 * w_y + w_y = w \quad w_y = w/(1 + k^4) \quad w_x = k^4 * w_y = w * k^4/(1 + k^4)$$

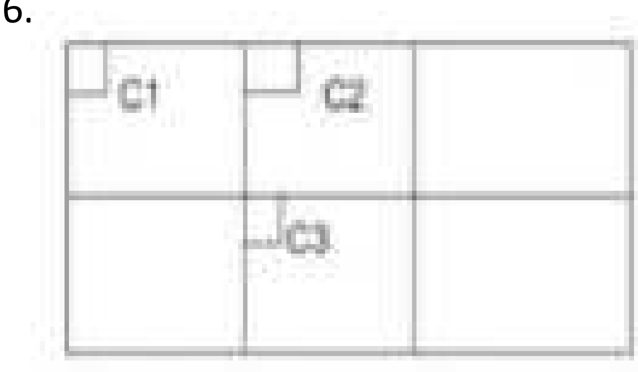
$$M_x = w_x * \frac{l_x^2}{8} = w * \frac{k^4}{1 + k^4} * \frac{l_x^2}{8} = \alpha_x * w * l_x^2$$

$$M_y = w_y * \frac{l_y^2}{8} = \frac{w}{1+k^4} * \frac{l_y^2}{8} = \frac{w}{1+k^4} * k^2 * l_x^2$$
$$= \frac{k^2}{1+k^4} * w * l_x^2 = \alpha_y * w * l_x^2$$

- α_x and α_y can be determined for different k values.
- Effective span to effective depth ratio (cl. 24.1 of IS 456)
- The following are the relevant provisions given in Notes 1 and 2 of cl. 24.1.
- • The shorter of the two spans should be used to determine the span to effective depth ratio.
- • For spans up to 3.5 m and with mild steel reinforcement, the span to overall depth ratios satisfying the limits of vertical deflection for loads up to 3 kN/m^2 are as follows:
- Simply supported slabs 35
- Continuous slabs 40
- The same ratios should be multiplied by 0.8 when high strength deformed bars (Fe 415) are used in the slabs.

- Design of Two-way Slabs:
- **Step 1: Selection of preliminary depth of slab**
- **Step 2: Design loads, bending moments and shear forces**
- **Step 3: Determination/checking of the effective and total depths of slabs**
- **Step 4: Depth of the slab for shear force**
- **Step 5: Determination of areas of steel**
- **Step 6: Selection of diameters and spacing's of reinforcing bars (cls.26.5.2.2 and 26.3.3 of IS 456)**
- **Step 7: Determination of torsional reinforcement .**
- (a) At corner C1 where the slab is discontinuous on both sides, the torsion reinforcement shall consist of top and bottom bars each with layers of bar placed parallel to the sides of the slab and extending a minimum distance of one-fifth of the shorter span from the edges. The amount of reinforcement in each of the four layers shall be 75 per cent of the area required for the maximum mid-span moment in the slab. This provision is given in cl. D-1.8 of IS 456.
- (b) At corner C2 contained by edges over one of which is continuous, the torsional reinforcement shall be half of the amount of (a) above. This provision is given in cl. D-1.9 of IS 456.
- (c) At corner C3 contained by edges over both of which the slab is continuous, torsional reinforcing bars need not be provided, as stipulated in cl. D-1.10 of IS 456.





Case 4 Two adjacent edges discontinuous	Case 3 One long edge discontinuous	Case 4 Two adjacent edges discontinuous
Case 2 One short edge discontinuous	Case 1 Interior panel Four edges continuous	Case 2 One short edge discontinuous
Case 4 Two adjacent edges discontinuous	Case 3 One long edge discontinuous	Case 4 Two adjacent edges discontinuous

Case 8 Three edges discontinuous One short edge continuous	Case 6 Two long edges discontinuous	Case 8 Three edges discontinuous One short edge continuous

DETAILING OF REINFORCEMENT

- **(i) Restrained slabs**
- The maximum positive and negative moments per unit width of the slab calculated are applicable only to the respective middle strips. There shall be no redistribution of these moments. The reinforcing bars so calculated from the maximum moments are to be placed satisfying the following stipulations of IS 456.
- Bottom tension reinforcement bars of mid-span in the middle strip shall extent in the lower part of the slab to within 0.25*l of a continuous edge, or 0.15l of a discontinuous edge (cl. D-1.4 of IS 456). Bars marked as B1, B2, B5 and B6 in FIGURE.*
- • Top tension reinforcement bars over the continuous edges of middle strip shall extend in the upper part of the slab for a distance of 0.15*l from the support, and at least fifty per cent of these bars shall extend a distance of 0.3l (cl. D-1.5 of IS 456). Bars marked as T2, T3, T5 and T6 in are these bars.*
- • To resist the negative moment at a discontinuous edge depending on the degree of fixity at the edge of the slab, top tension reinforcement bars equal to fifty per cent of that provided at mid-span shall extend 0.1*l into the span (cl. D-1.6 of IS 456). Bars marked as T1 and T4 in are these bars.*
-

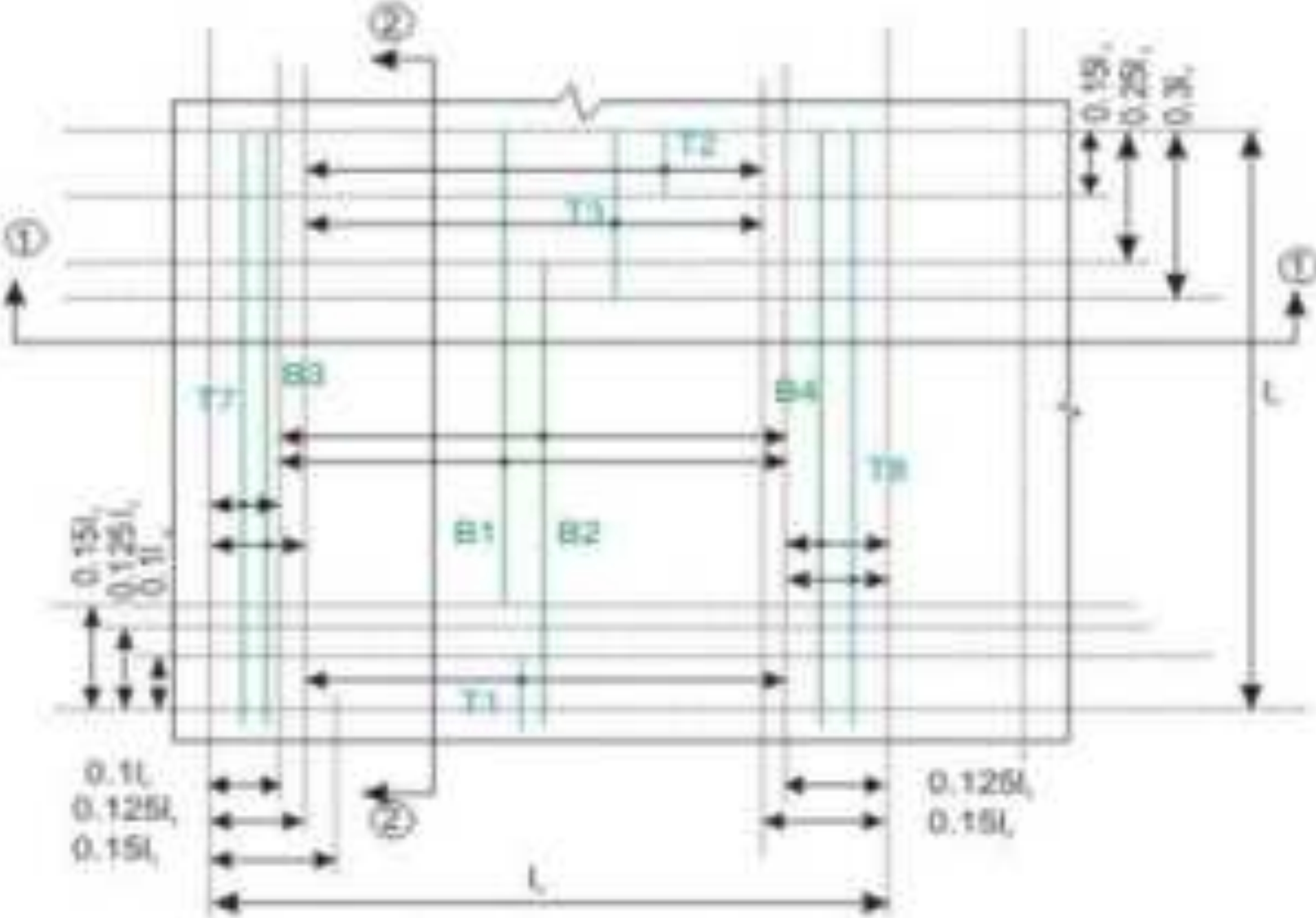
- The edge strip of each panel shall have reinforcing bars parallel to that edge satisfying the requirement of minimum amount as specified (cl. 26.5.2.1 of IS 456) and the requirements for torsion, explained in Step 7 of sec. 8.19.6 (cls. D-1.7 to D-1.10 of IS 456). The bottom and top bars of the edge strips are explained below.

- • Bottom bars B3 and B4 are parallel to the edge along l_x *for the edge strip for span* l^y*, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).*

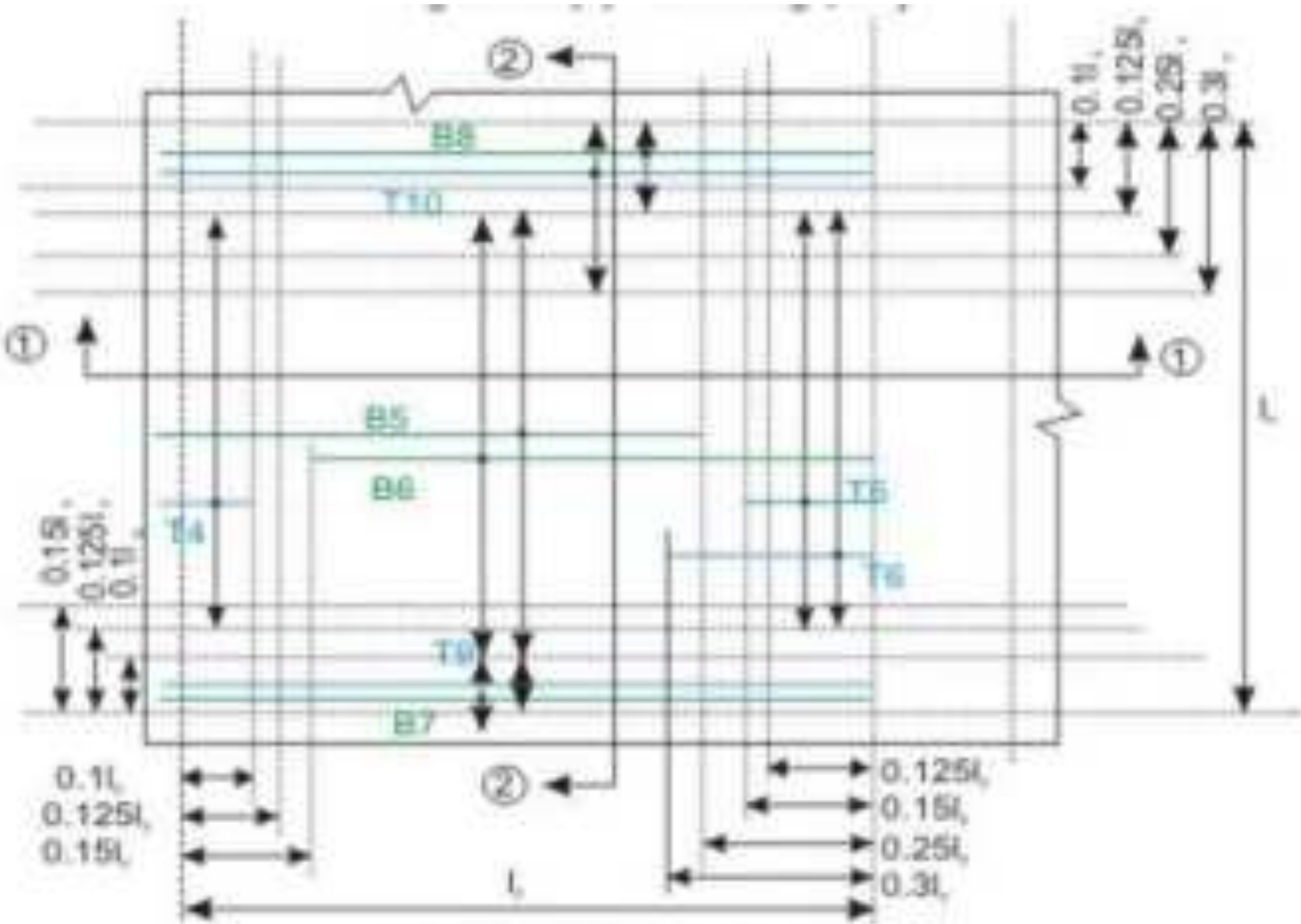
- • Bottom bars B7 and B8 are parallel to the edge along l^y *for the edge strip for span* l^x*, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).*

- • Top bars T7 and T8 are parallel to the edge along l^x *for the edge strip for span* l^y*, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).*

