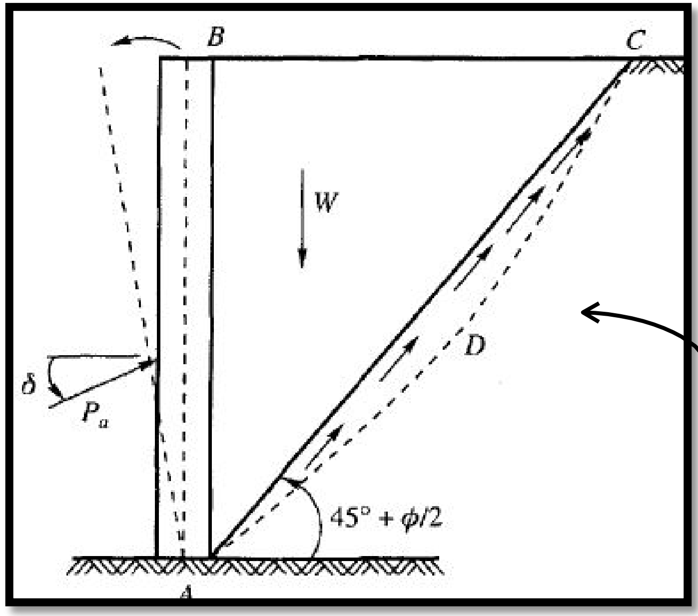
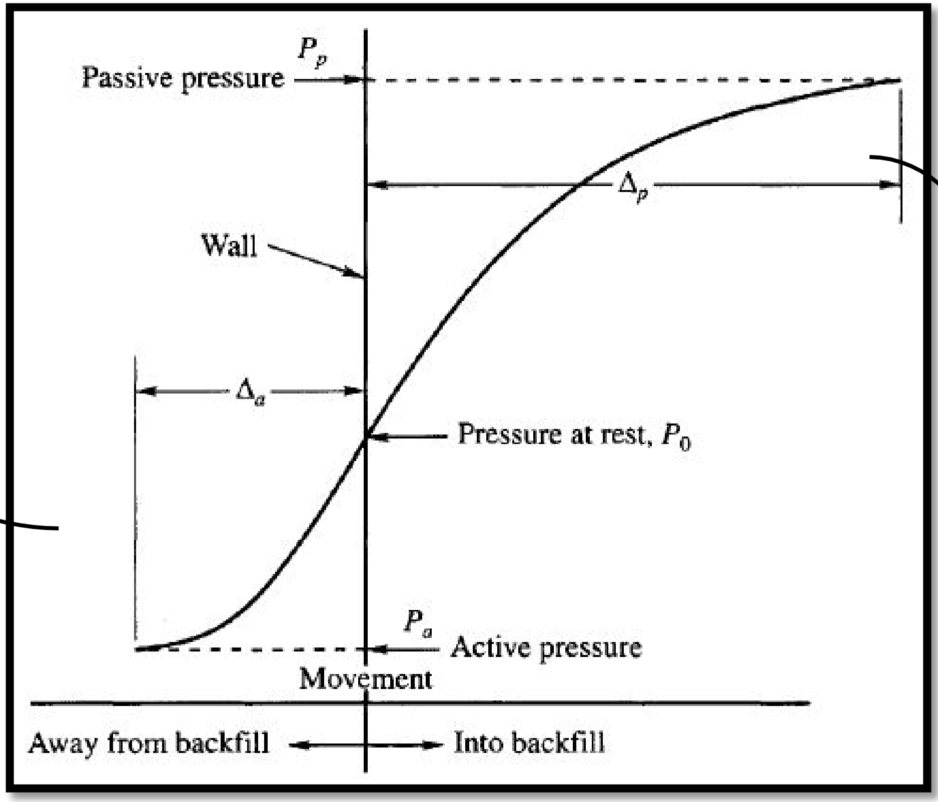
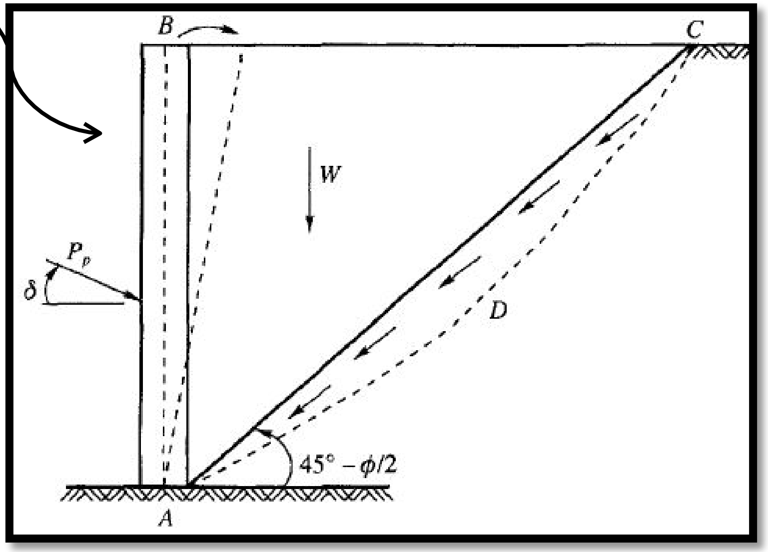
Earth Pressure I



Active stage





Passive stage

Earth Pressure at rest

Let ε_x be the strain in the horizontal direction at depth z on an element of soil and let the Poisson's ratio and elastic modulus be μ and E, respectively. The earth pressure at rest corresponds to a state of zero lateral strain (ε_x =0).

For plane strain condition, ε_x is given by

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \mu\left(\sigma_y + \sigma_z\right)\right]$$

$$When\ \varepsilon_x = 0,\ \sigma_x = \frac{\mu}{1-\mu}\sigma_z$$

σ_x is designated as p_0.

Hence, coefficient of earth pressure at rest,

$$k_0 = \frac{p_0}{\sigma_z} = \frac{\mu}{1-\mu}$$

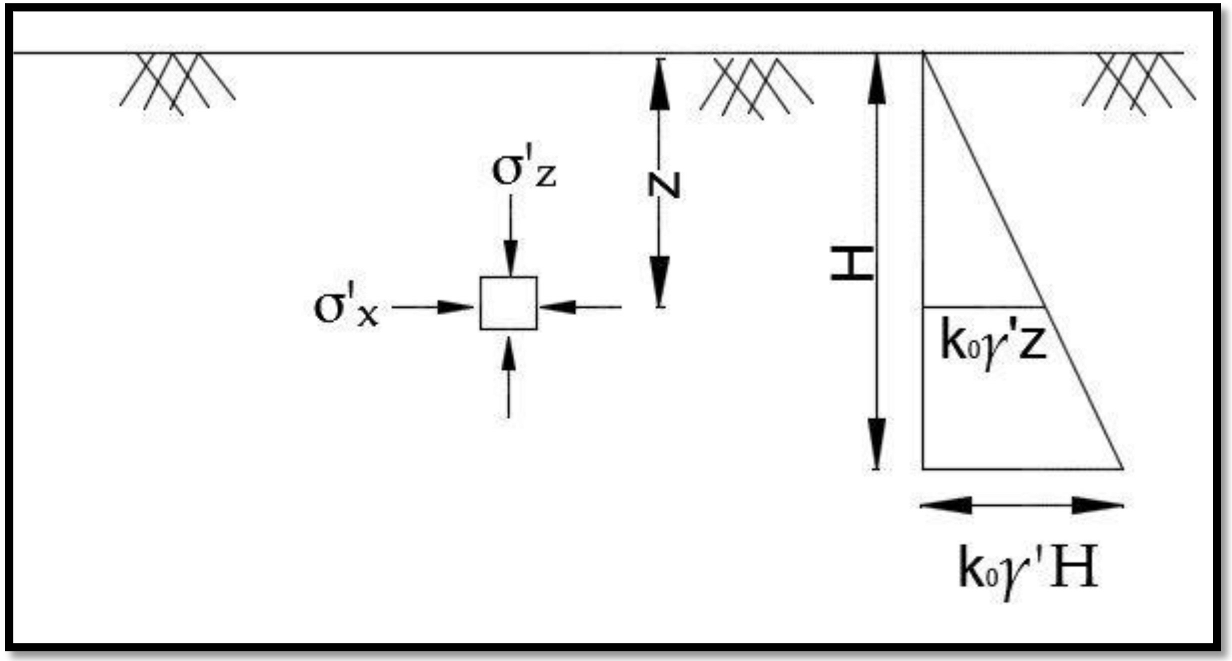
- In the natural state, an element of soil at a depth z below ground surface is not subjected to any strain-the element is in a condition known as 'at rest 'condition and can be expressed as

$$p_0 = k_0 \sigma_z$$

- The total pressure per length acting over a height of H of retaining wall is equal to the area of lateral pressure distribution diagram.

$$P_0 = \frac{1}{2} k_0 \gamma H^2$$

P_0 acts at a height of H/3 from base.



Soil	k_0
Dense sand	0.4-0.45
Loose sand	0.45-0.5
Mechanically compacted sand	0.8-1.5
Normally consolidated clay	0.5-0.6
Over consolidated clay	1.0-4.0

- For sands and normally consolidated clays, Jaky (1944) gave following equation:

$$k_0 = 1 - \sin\phi'$$

Rankine's theory of earth pressure

- Salient assumptions of Rankine's earth pressure can be summarized as

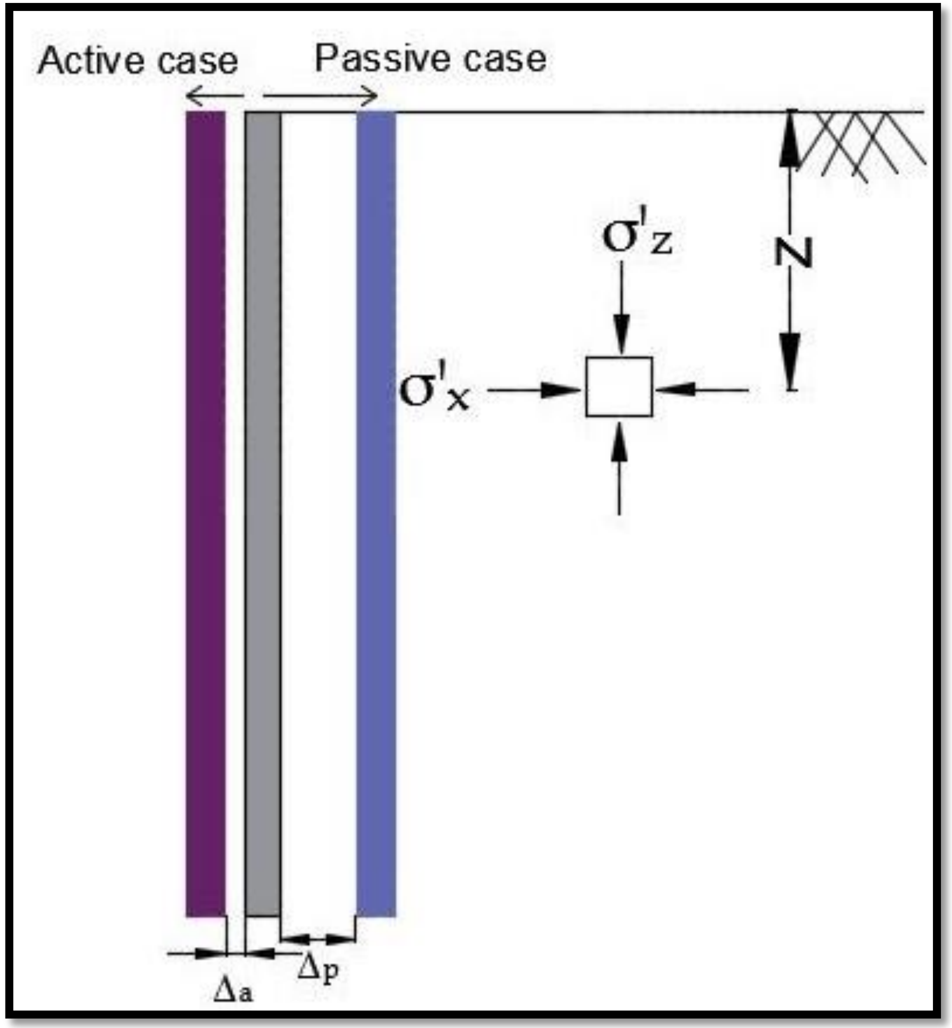
✓ The backfill is isotropic, homogenous and cohesionless.

