SOIL WATER AND WATER FLOW

6.0 ILLUSTRATIVE EXAMPLES

Example 6.1 Compute the maximum capillary tension for a tube 0.05 mm in diameter.

Solution:

The maximum capillary height at 4° C is given by

$$(hc)_{max} = \frac{0.3084}{d} = \frac{0.3084}{0.005} = 61.7cm = 0.617m$$

$\therefore$ Capillary tension = $(hc)_{max}\ \gamma_\omega = 0.617 \times 9.81$
$= 6.05 \times KN/m^3$

Example 6.2 Compute the height of capillary rise in a soil whose D_{10} is 0.1 mm and voids ratio is 0.60.

Solution:

Let the average size of the void be d mm.

Volume of each sphere of solids maybe assumed proportional to D_{10}^3. Since the voids ratio is 0.6, the volume of rods space, corresponding to the unit of volume of solids, will be proportional to 0.60 D_{10}^3 . But volume of each void space is also proportional to d^3.

Hence $d^3 = 0.60\ D_{10}^3$

$$d = (0.60)^{1/3} D_{10}$$

$= 0.845 \times D_{10}$

$= 0.845 \times 0.1$

d = 0.0845 mm = 0.00845 cm

$$hc = \frac{0.3084}{d} cm$$ at 4° C.

$$hc = \frac{0.3084}{0.00845} = 36.5cm$$

Example 6.3 When water at 20° is added to a fine sand and to a silt, a difference in capillary rise of 25 cm is observed between the two soils. If the capillary rise in fine sand is 25 cm, calculate the difference in the size of the voids of the two soils.

Solution:

Using suffix 1 for sand 2 for silt,

$hc_1 = 25$ cm

$hc_2 = 25+25 = 50$ cm

$$d_1 = \frac{0.2975}{hc_1} = \frac{0.2975}{25} = 0.0119\ cm$$

$$d_2 = \frac{0.2975}{hc_2} = \frac{0.2975}{50} = 0.00595\ cm$$

∴ Difference in the size of the voids

$d_1 - d_2 = 0.0119 - 0.00595$

$= 0.00595$ cm

Example 6.4 Estimate the coefficient of permeability for a uniform sand where a sieve analysis indicates that the D_{10} size is 0.12 mm

Solution:

$$D_{10} = 0.12 \text{ mm} = 0.012 \text{ cm}.$$

According to Allen Hazen's relationship,

$$k = 100\, D_{10}^2$$

where k is permeability in cm/s and D_{10} is effective size in cm.

∴ $k = 100 \times (0.012)^2 = 100 \times 0.000144 = 0.0144$ cm/s

∴ Permeability coefficient = **1.44×10^{-1} mm/s.**

Example 6.5 The capillary rise in soil A with D_{10} = 0.06 mm is 60 cm. Estimate the Capillary rise in soil B with D_{10} = 0.1 mm, assuming the same voids ratio in both the soils.

Solution:

Let the size of voids be d.

Now $V_S \propto D_{10}^{\ 3}$

$$V_v = eV_s$$

$$e = \left(\frac{d}{D_{10}}\right)^3 \qquad \text{(i)}$$

For soil A,

$$d = \frac{0.3084}{h_c} = \frac{0.3084}{60} = 5.14 \times 10^{-3} cm$$

$= 5.14 \times 10^{-2}$ mm

Substitute it in (i), we get,

$$e = \left(\frac{d}{D_{10}}\right) = \left(\frac{5.14 \times 10^{-2}}{0.06}\right)^3 = 0.629$$

Now for soil B $d = (e)^{1/3} D_{10}$

$$= (0.629)^{1/3} \times 0.1$$

$$= 0.857 \times 0.1 = 0.0857\, mm$$

$= 0.00857$ cm

Example 6.6 What is the height of capillary rise in a soil with an effective size of 0.06 mm and void ratio of 0.63?

Solution:

Effective size = 0.06 mm

Solid volume $\propto (0.06)^3$

$\therefore$ Void volume per unit of solid volume $\propto 0.63(0.06)^3$

Average void size, $d_c = (0.63)^{1/3} \times 0.06$ mm $= 0.857 \times 0.06 = 0.0514$ mm

Capillary rise, $h_c = \frac{4T_s}{\gamma_w d_c}$

$$= \frac{4 \times 73 \times 10^{-6}}{9.81 \times 10^{-6} \times 0.0514} \text{ mm}$$

$=$ **0.58 m.**

Example 6.7 The effective sizes of two soils are 0.05 mm and 0.10 mm, the void ratio being the same for both. If the capillary rise in the first soil is 72 cm, what would be the capillary rise in the second soil?

Solution:

Effective size of first soil = 0.05 mm

$\therefore$ Solid volume $\propto (0.05)^3$

$\therefore$ Void volume $\propto e(0.05)^3$

Average pore size, $d_c = e^{1/3} \times 0.05$ mm

Capillary rise of $h_c = \frac{4T_s}{\gamma_w d_c}$

$\therefore$ $h_c \propto 1/d_c$

Since the void ratio is the same for the soils, average pore size for the second soil $= e^{1/3} \times 0.10$ mm.

Substituting, $h_c = \frac{4T_s}{\gamma_w d_c} =$ **36 cm,**

since d_c for the second soil is double that of the first soil and since $h_c \propto \frac{1}{d_c}$.