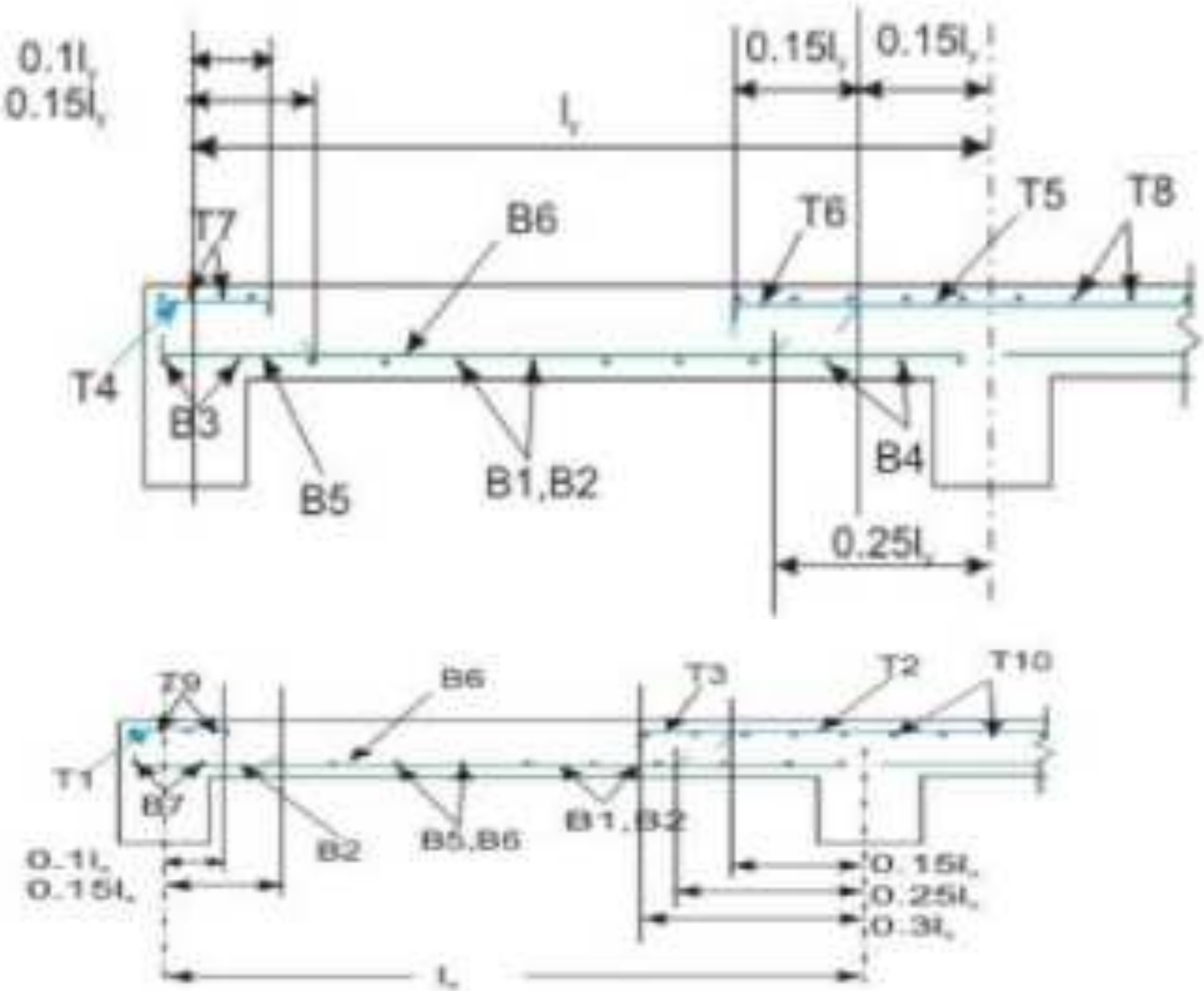
- • Top bars T9 and T10 are parallel to the edge along l^y *for the edge strip for span* l^x*, satisfying the requirement of minimum amount of steel (cl. D-1.7 of IS 456).*



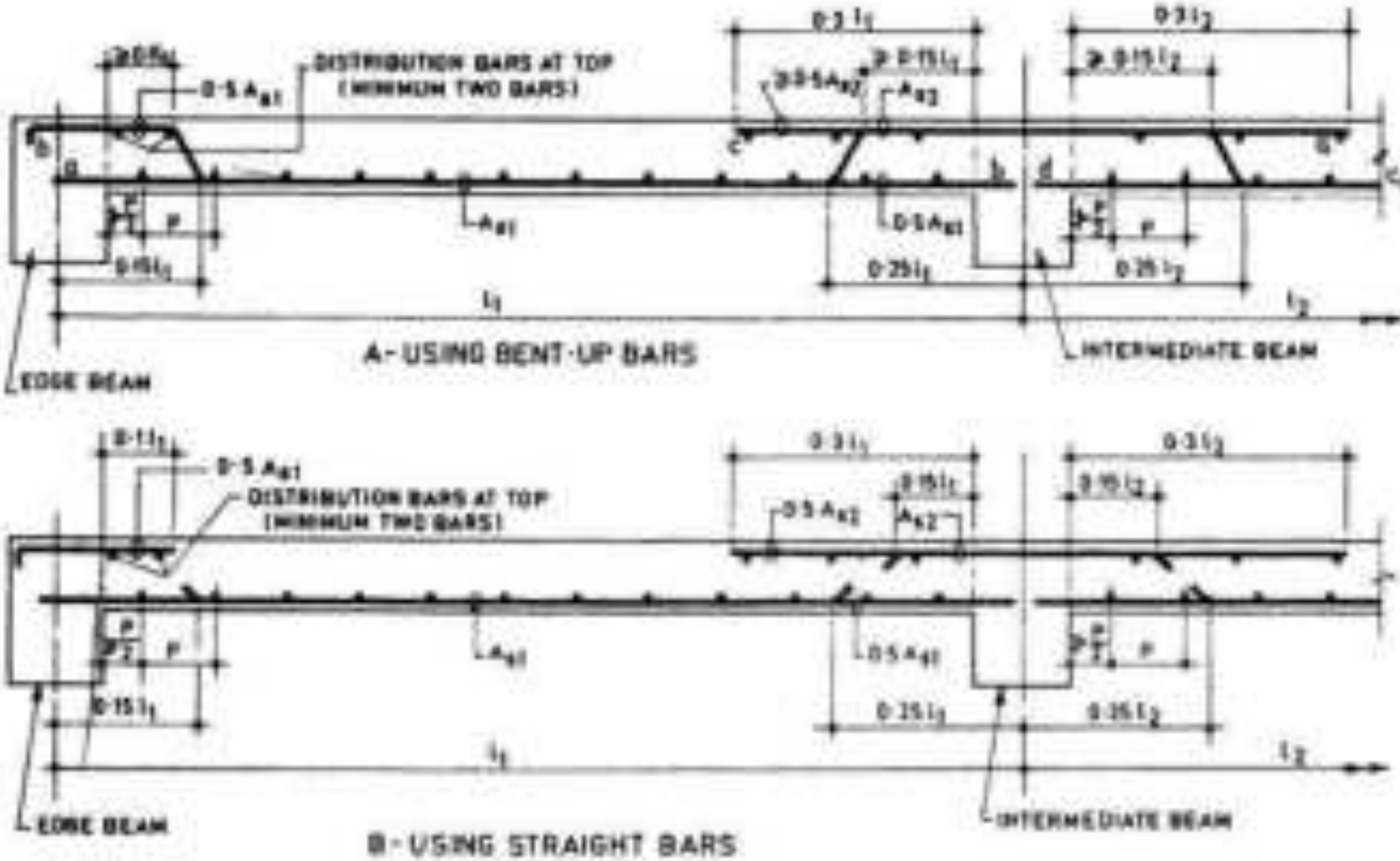
T2
T3
B3
T7
B4
T8
B1
B2
T1
0.1l
0.125l
0.15l
0.125l
0.15l



B8
T10
B5
B6
T4
T5
T6
T9
B7
0.1l,
0.125l,
0.15l,
0.25l,
0.3l,
l,

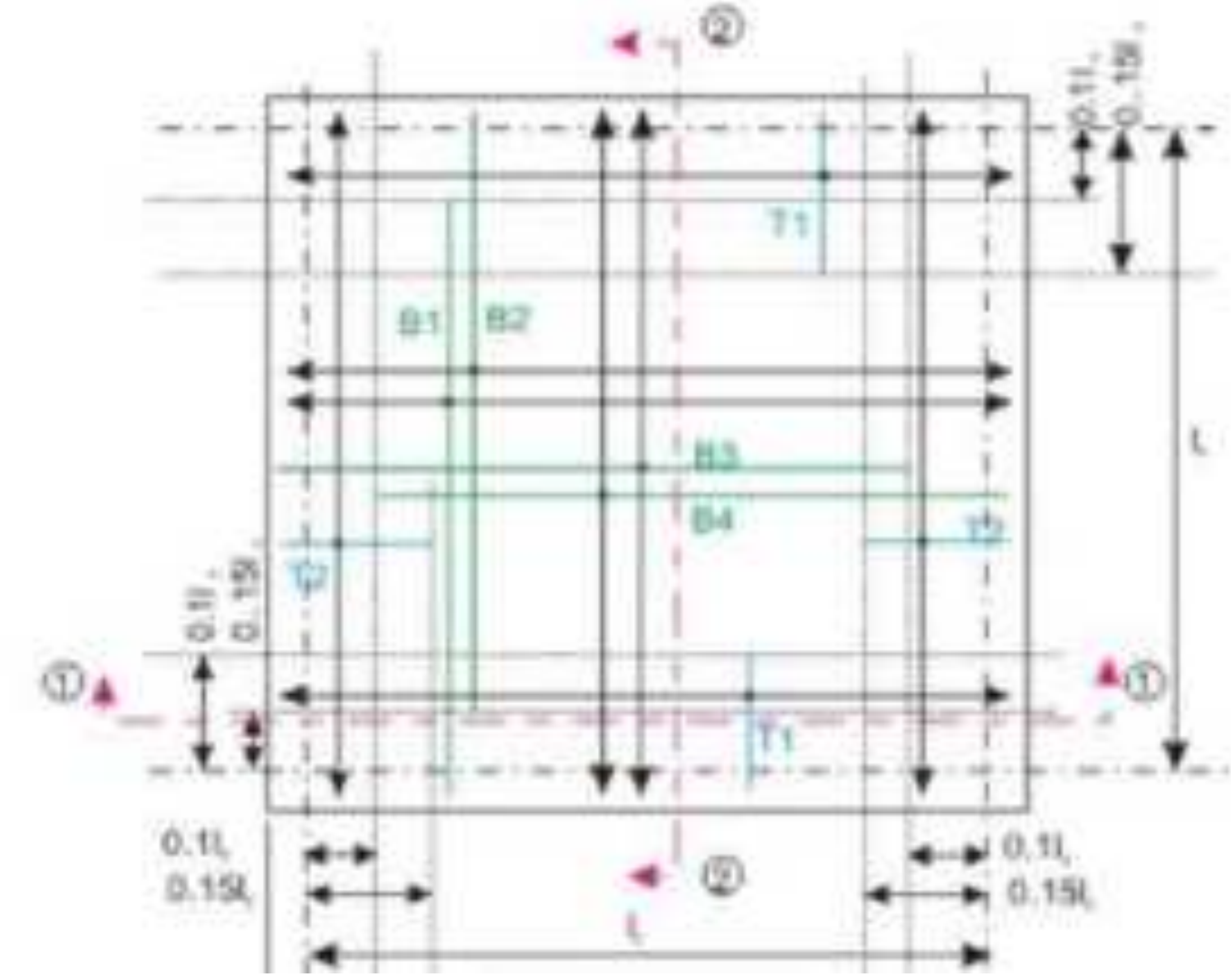


0.1l
0.15l
l
0.15l
0.15l
T7
B6
T6
T5
T8
T4
B3
B5
B1,B2
B4
0.25l
T9
B6
T3
T2
T10
T1
B7
B2
B5,B6
B1,B2
0.1l
0.15l
0.15l
0.25l
0.3l
l



DISTRIBUTION BARS AT TOP
(MINIMUM TWO BARS)
0·5 A
EDGE BEAM
INTERMEDIATE BEAM
A- USING BENT-UP BARS
B- USING STRAIGHT BARS

SIMPLY SUPPORTED SLABS



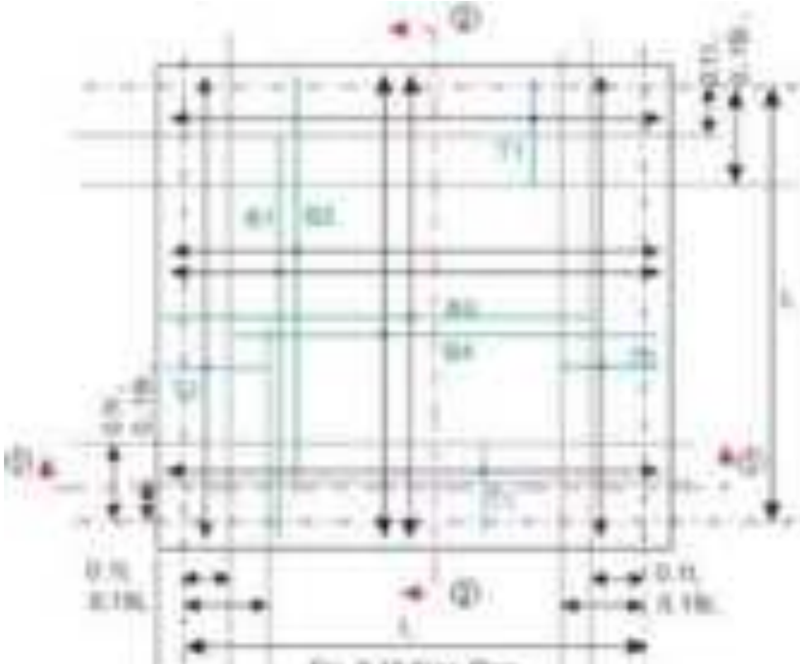


Fig. 8.19.4(a): Plan

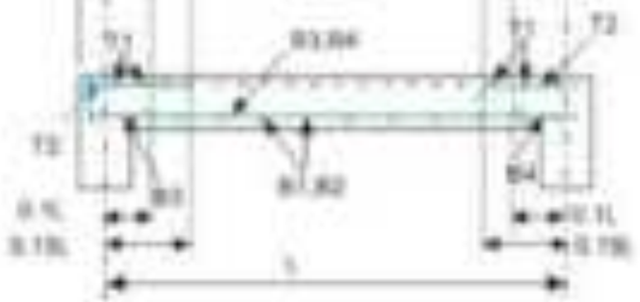
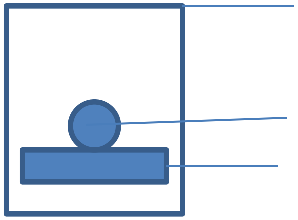


Fig. 8.19.4(b): Section 1-1

- Design a slab over a room 4 m*6 m according to IS code method simply supported and corners are not held down. LL=3kN/m^2.Slab is supported over 150 mm walls.
- ANS:
- Assuming effective length 4.15 m, taking L_{eff}/d_{eff}=30, Let us take d_{eff}=150 mm. D=170 mm.
- Assuming effective cover 19 mm, deff along short span d_x==170-19=151 mm.
- d_y=151-4-4=143 mm assuming 8 mm dia bar.



