# MODULE-I

**Three Phase Diagram In Terms Of Void Ratio**

| Mass | Unit weight |
|---|---|
| (i) $\rho = \frac{(G+Se)\rho_w}{1+e}$ | (i) $\gamma = \frac{(G+Se)\gamma_w}{1+e}$ |
| (ii) $\rho_d = \frac{G\rho_w}{1+e}$ | (ii) $\gamma_d = \frac{G\gamma_w}{1+e}$ |
| (iii) $\rho_{sat} = \frac{(G+e)\rho_w}{1+e}$ | (iii) $\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$ |
| (iv) $\rho' = \frac{(G-1)\rho_w}{1+e}$ | (iv) $\gamma' = \frac{(G-1)\gamma_w}{1+e}$ |
| (v) $\rho' = \frac{(G-1)-e\ (1-S)\rho_w}{1+e}$ | (v) $\gamma' = \frac{(G-1)-e\ (1-S)\gamma_w}{1+e}$ |

**Three Phase Diagram In Terms Of Porosity**

| Mass | Unit weight |
|---|---|
| (i) $\rho = (G(1-n)+Sn)\rho_w$ | (i) $\gamma = (G(1-n)+Sn)\gamma_w$ |
| (ii) $\rho_d = G\rho_w\ (1-n)$ | (ii) $\gamma_d = G\gamma_w\ (1-n)$ |
| (iii) $\rho_{sat} = (G(1-n)+n)\rho_w$ | (iii) $\gamma_{sat} = (G(1-n)+n)\gamma_w$ |
| (iv) $\rho' = (G-1).(1-n)\rho_w$ | (iv) $\gamma' = (G-1).(1-n)\gamma_w$ |

**Relationship between the void ratio and water content**

$w = \frac{(Se)}{G}$

Or $e = \frac{wG}{S}$

For fully saturated soil S=1, i.e. $w = \frac{e}{G}$ ; $e = wG$

**Expression for mass density in terms of water content**

| Mass | Unit weight |
|---|---|
| (i) $\rho = \frac{(G+w)G\rho_w}{1+\frac{wG}{S}}$ | (i) $\gamma = \frac{(G+w)G\gamma_w}{1+\frac{wG}{S}}$ |

| | |
|---|---|
| (ii) $\rho_{sat} = \frac{(1+w)G\rho_w}{1+wG}$ | (ii) $\gamma_{sat} = \frac{(1+w)G\gamma_w}{1+wG}$ |
| (iii) $\rho_{sub} = \frac{(G-1)\rho_w}{1+wG}$ | (iii) $\gamma_{sub} = \frac{(G-1)\gamma_w}{1+wG}$ |
| (iv) $\rho_d = \frac{G\rho_w}{1+\frac{wG}{S}}$ | (iv) $\gamma_d = \frac{G\gamma_w}{1+\frac{wG}{S}}$ |
| (v) $\rho_d = \frac{\rho}{1+w}$ | (v) $\gamma_d = \frac{\gamma}{1+w}$ |
| (vi) $(\rho_d)_{sat} = \frac{G\rho_w}{1+wG}$ | (vi) $(\gamma_d)_{sat} = \frac{G\gamma_w}{1+wG}$ |

**Relationship between dry mass density and percentage air voids**

$$\rho_d = \frac{(1-n_a)G\rho_w}{1+wG} \quad or \quad \frac{(1-n_a)G\rho_w}{1+e}$$

$$\gamma_d = \frac{(1-n_a)G\gamma_w}{1+wG} \quad or \quad \frac{(1-n_a)G\gamma_w}{1+e}$$

## 2. ILLUSTRATIVE EXAMPLES

**Example 2.1** One cubic metre of wet soil weighs 19.80 kN. If the specific gravity of soil particles is 2.70 and water content is 11%, find the void ratio, dry density and degree of saturation?