Example 6.8 The water table in a certain area is at a depth of 4m below the ground surface. To a depth of 12m, the soil consists of every fine sand having an average voids ratio of 0.7. Above the water table the sand has an average degree of saturation of 50%. Calculate the effective pressure on a horizontal plane at a depth 10 meters hollow the ground surface. What will be the increase in the effective pressure if the soil gets saturated by capillarity up to a height of 1m above the water table? Assume G = 2.65

Solution:

Height of sand layer above water table = Z_1 = 4 m

Height of saturated layer above water table = 12 - 4 = 8 m

Depth of point X, where pressure is to be computed = 10 m

Height of saturated layer above X = Z_2 = 10 - 4 = 6 m

Now

$$\gamma d = \frac{G\gamma\omega}{1+e} = \frac{2.65 \times 9.81}{1+0.7} = 15.29 KN/m^3$$

i. For sand above water table:-

$$e = \frac{\omega G}{Sr}$$

$$\omega = \frac{eS_r}{G} = \frac{0.7 \times 0.5}{2.65} = 0.132$$

$$\gamma_1 = \gamma d(1+\omega) = 15.29 \times 1.132 = 17.31 KN/m^3$$

ii. For saturated sand below water table

$$\omega_{sat} = \frac{e}{G} = \frac{0.7}{2.65} = 0.264$$

$$\gamma_2 = \gamma d(1+\omega_{sat})$$

$$15.29(1+0.264)$$

$$\gamma_2 = 19.33 KN/m^3$$

$$\gamma_2^1 = 19.33 - 9.81 = 9.52\ KN/m^3$$

Effective pressure at X

$$\sigma = Z_1\gamma_1 + Z_2\gamma_2$$

$$\sigma = 4\times 17.31 + 6\times 19.33$$

$$= 185.22 \text{ KN/m}^2$$

$$u = h_\omega \gamma_\omega = 6\times 9.81 = 58.86 KN/m^2$$

$$\sigma^1 = \sigma - u = 185.22 - 58.86 = 126.36 KN/m^2$$

Effective stress at x after capillary rise

$$\sigma^1 = 3\gamma_1 + (6+1)\gamma_2^{\,1} + h_c\gamma_\omega$$

$$= (3\times 17.31) + (7\times 9.52) + (1\times 9.81)$$

$$= 128.38 \text{ KN/m}^2$$

Increase in pressure

$$= 128.38 - 126.36 = 2.02 \text{ KN/m}^2$$

Result:

i. Effective pressure at a depth of 10m = 128.38 KN/m^2

ii. Increase in pressure = 2.02 KN/m^2

Example 6.9 A 10m thick bed of sand is underlain by a layer of clay of 6 m thickness. The water table which was originally at the ground surface is lowered by drainage to a depth of 4m, where upon the degree of saturation above the lowered water table reduces to 20%. Determine the increase in the magnitude of the vertical effective pressure at the middle of the day layer due to lowering of water table, the saturated unit weights of sand and clay are respectively 20.6 KN/m^3 and 17.6 KN/m^3 and the dry unit weight of sand is 16.7 KN/m^3.

Solution:

i) Before lowering the water table, the pressures at the middle of the clay layer are

$$\sigma = (10\times 20.6) + (3\times 17.6)$$

$$= 258.8 \text{ KN/m}^2$$

$$u = 13 \times 9.81 = 127.53 \text{ KN/m}^2$$

$$\sigma^1 = \sigma - u$$

$$= 258.8 - 127.53 = 131.27 \text{ KN/m}^2$$

ii) After lowering the water table, the unit weight of sand is given by

$$\gamma = \gamma_d + S_r(\gamma_{sat} - \gamma_d)$$
$= 16.7+0.2\ (20.6-16.7)$
$= 17.48\ KN/m^3$

$$\sigma = (4 \times 17.48) + (6 \times 20.6) + (3 \times 17.6)$$

$= 246.32\ KN/m^2$

$u = 9 \times 9.81 = 88.29\ KN/m^2$

$\sigma^1 = 246.32 - 88.29 = 158.03\ KN/m^2$

∴ Increase in effective pressure

$= 158.03 - 131.27$
$= 26.76\ KN/m^2$

Example 6.10 The water table in a deposit of sand 8 m thick is at a depth of 3m below the surface. Above the water table, the sand is saturated with capillary water. The bulk density of sand is 19.62 KN/m^3. Calculate the effective pressure of 1m, 3m and 8m below the surface. Hence plot the variation of total pressure, neutral pressure and effective pressure over the depth of 8 m.

Solution:

a. Stresses at D, & 8 m below ground:

If we insert a piezometric tube at D, water will rise through a height $h\omega = 5m$ in it.