- l_x=4+.151=4.151 m or c/c=4.15 m l_x=4.15 m
- l_y=6+.143=6.143 or c/c=6.15m l_y=6.143m
- l_y/l_x=k=1.48
- As per IS table

$$\alpha_x = 0.099 + \frac{0.104 - 0.099}{1.5 - 1.4} * (1.48 - 1.4) = 0.103$$

$$\alpha_y = 0.051 + \frac{0.046 - 0.051}{1.5 - 1.4} * (1.48 - 1.4) = 0.047$$

- DL of slab=1m*1m*.17*25=4.25 kN/m
- FF=1*1*0.02*24=0.48 kN/m
- LL=3 kN/m
- w=7.73 kN/m w_u=1.5*7.73=11.595 kN/m

Table 27 Bending Moment Coefficients for Slabs Spanning in Two Directions at Right Angles, Simply Supported on Four Sides

(Clause D-2.1)

l_y/l_x	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	2.5	3.0
α_x	0.062	0.074	0.084	0.093	0.099	0.104	0.113	0.118	0.122	0.124
α_y	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029	0.020	0.014

- # Calculation of Moment

$$M_{ux} = \alpha_x * w_u * l_x^2 = 20.568\ kNm$$

$$M_{uy} = \alpha_y * w_u * l_x^2 = 9.385\ kNm$$

- For the maximum moment depth required is

$$M_{ux} = 20.568 * 10^6 = 0.138 * f_{ck} * bd^2$$

Depth required for a balance section d_{req}=64.81 mm.

Depth is taken 151 mm.

Design as under reinforced.

Steel for short span direction:

$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6}{f_{ck}}*\frac{M_u}{bd^2}}}{\frac{f_y}{f_{ck}}}\right] = 0.264$$

Pt=0.264%

$A_{st\ xx}$=.264*1000*151/100=399.37 mm^2

Minimum steel=.12*1000*170/100=204 mm^2

Spacing of 8 mm bars along short span direction

$$spacing\ s_x = \frac{1000}{\frac{399.37}{50}} = 125.19\ mm$$

Provide 8mm dia. Bars 120 mm c/c.

- Steel for Long span direction:

$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6}{f_{ck}}*\frac{M_y}{bd^2}}}{\frac{f_y}{f_{ck}}}\right]$$

$$= 50\left[\frac{1-\sqrt{1-\frac{4.6}{20}*\frac{9385660}{1000*143*143}}}{\frac{415}{20}}\right] = 0.13$$

- Steel for long span direction
- A_{styy}=.13*1000*143=186.94 mm^2
- Minimum steel=.12*1000*170/100=204 mm^2
- Spacing of 8 mm bars along long span direction

$$spacing\ s_y = \frac{1000}{\frac{204}{50}} = 245.10\ mm$$

- Provide 8 mm dia 240 mm c/c.

CHECK FOR SHEAR

- Taking l=4 m

$$Maximum\ SF = V_u = * w_u * \frac{l_x}{2} = 11.595 * \frac{4}{2} =$$
$$= 23.10\ kN\ taking\ l = 4\ m$$

$$Nominal\ shear\ stress = \tau_{vu} = \frac{V_u}{bd} = \frac{23100}{(1000 * 151)}$$
$$= 0.153\ N/mm^2$$

$$Area\ of\ steel\ available\ near\ support = \left(\frac{1000}{120}\right) 0.5 * 50$$
$$= 208.35 mm^2$$

$$p_t = 208.35 * \frac{100}{1000 * 151} = 0.14$$

$$\tau_c = 0.28\ N/mm^2$$

$$Design\ shear\ stress = k\tau_c = 1.25 * 0.28 = 0.35 N/mm^2$$

$$(Nominal\ shear\ stress = 0.153) < (Design\ shear\ stress$$
$$= k\tau_c = 1.25 * 0.28 = 0.35 N/mm^2)$$

CHECK FOR DEVELOPMENT LENGTH

- A_{st}=208.53 mm^2

$$M_1 = A_{st} * 0.87 * f_y * d * \left(1 - \frac{A_{st} * f_y}{f_{ck}bd}\right)$$

$$= 208.35 * 0.87 * 415 * 151 * \left(1 - \frac{208.35 * 415}{20 * 1000 * 151}\right) = 11.034\ kNm$$

$$V_u = 23.10\ kN \qquad Taking\ l_0 = \frac{150}{2} - 15 = 60mm$$

$$\frac{1.3 * M_1}{V_u} + l_0 = \frac{1.3 * 11.034 * 10^6}{23100} + 60 = 680.95\ mm$$

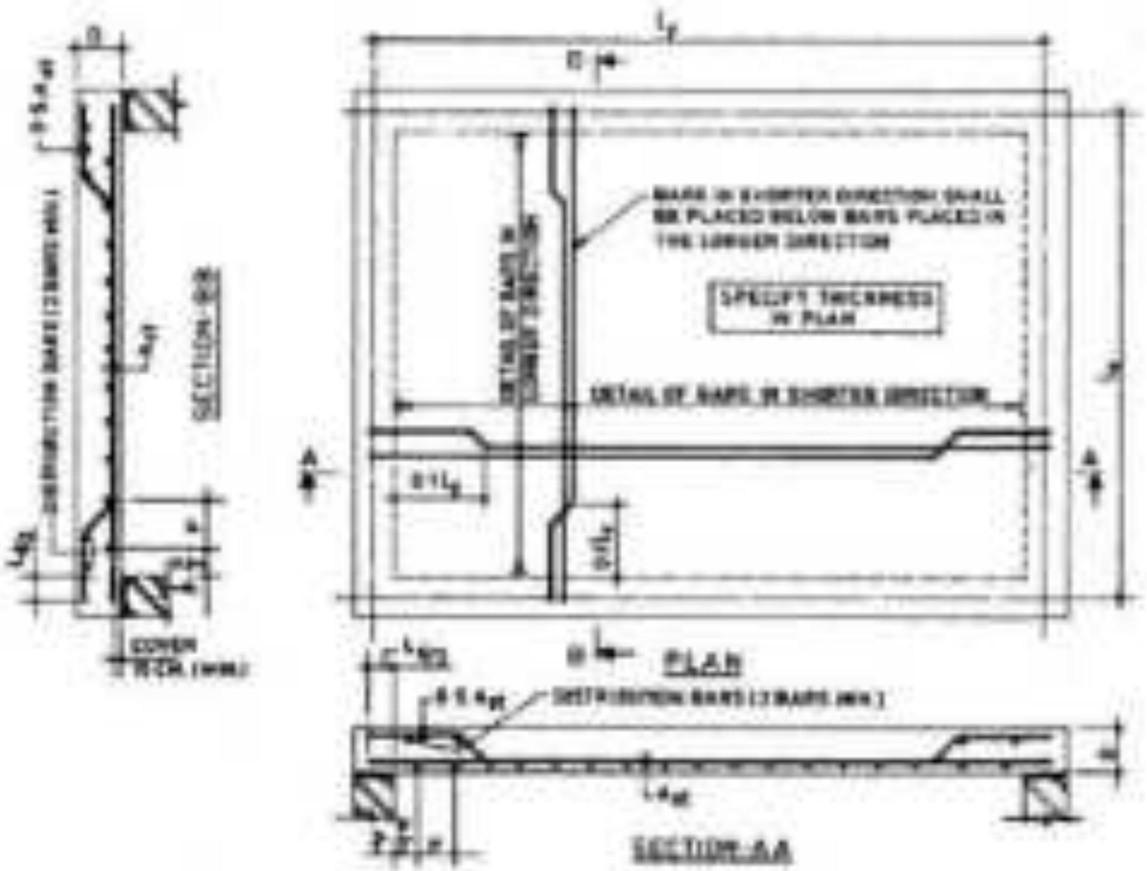
$$l_d = \frac{0.87 * f_y * \varphi}{4 * \tau_{bd}} = 376.19\ mm$$

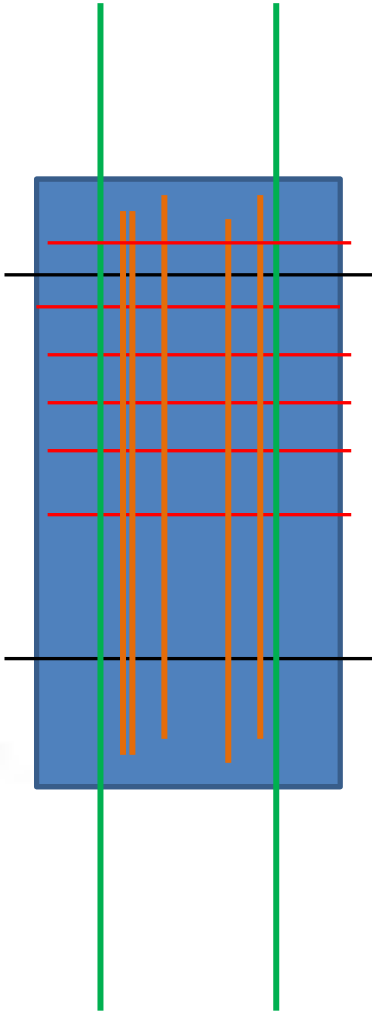
$$(l_d = 376.19\ mm) < [(\frac{1.3 * M_1}{V_u} + l_0) = 680.95\ mm]$$

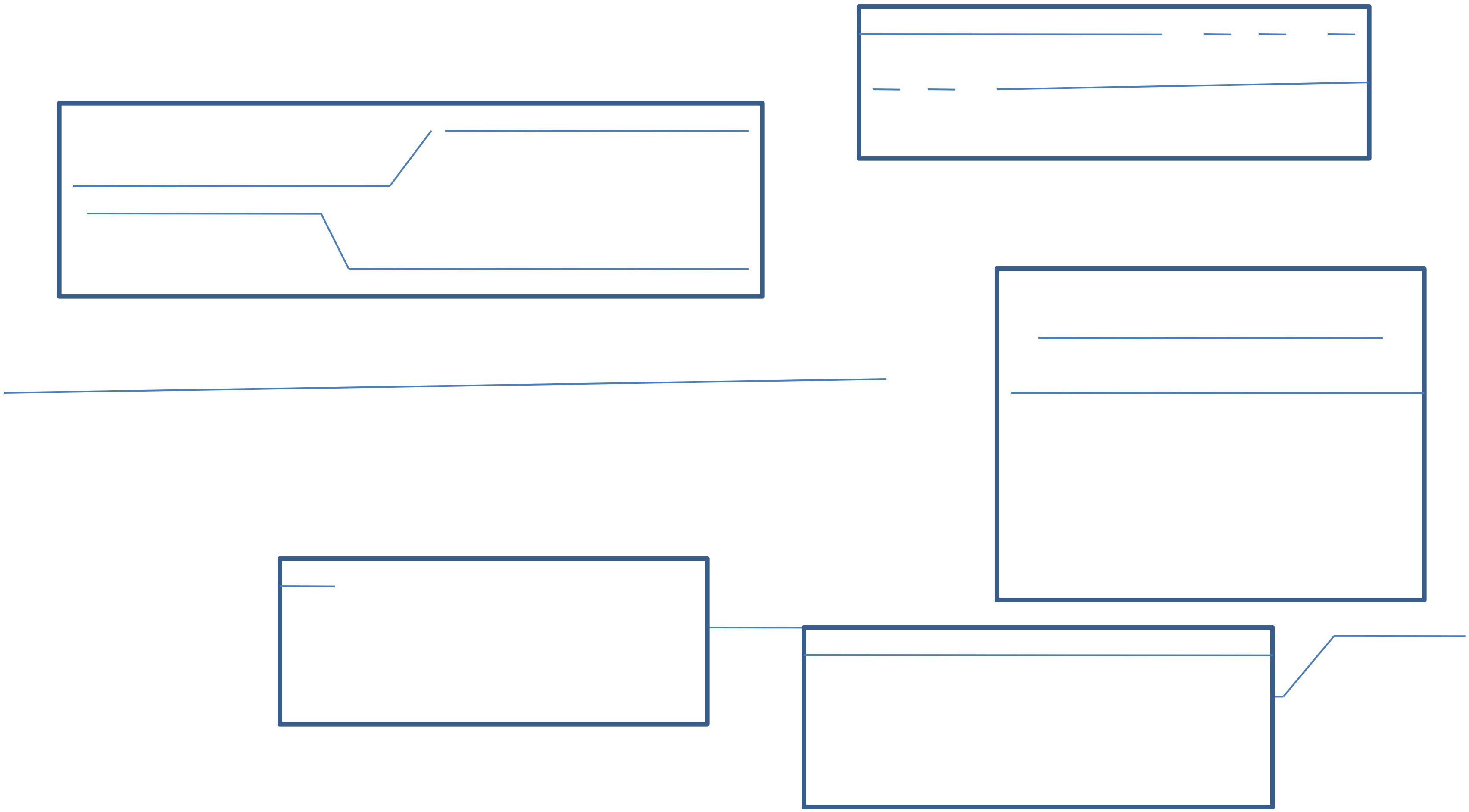
- So, it is ok.