Solution:

Bulk unit weight, = 19.80 kN/m$^3$

Water content, $w$ = 11% = 0.11

Dry unit weight, $\gamma_d = \frac{\gamma}{(1+w)} = \frac{19.80}{(1+0.11)}$ kN/m$^3$ = **17.84 kN/m$^3$**

Specific gravity of soil particles $G$ = 2.70

$$\gamma_d = \frac{G.\gamma_w}{1+e}$$

Unit weight of water, $\gamma_w$ = 9.81 kN/m$^3$

$$\therefore \quad 17.84 = \frac{2.70 \times 9.81}{(1+e)}$$

$$(1+e) = \frac{2.70 \times 9.81}{17.84} = 1.485$$

Void ratio, $e$ = **0.485**

Degree of Saturation, $S = wG/e$

$$\therefore \quad S = \frac{0.11 \times 2.70}{0.485} = 0.6124$$

$\therefore$ Degree of Saturation = **61.24%.**

**Example 2.2** A soil has bulk density of 20.1 kN/m$^3$ and water content of 15%. Calculate the water content if the soil partially dries to a density of 19.4 kN/m$^3$ and the void ratio remains unchanged?

Solution:

Bulk unit weight, $\gamma = 20.1$ kN/m$^3$

Water content, $w = 15\%$

Dry unit weight, $\gamma_d = \dfrac{\gamma}{(1+w)} = \dfrac{20.1}{(1+0.15)}$ kN/m$^3$ = 17.5 kN/m$^3$

But $\gamma_d = \dfrac{G.\gamma_w}{(1+e)}$ ;

if the void ratio remains unchanged while drying takes place, the dry unit weight also remains unchanged since $G$ and $\gamma_w$ do not change.

New value of $\gamma = 19.4$ kN/m$^3$

$$\gamma_d = \frac{\gamma}{(1+w)}$$

$$\therefore \quad \gamma = \gamma_d(1+w)$$

or

$$19.4 = 17.5 \quad (1+w)$$

$$(1+w) = \frac{19.4}{17.5} = 1.1086$$

$$w = 0.1086$$

Hence the water content after partial drying = **10.86%.**

**Example 2.3** The porosity of a soil sample is 35% and the specific gravity of its particles is 2.7. Calculate its void ratio, dry density, saturated density and submerged density?

Solution:

Porosity, $n = 35\%$

Void ratio, $e = n/(1-n) = 0.35/0.65 = \mathbf{0.54}$

Specific gravity of soil particles = 2.7

Dry unit weight, $\gamma_d = \dfrac{G.\gamma_w}{(1+e)}$

$= \dfrac{2.7 \times 9.81}{1.54}$ kN/m$^3$ = **17.20 kN/m$^3$**

Saturated unit weight, $\gamma_{sat} = \dfrac{(G+e)}{(1+e)} \cdot \gamma_w$

$= \dfrac{(2.70+0.54)}{1.54} \times 9.81$ kN/m$^3$

= **20.64 kN/m$^3$**

Submerged unit weight, $\gamma' = \gamma_{sat} - \gamma_w$

= (20.64 – 9.81) kN/m$^3$

= **10.83 kN/m$^3$.**

**Example 2.4** (i) A dry soil has a void ratio of 0.65 and its grain specific gravity is = 2.80. What is its unit weight?

(ii) Water is added to the sample so that its degree of saturation is 60% without any change in void ratio. Determine the water content and unit weight.

(iii) The sample is next placed below water. Determine the true unit weight (not considering buoyancy) if the degree of saturation is 95% and 100% respectively.

Solution:

(*i*) **Dry Soil**

Void ratio, $e = 0.65$

Grain specific gravity, $G = 2.80$

Unit weight, $\gamma_d = \dfrac{G.\gamma_w}{(1+e)} = \dfrac{2.80 \times 9.8}{1.65}$ kN/m$^3$ = **16.65 kN/m³.**

(*ii*) **Partial Saturation of the Soil**

Degree of saturation, $S = 60\%$

Since the void ratio remained unchanged, $e = 0.65$

Water content, $w = \dfrac{S.e}{G} = \dfrac{0.60 \times 0.65}{2.80} = 0.1393$

= **13.93%**

$$\text{Unit weight} = \frac{(G + Se)}{(1+e)} \cdot \gamma_w = \frac{(2.80 + 0.60 \times 0.65)}{1.65} 9.81 \text{ kN/m}^3$$