$\sigma = (3+5)\ \gamma_{sat}$
$= 8 \times 19.62$
$\sigma = 156.96\ KN/m^2$

$u = h_\omega \gamma_\omega$

$= 5 \times 9.81 = 49.05\ KN/m^2$

$$\sigma^1 = \sigma - u = 156.96 - 49.05 = 107.91 KN/m^2$$

b. Stresses at C, 3m below ground level:

$\sigma = 3\gamma_{sat} = 3 \times 19.62 = 58.86 KN/m^2$
$u = 0$
$\sigma^1 = \sigma - u$

$\sigma^1 = 58.86\ KN/m^2$

c Stress at A, at ground level

$\sigma = 0$

$u = -h_c \gamma_\omega = -3(9.81) = -29.43\ KNm^2$

$\sigma^1 = \sigma - u = 29.43\ KN/\ m^2$

d. Stresses at B, 1m below ground level

$\sigma = 1\gamma_{sat} \quad = 1 \times 19.62 = 19.62$ KN/m²

$u = -2\gamma_{\omega} \quad = -2 \times 9.81 = -19.62$ KN/m²

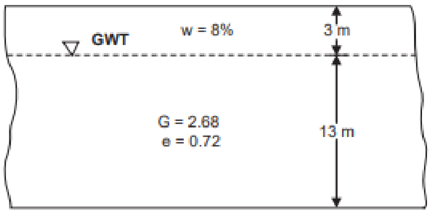
(i.e.) Pressure due to weight of water hanging below that level

$$\sigma^1 = (\sigma - u) = 19.62 - (-19.62)$$
$$= 19.62 + 19.62$$
$$\sigma^1 = 39.34 \text{ KN/m}^2$$

The total stress, effective stress and pore pressure distribution are shown in fig.

Example 6.11 Determine the neutral and effective stress at a depth of 16 m below the ground level for the following conditions: Water table is 3 m below ground level; G = 2.68; e = 0.72; average water content of the soil above water table is 8%.

Solution:



Soil profile

$G = 2.68$

$e = 0.72$

$w = 8\%$ for soil above water table.

$$\gamma = \frac{G(1+w)}{(1+e)} \cdot \gamma_w$$
$$= 2.68 \times \frac{1.08}{1.72} \times 9.81 \text{ kN/m}^3$$
$$= 16.51 \text{ kN/m}^3.$$

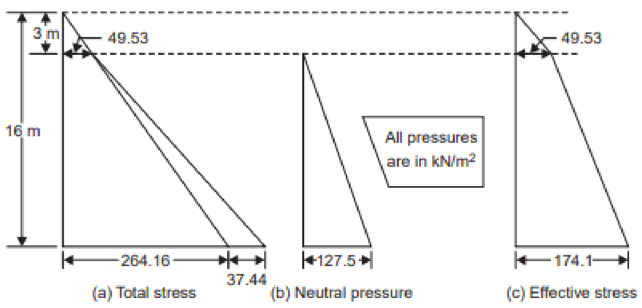
$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right) \cdot \gamma_w$$
$$= \frac{(2.68 + 0.72)}{1.72} \times 9.81 \text{ kN/m}^3 = 19.39 \text{ kN/m}^3$$

Total pressure at a depth of 16 m : $\sigma = (3 \times 16.51 + 13 \times 19.39) = 301.6$ kN/m².

Neutral pressure at this depth : $u = 13 \times 9.81 =$ **127.5 kN/m²**

∴ Effective stress at 16 m below the ground level :

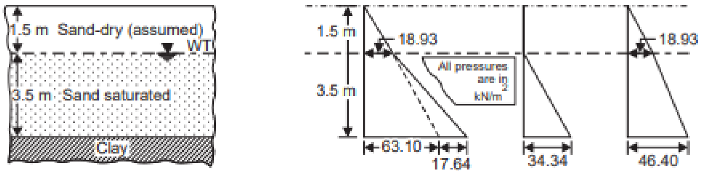
$$\bar{\sigma} = (\sigma - u) = (301.6 - 127.5)$$
$$= \mathbf{174.1 \text{ kN/m}^2}.$$



Example 6.12 A saturated sand layer over a clay stratum is 5 m in depth. The water is 1.5 m below ground level. If the bulk density of saturated sand is 17.66 kN/m³, calculate the effective

and neutral pressure on the top of the clay layer.

Solution:



(a) Total pressure (b) Neutral pressure (c) Effective pressure

Let us, in the absence of data, assume that the sand above the water table is dry. Bulk density of saturated sand,

$$\gamma_{sat} = 17.66 \text{ kN/m}^3.$$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$$

Let us assume: $G = 2.65$

or $$17.66 = \frac{(2.65+e)}{(1+e)}(9.81)$$

whence $e = 1.06$

$$\gamma_d = \frac{G\gamma_w}{(1+e)} = \frac{2.65}{(1+1.06)} \times 9.81 \text{ kN/m}^3 = 12.62 \text{ kN/m}^3$$

Total stress at the top of clay layer:

$$\sigma = 1.5 \times 12.62 + 3.5 \times 17.66 = 80.74 \text{ kN/m}^2$$

Neutral stress at the top of clay layer:

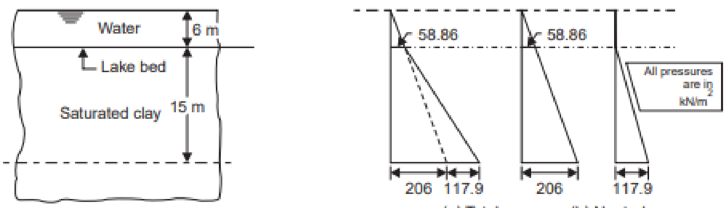
$$u = 3.5 \times 9.81 = \mathbf{34.34 \text{ kN/m}^2}$$

Effective stress at the top of clay layer:

$$\bar{\sigma} = (\sigma - u) = 80.74 - 34.34 = \mathbf{46.40 \text{ kN/m}^2.}$$

Example 6.13 Compute the total, effective and pore pressure at a depth of 15 m below the bottom of a lake 6 m deep. The bottom of the lake consists of soft clay with a thickness of more than 15 m. The average water content of the clay is 40% and the specific gravity of soils may be assumed to be 2.65?