Check for deflection

- l_{eff}/d_{eff}=4.15/.151=27.85 <(35*0.8=28)

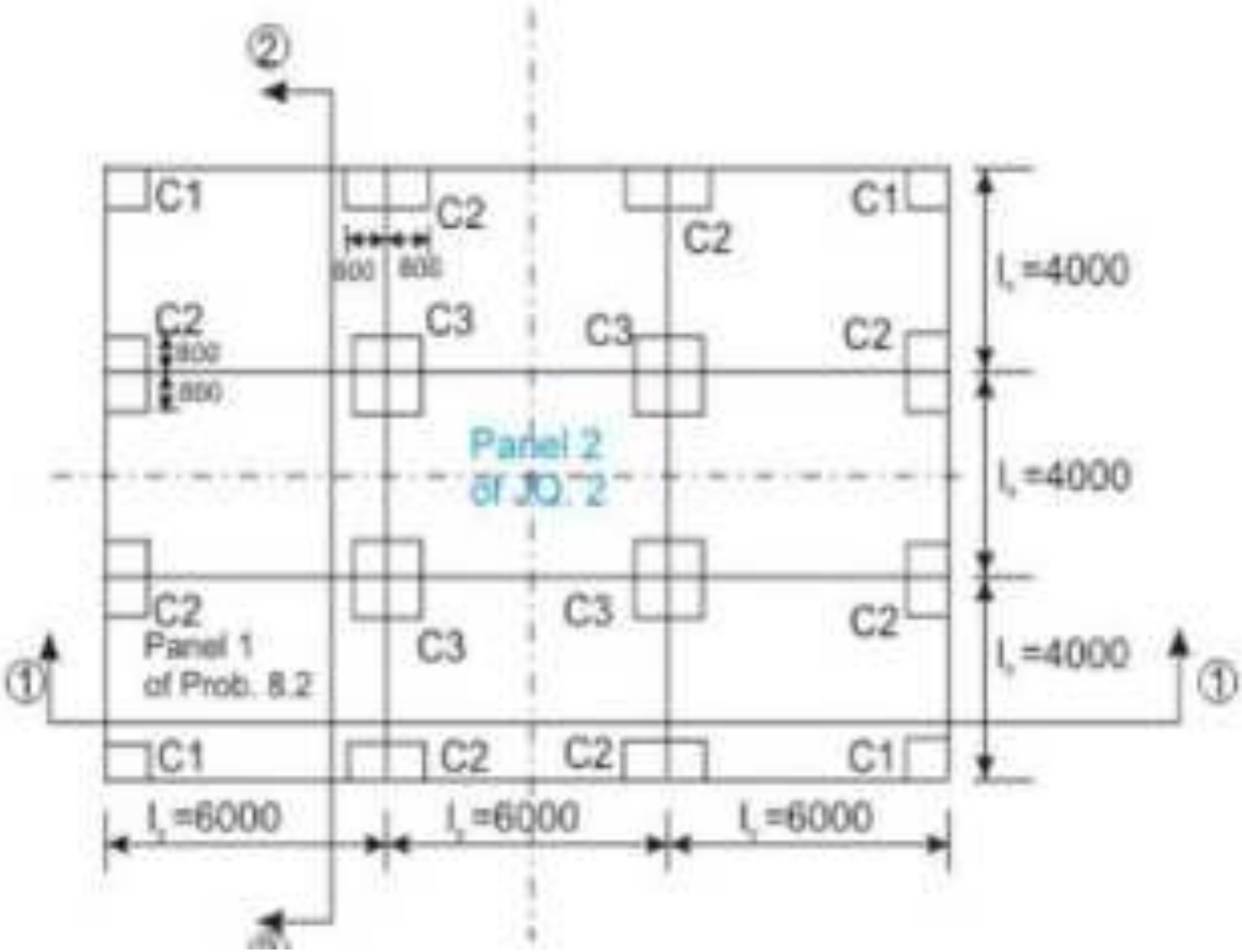

BARS IN SHORTER DIRECTION SHALL BE PLACED BELOW BARS PLACED IN THE LONGER DIRECTION
SPECIFY THICKNESS IN PLAN
DETAIL OF BARS IN SHORTER DIRECTION
PLAN
DISTRIBUTION BARS (3 BARS MIN.)
SECTION-AA
SECTION-BB





DESIGN OF RESTRAINED SLABS

- Design the slab panel 1 of Fig. subjected to factored live load of 8 kN/m^2 in addition to its dead load using M 20 and Fe 415. The load of floor finish is 1 kN/m^2. The spans shown in figure are effective spans. The corners of the slab are prevented from lifting.
- Ans:
- **Step 1: Selection of preliminary depth of slab**
- The span to depth ratio with Fe 415 is taken from cl. 24.1, Note 2 of IS 456 as 0.8 (35 + 40) / 2 = 30. This gives the minimum effective depth *d = 4000/30 = 133.33 mm, say 135 mm. The total depth D is thus 160 mm.*
- **Step 2: Design loads, bending moments and shear forces**
- Dead load of slab (1 m width) = 0.16*(25) = 4.0 kN/m^2
- Dead load of floor finish (given) = 1.0 kN/m^2
- Factored dead load = 1.5(5) = 7.5 kN/m^2
- Factored live load (given) = 8.0 kN/m^2
- Total factored load = w_u=15.5 kN/m^2
- The coefficients of bending moments and the bending moments M_x *and* M_y *per unit width (positive and negative) are determined as per cl. D-1.1 and Table 26 of IS 456 for the case 4, "Two adjacent edges discontinuous" and presented in Table. The* l^y / *l for this problem is 6/4 = 1.5.*



2
C1
C2
C2
C1
800
800
l₁=4000
C2
C3
C3
C2
800
800
Panel 2
of JQ. 2
l₁=4000
C2
C3
C3
C2
Panel 1
of Prob. 8.2
l₁=4000
1
1
C1
C2
C2
C1
l₂=6000
l₂=6000
l₂=6000

- Table Maximum bending moments of Problem

For	Short span		Long span	
	α_x	M_x (kNm/m)	α_y	M_y (kNm/m)
Negative moment at continuous edge	0.075	18.6	0.047	11.66
Positive moment at mid-span	0.056	13.89	0.035	8.68

Maximum shear force in either direction is determined from

$V_u = w(l_x/2) = 15.5\ (4/2) = 31\ kN/m$

Step 3: Determination/checking of the effective depth and total depth of slab

$$M_{ulimit} = 0.138 * f_{ck} * bd_{req}^2$$

$$d_{req} = sqrt\left[18.6 * \frac{10^6}{0.138} * 20 * 1000\right] = 82.09\ mm$$

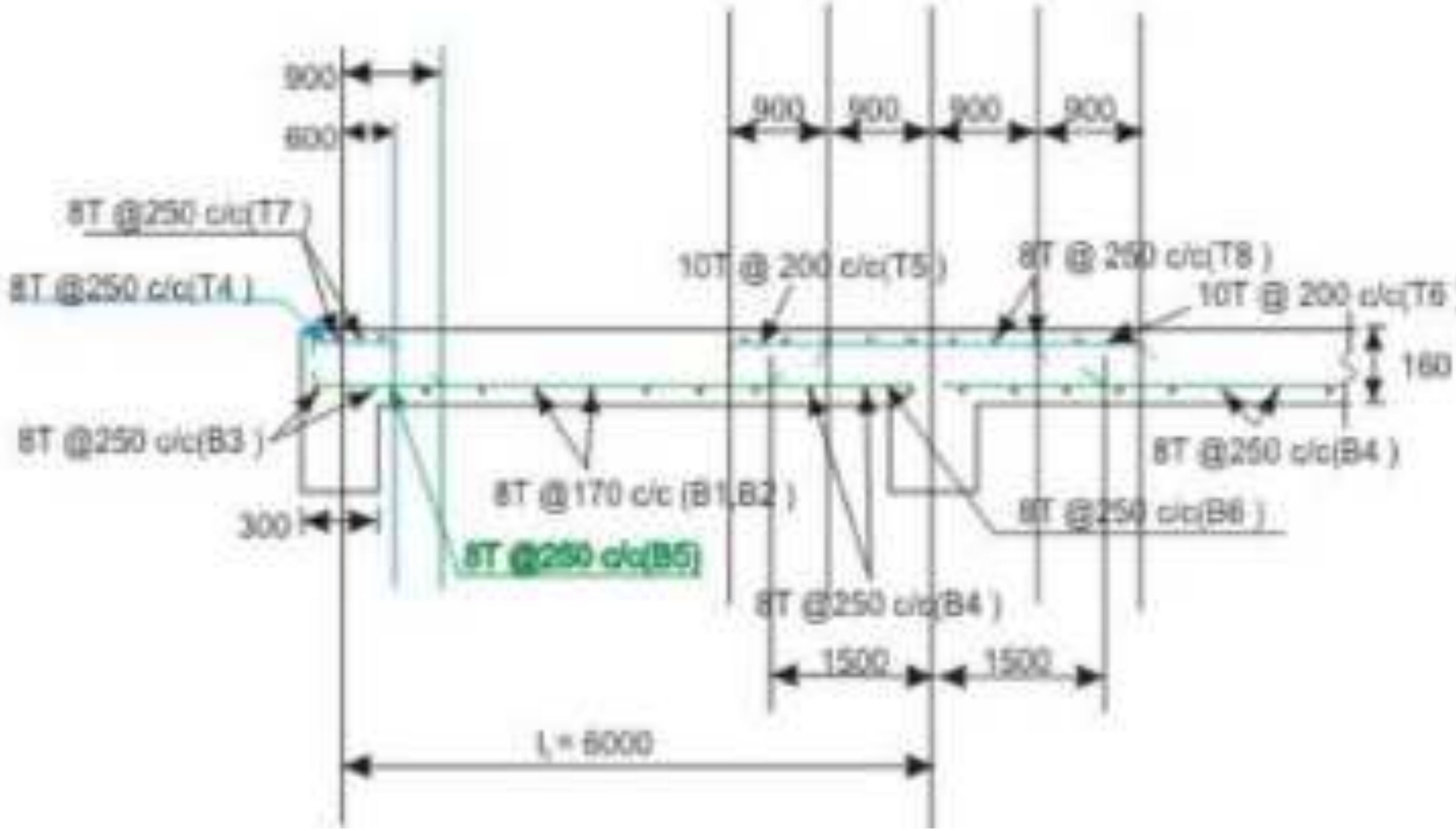
Since, this effective depth is less than 135 mm assumed in Step 1, we retain *d = 135 mm and D = 160 mm.*

- **Step 4: Depth of slab for shear force**
- Table 19 of IS 456 gives the value of τ_c = *0.28 N/mm^2 when the lowest percentage of steel is provided in the slab. However, this value needs to be modified by multiplying with k of cl. 40.2.1.1 of IS 456. The value of k for the total depth of slab as 160 mm is 1.28.*

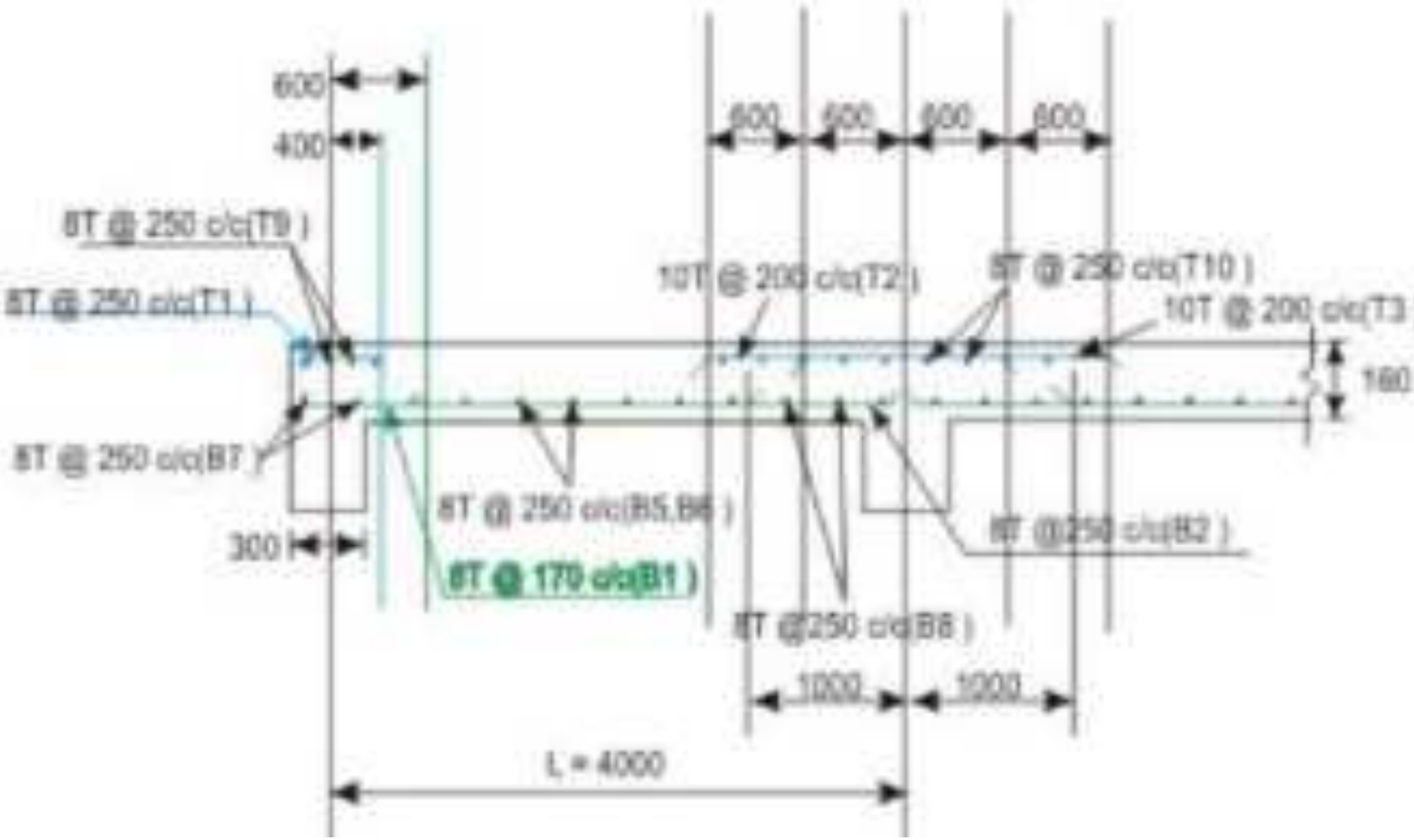
  So, the value of τ_c is 1.28(0.28) = 0.3584 N/mm^2.
- Table 20 of IS 456 gives max τ_c = *2.8 N/mm^2. The computed shear stress $\tau_{vu} = V_u/bd$ = 31*1000/(1000*135) = 0.229 N/mm^2.*
- Since, $\tau_{vu} < \tau_c < \tau_{cmax}$, *the effective depth of the slab as 135 mm and the total depth as 160 mm are safe.*