= **18.97 kN/m³.**

(*iii*) **Sample below Water**

High degree of saturation $S = 95\%$

$$\text{Unit weight} = \frac{(G + Se)}{(1+e)} \cdot \gamma_w = \frac{(2.80 + 0.95 \times 0.65)}{1.65} 9.81 \text{ kN/m}^3$$

= **20.32 kN/m³**

Full saturation, $S = 100\%$

$$\text{Unit weight} = \frac{(G + e)}{(1+e)} \cdot \gamma_w = \frac{(2.80 + 0.65)}{1.65} 9.81 \text{ kN/m}^3$$

= **20.51 kN/m³.**

**Example 2.5** A sample of saturated soil has a water content of 35%. The specific gravity of solids is 2.65. Determine its void ratio, porosity, saturated unit weight and dry unit weight?

Solution:

**Saturated soil**

Water content, $w = 35\%$

specific gravity of solids, $G = 2.65$

Void ratio, $e = wG$, in this case.

∴ $e = 0.35 \times 2.65 = \mathbf{0.93}$

Porosity, $n = \dfrac{e}{1+e} = \dfrac{0.93}{1.93} = 0.482 = \mathbf{48.20\%}$

Saturated unit weight, $\gamma_{Sat} = \dfrac{(G+e)}{(1+e)} \cdot \gamma_w$

$= \dfrac{(2.65 + 0.93)}{(1 + 0.93)} \times 9.81$

$= 18.15$ kN/m$^3$

Dry unit weight, $\gamma_d = \dfrac{G.\gamma_w}{(1+e)}$

$= \dfrac{2.65 \times 9.81}{1.93}$

= **13.44 kN/m³.**

**Example 2.6** A saturated clay has a water content of 39.3% and a bulk specific gravity of 1.84. Determine the void ratio and specific gravity of particles?

Solution:

**Saturated clay**

Water content, $w = 39.3\%$

Bulk specific gravity, $G_m = 1.84$

Bulk unit weight, $\gamma = G_m \cdot \gamma_w$

$= 1.84 \times 9.81 = 18.05 \text{ kN/m}^3$

In this case, $\gamma_{sat} = 18.05 \text{ kN/m}^3$

$$\gamma_{sat} = \frac{(G+e)}{(1+e)} \cdot \gamma_w$$

For a saturated soil,

$$e = wG$$

or $e = 0.393\, G$

$\therefore$
$$18.05 = \frac{(G + 0.393\, G)}{(1 + 0.393\, G)} \cdot (9.81)$$

whence $G = 2.74$

Specific gravity of soil particles = **2.74**

Void ratio = 0.393 × 2.74 = **1.08.**

**Example 2.7** The mass specific gravity of a fully saturated specimen of clay having a water content of 30.5% is 1.96. On oven drying, the mass specific gravity drops to 1.60. Calculate the specific gravity of clay?

Solution:

**Saturated clay**

Water content, $w = 30.5\%$

Mass specific gravity, $G_m = 1.96$

$\therefore$ $\gamma_{sat} = G_m \cdot \gamma_w = 1.96\, \gamma_w$

On oven-drying, $G_m = 1.60$

$\therefore$ $\gamma_d = G_m \cdot \gamma_w = 1.60 \gamma_w$

$$\gamma_{sat} = 1.96 . \gamma_w = \frac{(G+e)\gamma_w}{(1+e)} \quad ...(i)$$

$$\gamma_d = 1.60 . \gamma_w = \frac{G . \gamma_w}{(1+e)} \quad ...(ii)$$

For a saturated soil, $e = wG$

$\therefore$ $e = 0.305G$

From ($i$),

$$1.96 = \frac{(G + 0.305G)}{(1+0.305G)} = \frac{1.305G}{(1+0.305G)}$$

$\Rightarrow$ $1.96 + 0.598G = 1.305G$

$\Rightarrow$ $G = \frac{1.960}{0.707} = \mathbf{2.77}$

From ($ii$),

$1.60 = G/(1 + e)$

$\Rightarrow$ $G = (1 + 0.305G)\, 1.6$

$\Rightarrow$ $G = 1.6 + 0.485G$

$\Rightarrow$ $0.512G = 1.6$

$\Rightarrow$ $G = 1.6/0.512 = \mathbf{3.123}$

The latter part should not have been given (additional and inconsistent data).