Solution:



(a) Total pressure (b) Neutral pressure (c) Effective pressure

Water content $w_{sat} = 40\%$

Specific gravity of solids, $G = 2.65$

Void ratio, $e = w_{sat} \cdot G$

$= 0.4 \times 2.65$

$= 1.06$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$$

$$= \frac{(2.65+1.06)}{(1+1.06)} \times 9.81 \text{ kN/m}^3$$

$$= 17.67 \text{ kN/m}^3$$

Total stress at 15 m below the bottom of the lake:

$$\sigma = 6 \times 9.81 + 15 \times 17.67 = 323.9 \text{ kN/m}^2$$

Neutral stress at 15 m below the bottom of the lake:

$$u = 21 \times 9.81 \text{ kN/m}^3 = \mathbf{206.0 \text{ kN/m}^2}$$

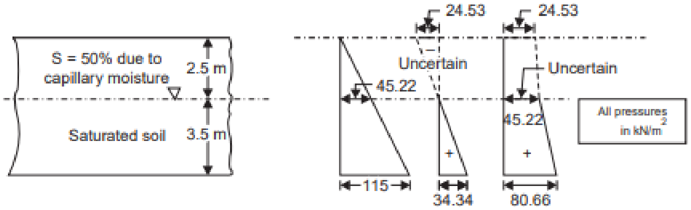
Effective stress at 15 m below the bottom of the lake:

$$\sigma' = 323.9 - 206.0 = \mathbf{117.9 \text{ kN/m}^2}$$

Also,
$$\bar{\sigma} = 15 \times \gamma' = 15 \times (\gamma_{sat} - \gamma_w)$$
$$= 15(17.67 - 9.81)$$
$$= 117.9 \text{ kN/m}^2$$

Example 6.14 A uniform soil deposit has a void ratio 0.6 and specific gravity of 2.65. The natural ground water is at 2.5 m below natural ground level. Due to capillary moisture, the average degree of saturation above ground water table is 50%. Determine the neutral pressure, total pressure and effective pressure at a depth of 6 m. Draw a neat sketch.

Solution:



(a) Total pressure (b) Neutral pressure (c) Effective pressure

Void ratio, $e = 0.6$

Specific gravity $G = 2.65$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\cdot\gamma_w = \frac{(2.65+0.60)}{(1+0.60)} \times 9.81 \text{ kN/m}^3$$
$$= 19.93 \text{ kN/m}^3$$

γ at 50% saturation

$$= \left(\frac{G+Se}{1+e}\right)\cdot\gamma_w = \frac{(2.65\times 0.5+0.60)}{(1+0.60)} \times 9.81 \text{ kN/m}^3 = 18.09 \text{ kN/m}^3.$$

Total pressure, σ at 6 m depth $= 2.5 \times 18.09 + 3.5 \times 19.93$

$= 115 \text{ kN/m}^2$

Neutral pressure, u at 6 m depth $= 3.5 \times 9.81 = \mathbf{34.34 \text{ kN/m}^2}$

Effective pressure, $\bar{\sigma}$ at 6 m depth $= (\sigma - u)$

$= 115.00 - 34.34 = \mathbf{80.66 \text{ kN/m}^2}$

PERMEABILITY

7.0 ILLUSTRATIVE EXAMPLES

Example 7.1 Determine the coefficient of permeability from the following data:

Length of sand sample = 25 cm

Area of cross section of the sample = 30 cm^2

Head of water = 40 cm

Discharge = 200 ml in 110 s.

Solution:

$L = 25$ cm

$A = 30\ cm^2$

$h = 40$ cm (assumed constant)

$Q = 200$ ml. $t = 110$ s

$q = Q/t = 200/110$ ml/s $= 20/11 = 1.82\ cm^3/s$

$i = h/L = 40/25 = 8/5 = 1.60$

$q = k \cdot i \cdot A$

$$k = q/iA = \frac{20}{11 \times 1.6 \times 30} \text{ cm/s}$$

$= 0.03788$ cm/s

$= \mathbf{3.788 \times 10^{-1}}$ **mm/s.**

Example 7.2 The discharge of water collected from a constant head permeameter in a period of 15 minutes is 500 ml. The internal diameter of the permeameter is 5 cm and the measured difference in head between two gauging points 15 cm vertically apart is 40 cm. Calculate the coefficient of permeability.

If the dry weight of the 15 cm long sample is 4.86 N and the specific gravity of the solids is 2.65, calculate the seepage velocity.

Solution:

$Q = 500$ ml ; $t = 15 \times 60 = 900$ s.