✓ The soil is in state of plastic equilibrium during active and passive earth pressure conditions.

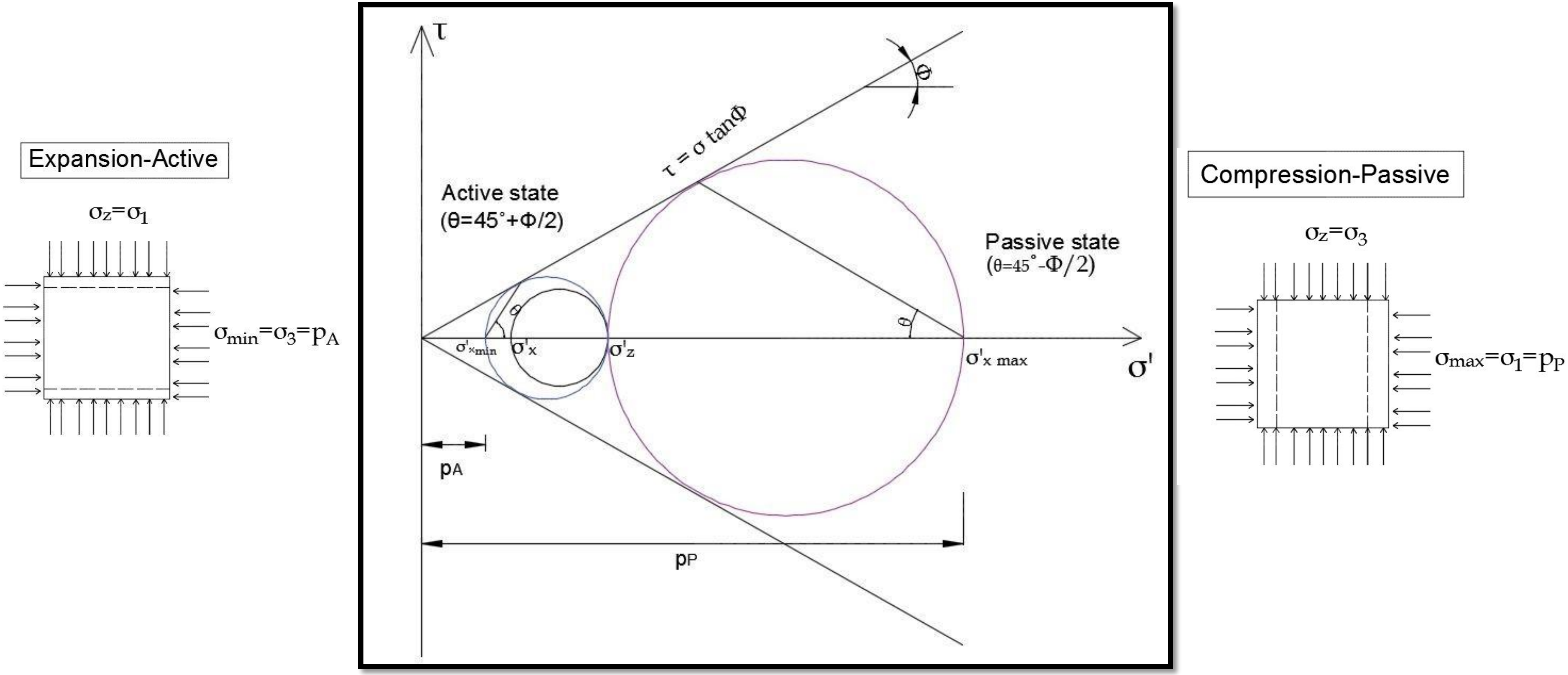
✓ The rupture surface is planar surface which is obtained by considering plastic equilibrium of soil.

✓ The backfill surface is horizontal.

✓ The back of the wall is vertical and smooth.

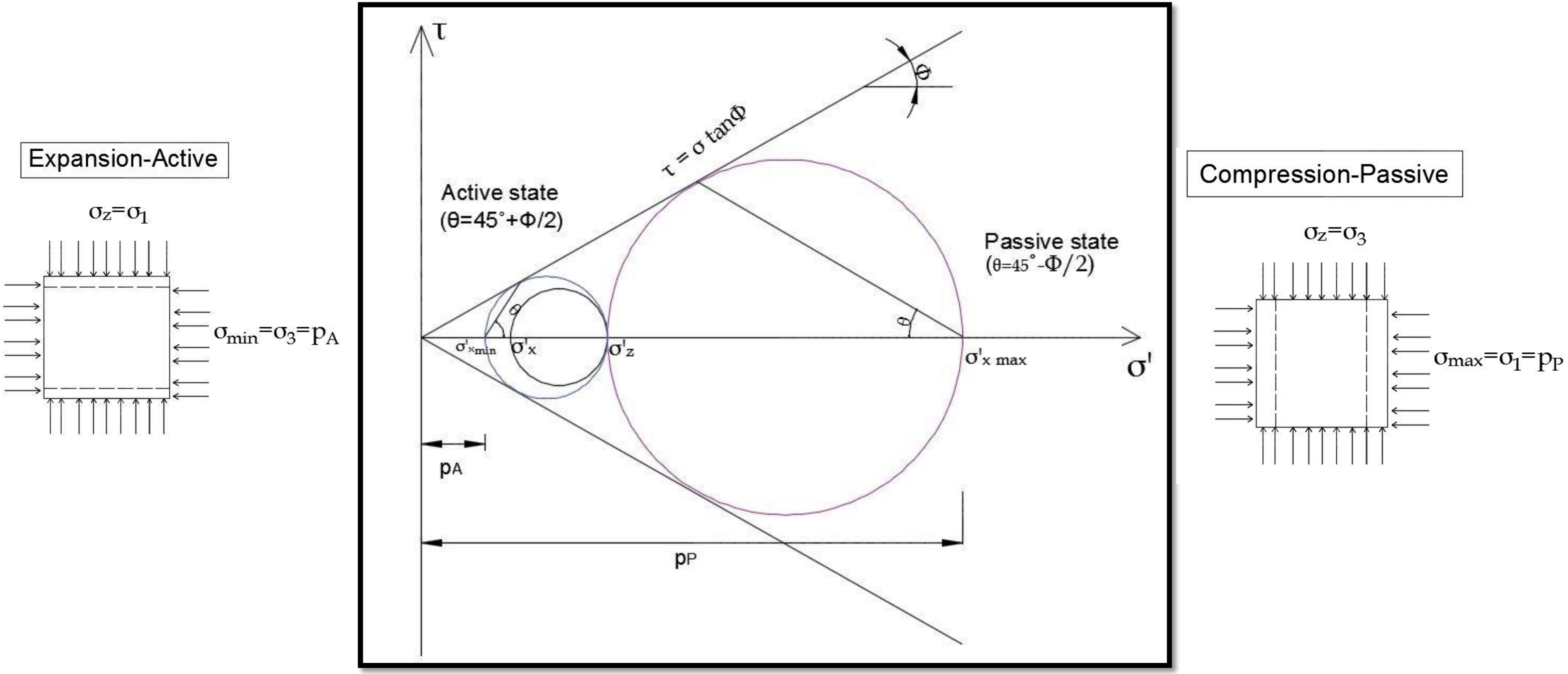


Active case
Passive case
σ'z
σ'x
Z
Δa
Δp



Expansion-Active
σz=σ1
σmin=σ3=pA
τ
τ = σ tanΦ
Φ
Active state
(θ=45°+Φ/2)
Passive state
(θ=45°-Φ/2)
θ
σ'x min
σ'x
σ'z
σ'x max
σ'
pA
pP
Compression-Passive
σz=σ3
σmax=σ1=pP

Earth Pressure II


Expansion-Active
σz=σ1
σmin=σ3=pA
τ
τ = σ tanΦ
Φ
Active state
(θ=45°+Φ/2)
θ
σ'x min
σ'x
σ'z
Passive state
(θ=45°-Φ/2)
θ
σ'x max
σ'
pA
pP
Compression-Passive
σz=σ3
σmax=σ1=pP

Active earth pressure

- If the wall moves away from the backfill, the soil element expands and the horizontal pressure decreases to a minimum value so that a state of plastic equilibrium is developed.

$$\sin\phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

$$\Rightarrow \frac{\sigma_1}{\sigma_3} = \frac{1+\sin\phi}{1-\sin\phi}$$

Here, $\sigma_1 = \sigma_z =$ weight of soil at depth z, that is $\sigma_1 = \gamma z$.
The minimum value of σ_x is defined as the active earth pressure p_A; that is
$\sigma_3 = (\sigma_x)_{min} = p_a$

$$\boxed{p_A = K_A \gamma z}$$

where K_A is the coefficient of active earth pressure $= \frac{1-\sin\phi}{1+\sin\phi} = \tan^2\left(45° - \frac{\phi}{2}\right)$

Passive earth pressure

- If the wall moves towards from the backfill, there will be a uniform compression in horizontal direction. There will be an increase in the horizontal pressure while σ_z remains constant.