**Example 2.8** A soil sample in its undisturbed state was found to have volume of 105 $cm^3$ and mass of 210 g. After oven drying the mass got reduced to 168 g. Compute (i) water content, (ii) void ratio, (iii) porosity (iv) degree of saturation and (v) air content. Take G =2.7

Solution:

Volume of soil mass, V = 105 cm$^3$
Mass of soil mass, M = 201 g
Mass of dry soil mass, $M_d$ = 168 g
Specific gravity of soil particles, G = 2.7

(*i*) Water content, $w = \frac{M_w}{M_s} = \frac{M - M_d}{M_d} = \frac{201 - 168}{168} = 0.196 = 19.6\%$

Dry density, $\gamma_d = \frac{M_s}{V} = \frac{M_d}{V} = \frac{168}{105} = 1.6 g/cm^3$

(*ii*) Void ratio, $e = \frac{G\gamma_m}{\gamma_d} - 1 = \frac{2.7(1)}{1.6} - 1 = 0.69$

(*ii*) Potosity, $n = \frac{e}{1+e} = \frac{0.69}{1.69} = 0.41$

(iv) Degree of saturation Si $\frac{wG}{e} = \frac{(0.196)(2.7)}{0.69}$
$= 0.777 = 77.7\%$

(v) Air content, $a_c = 1 - Si = 1 - 0.77 = 0.23$

**Example 2.9** For a soil sample the specific gravity of soil mass is 1.7 and specific gravity of soil particles is 2.7. Determine the void ratio (i) assuming the soil sample is dry and (ii) the sample has a water content of 12 percent?

Solution:

$G_m = 1.7$ $\quad G = 2.7$

(*i*) For dry soil mass $\gamma = \gamma_d$

$$G_m = \frac{\gamma}{\gamma_w} = \frac{\gamma_d}{\gamma_w} \quad \gamma_d = G_m \gamma_w$$

Void ratio, $e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{G\gamma_w}{G_m\gamma_w} - 1 = \frac{G}{G_m} - 1$

$$= \frac{2.7}{1.7} - 1 = 0.59$$

(*ii*) When $w = 12\% = 0.12$

$$G_m = \frac{\gamma}{\gamma_w} \text{ or } \gamma = G_m\gamma_w = (1.7)(9.81) = 16.68 kN/m^3$$

$$\gamma_d = \frac{\gamma}{1+w} = \frac{16.68}{1+0.12} = 14.89 kN/m^3$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{(2.7)(9.81)}{14.89} - 1 = 0.78$$

**Example 2.10** A soil sample assumed to consist of spherical grains all of same diameter will have maximum void ratio when the grains are arranged in a cubical array. Find the void ratio and dry unit weight. Take unit weight of grains as 20kN/m$^3$.

Solution:

We consider a unit cube packed with the spherical grains of diameter d.

Number of spherical grains in the container $= \frac{1}{d} \times \frac{1}{d} \times \frac{1}{d} = \frac{1}{d^3}$

| | |
|---|---|
| Volume of each spherical grain | $= \frac{\pi d^3}{6}$ |
| Volume of soil solids, | $V_s = \frac{1}{d^3} \times \frac{\pi d^3}{6} = \frac{\pi}{6}$ |
| Total volume of cube, | $V = 1 \times 1 \times 1 = 1 \text{ m}^3$ |
| Volume of voids, | $V_v = V - V_s = 1 - \frac{\pi}{6} = \frac{6-\pi}{6}$ |
| Void ratio | $e = \frac{V_v}{V_s} = \frac{(6-\pi)(6)}{6(\pi)} = \frac{6-\pi}{\pi} = 0.91$ |
| Dry unit weight | $\gamma_d = \frac{W_s}{V} = \frac{V_s \gamma_s}{V} = \frac{\pi}{6}(20) = 10.47 \, kN/m^3$ |

**Example 2.11** 1000m$^3$ of earthfill is to be constructed. How many cubic meters of soil is to be excavated from borrow pit in which the void ratio is 0.95, if the void ratio of earthfill is to be 0.7?