$A = (\pi/4) \times 5^2 = 6.25\pi\ cm^2$; $L = 15$ cm ; $h = 40$ cm;

$$k = \frac{QL}{Ath} = \frac{500 \times 15}{6.25 \times \pi \times 900 \times 40} \text{ cm/s} = \mathbf{0.106\ mm/s}$$

Superficial velocity $$v = Q/At = \frac{500}{900 \times 6.25\pi} \text{ cm/s}$$

$= 0.0283$ cm/s

$= 0.283$ mm/s

Dry weight of sample $= 4.86$ N

Volume of sample $= A \cdot L = 6.25 \times \pi \times 15\ cm^3 = 294.52\ cm^3$

Dry density, $$\gamma_d = \frac{4.86}{294.52} \text{ N/cm}^3 = 16.5 \text{ kN/m}^3$$

$$\gamma_d = \frac{G\gamma_w}{(1+e)}$$

$$(1+e) = \frac{2.65 \times 10}{16.5} = 1.606, \quad \text{since} \quad \gamma_w = 10 \text{ kN/m}^3$$

$e = 0.606$

$$n = \frac{e}{(1+e)} = 0.3773 = 37.73\%$$

$$\therefore \quad \text{Seepage velocity, } v_s = v/n = \frac{0.283}{0.3773} = \mathbf{0.750\ mm/s.}$$

Example 7.3 A glass cylinder 5 cm internal diameter and with a screen at the bottom was used as a falling head permeameter. The thickness of the sample was 10 cm. With the water level in the tube at the start of the test as 50 cm above the tail water, it dropped by 10 cm in one minute, the tail water level remaining unchanged. Calculate the value of k for the sample of the soil. Comment on the nature of the soil.

Solution:

Falling head permeability test:

$$h_1 = 50 \text{ cm}; \qquad h_2 = 40 \text{ cm}$$
$$t_1 = 0; \qquad t_2 = 60\text{s} \ldots t = t_2 - t_1 = 60\text{s}$$
$$A = (\pi/4) \times 5^2 = 6.25\pi \text{ cm}^2 ; L = 10 \text{ cm}$$

Since a is not given, let us assume $a = A$.

$$k = 2.303 \frac{aL}{At} . \log_{10} (h_1/h_2)$$
$$= 2.303 \times (10/60) \log_{10} (50/40) \text{ cm/s}$$
$$= 0.0372 \text{ cm/s}$$
$$= \mathbf{3.72 \times 10^{-1} \text{ mm/s}}$$

The soil may be coarse sand or fine graved.

Example 7.4 In a falling head permeability test, head causing flow was initially 50 cm and it drops 2 cm in 5 minutes. How much time required for the head to fall to 25 cm?

Solution:

Falling head permeability test:

We know: $$k = 2.303 \frac{aL}{At} . \log_{10} (h_1/h_2)$$

Designating $2.303 \frac{aL}{At}$ as a constant C

$$k = C . \frac{1}{t} . \log_{10} (h_1/h_2)$$

When $h_1 = 50$; $h_2 = 48$, $t = 300$ s

$$\therefore \quad \frac{k}{C} = \frac{1}{300} \log_{10}(50/48)$$

When $h_1 = 50$; $h_2 = 25$; substituting:

$$\frac{1}{300} \log_{10} (50/48) = (1/t) \log_{10} (50/25)$$

$$\therefore \quad t = 300 \frac{\log_{10} 2}{\log_{10} (25/24)} = 5093.55\text{s} = \mathbf{84.9 \text{ min.}}$$

Example 7.5 A sample in a variable head permeameter is 8 cm in diameter and 10 cm high. The permeability of the sample is estimated to be 10×10^{-4} cm/s. If it is desired that the head in the stand pipe should fall from 24 cm to 12 cm in 3 min., determine the size of the standpipe which should be used.

Solution:

Variable head permeameter:
Soil sample diameter = 8 cm
height (length) = 10 cm
Permeability (approx.) = 10×10^{-4} cm/s

$$h_1 = 24 \text{ cm}, h_2 = 12 \text{ cm}, t = 180 \text{ s}$$

Substituting in the equation

$$k = 2.303 \frac{aL}{At} \log_{10} (h_1/h_2),$$

$$10^{-3} = \frac{2.303 \times a \times 10}{\pi \times 16 \times 180} \log_{10}(24/12)$$

$$\therefore \quad a = \frac{\pi \times 16 \times 180}{2.303 \times 10^4 (\log_{10} 2)} \text{cm}^2 = 1.305 \text{ cm}^2$$

If the diameter of the standpipe is d cm

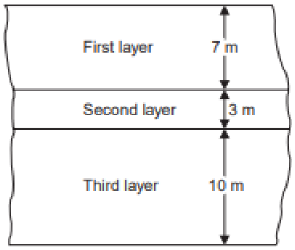
$$a = (\pi/4)\, d^2$$

$$\therefore \quad d = \sqrt{\frac{4 \times 1.305}{\pi}} \text{ cm} = 1.29 \text{ cm}$$

$\therefore$ The standpipe should be **13 mm** in diameter.

Example 7.6 A horizontal stratified soil deposit consists of three layers each uniform in itself. The permeabilities of these layers are 8×10^{-4} cm/s, 52×10^{-4} cm/s, and 6×10^{-4} cm/s, and their thicknesses are 7, 3 and 10 m respectively. Find the effective average permeability of the deposit in the horizontal and vertical directions.