Passive earth pressure can be estimated as

$$p_P = K_P \gamma z$$

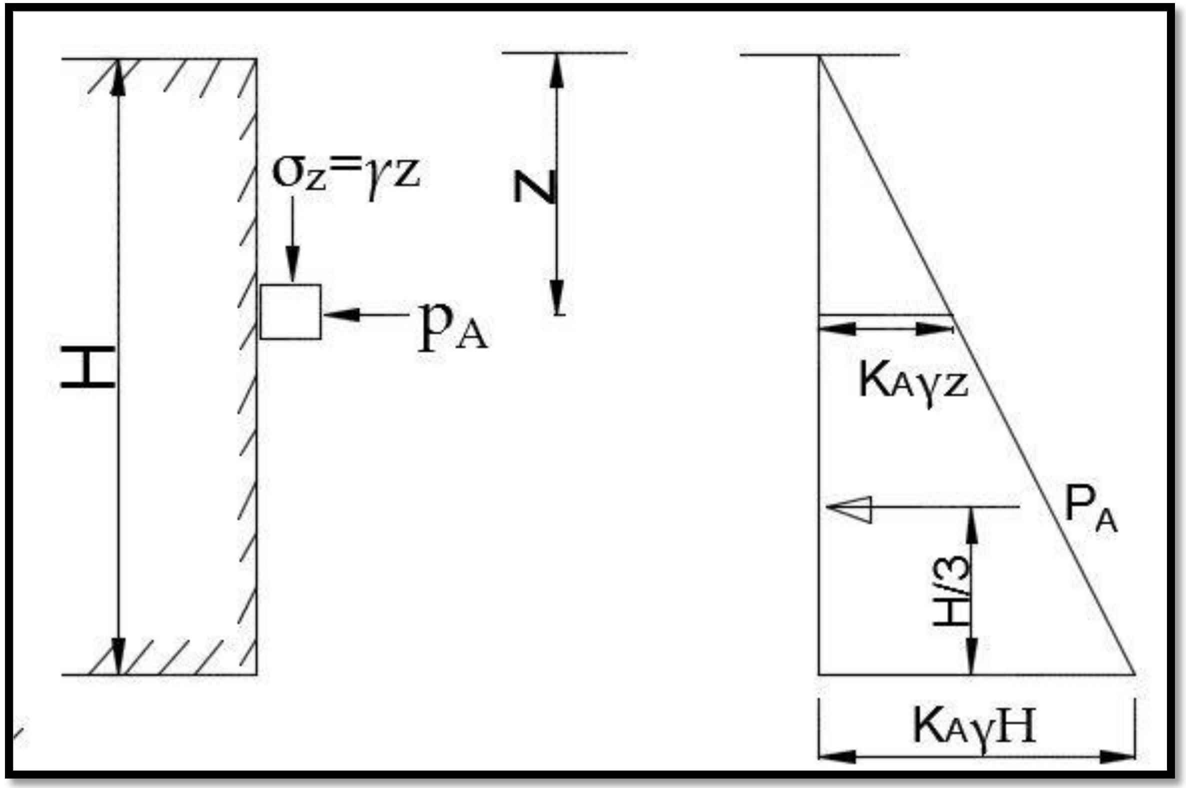
Where K_P is the coefficient of active earth pressure $= \dfrac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(45 + \dfrac{\phi}{2}\right)$

Active pressure on retaining walls: Cohesionless Backfill

a) Dry backfill with no surcharge

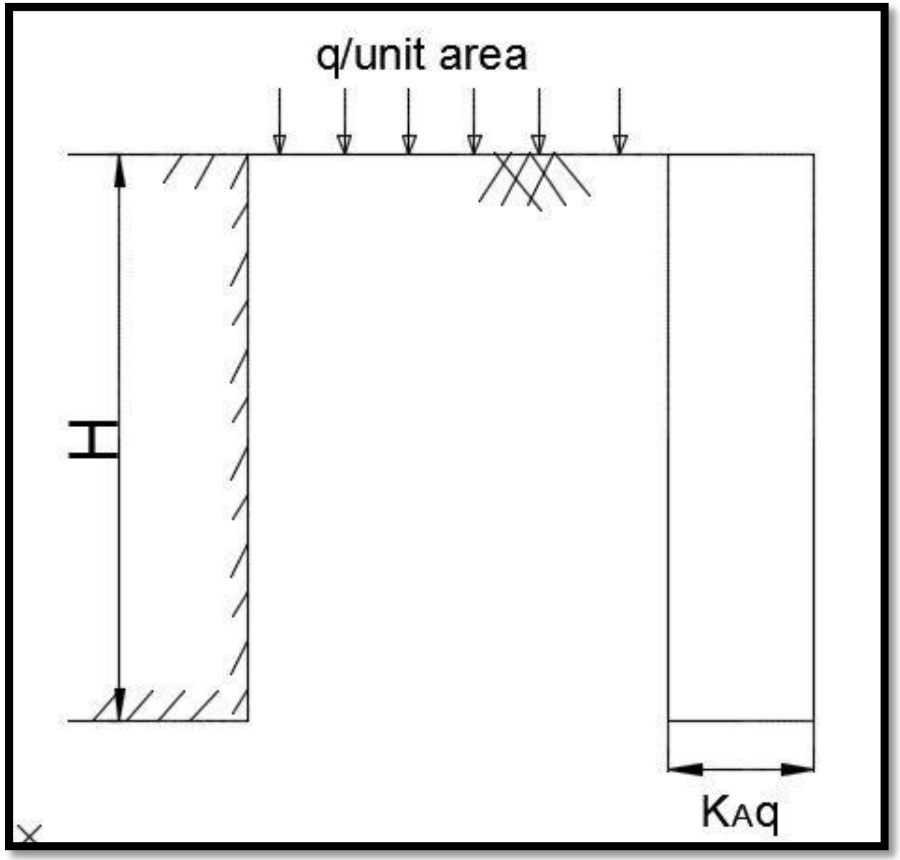
Total Active thrust

$$P_A = \frac{1}{2} K_A \gamma H^2$$



b) Effect of uniform surcharge

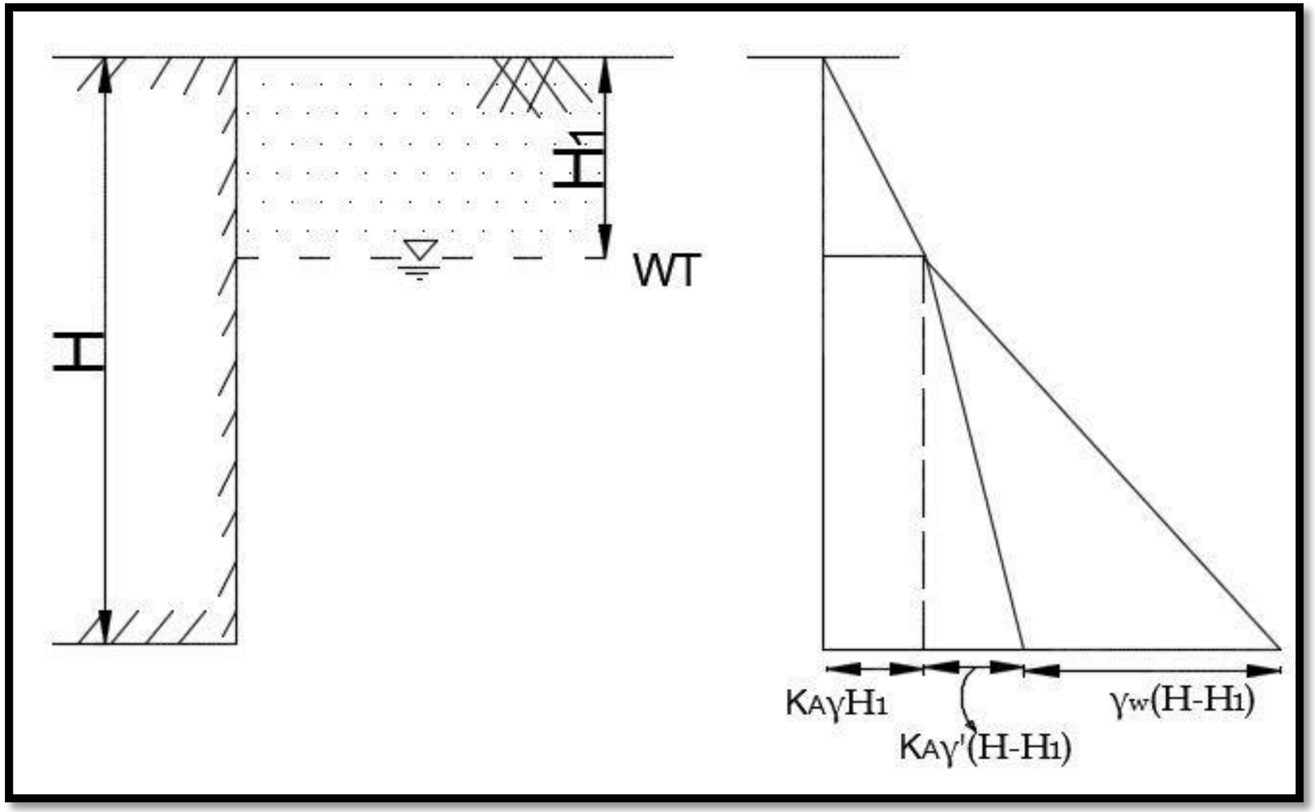
- If a uniformly distributed load of intensity q/unit area is acting over the entire surface of backfill, then effective stress at any depth is increased by $K_A q$.



c) Submerged backfill

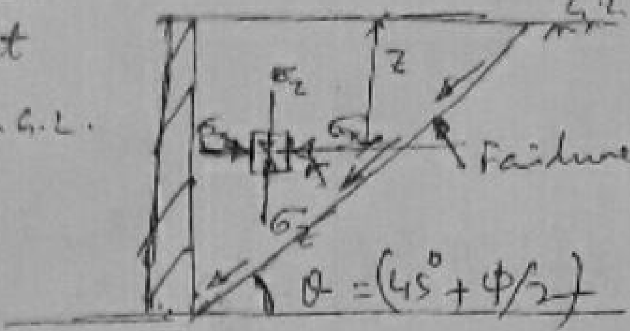
Active earth pressure at base of wall

$$p_A = K_A \gamma H_1 + K_A \gamma' (H - H_1) + \gamma_w (H - H_1)$$



(8) **Rankine's Active & Passive Earth Pressure.**

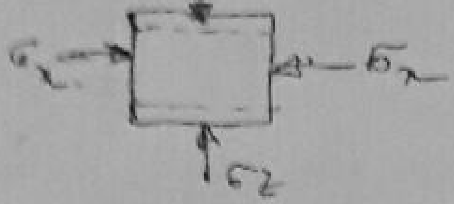
Consider the soil element at pt A at a dist z from G.L.



$\sigma_z = \gamma z$

Rankine Active Earth Pressure

Active Case
σ_z Expansion of soil.