Particulars	**Short Span l_x** **Dia. & spacing**	**Long span l_y** **Dia. & spacing**
Top steel for negative moment	10 mm @ 200 mm c/c	8 mm @ 200 mm c/c
Bottom steel for positive moment	8 mm @ 170 mm c/c	8 mm @ 250 mm c/c

- The minimum steel is determined from the stipulation of cl. 26.5.2.1 of IS 456 and is
- $A_{s\,t} = (0.12/100)(1000)(160) = 192\ mm^2$ and 8 mm bars @ 250 mm c/c (= 201 mm^2) is acceptable. It is worth mentioning that the areas of steel as shown in Table are more than the minimum amount of steel.
- **Step 6: Selection of diameters and spacings of reinforcing bars**
- The advantages of using the tables of SP-16 are that the obtained values satisfy the requirements of diameters of bars and spacings. However, they are checked as ready reference here. Needless to mention that this step may be omitted in such a situation.
- Maximum diameter allowed, as given in cl. 26.5.2.2 of IS 456, is 160/8 = 20 mm, which is more that the diameters used here.
- The maximum spacing of main bars, as given in cl. 26.3.3(1) of IS 456, is the lesser of 3(135) and 300 mm. This is also satisfied for all the bars.
- The maximum spacing of minimum steel (distribution bars) is the lesser of 5(135) and 450 mm. This is also satisfied.



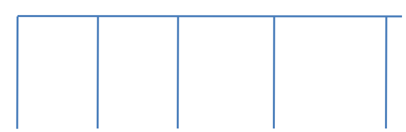
900
600
900
900
900
900
8T @250 c/c(T7)
8T @250 c/c(T4)
10T @ 200 c/c(T5)
8T @ 250 c/c(T8)
10T @ 200 c/c(T6)
160
8T @250 c/c(B3)
8T @170 c/c (B1,B2)
8T @250 c/c(B4)
8T @250 c/c(B6)
300
8T @250 c/c(B5)
8T @250 c/c(B4)
1500
1500
L = 6000

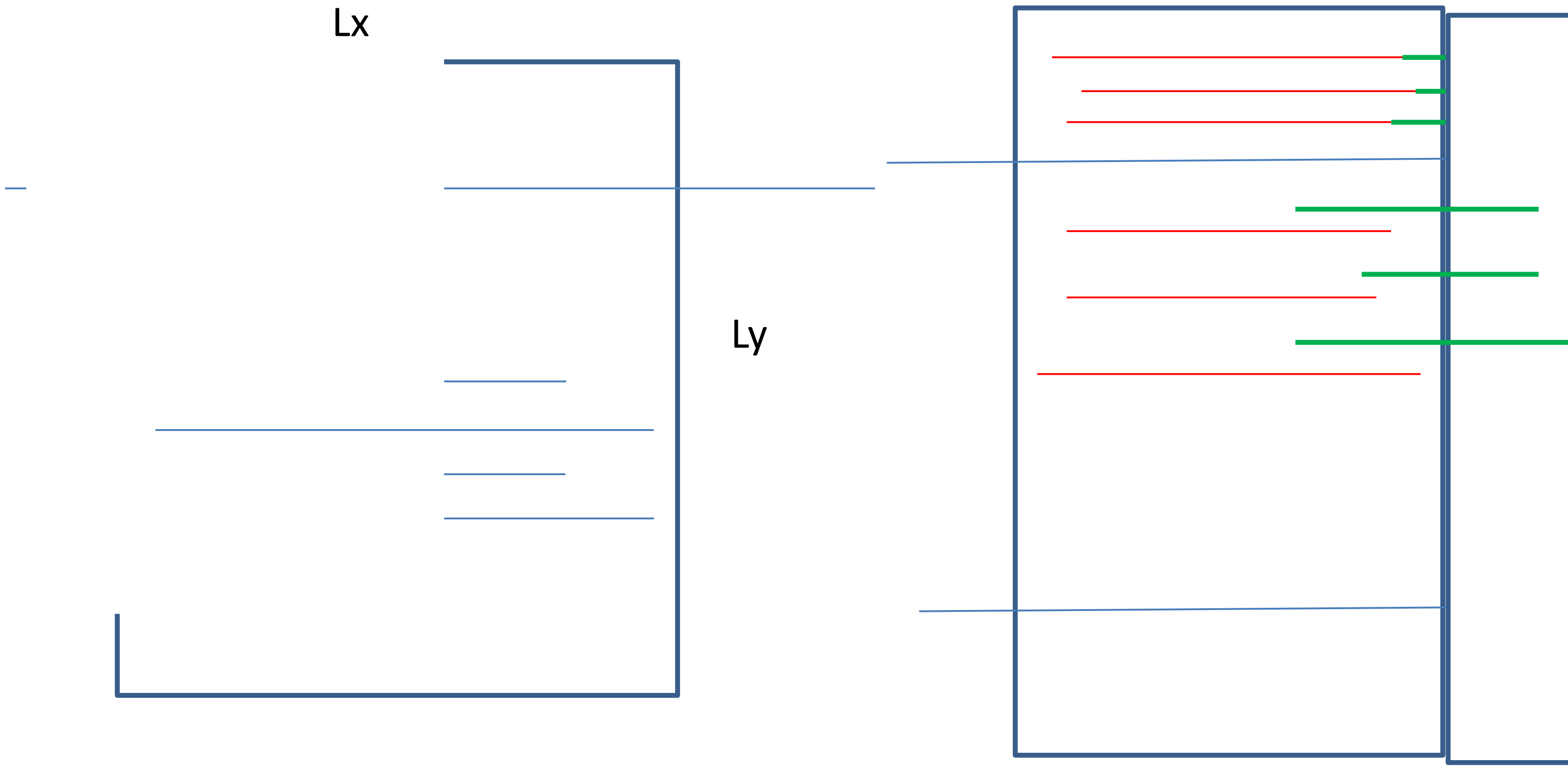


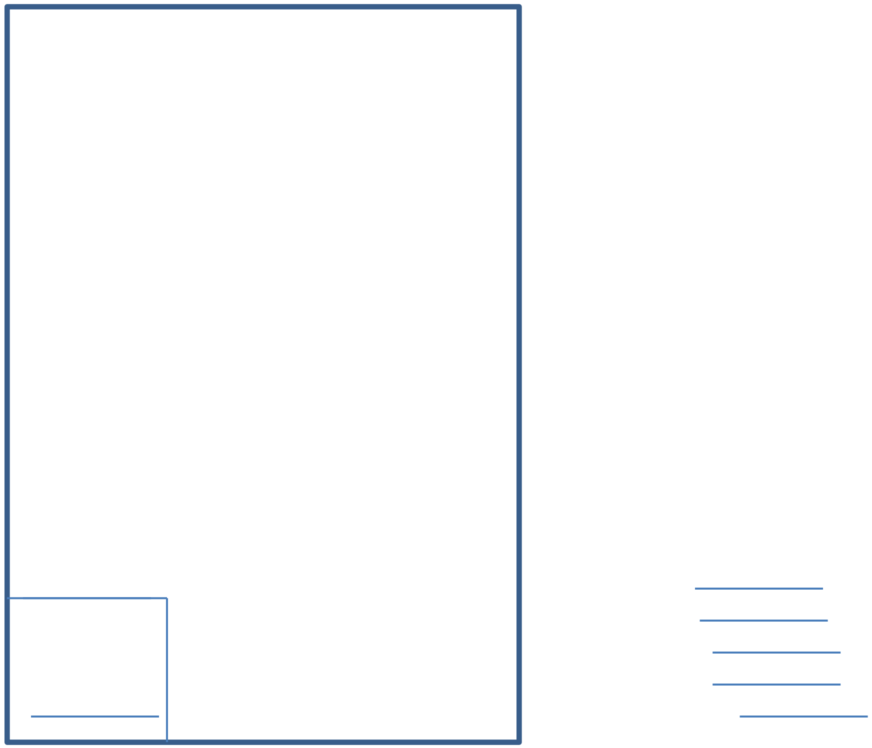
600
400
600
600
600
600
8T @ 250 c/c(T9)
8T @ 250 c/c(T1)
10T @ 200 c/c(T2)
8T @ 250 c/c(T10)
10T @ 200 c/c(T3)
160
8T @ 250 c/c(B7)
8T @ 250 c/c(B5,B6)
8T @250 c/c(B2)
300
8T @ 170 c/c(B1)
8T @250 c/c(B8)
1000
1000
L = 4000

- **Step 7: Determination of torsional reinforcement**
- Torsional reinforcing bars are determined for the three different types of corners as explained earlier. The length of torsional strip is 4000/5 = 800 mm and the bars are to be provided in four layers. Each layer will have 0.75 times the steel used for the maximum positive moment. The C1 type of corners will have the full amount of torsional steel while C2 type of corners will have half of the amount provided in C1 type. The C3 type of corners do not need any torsional steel. The results are presented in Tabular form.

Type	Dimensions along		Bar diameter & spacing	No. of bars along		Cl. no. of IS 456
	x (mm)	*y* (mm)		*x*	*y*	
C1	800	800	8 mm @ 200 mm c/c	5	5	D-1.8
C2	800	1600	8 mm @ 250 mm c/c	5	8	D-1.9
C2	1600	800	8 mm @ 250 mm c/c	8	5	D-1.9




Lx
Ly

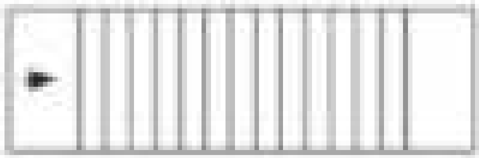


STAIRCASES

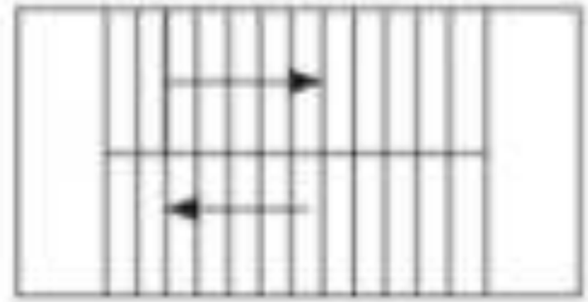
Dr. G.C.Behera

INTRODUCTION

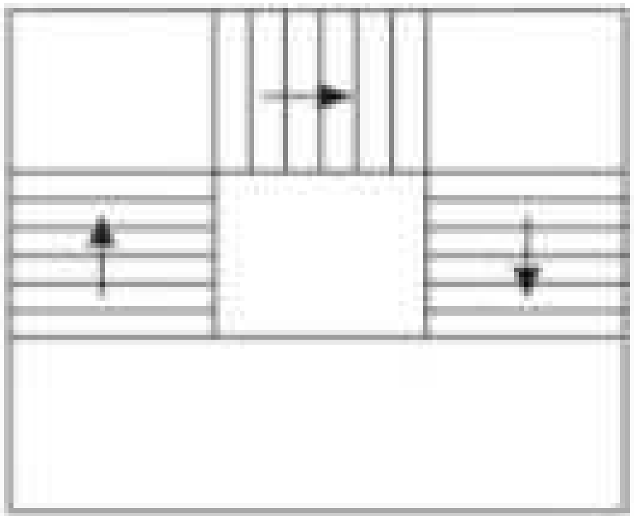
- Staircase helps in accessioning to different floors and roof of the building. It consists of a flight of steps (stairs) and one or more intermediate landing slabs between the floor levels. Different types of staircases can be made by arranging stairs and landing slabs.
- The design of the main components of a staircase-
- stair,
- landing slabs
- and supporting beams or wall –