Solution:

Volume of earth fill, $V_1 = 1000 \text{ m}^3$

Void ratio of earth fill, $e_1 = 0.7$

Void ratio in borrow pit, $e_2 = 0.95$

Let volume of soil to be excavated from borrow pit $= V_2$

We have $$e = \frac{V_v}{V_s}$$

Adding 1 to both sides, $$1 + e = \frac{V_v}{V_s} + 1 = \frac{V_v + V_s}{V_s} = \frac{V}{V_s}$$

For soil in earth fill, $$1 + e_1 = \frac{V_1}{V_s} \quad \ldots(i)$$

For soil to be excavated from borrow pit $$1 + e_2 = \frac{V_2}{V_s} \quad \ldots(ii)$$

Dividing (ii) by (i), $\frac{V_2}{V_1} = \frac{1+e_2}{1+e_1}$ ($\because$ $V_s$ is same for earthfill and soil excavated from borrow pit)

$$V_2 = \left(\frac{1+e_2}{1+e_1}\right) V_1 = \left(\frac{1+0.95}{1+0.7}\right)(1000) = 1147 \, m^3$$

**Example 2.12** In an earthen embankment under construction the bulk unit weight is 16.5 kN/m$^3$ at water content of 11 %. If the water content is to be raised to 15 %, compute the quantity of water required to be added per cubic meter of soil? Assume no change in the void ratio.

Solution:

$$\gamma_1 = 16.5\ kN/m^3$$
$$w_1 = 11\%$$
$$w_2 = 15\%$$
$$V = 1 m^3$$

$$(\gamma_d)_1 = \frac{\gamma_1}{1+w_1} = \frac{16.5}{1+0.11} = 14.86\ kN/m^3$$

$$W_s = (\gamma_d)_1 . V = (14.86)(1) = 14.86\ kN$$

$$w_1 = \frac{Ww_1}{W_s}$$

$$Ww_1 = w_1 . W_s = (0.11)(14.86) = 1.635\ kN$$

$$Vw_1 = \frac{Ww_1}{\gamma_w} = \frac{1.635}{9.81} = 0.167\ m^3 = 167\ litres$$

Similarly,

$$Ww_2 = w_2 . W_s = (0.15)(14.86) = 2.229\ kN$$

$$Vw_2 = \frac{Ww_2}{\gamma_w} = \frac{2.229}{9.81} = 0.227\ m^3 = 227\ litres$$

Answer: Required quantity of water to be added per cubic meter of soil

= 227 – 167 = 60 litres.

**Example 2.13** A soil sample has a porosity of 40%, the specific gravity of solids 2.70, Calculate (a) void ratio (b) Dry density (c) Unit weight if the soil is 50% saturated (d) Unit weight if the soil is completely saturated

Solution:

(a) $e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = 0.667$

(b) $\gamma_d = \frac{G.\gamma_w}{1+e} = \frac{2.7*9.81}{1+0.667} = 15.89$ KN / m$^3$

(c) $e = \frac{w.G}{Sr}$

$w = \frac{eSr}{G} = \frac{.667*0.5}{2.7} = 0.124$

$\gamma = \gamma_d . (1+w) = 15.89*1.124 = 17.85$ KN / m$^3$

(d) When the soil is fully saturated

$e = w_{sat}.G$

$w_{sat} = e / G = 0.667 / 2.7 = 0.247$

$\gamma_{sat} = G\ \gamma_w (1-n) + \gamma_w . n$

$= 2.7 * 9.81(1- 0.4) + 9.81*0.4 = 19.81$ KN / m$^3$

**Example 2.14** An undisturbed sample of soil has a volume of 100 $cm^3$ and mass of 190g. On oven drying for 24 hrs, the mass is reduced to 160 g. If the specific gravity grain is 2.68, determine the water content, voids ratio and degree of saturation of the soil.