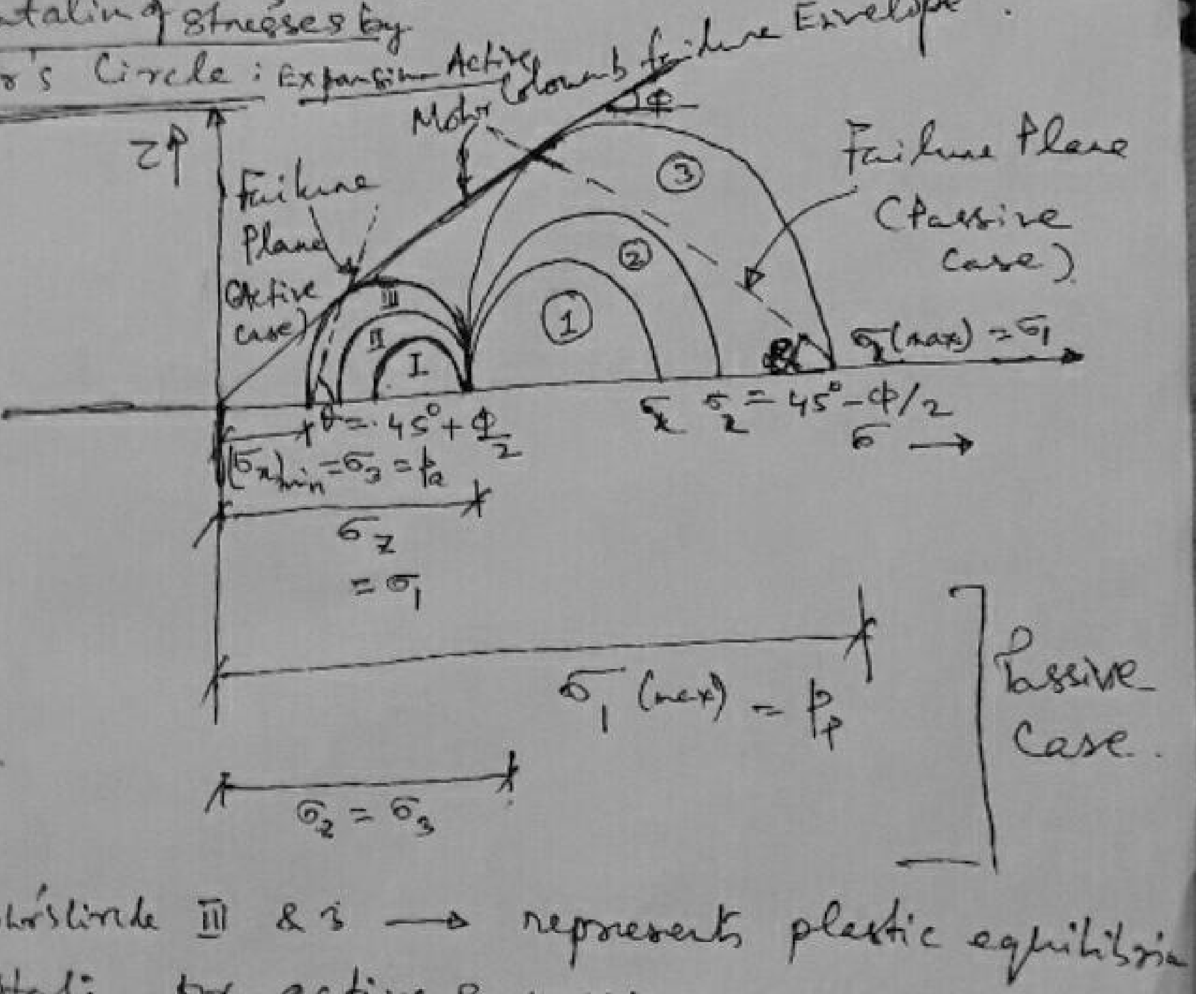
As no shear stress acting on the planes (two mutually ⊥r direction x & z) the stresses are principal stresses.

$$\sigma_1 = \sigma_z = \gamma z$$

$$\sigma_3 = \sigma_x = K_a \gamma z$$

Representation of stresses by Mohr's Circle: Expansion–Active

Compression–Passive



Mohr's circle III & 3 → represents plastic equilibrium stress state for active & passive earth pressure respectively

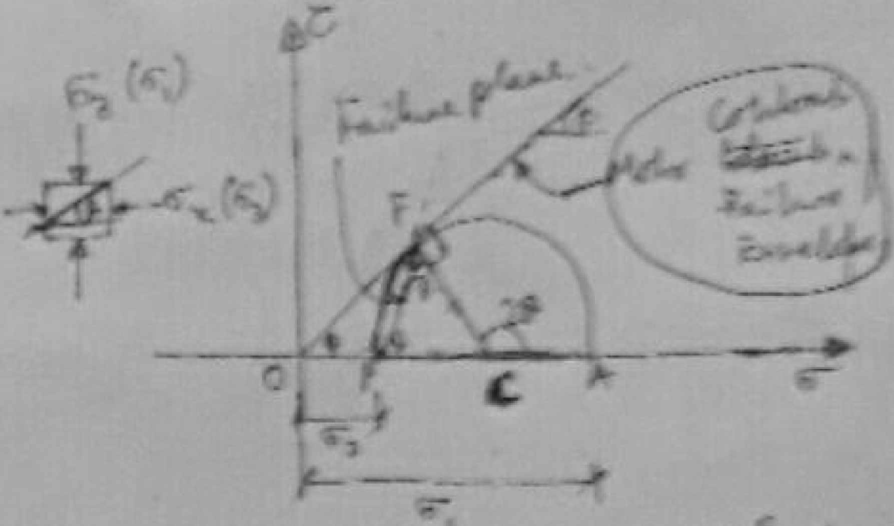
⑤

Earth Active Pressure Case

θ = Angle which the failure plane makes with x-direction (major principal plane)

$\sigma_1 = \sigma_z = \gamma z$

$\sigma_3 = \sigma_x = \frac{p}{\,}$



$\frac{\sigma_x}{\sigma_z} = \frac{\sigma_3}{\sigma_1} = k_a = \frac{p}{\gamma z}$

k_a = Coeff of earth pressure

From Δ FOC, $\angle FCA = 2\theta = 90° + \phi \Rightarrow \boxed{\theta = 45° + \frac{\phi}{2}}$

$PC = \frac{\sigma_1 - \sigma_3}{2}$, $OC = \sigma_3 + \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2}$

From Δ OCF,

$\sin\phi = \frac{FC}{OC} = \frac{PC}{OC} = \frac{\frac{\sigma_1 - \sigma_3}{2}}{\frac{\sigma_1 + \sigma_3}{2}} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$

$\Rightarrow \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \sin\phi$

$\frac{a}{b} = \frac{c}{d}$

$\Rightarrow \frac{a-b}{b+a} = \frac{c-d}{d+c}$

$\Rightarrow \frac{\sigma_1 + \sigma_3}{\sigma_1 - \sigma_3} = \frac{1}{\sin\phi}$

$\Rightarrow \frac{\sigma_1 + \sigma_3 - \sigma_1 + \sigma_3}{\sigma_1 - \sigma_3 + \sigma_1 + \sigma_3} = \frac{1 - \sin\phi}{\sin\phi + 1}$

$\Rightarrow \frac{2\sigma_3}{2\sigma_1} = \frac{1 - \sin\phi}{1 + \sin\phi}$

$\Rightarrow \frac{\sigma_3}{\sigma_1} = \frac{1 - \sin\phi}{1 + \sin\phi}$

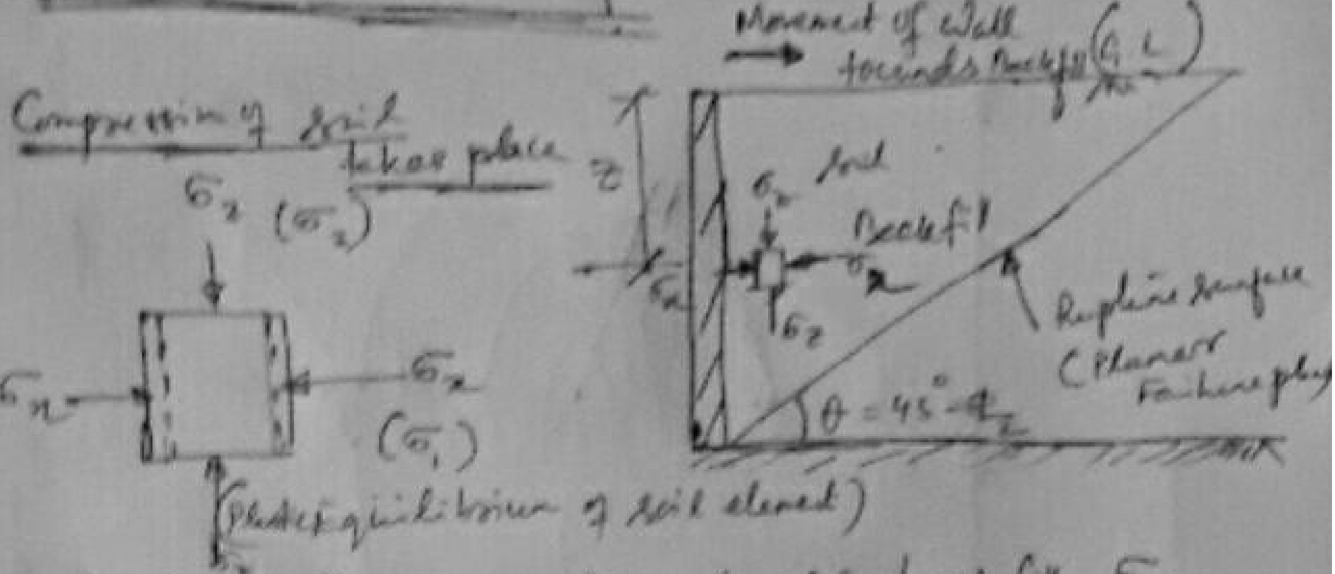
$\Rightarrow \frac{\sigma_3}{\sigma_1} = \boxed{k_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \tan^2\left(45° - \frac{\phi}{2}\right)}$

$\Rightarrow \frac{p_a}{\gamma z} = k_a \Rightarrow \boxed{p_a = k_a \gamma z}$

(2)

Passive Earth Pressure

Compression of soil takes place



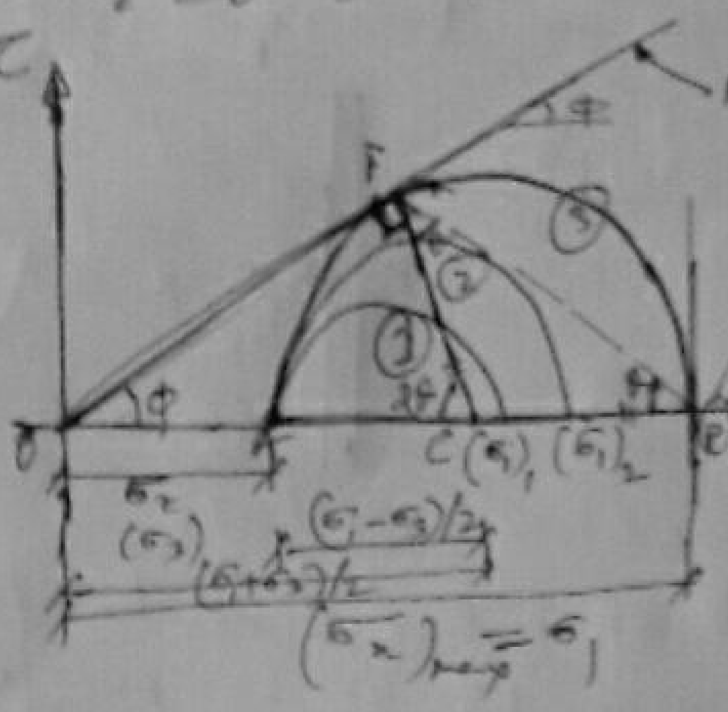
As the wall moves towards the backfill σ_x goes on increasing till the maximum displacement/deformation takes place, any further movement may cause the failure along the planar failure/rupture surface in the backfill/soilmass behind the wall.

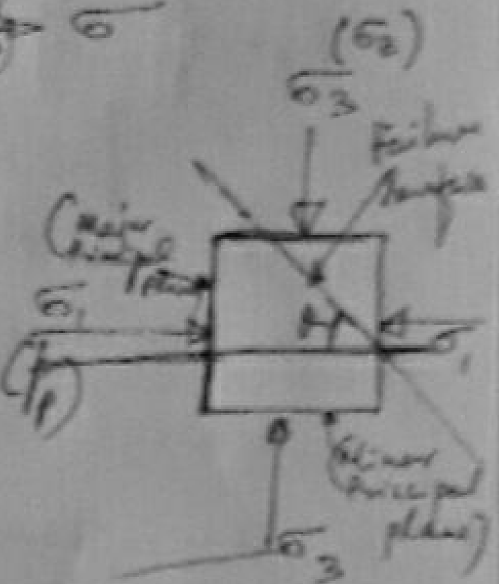
Here, σ_x goes on increasing from its original earth pressure at rest position (elastic equilibrium condition) and becomes equal [at one stage of yielding of the wall towards backfill] to σ_z & then becomes more than σ_z with further movement of the wall, which causes more deformation of soil/backfill. This goes on till the plastic equilibrium stage is reached. The soil is on the verge of failure along the failure/rupture surface.