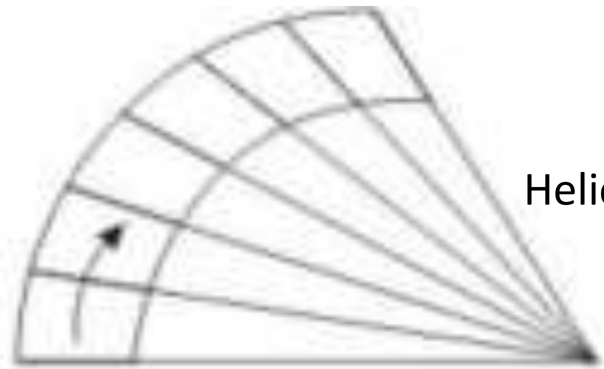
Single Flight



Two Flight

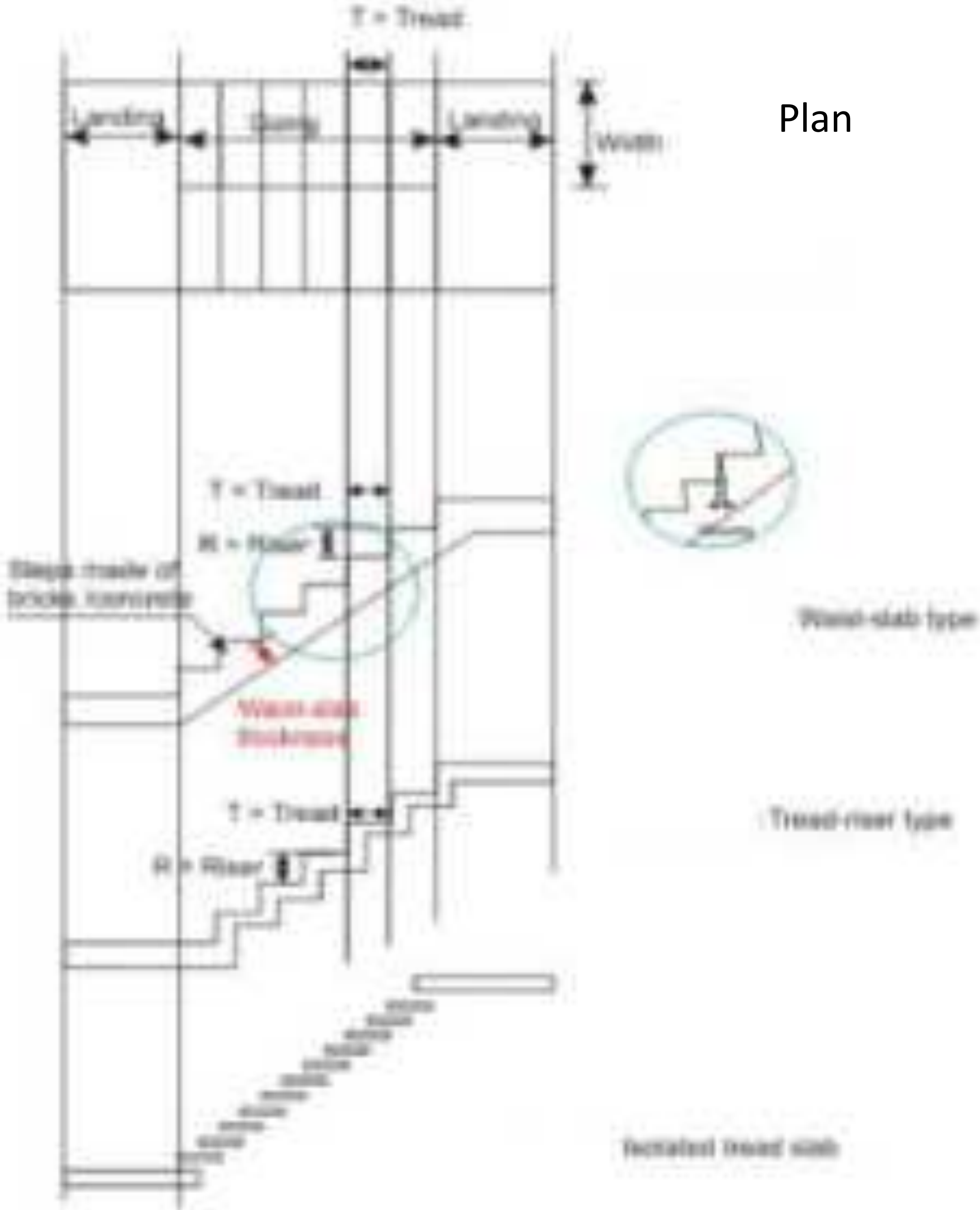


Open well



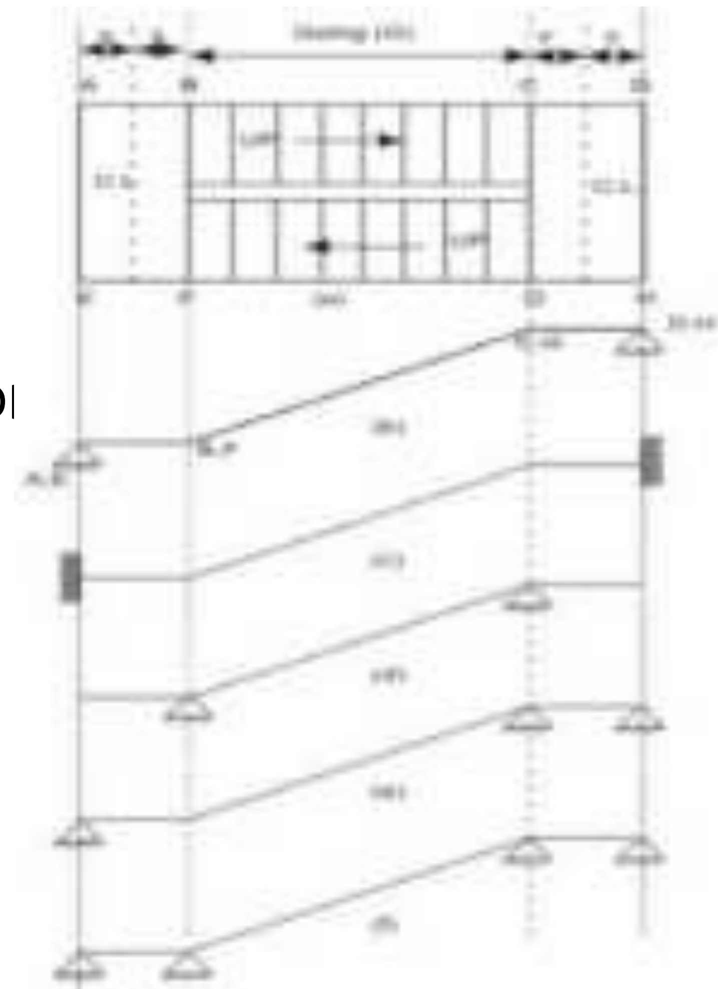
Helicoidal Type

Spiral Type

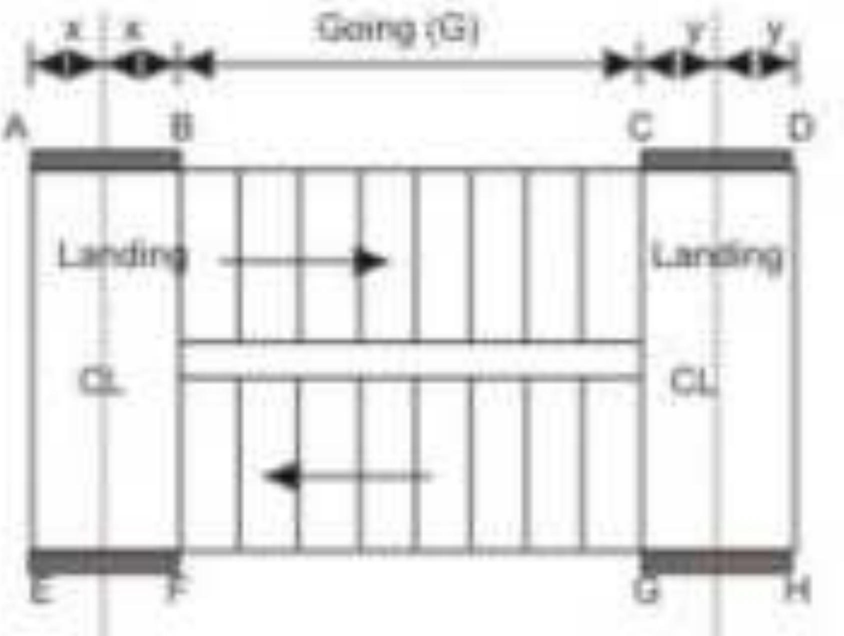

Plan
T = Tread
Landing
Going
Landing
Width
T = Tread
R = Riser
Steps made of bricks, concrete
Waist slab thickness
Waist-slab type
T = Tread
R = Riser
Tread-riser type
Isolated tread slab

- (a) Tread: The horizontal top portion of a step where foot rests is known as tread. The dimension ranges from 270 mm for residential buildings and factories to 300 mm for public buildings where large number of persons use the staircase.
- (b) Nosing: In some cases the tread is projected outward to increase the space. This projection is designated as nosing.
- (c) Riser: The vertical distance between two successive steps is termed as riser. The dimension of the riser ranges from 150 mm for public buildings to 190 mm for residential buildings and factories.
- (d) Waist: The thickness of the waist-slab on which steps are made is known as waist . The depth (thickness) of the waist is the minimum thickness perpendicular to the soffit of the staircase (cl. 33.3 of IS 456). The steps of the staircase resting on waist-slab can be made of bricks or concrete.
- (e) Going: Going is the horizontal projection between the first and the last riser of an inclined flight .
- The flight shown in Fig.a has two landings and one going. Figures b to d present the three ways of arranging the flight as mentioned below:
- ## General Guidelines
- The following are some of the general guidelines to be considered while planning a staircase:
- • The respective dimensions of tread and riser for all the parallel steps should be the same in consecutive floor of a building.
- • The minimum vertical headroom above any step should be 2 m.
- • Generally, the number of risers in a flight should be restricted to twelve.
- • The minimum width of stair should be 850 mm, though it is desirable to have the width between 1.1 to 1.6 m. In public building, cinema halls etc., large widths of the stair should be provided.

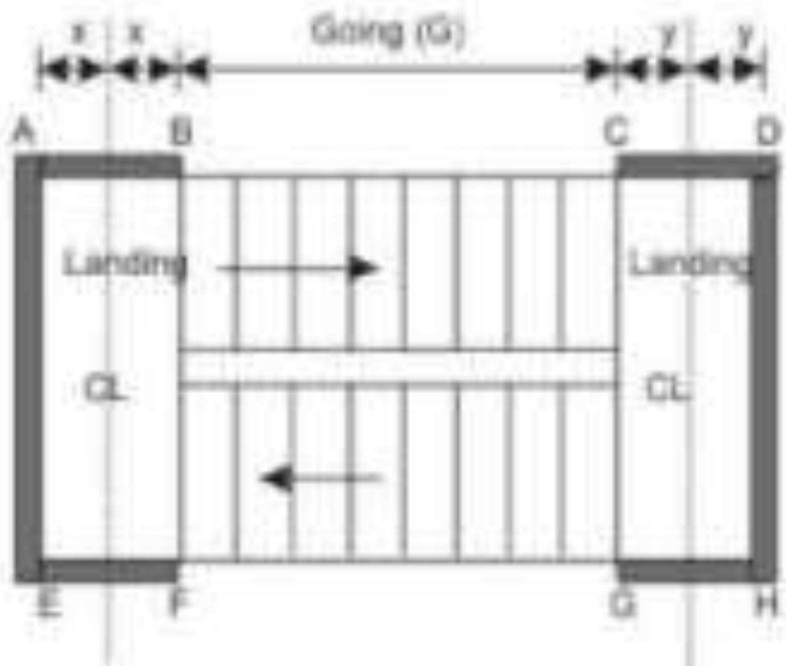
- The slab component of the stair spans either in the direction of going i.e., longitudinally or in the direction of the steps, i.e., transversely. The systems are discussed below:
- **(a) Stair slab spanning longitudinally**
- b) Supported on edges AE and DH
- (c) Clamped along edges AE and DH
- (d) Supported on edges BF and CG
- (e) Supported on edges AE, CG (or BF) and DH
- (f) Supported on edges AE, BF, CG and DH



- In the case of two flight stair, sometimes the flight is supported between the landings which span transversely as shown in Fig. It is worth mentioning that some of the above mentioned structural systems are statically determinate while others are statically indeterminate where deformation conditions have to taken into account for the analysis.