Solution:

Mw= 190-160= 30 g

Md = 160 g

W= Mw / Md= 30 / 160 = 0.188 = 18.8 %

Mass of moist soil = M = 190 g

Bulk density = M/ V = 190 / 100= 1.9 g/ $cm^3$

$\gamma = 9.81 * \rho = 9.81 * 1.9 = 18.64$ KN / $m^3$

$$\gamma_d = \frac{\gamma}{1+w}$$

$$= \frac{18.64}{1+.188} = 15.69 \text{ KN / m}^3$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1$$

$$= \frac{2.68 * 9.81}{15.69} - 1 = 0.67$$

$$Sr = \frac{w.G}{e} = \frac{0.188 * 2.68}{0.67} = .744 = 74.45\%$$

**Example 2.15** The in-situ density of an embankment, compacted at a water content of 12 % was determined with the help of core cutter. The empty mass of the cutter was 1286 g and the cutter full of soil had a mass of 3195 g, the volume of the cutter being 1000 $cm^3$. Determine the bulk density, dry density and the degree of saturation of the embankment.

If the embankment becomes fully saturated during rains, what would be its water content and saturated unit weight / assume no volume change in soil on saturation. Take the specific gravity of the soil as 2.70.

Solution:

Mass of soil in cutter

M= 3195- 1286= 1909 g

Bulk density $\rho$ = M/ V = 1909 / 1000 = 1.909 g/ $cm^3$

Bulk unit weight $\gamma = 9.81 * \rho$

$= 9.81 * 1.909 = 18.73 \text{ KN/m}^3$

$$\gamma_d = \frac{\gamma}{1+w} = \frac{18.73}{1+0.12} = 16.72 \text{ KN/m}^3$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 * 9.81}{16.72} - 1 = .0.584$$

$$Sr = \frac{w.G}{e} = \frac{0.12 * 2.7}{0.584} = .555 = 55.5\%$$

At saturation:

Since the volume remains the same, the voids ratio also remains unchanged.

$e = w_{sat.} G$

$w_{sat.} = e/G = 0.584 / 2.7 = 0.216 = 21.6\%$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.7+.584)9.81}{1+.584} = 20.34 \text{ KN/m}^3$$

**Example 2.16** The in-situ percentage voids a sand deposit is 34 percent, for determining the density index, dried sand from the stratum was first filled loosely in a 1000 $cm^3$ mould and was then vibrated to give a maximum density. The loose dry mass in the mould was m1610 g and dense dry mass at maximum compaction was found to be 1980 g. Determine the density index if the specific gravity of the sand particles 2.67.

Solution:

$n = 34\%$

$e = n / (1-n) = 0.34 / (1-0.34) = 0.515$

$$(\gamma_d)_{max} = \frac{1980}{1000} * 9.81 = 19.42 \text{ KN/m}^3$$

$$(\gamma_d)_{min} = \frac{1610}{1000} * 9.81 = 15.79 \text{ KN/m}^3$$

$$e_{min} = \frac{G\gamma_w}{(\gamma_d)_{min}} - 1 = \frac{2.67 * 9.81}{19.42} - 1 = 0.349$$

$$e_{max} = \frac{G\gamma_w}{(\gamma_d)_{max}} - 1 = \frac{2.67 * 9.81}{15.79} - 1 = 0.659$$

$I_D = (e_{max} - e) / (e_{max} - e_{min}) = (0.659 - 0.515) / (0.659 - 0.349)$

$= 0.465 = 46.5\ \%$

**Example 2.17** The mass specific gravity (apparent gravity) of a soil equals 1.64. The specific gravity of solids is 2.70. Determine the voids ratio under assumption that the soil is perfectly dry. What would be the voids ratio, if the sample is assumed to have a water content of 8 percent?

Solution: When the sample is dry

$$Gm = \frac{\gamma_d}{\gamma_w} = 1.64$$

$$\gamma_d = 1.64 * \gamma_w = 1.64 * 9.81 = 16.09 \text{ KN/ m}^3$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 * 9.81}{16.09} - 1 = 0.646$$

When the sample has water content

w = 8 %

$$\gamma = 1.64 * \gamma_w = 1.64 * 9.81 = 16.09 \text{ KN/ m}^3$$

$$\gamma = \frac{\gamma}{1+w} = \frac{16.09}{1+0.08} = 14.9 \text{ KN/ m}^3$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 * 9.81}{14.9} - 1 = 0.78$$

**Example 2.18** A natural soil deposit has a bilk unit weight of 18.44 KN/ $m^3$, water content of 5 %. calculate the amount of water required to the added to 1 m3 of soil to raise the water content to 15 %. Assume the void ratio to remain content. What will then be the degree of saturation? Assume G= 2.67

Solution:

$\gamma$ = 18.44 KN/ $m^3$; w = 5%

$$\gamma_d = \frac{\gamma}{1+w} = \frac{18.44}{1+0.05} = 17.56 \text{ KN/ m}^3$$

w = Ww / Wd = 0.05

For one cubic meter of soil, v = 1 $m^3$

Wd = $\gamma_d$ . V= 17.56 * 1 = 17.56 KN.