$\sigma_x \longrightarrow (\sigma_x)_{max} \longrightarrow \sigma_1$ (major principal stress)

(4)

$\sigma_x \Rightarrow \sigma_3 \Rightarrow \gamma z$ (minor principal stress)





Δ OFC,

$$\angle FCO = 180^\circ - 90^\circ - \varphi$$
$$= 90^\circ - \varphi$$
$$= 2\theta$$
$$\Rightarrow \boxed{\theta = 45^\circ - \frac{\varphi}{2}}$$

θ is angle which the failure surface makes with minor principal plane. (i.e. x-direction)

Δ OFC,

$$\sin\varphi = \frac{CF}{OC} = \frac{AC}{OC} = \frac{\frac{\sigma_1 - \sigma_3}{2}}{\frac{\sigma_1+\sigma_3}{2}} = \frac{\sigma_1 - \sigma_3}{\sigma_1+\sigma_3}$$

$$\Rightarrow \frac{\sigma_1 - \sigma_3}{\sigma_1+\sigma_3} = \sin\varphi$$

$$\Rightarrow \frac{\sigma_1}{\sigma_3} = \frac{1+\sin\varphi}{1-\sin\varphi}$$

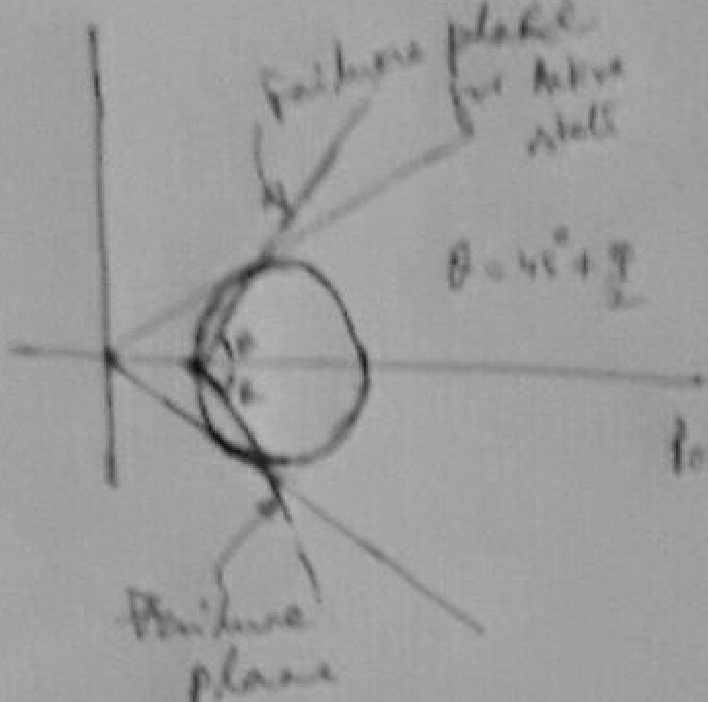
$$\Rightarrow \boxed{\frac{P_p}{\sigma_z} = K_p = \frac{1+\sin\varphi}{1-\sin\varphi} = \tan^2\left(45^\circ + \frac{\varphi}{2}\right)}$$

K_p = coefficient of Passive Earth Pressure

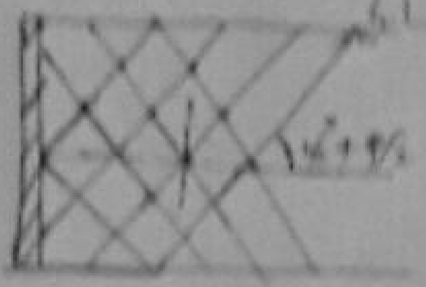
$$\boxed{P_p = K_p \sigma_z = K_p \gamma z}$$

Rankine's Active State

Active Case



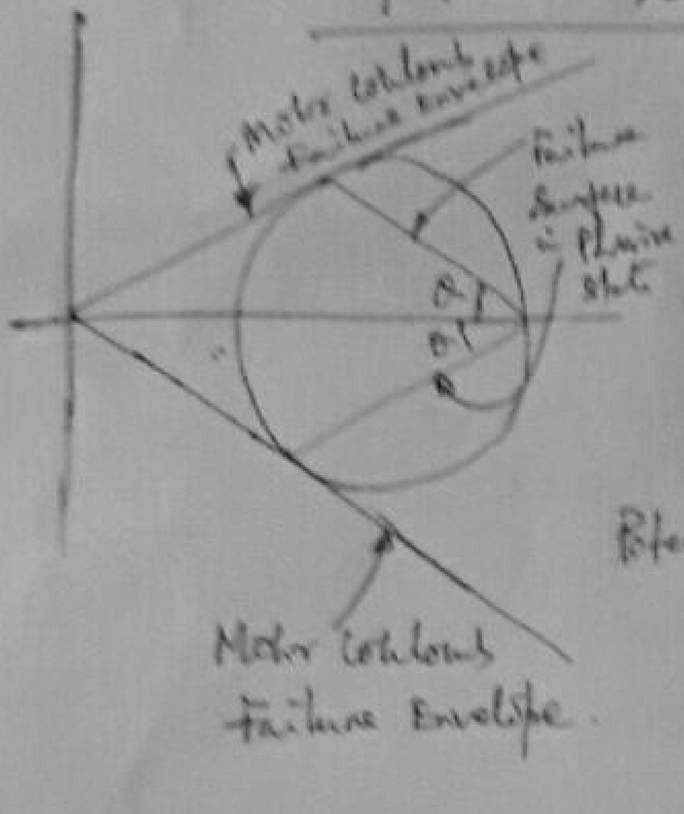
$\theta = 45° + \frac{\phi}{2}$



Potential failure plane within soils in Rankine Active case.

II

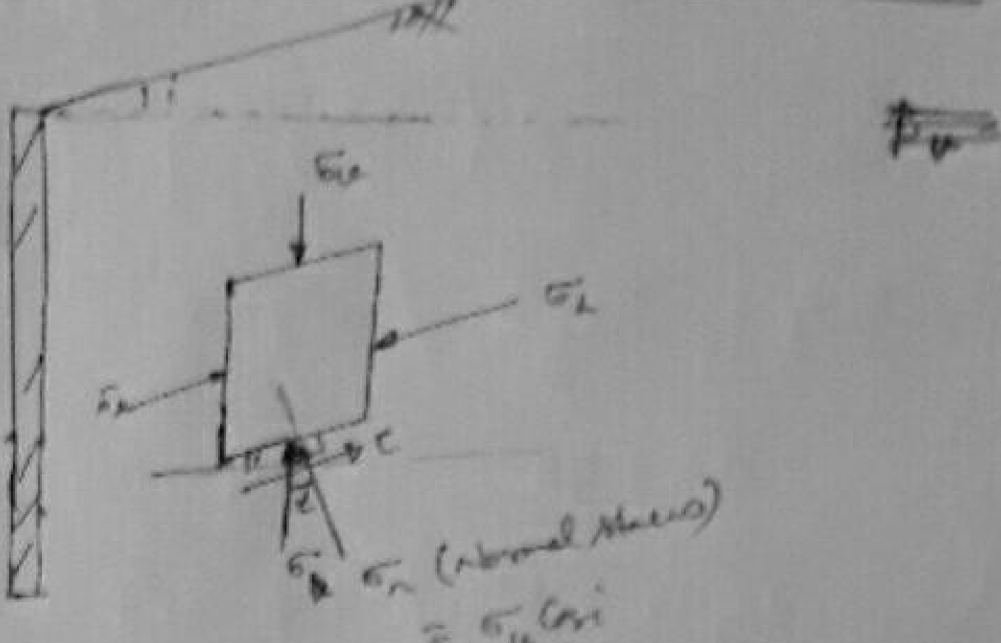
Passive Rankine State

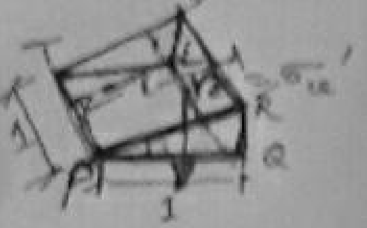


Potential failure planes within the soil/backfill in Rankine Passive Pressure State

$\theta = 45° - \frac{\phi}{2}$

Effect of sloping ground surface (Inclined backfill) 24.04.21





Area of TPQL = 1×1
Area of TPRS = l×1

$\frac{1}{l} = \cos i \Rightarrow l = \frac{1}{\cos i}$

$\sigma_v' \times (1\times1) = \sigma_v^{\bullet} \times (l \times 1)$

$\Rightarrow \sigma_v^{\bullet} = \frac{\sigma_v'}{l} = \frac{\gamma z}{1/\cos i} = \gamma z \cos i$

$\sigma_n = \sigma_v \cos i$
$= (\gamma z \cos i) \cos i$
$= \gamma z \cos^2 i$

$\tau = \sigma_v \sin i$
$= \gamma z \cos i \sin i$
$= \gamma z \cos i \sin i$

Mohr Circle

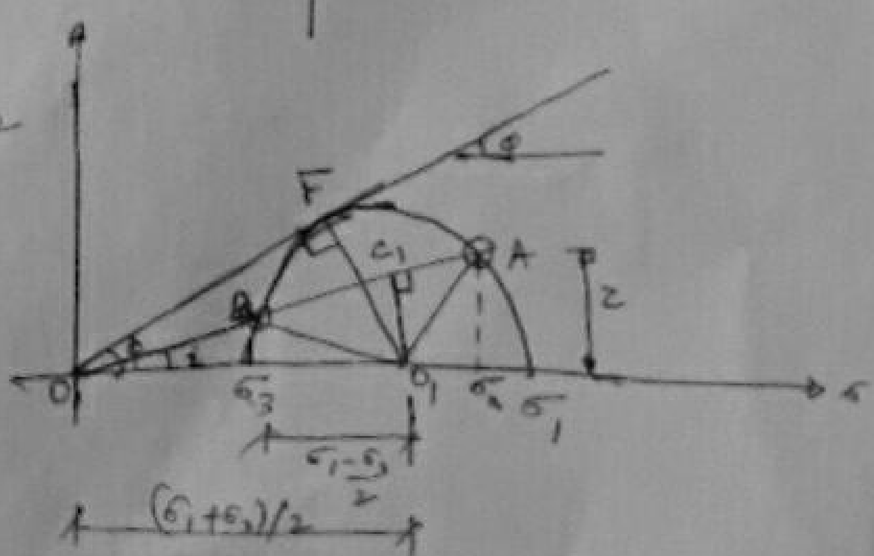
ΔOFO_1, $\sin\varphi = \frac{(\sigma_1 - \sigma_3)/2}{(\sigma_1 + \sigma_3)/2}$