Solution:



Soil profile

$$k_1 = 8 \times 10^{-4} \text{ cm/s} \quad h_1 = 7 \text{ m}$$
$$k_2 = 52 \times 10^{-4} \text{ cm/s} \quad h_2 = 3 \text{ m}$$
$$k_3 = 6 \times 10^{-4} \text{ cm/s} \quad h_3 = 10 \text{ m}$$

$$k_h \text{ (or } k_x) = \frac{(k_1 h_1 + k_2 h_2 + k_3 h_3)}{(h_1 + h_2 + h_3)}$$

$$= \frac{(8 \times 7 + 52 \times 3 + 6 \times 10)}{20} \times 10^{-4}$$

$$= 13.6 \times 10^{-4} \text{ cm/s}$$

∴ Effective average permeability in the horizontal direction

$$\mathbf{= 13.6 \times 10^{-3} \text{ mm/s}}$$

$$k_v \text{(or } k_z) = \frac{h}{\left(\frac{h_1}{k_1} + \frac{h_2}{k_2} + \frac{h_3}{k_3}\right)}$$

$$= \frac{20}{\frac{1}{10^{-4}}[7/8 + 3/52 + 10/6]}$$

$$= 7.7 \times 10^{-4} \text{ cm/s}$$

∴ Effective average permeability in the vertical direction

$$\mathbf{= 7.7 \times 10^{-3} \text{ mm/s.}}$$

Example 7.7 Calculate the co-efficient of permeability of a soil sample, 6 cm in height and 50 cm2 in cross-sectional area, if a quantity of water equal to 430 ml passed down in 10 min. Under an effective constant head of 40 cm.

On oven-drying, the test specimen has mass of 498 g. Taking the specific gravity of soil solids as 2.65, calculate the seepage velocity of water during the test.

Solution:

Q = 430 ml ; t = 10 x 60 = 600 seconds
A = 50 cm^2 : L = 6 cm ; h = 40 cm

From the equation for constant head permeability test

$$K = \frac{430}{600} \times \frac{6}{40} \times \frac{1}{50}$$

K = 2.15 x 10^{-3} cm/sec
= 2.15 x 10^{-3} x 864 = 1.86 m/day
(Since 1cm/sec=864 m/day)

Now

$$V = \frac{q}{A} = \frac{430}{600 \times 50} = 1.435 \times 10^{-2} cm/\sec$$

Now

$$\rho_d = \frac{498}{50 \times 6} = 1.66 g/cm^3$$

$$e = \frac{G\rho_\omega}{\rho_d} - 1 = \frac{2.65 \times 1}{1.66} - 1 = 0.595$$

$$n = \frac{\rho}{1+\rho} = \frac{0.595}{1.595} = 0.373$$

$$v_s = \frac{v}{n} = \frac{1.435 \times 10^{-2}}{0.373} = 3.85 \times 10^{-2} cm/\sec.$$

Example 7.8 In a falling head permeameter test, the initial head (t = 0) is 40 cm. The head drops by 5 cm in 10 minutes. Calculate the time required to run the test for the final head to be at 20cm. If the sample is 6 cm is height and 50 cm^2 in cross-sectional area, calculate the coefficient of permeability, taking area of stand pipe = 0.5 cm^2.

Solution:

In a time interval t = 10 minutes, the head drops from initial value of h_1 = 40 to h_2 = 40 – 5 = 35 cm

From the equation for falling head permeameter

$$K = 2.3 \frac{aL}{At} \log_{10} \frac{h_1}{h_2}$$

$$t = \frac{2.3aL}{AK}\log_{10}\frac{h_1}{h_2} = m\log_{10}\frac{h_1}{h_2}$$

Where $m = \frac{2.3aL}{AK} =$ Constant for the set up

$$10 = m \log_{10}\frac{40}{35}$$

$$m = \frac{10}{\log_{10}\frac{40}{35}} = \frac{10}{0.058} = 172 units$$

$$t = m\log_{10}\frac{h_1}{h_2}$$

$$= 172.5\log_{10}\frac{h_1}{h_2}$$

Now, let the time interval required for the head to drop from initial value of $h_1$40cm to a final value of h_2 = 20cm, be t minutes.

$$t = 172.5 \ \log_{10}\frac{40}{20} = 172.5\times 0.301 = 5.19 \min utes$$

Again $m = \frac{2.3aL}{AK} = 172.5\ Units$

$$K = \frac{2.3aL}{4\times 172.5} Cm/\min$$

(Since t used to compute m was in minutes)

$$K = \frac{2.3\times 0.5\times 6}{50\times 172.5\times 6} Cm/\sec.$$

$$= 1.33\times 10^{-5} Cm/\sec.$$

Example 7.9 A cohesionless soil has a permeability of 0.036 cm per second at a void ratio of 0.36. Make predictions of the permeability of this soil when at a void ratio of 0.45 according to the two functions of void ratio that are proposed.

Solution:

$$k_1 : k_2 = \frac{e_1^3}{(1+e_1)} : \frac{e_2^3}{(1+e_2)}$$

$$0.036 : k_2 = \frac{(0.36)^3}{1.36} : \frac{(0.45)^3}{1.45} = 0.546 : 1$$

∴ $k_2 = \frac{1}{0.546} \times 0.36$ mm/s = **6.60 × 10^{-1} mm/s**

Also, $k_1 : k_2 = e_1^2 : e_2^2$

$$0.036 : k_2 = (0.36)^2 : (0.45)^2$$

$$= 0.1296 : 0.2025$$

∴ $k_2 = \frac{0.2025}{0.1296} \times 0.36$ = **5.625 × 10^{-1} mm/s.**

Example 7.10 In a falling head permeability test on a specimen 6 cm high and 50 cm^2 in cross-sectional area, the water level in the stand pipe, 0.8 cm^2 in sectional area, dropped from a height of 60 cm to 20 cm in 3 min 20 sees. Find the permeability.