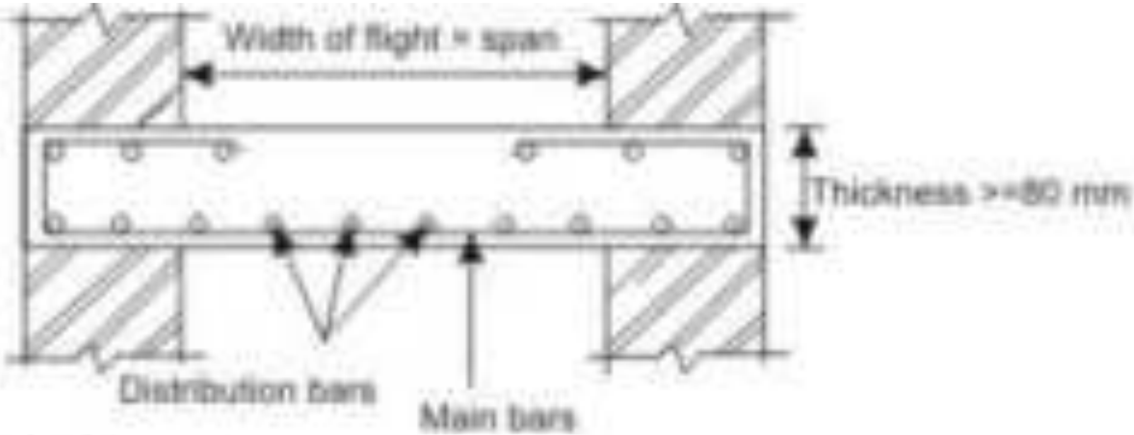


BEAMS AT TWO ENDS OF LANDING

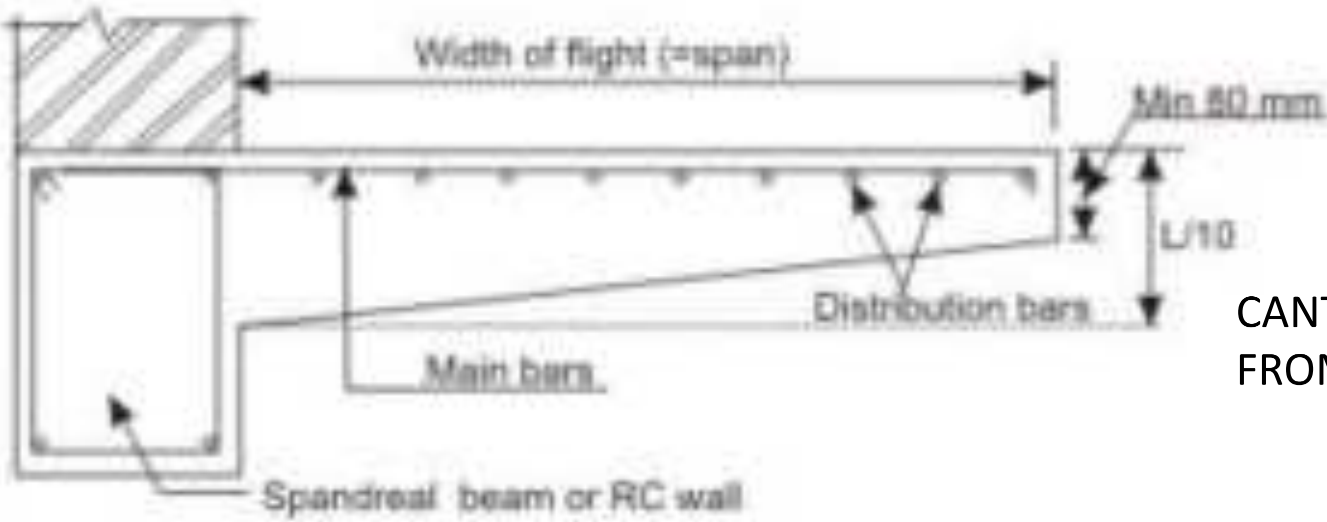


BEAMS AT THREE ENDS OF LANDING

- **(B) Stair slab spanning transversely**



SLABS SUPPORTED BETWEEN TWO STRINGER BEAMS



CANTILEVER SLAB FROM WALL

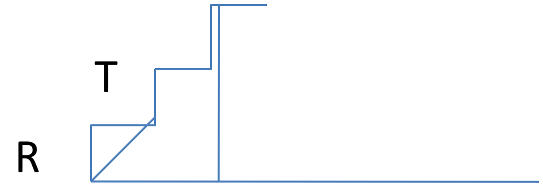
• Effective Span of Stairs

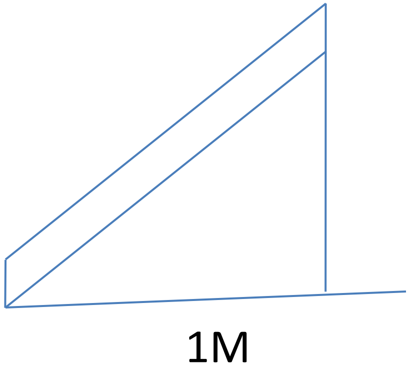
- The stipulations of clause 33 of IS 456 are given below as a ready reference regarding the determination of effective span of stair. Three different cases are given to determine the effective span of stairs without stringer beams.
- (i) The horizontal centre-to-centre distance of beams should be considered as the effective span when the slab is supported at top and bottom risers by beams spanning parallel with the risers.
- (ii) The horizontal distance equal to the going of the stairs plus at each end either half the width of the landing or one meter, whichever is smaller when the stair slab is spanning on to the edge of a landing slab which spans parallel with the risers.
- (III) Where the landing slab spans in the same direction as the stairs, they shall be considered as acting together to form a single slab and the span determined as the distance centre-to-centre of the supporting beams or walls, the going being measured horizontally.

Effective span of stairs

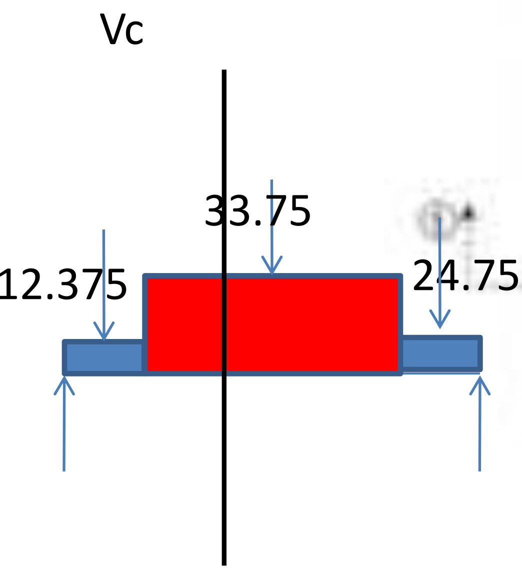
Sl. No.	x	y	Effective span in metres
1	< 1 m	< 1 m	$G + x + y$
2	< 1 m	≥ 1 m	$G + x + 1$
3	≥ 1 m	< 1 m	$G + y + 1$
4	≥ 1 m	≥ 1 m	$G + 1 + 1$

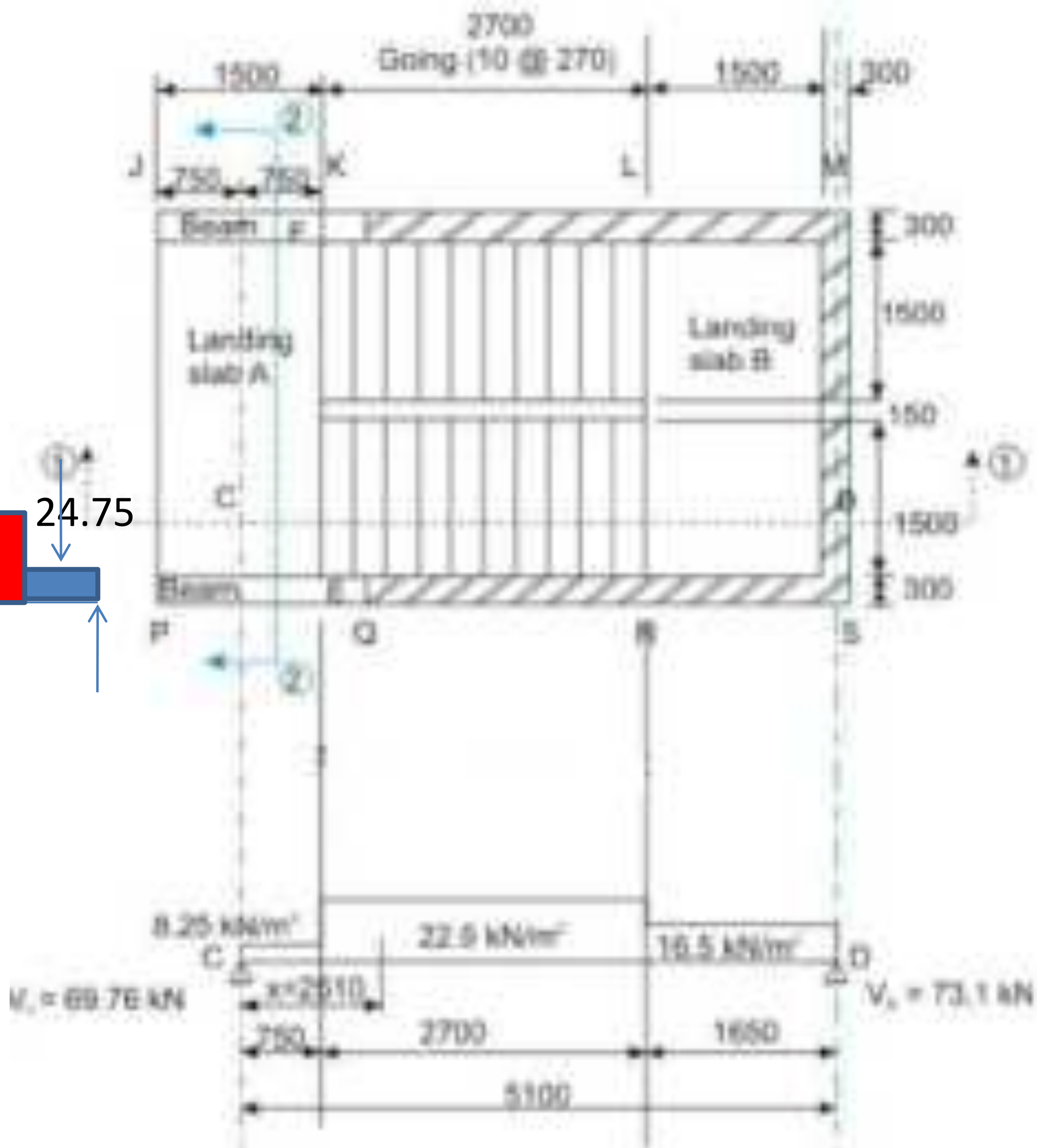
- Design the waist-slab type of the staircase of Fig. Landing slab A is supported on beams along JK and PQ, while the waist-slab and landing slab B are spanning longitudinally as shown in Fig. The finish loads and live loads are 1 kN/m^2 and 5 kN/m^2, respectively. Use riser *R = 160 mm, trade T = 270 mm, concrete grade = M 20 and steel grade = Fe 415.*
- *SOLUTION:*





- *When T is horizontal, inclined length=*$(R^2+T^2)^{1/2}$
- *Foe 1 m. horizontal length,*
- *inclined length will be* $[(R^2+T^2)^{1/2}\ /T]=313.84$
- **(A) Design of going and landing slab B**
- **Step 1: Effective span and depth of slab**
- **TAKING 1 M WIDTH FIND VOLUME OF ONE STEP=(1/2)*R*T*1**
- **NO. OF STEPS IN IM HORIZONTAL LENGTH=1/T**


Vc
33.75
12.375
24.75


2700
Going (10 @ 270)
1500
1500
300
J
K
L
M
750
750
Beam
Beam
300
1500
150
1500
300
Landing slab A
Landing slab B
C
D
P
Q
R
S
8.25 kN/m²
22.5 kN/m²
16.5 kN/m²
x=2510
750
2700
1650
5100

- The effective span (cls. 33.1b and c) = 750 + 2700 + 1500 + 150 = 5100 mm. The depth of waist slab = 5100/20 = 255 mm. Let us assume total depth of 250 mm and effective depth = 250 – 20 – 6 = 224 mm (assuming cover = 20 mm and diameter of main reinforcing bar = 12 mm). The depth of landing slab is assumed as 200 mm and effective depth = 200 – 20 – 6 = 174 mm.
- **Step 2: Calculation of loads**
- (i) Loads on going (on projected plan area)
- (a) Self-weight of waist-slab =1* 25*(0.25)*(313.85)/270 = 7.265 kN/m^2
- (b) Self-weight of steps = 24*(0.5)(0.16) = 2.0 kN/m^2
- (c) Finishes (given) = 1.0 kN/m^2
- (d) Live loads (given) = 5.0 kN/m^2
- Total = 15.265 kN/m^2

Total factored loads = 1.5(15.265) = 22.9 kN/m^2

(ii) Loads on landing slab A (50% of estimated loads)

(a) Self-weight of landing slab = 25(0.2) = 5 kN/m^2

(b) Finishes (given) = 1 kN/m^2

(c) Live loads (given) = 5 kN/m^2

Total = 11 kN/m^2

Factored loads on landing slab A = 0.5(1.5)(11) = 8.25 kN/m^2

(iii) Factored loads on landing slab B = (1.5)(11) = 16.5 kN/m^2

Step 3: Bending moment and shear force (Fig.)

Total loads for 1.5 m width of flight = 1.5{8.25(0.75) + 22.9(2.7) + 16.5(1.65)} = 142.86 kN

V_C = *1.5{8.25(0.75)(5.1 – 0.375) + 22.9(2.7)(5.1 – 0.75 – 1.35)* + 16.5(1.65)(1.65)(0.5)}/5.1 = 69.76 kN

V_D = *142.86 – 69.76 = 73.1 kN*

The distance *x from the left where shear force is zero is obtained from:*

69.76-12.375.75-33.75*(X-.75)=0*

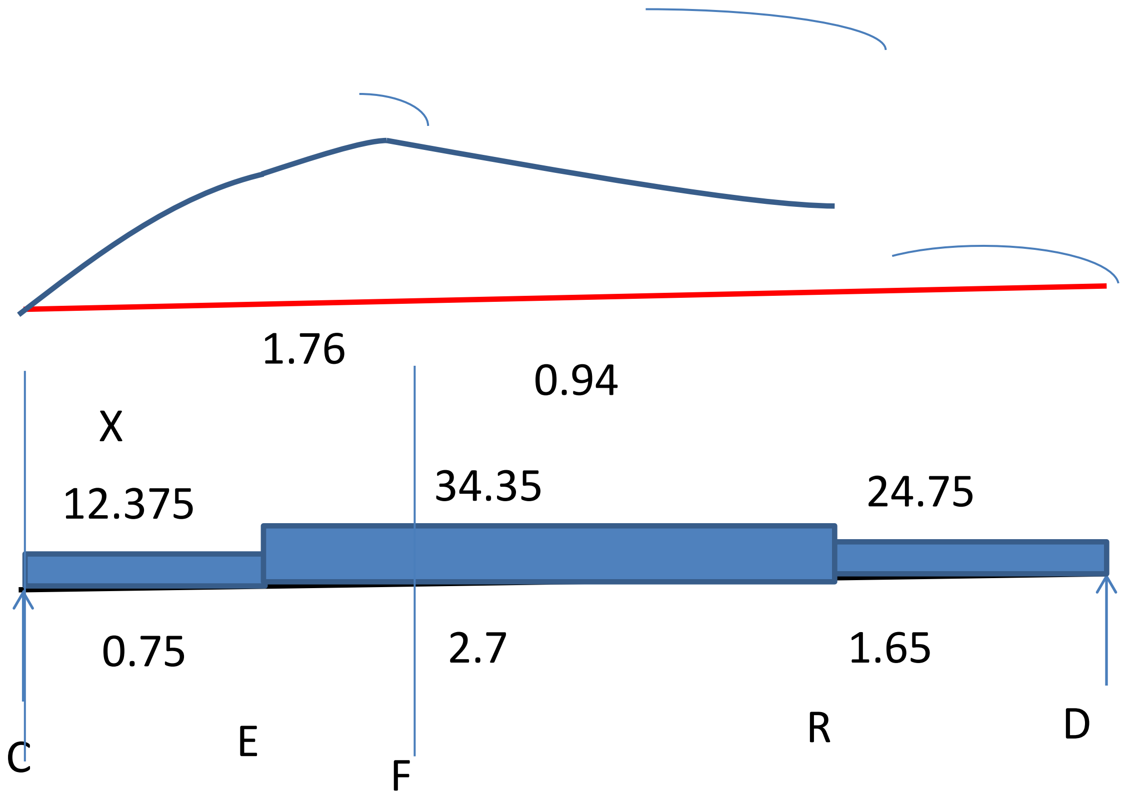
x = {69.76 – 1.5(8.25)(0.75) + 1.5(22.9)(0.75)}/(1.5)(22.9) = 2.51 m

The maximum bending moment at *x = 2.51 m is*

= 69.76(2.51) – (1.5)(8.25)(0.75)(2.51 – 0.375) - (1.5)(22.9)(2.51 – 0.75)(2.51 – 0.75)(0.5) = 102.08 kNm.