$\Delta O C C_1$

$\sin i = \frac{O_1 C_1}{O O_1}$

$\Rightarrow O_1 C_1 = O O_1 \sin i$

$= \frac{\sigma_1 + \sigma_3}{2} \sin i$



ΔO_1C_1B, $BC_1^2 = O_1B^2 - O_1C_1^2$

$$= \left(\frac{\sigma_1 - \sigma_3}{2}\right)^2 - \left(\frac{\sigma_1 + \sigma_3}{2}\right)^2 \sin^2 i$$

$$= \left(\frac{\sigma_1 + \sigma_3}{2}\right)^2 \sin^2\varphi - \left(\frac{\sigma_1 + \sigma_3}{2}\right)^2 \sin^2 i$$

$$BC_1 = \frac{\sigma_1 + \sigma_3}{2}\sqrt{\sin^2\varphi - \sin^2 i}$$

$O_1B = O_1A$, $\Rightarrow$ $\underline{AC_1 = BC_1}$

$$AC_1 = \frac{\sigma_1 + \sigma_3}{2}\sqrt{\sin^2\varphi - \sin^2 i}$$

$$\sigma_v = OA$$

$$= OC_1 + AC_1$$

$$= \left(\frac{\sigma_1 + \sigma_3}{2}\right)\cos i + AC_1$$

$$\sigma_v = \frac{\sigma_1 + \sigma_3}{2}\cos i + \frac{\sigma_1 + \sigma_3}{2}\sqrt{\sin^2\varphi - \sin^2 i}$$

$$\sigma_h = p_a = OB = OC_1 - BC_1$$

$$= \left(\frac{\sigma_1 + \sigma_3}{2}\right) \times \cos i - \left(\frac{\sigma_1 + \sigma_3}{2}\right)\sqrt{\sin^2\varphi - \sin^2 i}$$

$$\frac{\sigma_h}{\sigma_v} = \frac{p_a}{\gamma z \cos i} = \frac{\frac{\sigma_1 + \sigma_3}{2}\left[\cos i - \sqrt{\sin^2\varphi - \sin^2 i}\right]}{\frac{\sigma_1 + \sigma_3}{2}\left[\cos i + \sqrt{\sin^2\varphi - \sin^2 i}\right]}$$

$$= \frac{\cos i - \sqrt{\sin^2\varphi - \sin^2 i}}{\cos i + \sqrt{\sin^2\varphi - \sin^2 i}}$$

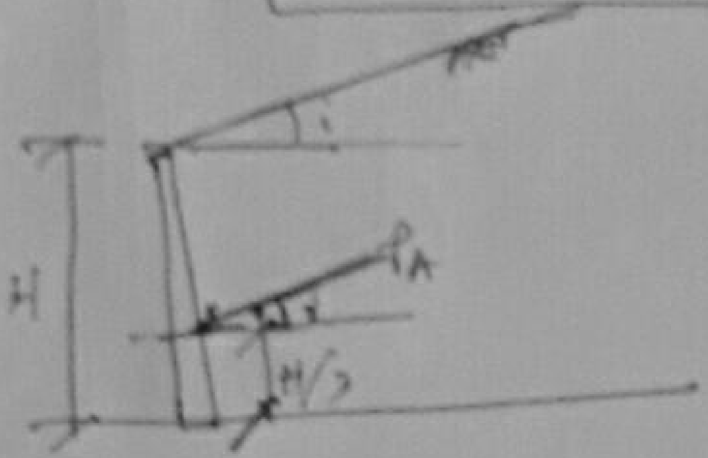
$$= \frac{\cos i - \sqrt{\cos^2 i - \cos^2\varphi}}{\cos i + \sqrt{\cos^2 i - \cos^2\varphi}}$$

$$p_a = \gamma z \cos i \times \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}$$

$$\Rightarrow \boxed{p_a = k_a \gamma z}$$

where,

$$\boxed{k_a = \cos i \times \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}}$$

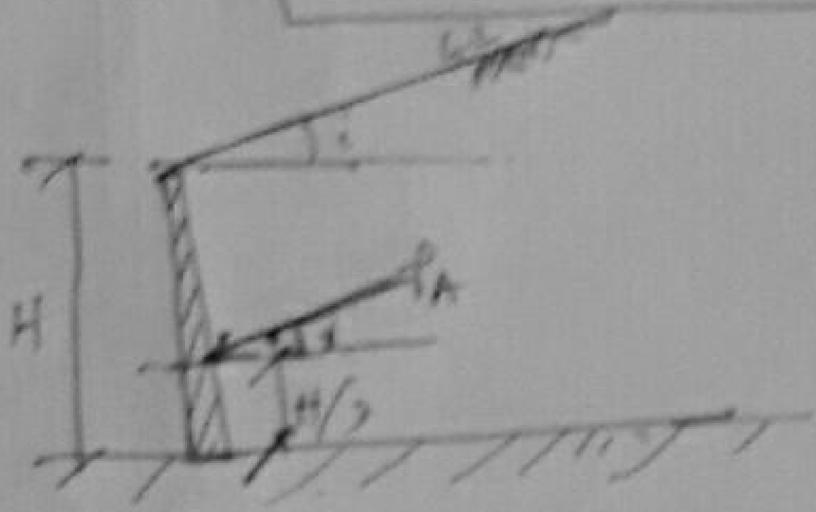


$$p_a = \gamma z \cos i \cdot \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}$$

$$\Rightarrow \boxed{p_a = k_a \gamma z}$$

where,

$$\boxed{k_a = \cos i \times \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}}$$



Similarly you can derive the coefficient of earth pressure (Passi[ve])
K_p for an inclined backfill as

$$K_p = \cos i \times \frac{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}$$

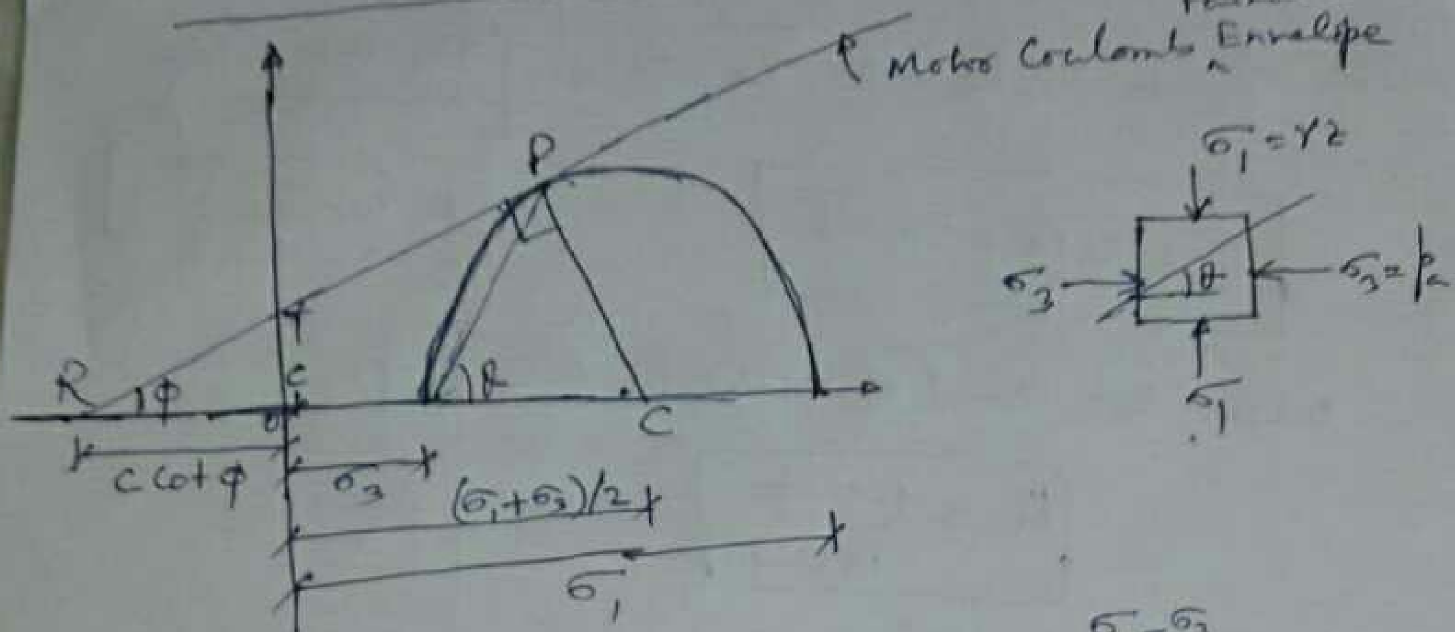
p_p = Passive earth pressure on the wall at a depth z from ground level $= K_p \gamma z$

[...]e $= P_p = \frac{1}{2} K_p \gamma H^2$

[...]ssure [...] wall

→ point of applica[tion] will be at a dist[ance] $\frac{H}{3}$ from the base [of] the wall.

(10) Rankine's active earth pressure — Cohesive soil (c-φ soil)



In RPC Δ, $\sin\varphi = \dfrac{PC}{RC} = \dfrac{\frac{\sigma_1 - \sigma_3}{2}}{\left(c\cot\varphi\right) + \left(\frac{\sigma_1+\sigma_3}{2}\right)}$

$\Rightarrow c\sin\varphi\cot\varphi + \dfrac{\sigma_1+\sigma_3}{2}\sin\varphi = \dfrac{\sigma_1-\sigma_3}{2}$

$\Rightarrow c\cos\varphi + \dfrac{\sigma_1+\sigma_3}{2}\sin\varphi = \dfrac{\sigma_1-\sigma_3}{2}$

$\Rightarrow 2c\cos\varphi + (\sigma_1+\sigma_3)\sin\varphi = \sigma_1 - \sigma_3$

$\Rightarrow 2c\cos\varphi + \sigma_1\sin\varphi + \sigma_3\sin\varphi = \sigma_1 - \sigma_3$

$\Rightarrow 2c\cos\varphi + \sigma_1\sin\varphi - \sigma_1 = -\sigma_3 - \sigma_3\sin\varphi$

$\Rightarrow \sigma_1(\sin\varphi - 1) = -\sigma_3(1+\sin\varphi) - 2c\cos\varphi$

$\Rightarrow \sigma_1(1-\sin\varphi) = \sigma_3(1+\sin\varphi) + 2c\cos\varphi$

$= \sigma_3(1+\sin\varphi) + 2c\sqrt{(1+\sin\varphi)(1-\sin\varphi)}$

$\Rightarrow \sigma_1 = \sigma_3\left(\dfrac{1+\sin\varphi}{1-\sin\varphi}\right) + 2c\sqrt{\dfrac{1+\sin\varphi}{1-\sin\varphi}}$ —— (1)