Solution:

Given $A = 50\ cm^2,\ L = 6\ cm$

$a = 0.8\ cm^2$

$h_1 = 60\ cm,\ h_2 = 20\ cm$

$t = 3\ min\ 20\ sec = 200\ sec$

At

$$K = 2.3 \frac{aL}{At} \log_{10} \frac{h_1}{h_2} = 2.3 \times \frac{0.8 \times 6}{50 \times 200} \log_{10} \frac{60}{20}$$

$$= 2.3 \times \frac{4.8}{10^4} \log_{10} 3$$

$$= \frac{2.3 \times 4.8 \times 0.477}{10^4}$$

$$= 5.27 \times 10^{-4}\ cm/s \qquad [\because \log_{10} 3 = 0.477]$$

Example 7.11 During a constant head permeameter test, a flow Q of 160 cm^3 is measured in 5 mins under a const, head of 15 cm. The specimen is 6 cm long and has a sectional area of 50 cm^2.The porosity n_1 of specimen is 42%. Determine the permeability, the flow velocity V and seepage velocity V_s. Estimate K_2 for $n_2 = 35\%$.

Solution: Given Q = 160 cm^3 L = 6 cm

$$A = 50 \text{ cm}^2$$
$$t = 5 \text{ min} = 300 \text{ secs}$$
$$h = 15 \text{ cm}$$

$$K = \frac{QL}{Ath} = \frac{160 \times 6}{50 \times 300 \times 15} = 4.27 \times 10^{-3} \text{ cm/s}$$

$$V = \frac{Kh}{L} = \frac{4.27 \times 10^{-3} \times 15}{6} = 10.67 \times 10^{-2} \text{ cm/s}$$

$$n_1 = 0.42,\ e_1 = 0.724$$

$$V_s = \frac{V}{n_1} = \frac{1.067 \times 10^{-2}}{0.42} = 2.54 \times 10^{-2} \text{ cm/s}$$

$$n_2 = 0.35,\ e_2 = 0.538$$

We know $K \propto \dfrac{e^3}{1+e}$

or $K_2 \propto \dfrac{e_2^3}{1+e_2}$

Similarly $K_1 \propto \dfrac{e_1^3}{1+e_1}$

or $$\frac{K_2}{K_1} = \frac{\frac{e_2^3}{1+e_2}}{\frac{e_1^3}{1+e_1}} = \frac{\frac{(0.538)^3}{1+0.538}}{\frac{(0.724)^3}{1+0.724}} = \frac{0.101}{0.220} = 0.459$$

$$K_2 = 0.459 \times 4.27 \times 10^{-3} = 1.96 \times 10^{-3} \text{ cm/s} \quad \textbf{Ans.}$$

Example 7.12 A sand deposit is made up of three horizontal layers of equal thickness. The permeability of the top and bottom layers is 2 x10^{-4} cm/s and that of middle layer is 3.2 x 10^{-2} cm/s. Find the equivalent permeability in the horizontal and vertical direction and their ratio.

Solution:

Given $H_1 = H_3$

$K_1 = K_3 = 2 \times 10^{-4}$ cm/s

$K_x = 3.2 \times 10^{-2}$

$$K_x = \frac{K_1H_1 + K_2H_2 + K_3H_3}{H}$$

$$= \frac{2 \times 10^{-4} \times H_1 + 3.2 \times 10^{-2} \times H_1 + 2 \times 10^{-4} \times H_1}{3H_1}$$

$[\because H_1 = H_2 = H_3]$

$K_x = 1.08 \times 10^{-2}$ cm/s **Ans**

$$K_z = \frac{H}{\frac{H_1}{K_1} + \frac{H_2}{K_2} + \frac{H_3}{K_3}}$$

$$= \frac{3H_1}{\frac{H_1}{2 \times 10^{-4}} + \frac{H_1}{3.2 \times 10^{-2}} + \frac{H_1}{2 \times 10^{-4}}}$$

$= 2.99 \times 10^{-4}$ cm/s

$\frac{K_x}{K_z} = 36.1$ **Ans.**

Example 7.13 Calculate the value of coefficient of permeability of soil with their effective dia 0.5 mm.

Solution:

Hazen's correlation $K = CD^2_{10}$ cm/s

$C = 1.0$

$D_{10} = 0.5$ mm

$K = ?$

$K = 1.0 \text{ X } (0.5)^2$ cm/s = 0.25 cm/s Ans.

Example 7.14 A soil sample was tested in a constant head permeameter. The diameter and length of the sample was 3 cm and 15 cm respectively. Under a head of 30 cm, the discharge was found to be 80 cc in 15 minutes.

Calculate:

(i) Coefficient of permeability

(ii) Type of soil used in the test

Solution:

Given $d = 3$ cm

$L = 15$ cm

$h = 30$ cm

$Q = 80$ c.c.