For the landing slab B, the bending moment at a distance of 1.65 m from D

- = 73.1(1.65) – 1.5(16.5)(1.65)(1.65)(0.5) = 86.92 kNm



VC*5.1-12.375*0.75*(5.1-.75/2)-34.35*2.7*(1.65+2.7/2)-24.75*1.65*1.65/2=0
VC=69.76kN
VD=12.375*.75+34.35*2.7+24.75*1.65-69.76=73.1kN
MC=0
ME=69.76*0.75-12.375*0.75*0.75/2=48.83
SF at F=0=69.76-12.375*0.75-34.35*(X-0.75)=0
X=2.51m
MF=69.76*2.51-12.375*0.75*(2.51-0.75/2)-34.75*1.76*1.76/2=102.08
MR=73.1*1.65-24.75*1.65*1.65/2=86.92

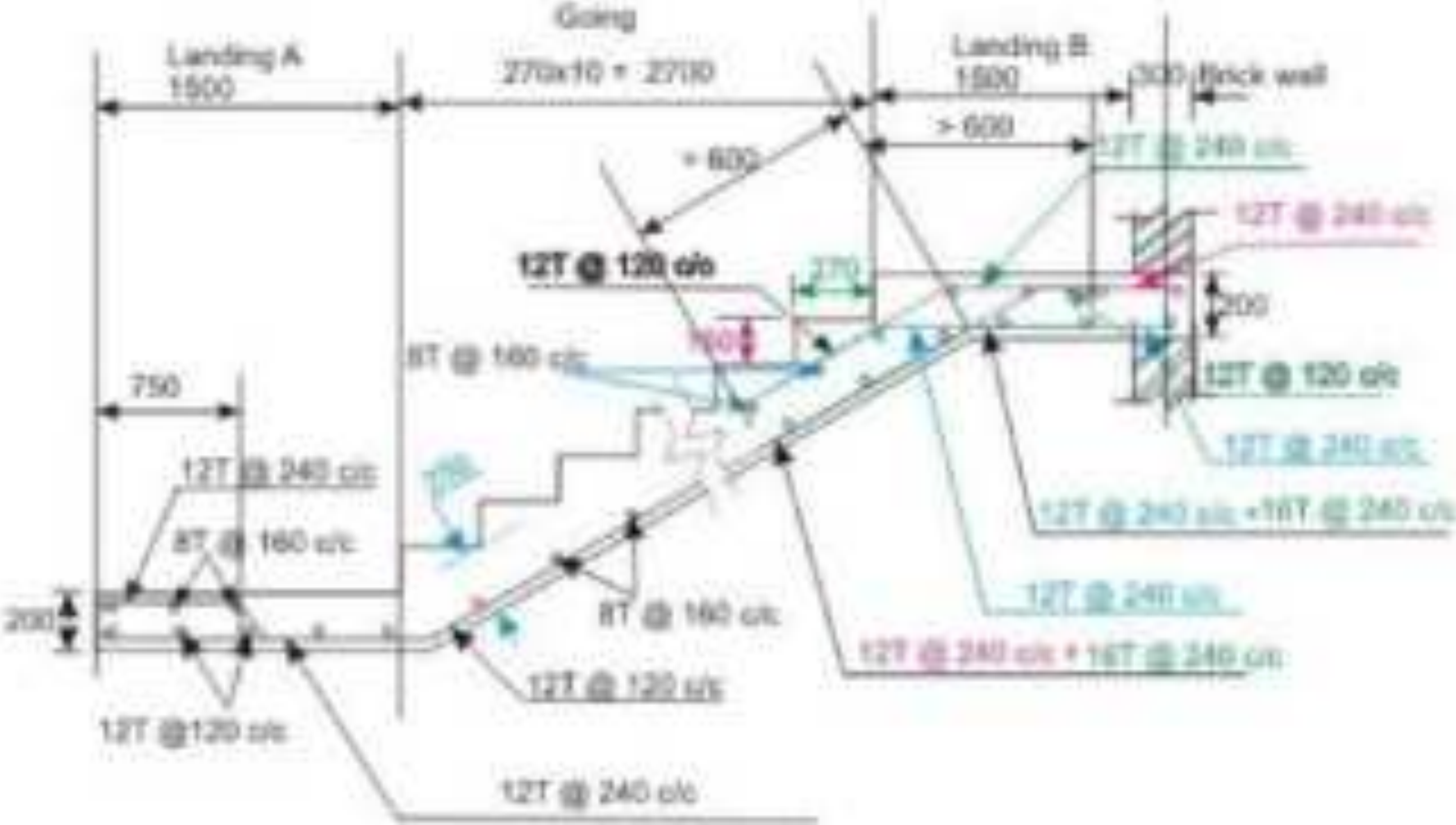
MF=73.1*(1.65+0.94)-24.75*1.65*(0.94+1.65/2)-34.35**0.94*0.94/2=

$$M_u = 0.138 * f_{ck} * bd^2$$

- From the maximum moment, we get *d = {102080000/(0.138*20*1500)}½ = 157.02 mm < 224 mm for waist-slab and < 174 mm for landing slabs. Hence, both the depths of 250 mm and 200 mm for waist-slab and landing slab are more than adequate for bending.*
- For the waist-slab, τ_{vu} *= 73100/[1500*(224)] = 0.217 N/mm*^{2}*. For the waist-slab of depth 250 mm, k = 1.1 (cl. 40.2.1.1 of IS 456) and from Table 19 of IS 456,* τ_c *= 1.1(0.28) = 0.308 N/mm*^{2}*. Table 20 of IS 456,* τ_{cmax} *= 2.8 N/mm*^{2}*. Since* $\tau_{vu} < \tau_c < \tau_{cmax}$*, the depth of waist-slab as 250 mm is safe for shear.*
- For the landing slab, τ_{vu} *= 73100/(1500*174) = 0.28 N/mm*^{2}*. For the landing slab of depth 200 mm, k = 1.2 (cl. 40.2.1.1 of IS 456) and from Table 19 of IS 456,k** τ_c*= 1.2(0.28) = 0.336 N/mm*^{2} *and from Table 20 of IS 456,* τ_{cmax} *= 2.8 N/mm*^{2}*. Here also* $\tau_{vu} < \tau_c < \tau_{cmax}$ *, so the depth of landing slab as 200 mm is safe for shear.*

$$M_u = A_{st} * 0.87 * f_y * d * \left(1 - \frac{A_{st} * f_y}{f_{ck} bd}\right)$$

- **Step 5: Determination of areas of steel reinforcement**
- (i) Waist-slab: *M_u/bd^2 = 10208000/[1500*224*224]= 1.356 N/mm^2. Table 2 of SP-16 gives p = 0.411.*
- The area of steel = 0.411(1000)(224)/(100) = 920.64 mm^2. Provide 12 mm diameter @ 120 mm c/c (= 942 mm^2/m). [Calculated for 1m width)
- (ii) Landing slab B: *M_u/bd^2 at a distance of 1.65 m from V_D = 86920000/(1500*174*174) = 1.91 N/mm^2. Table 2 of SP-16 gives: p = 0.606. The area of steel = 0.606(1000) (174)/100 = 1054 mm^2/m. Provide 16 mm diameter @ 240 mm c/c and 12 mm dia. @ 240 mm c/c (1309 mm^2) at the bottom of landing slab B of which 16 mm bars will be terminated at a distance of 500 mm from the end and will continue up to a distance of 1000 mm at the bottom of waist slab .*
- Distribution steel: The same distribution steel is provided for both the slabs as calculated for the waist-slab. The amount is = 0.12(250)(1000)/100 = 300 mm^2/m. Provide 8 mm diameter @ 160 mm c/c (= 314 mm^2/m).



Going
Landing A
1500
270x10 = 2700
Landing B
1500
300 Brick wall
> 600
> 600
12T @ 240 c/c
12T @ 240 c/c
12T @ 120 c/c
270
200
160
8T @ 160 c/c
12T @ 120 c/c
750
12T @ 240 c/c
12T @ 240 c/c
8T @ 160 c/c
12T @ 240 c/c +16T @ 240 c/c
200
8T @ 160 c/c
12T @ 240 c/c
12T @ 120 c/c
12T @ 240 c/c + 16T @ 240 c/c
12T @120 c/c
12T @ 240 c/c

- **Step 6: Checking of development length and diameter of main bars**
- Development length of 12 mm diameter bars = 47(12) = 564 mm, say 600 mm and the same of 16 mm dia. Bars = 47(16) = 752 mm, say 800 mm.
- (i) For waist-slab
- A_{st}=(1500/120)*113=1412.5 mm^2

$$M_1 = A_{st} * 0.87 * f_y * d * \left(1 - \frac{A_{st} * f_y}{f_{ck} bd}\right)$$

$$= 1412.5 * 0.87 * 415 * 224 * \left(1 - \frac{1412.5 * 415}{20 * 1500 * 224}\right) = 104.27\ kNm$$

V_u (shear force) = 73.1 kN

$$\frac{1.3 * M_1}{V_u} + l_0 = \frac{1.3 * 104.27 * 10^6}{73100} + 0 = 1854.35\ mm$$

$$l_d = \frac{0.87 * f_y * \varphi}{4 * \tau_{bd}} = \frac{0.87 * 415 * \varphi}{4 * 1.2 * 1.6} = 47\varphi = 47 * 12 = 564mm$$

$$(l_d = 564\ mm) < [(\frac{1.3 * M_1}{V_u} + l_0) = 1854.35\ mm]$$

- So, it is OK

- (ii) For landing-slab B

$$A_{st} = \left(\frac{1500}{120}\right) * 201 = 2512.5\ mm^2$$

$$M_1 = A_{st} * 0.87 * f_y * d * \left(1 - \frac{A_{st} * f_y}{f_{ck} bd}\right)$$
$$= 2512.5 * 0.87 * 415 * 174 * \left(1 - \frac{2512.5 * 415}{20 * 1500 * 174}\right) = 126.31\ kNm$$

$$\frac{1.3 * M_1}{V_u} + l_0 = \frac{1.3 * 126.31 * 10^6}{73100} + 0 = 2246.34\ mm$$

$$l_d = \frac{0.87 * f_y * \varphi}{4 * \tau_{bd}} = \frac{0.87 * 415 * \varphi}{4 * 1.2 * 1.6} = 47\varphi = 47 * 16$$
$$= 752 mm$$

$$(l_d = 752\ mm) < [(\frac{1.3 * M_1}{V_u} + l_0) = 2246.34\ mm]$$

- So, it is OK.

- **(B) Design of landing slab A**
- **Step 1: Effective span and depth of slab**

The effective span is lesser of (Taking Depth D=200 mm, d_{eff}=174 mm)

(i) (1500 + 1500 + 150 + 174)=3324 mm, $l_{cl}+d_{eff}$

(ii) (1500 + 1500 + 150 + 300) = 3450 mm, c/c

L_{eff}=3324 mm

The depth of landing slab = 3324/20 = 166 mm, < 200 mm already assumed.

So, the depth is 200 mm.

- **Step 2: Calculation of loads**
- The following are the loads:

(i) Factored load on landing slab A(see Step 2 of A @ 50%) = 8.25 kN/m^2

(ii) Factored reaction V_C *(see Step 3 of A) = 69.76 kN as the total load of one flight*

(iii) Factored reaction V_C *from the other flight = 69.76 kN*

- Thus, the total load on landing slab A
- = (8.25)(1.5)(3.324) + 69.76 + 69.76 = 180.65 kN

- Due to symmetry of loadings, $V^E = V^F = 90.33$ *kN. The bending moment is maximum at the centre line of EF.*
- **Step 3: Bending moment and shear force (width = 1500 mm)**
- Maximum bending moment = (180.65)(3.324)/8 = 75.06 kNm
- Maximum shear force = 0.5(180.65) = 90.33 kN
- **Step 4: Checking of depth of slab**
- In Step 3 of A, it has been observed that 135.98 mm is the required depth for bending moment = 102.08 kNm. So, the depth of 200 mm is safe for this bending moment of 75.06 kNm. However, a check is needed for shear force.
- $\tau_{vu} = 90330/1500(174) = 0.347\ N/mm^2 > 0.336\ N/mm^2$
- The above value of $\tau_c = 0.336\ N/mm^2$ *for landing slab of depth 200 mm has been obtained in Step 4 of A. However, here* τ_c *is for the minimum tensile steel in the slab. The checking of depth for shear shall be done after determining the area of tensile steel as the value of vτ is marginally higher.*
- For $M_u/bd^2 = 75060000/[(1500)*(174)*(174)] = 1.65\ N/mm^2$, *Table 2 of SP-16 gives p = 0.512.*
- The area of steel = (0.512)(1000)(174)/100 = 890.88 mm^2/m. Provide 12 mm diameter @ 120 mm c/c (= 942 mm^2/m). With this area of steel $p = 942(100)/1000(174) = 0.541$.

- Distribution steel = The same as in Step 5 of A i.e., 8 mm diameter @ 160 mm c/c.
- **Step 6: Checking of depth for shear**
- Table 19 and cl. 40.2.1.1 gives: τ_c= *(1.2)(0.493) = 0.5916 N/mm²*. τ_{vu} *= 0.347 N/mm² (see Step 3 of B) is now less than cτ (= 0.5916 N/mm²). Since,* $\tau_{vu} < \tau_c < \tau_{cmax}$*, the depth of 200 mm is safe for shear.*