(11)

$$\sigma_3 = \sigma_1 \left(\frac{1-\sin\varphi}{1+\sin\varphi}\right) - 2c\sqrt{\frac{1-\sin\varphi}{1+\sin\varphi}} \quad \text{——} (2)$$

$$\Rightarrow p_a = \gamma z k_a - 2c\sqrt{k_a}$$

[For Active case we have $\sigma_3 = p_a$ & $\sigma_1 = \gamma z$ For horizontal Backfill]

$$\boxed{p_a = k_a \gamma z - 2c\sqrt{k_a}}$$

where, k_a = coeff. of active earth

$$\boxed{k_a = \frac{1-\sin\varphi}{1+\sin\varphi}}$$

For Passive Case

$\sigma_1 = p_p$ & $\sigma_3 = \gamma z$

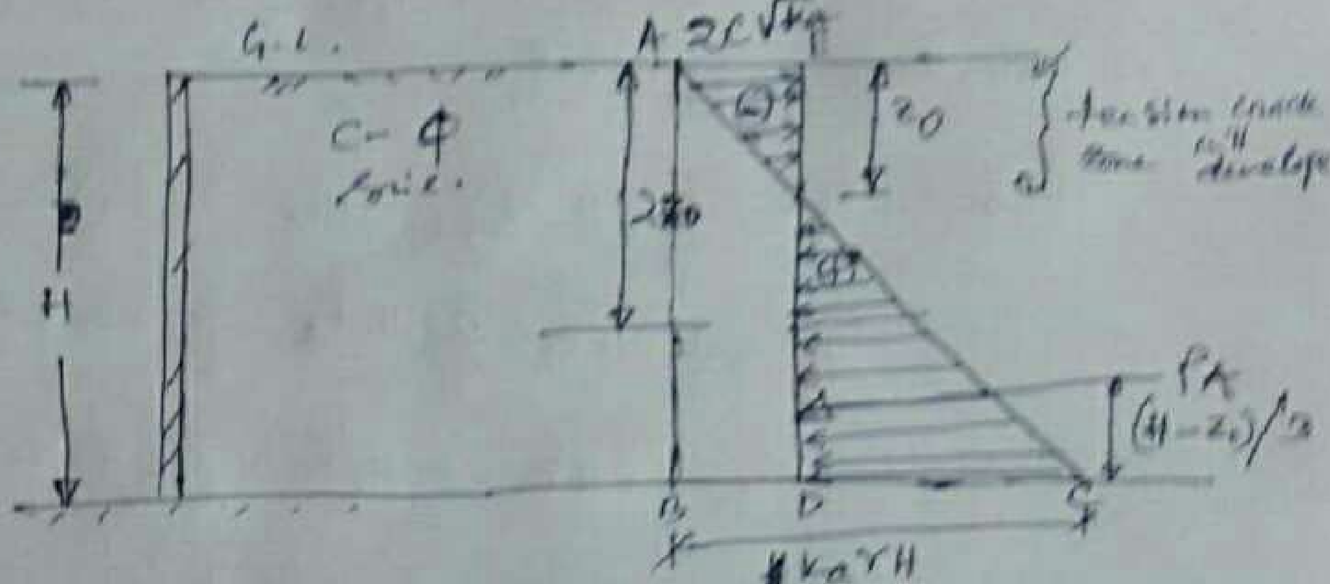
We have Eq (1) →

$$\sigma_1 = \sigma_3 \left(\frac{1+\sin\varphi}{1-\sin\varphi}\right) + 2c\sqrt{\frac{1+\sin\varphi}{1-\sin\varphi}}$$

$$\Rightarrow \boxed{p_p = \gamma z k_p + 2c\sqrt{k_p}}$$

where, $k_p = \frac{1+\sin\varphi}{1-\sin\varphi}$

(12)



at $z = 0$, $p_a = -2c\sqrt{K_a}$

But when, $p_a = 0$, $z = z_0$ $\Rightarrow \gamma z_0 K_a - 2c\sqrt{K_a} = 0$

$\Rightarrow z_0 = \frac{2c\sqrt{K_a}}{\gamma K_a}$

H_c = Critical depth of vertical cut

$$2z_0 = H_c = \frac{4c}{\gamma\sqrt{K_A}}$$

$$\Rightarrow z_0 = \frac{2c}{\gamma\sqrt{K_a}}$$

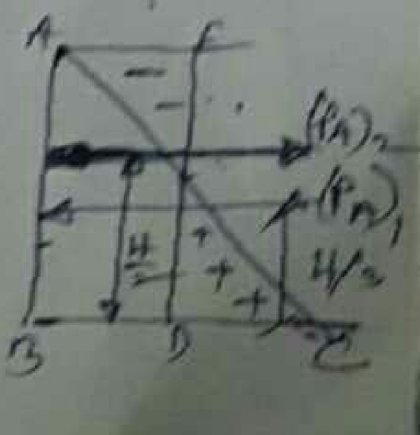
In cohesive soil a vertical cut can be made up to a depth $2z_0$ which is called critical depth of vertical cut.

Before, the tension crack occured, one has to consider both +ve & -ve earth pressure.

$$P_A = \frac{1}{2}(K_a \gamma H) \times H \ (\Delta ABC) - (2c\sqrt{K_a}) \times H \ (\square ABDE)$$

$$P_A = \frac{1}{2} K_a \gamma H^2 - 2cH\sqrt{K_A}$$

P_A is the resultant of $(P_A)_1$ & $(P_A)_2$.



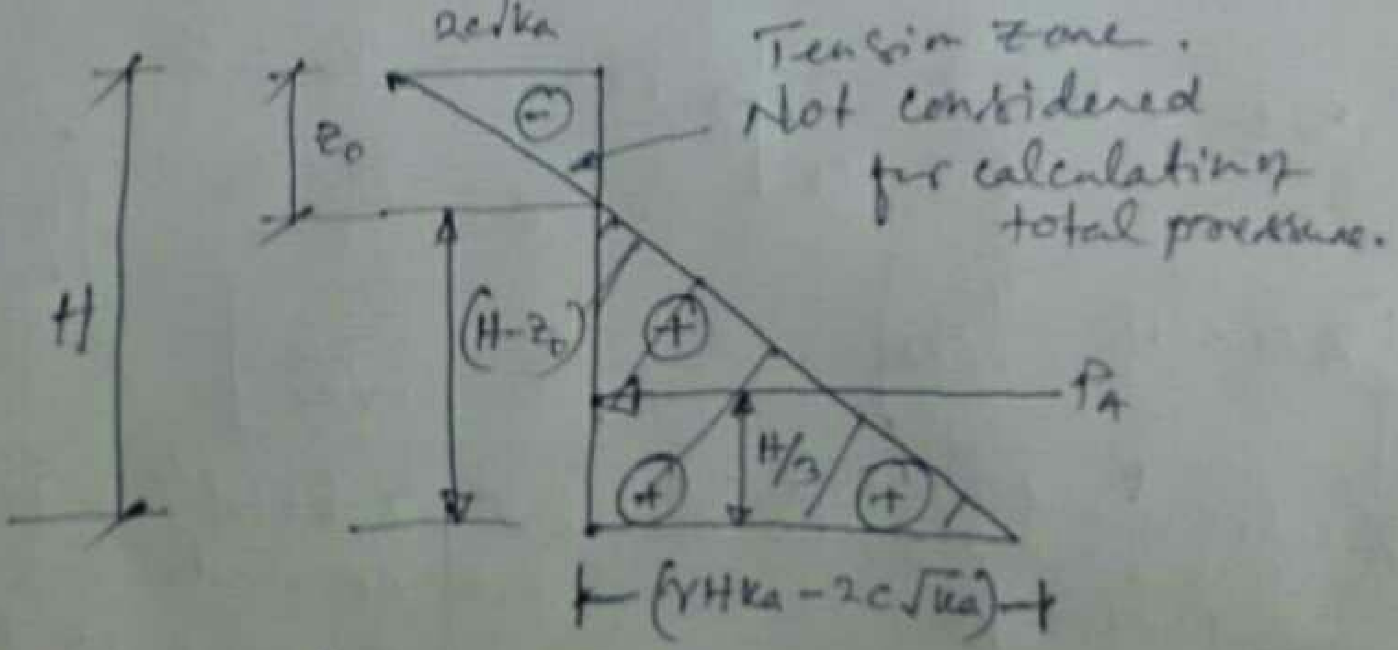
After Tensioning crack occurred, the tension zone is usually ignored.

Thus, $P_A = \frac{1}{2}(H - z_0)(\gamma H K_a - 2c\sqrt{K_a})$

$$P_A = \frac{1}{2}\gamma H^2 K_a - \frac{2cH\sqrt{K_a}}{2} - \frac{z_0}{2}\gamma H K_a + \frac{2cz_0\sqrt{K_a}}{2}$$

$$= \frac{1}{2}\gamma H^2 K_a - \frac{2c}{2}H\sqrt{K_a} - \frac{2c}{2\gamma\sqrt{K_a}}\gamma H K_a + \frac{2c}{2}\times\frac{2c\sqrt{K_a}}{\gamma\sqrt{K_a}}$$

$$\boxed{P_A = \frac{1}{2}\gamma H^2 K_a - 2cH\sqrt{K_a} + \frac{2c^2}{\gamma}}$$



(Only the shaded Area is considered)

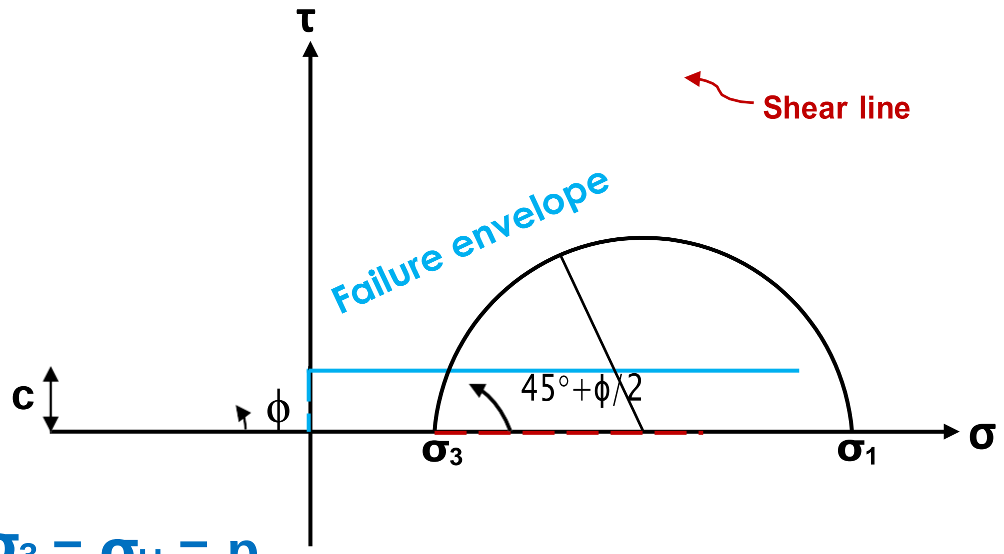
- In practice retaining wall is generally constructed then the soil backfilled .
- During the process of backfilling, a certain amount of wall-deformation away from backfill will have taken place.
- Since the minimum deformation required to produce the active case is quite small, a retaining wall is designed to resist only active thrust.