$t = 15 \times 60 = 900$ secs

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (3)^2 = 7.07 \text{ cm}^2$$

$$i = \frac{h}{L} = \frac{30}{15} = 2$$

(i) We know

$$K = \frac{Q}{tiA} = \frac{80}{900 \times 2 \times 7.07}$$

$$= 6.29 \times 10^{-3} \text{ cm/sec} \quad \textbf{Ans.}$$

(ii) The value of K lies between 10^{-1} to 10^{-1}. The soil consists of fine gravel coarse, medium and fine sands.

Example 7.15 The coefficient of permeability of a soil sample in found to be 1 X 10^{-3} cm/s at a void ratio of 0.4. Estimate its permeability at a void ratio of 0.6.

Solution:

$K \alpha\, e^3 / 1+e$

$$\Rightarrow \quad \frac{K_2}{K_1} = \frac{e_2^3}{e_1^3} \times \frac{1+e_1}{1+e_2}$$

Given $K_1 = 1 \times 10^{-3}$ cm/s

$e_1 = 0.4$

$K_2 = ?, \; e_2 = 0.6$

Putting the value of K_1, e_1 and e_2 in (*i*) we get

$$\frac{K_2}{1 \times 10^{-3}} = \frac{(0.6)^3}{(0.4)^3} \times \frac{1+0.4}{1+0.6}$$

$$\therefore \quad K_2 = \frac{0.216}{0.064} \times \frac{1.4}{1.6} \times 1 \times 10^{-3} = 2.953 \times 10^{-3} \text{ cm/s} \quad \textbf{Ans.}$$

Example 7.16 A soil sample 5 cm in length and 60 cm in cross-sectional area, water percolates through the sample in 10 minutes is 480 ml under a constant head of 40 cm. Weight of oven dried sample is 498 gm and specific gravity of soil = 2.65.

Calculate:

(i) Coefficient of permeability

(ii) Seepage velocity.

Solution : Given :

$$Q = 480 \text{ ml}$$
$$L = 5 \text{ cm}$$
$$A = 60 \text{ cm}^2$$
$$h = 40 \text{ cm}$$
$$W_d = 498 \text{ gm}$$
$$G = 2.65$$
$$t = 10 \times 60 = 600 \text{ secs}$$
$$i = \frac{h}{L} = \frac{40}{5} = 8$$

(i) We know $K = \frac{Q}{tiA} = \frac{480}{600 \times 8 \times 60} = 1.67 \times 10^{-3}$ cm/s **Ans.**

(ii) Discharge velocity, $V = \frac{q}{A} = \frac{Q}{tA} = \frac{480}{600 \times 60} = 1.33 \times 10^{-2}$ cm/s

Seepage velocity, $V_s = \frac{V}{n}$ ...(i)

where $n = \frac{e}{1+e}$

and $e = \frac{G\gamma_w}{\gamma_d} - 1$ ...(ii)

$$\gamma_d = \frac{W_d}{A \times L} = \frac{498}{60 \times 5} = 1.66 \text{ gm/c.c}$$

Putting the value of γ_d in (*ii*) we get

$$e = \frac{2.65 \times 1}{1.66} - 1 = 0.596$$

$\therefore$ $n = \frac{0.596}{1 + 0.596} = 0.373$

Putting the value of V and n in (*i*) we get

$$V_s = \frac{1.33 \times 10^{-2}}{0.373} = 3.56 \times 10^{-2} \text{ cm/s} \quad \textbf{Ans.}$$

Example 7.17 If during a permeability test on a soil sample with falling head permeameter, equal time intervals are noted for drops of head from h_1 and h_2 and again from h_1 to h_2, find a relationship between h_1, h_2 and h_3.

Solution:

For falling head from h_1 and h_2

$$K = 2.3\frac{aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right) \quad ...(i)$$

For falling head from h_2 to h_3

$$K = 2.3\frac{aL}{At}\log_{10}\left(\frac{h_2}{h_3}\right) \quad ...(ii)$$

a, L, A and t are same for both the tests.

From (i) and (ii) we get

$$2.3\frac{aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right) = 2.3\frac{aL}{At}\log_{10}\left(\frac{h_2}{h_3}\right)$$

$$\log_{10}\left(\frac{h_1}{h_2}\right) = \log_{10}\left(\frac{h_2}{h_3}\right)$$

$$\frac{h_1}{h_2} = \frac{h_2}{h_3}$$

$$h_2^2 = h_1 h_3$$

$$\therefore \quad h_2 = \sqrt{h_1 h_3} \quad \textbf{Ans.}$$

SEEPAGE AND FLOW NETS

8.0 ILLUSTRATIVE EXAMPLES

Example 8.1: For a homogeneous earth dam 52 m high and 2 m free board, a flow net was constructed and following results were obtained:

Number of potential drops = 25

Number of flow channels = 4

The dam has a horizontal filter of 40 m length at its downstream end. Calculate the discharge per metre length of the dam if the coefficient of permeability of the dam material is $3x10^{-3}$ cm/sec

Solution:

The dischrge per unit length is given by

$$q = kH\frac{N_f}{N_d}$$

Hence, H = water depth = 52-2 = 50 m

$k = 3\times 10^{-3}$ cm/sec = 3×10^{-5} m/sec

$N_f = 4$

$N_d = 25$

∴ $q = 3\times 10^{-5}\times 50 \times \frac{4}{25} = 24\times10^{-5}$ m^3/sec/m

= 0.00024 cumecs/metre length