Soil	Amount of translation at top
Cohesionless (dense)	0.001H-0.002H
Cohesionless (loose)	0.002H-0.004H
Cohesive (stiff)	0.01-0.02H
Cohesive (soft)	0.02-0.05H

Ranjan and Rao,1991 *H= height of the wall

Rankine's active earth pressure – Cohesive backfill

For c-ϕ soil, the relation ship between the major principal stress (σ_1) and minor principal stress (σ_3) at plastic equilibrium can be expressed as:



$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

For the case of active earth pressure $\sigma_1 = \sigma_v = \gamma z$ and $\sigma_3 = \sigma_H = p_A$

$$p_A = \gamma z K_A - 2c\sqrt{K_A}$$

$$where, K_A = \frac{1-\sin\phi}{1+\sin\phi}$$

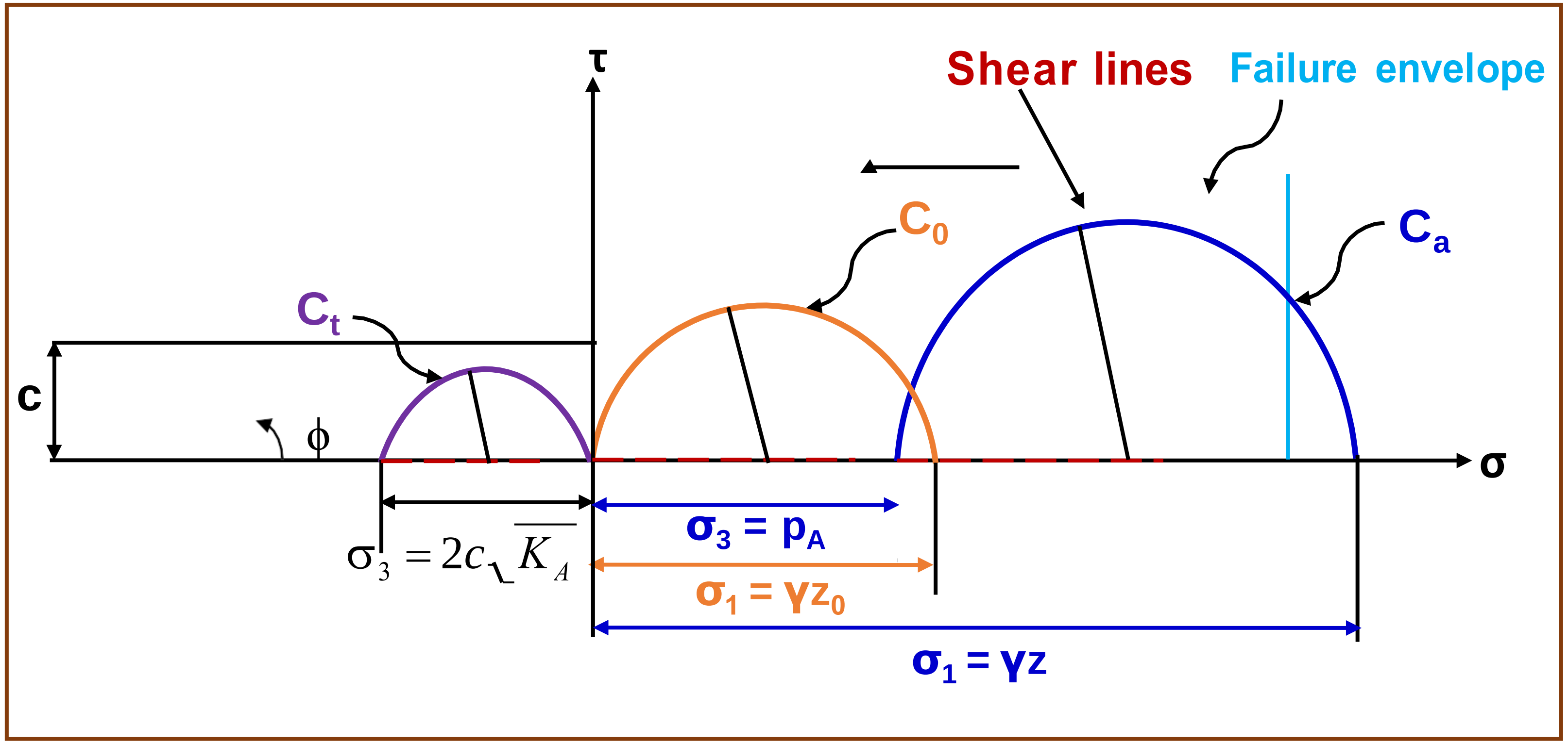
$$p_A = \gamma z K_A - 2c\sqrt{K_A}$$ **Mohr circle C_a**

The active pressure $p_A = 0$, when

$$z = z_0 = \frac{2c}{\gamma\sqrt{K_A}}$$ **Mohr circle C_0**

At depth z = 0

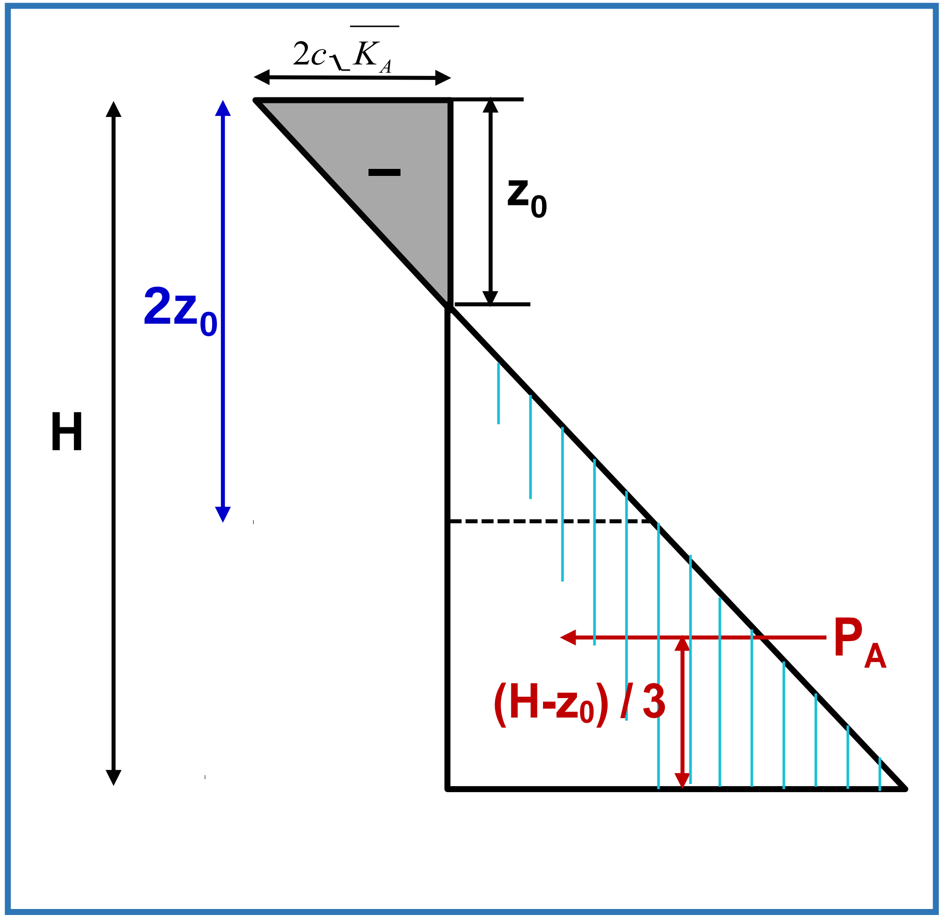
$$p_A = -2c\sqrt{K_A}$$ **Mohr circle C_t**



The soil is **in a state of tension** within the zone **between the ground surface and depth z_0.**

In calculating the total active thrust on the wall, **the tension zone is usually ignored.** Thus,

$$P_A = \frac{1}{2}(H - z_0)\left(\gamma H K_A - 2c\sqrt{K_A}\right) = \frac{1}{2}\gamma H^2 K_A - 2cH\sqrt{K_A} + \frac{2c^2}{\gamma}$$



The net total active thrust is zero for a depth equal to **$2z_0$.**

Thus, in cohesive soil a vertical can be made upto a depth of $2z_0$.

$$H_c = 2z_0 = \frac{4c}{\gamma\sqrt{K_A}}$$

H_c = critical depth of vertical cut

Effect of water table :

The lateral earth pressure due to partial submergence is due to **soil and water**

The total pressure due to soil (area of ocebo):

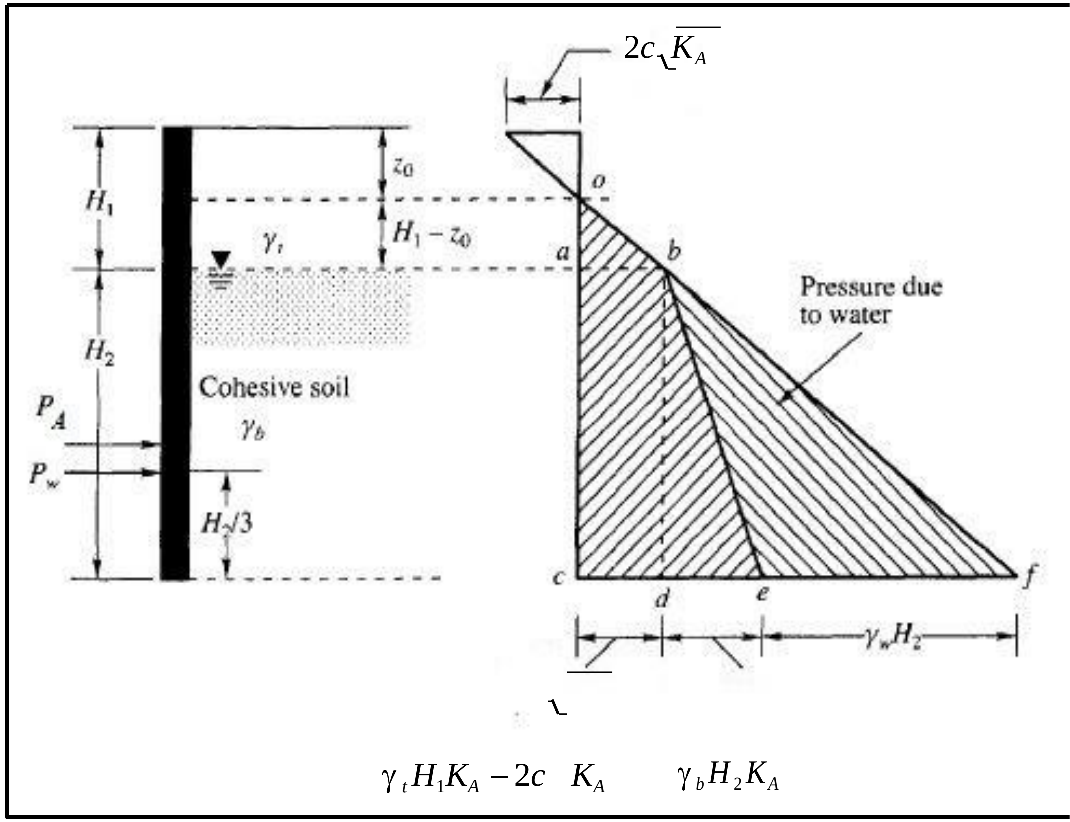
$$P_A = oab + acdb + bde$$

$$P_A = \frac{1}{2}(H_1 - z_0)\left(\gamma_t H_1 K_A - 2c\sqrt{K_A}\right) + \left(\gamma_t H_1 K_A - 2c\sqrt{K_A}\right)H_2 + \frac{1}{2}\gamma_b H_2^2 K_A$$

$$z_0 = \frac{2c}{\gamma_t \sqrt{K_A}}$$

The total pressure due to water (area of bfe):

$$P_w = \frac{1}{2}\gamma H^2$$



Murthy 2001

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w 2

Rankine's passive earth pressure – Cohesive backfill

$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

For the case of active earth pressure $\sigma_1 = \sigma_H = p_p$ and $\sigma_3 = \sigma_v = \gamma z$

$$p_P = \gamma z K_P + 2c\sqrt{K_P}$$

$$where, K_P = \frac{1+\sin\phi}{1-\sin\phi}$$

At depth z = 0, $p_P = 2c\sqrt{K_P}$

At depth z = H, $p_P = \gamma H K_P + 2c\sqrt{K_P}$

The total pressure $P_P = P'_P + P''_P$

$$P'_P = \int_0^H \gamma z K_p dz = \frac{1}{2}\gamma H^2 K_P$$ acts at a height H/3 from base

$$P''_p = \int_0^H 2c\sqrt{K_P}\, dz = 2cH\sqrt{K_P}$$ acts at a height H/2 from base

$$P_P = P'_P + P''_P = \frac{1}{2}\gamma H^2 K_P + 2cH\sqrt{K_P}$$