Ww= 0.05* Wd= 0.05 * 17.56= 0.878 KN

Vw = Ww / $\gamma_d$ =0.878 / 9.81 = 0.0895 $m^3$

Later, when w = 15 %

Ww = w. Wd = 0.15 * 17.56 = 2.634 KN

Vw = Ww / $\gamma_w$ = 2.534 / 9.81 = 0.2685 $m^3$

Hence additional water required to raise the water content from 5 % to15% = 0.2685 – 0.0895 = 0.179 m= 179 liters.

$$\text{Voids ratio, } e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.67 * 9.81}{17.56} - 1 = 0.49$$

After the water has been added 'e 'remains the same

$$Sr = w.G / e = 0.15 * 2.67 / 0.49 = 0.817 = 81.7\%$$

**Example 2.19** Calculate the unit weighs and specific gravities of solids of (a) soil composed of pure quartz and (b) a soil composed of 60 % quartz, 25% mica, and 15% iron oxide. Assume that both soils are saturated and have voids 0f 0.63. Take average and for iron oxide = 3.8

Solution:

a) For the soil composed of pure Quartz,

G for quartz = 2.66

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.66+0.63) * 9.81}{1+0.63} = 19.8 \text{ KN/ m}^3$$

b) for the composite soil,

G average= (2.66*0.6) + (3.* 0.25) + (3.8 *0.15)
=1.6 + 0.75 + 0.57= 2.92

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.92+0.63) * 9.81}{1+0.63} = 21.36 \text{ KN/ m}^3$$

**Example 2.20** A soil has a bulk unit weight of 20.22 KN/ m3 and water content of 15%. Calculate the water content if the soil partially dries to a unit weight of 19.42 KN/ m3.and voids ratio remains unchanged.

Solution:

Before drying,

$\gamma$ = 20.11 KN/ m$^3$

$\gamma_d$ = 20.11 / (1 + 0.15) = 17.49 KN/ m$^3$

Since after drying, e does not change, V and $\gamma_d$, are the same,

$\gamma = \gamma_d(1+ w)$

$1+w = \gamma / \gamma_d$ =19.42 / 17.49= 1.11

w= 1.11 – 1 = 11%

**Example 2.21** A cube of dried clay having sides 4 cm long has a mass of 110 g. The same cubes of soil, when saturated at unchanged volume, has mass of 135 g. Draw the soil element showing the volumes and weights of the constituents, and then determine the specific gravity of soil solids and voids ratio.

Solution:

Volume of soil = $(4)^3 = 64\ m^3$

Mass water after saturation

= 135- 110= 25 g

Volume of solids = 25 $cm^3$

Volume of solids = 64- 25 = 39 $m^3$

Ms = 110 g

$$G = \frac{\gamma_s}{\gamma_w} = \frac{\rho_s}{\rho_w} = \frac{Ms}{Vs*1} = \frac{110}{39*1} = 2.82$$

$$e = \frac{Vs}{Vd} = 25/39 = 0.64$$

**Example 2.22** A soil sample with a grain specific gravity of 2.67 was filled in a 1000 ml container in the loosest possible state and the dry weight of the sample was found to be 14.75 N. It was then filled at the densest state obtainable and the weight was found to be 17.70 N. The void ratio of the soil in the natural state was 0.63. Determine the density index in the natural state.

Solution:

$G = 2.67$

Loosest state:

Weight of soil = 14.75 N

Volume of solids $= \frac{14.75}{0.0267}\ cm^3 = 552.4\ cm^3$

Volume of voids $= (1000 - 552.4)\ cm^3 = 447.6\ cm^3$

Void ratio, $e_{max}$ = 447.6/552.4 = 0.810

Densest state:

Weight of soil = 17.70 N

Volume of solids $= \frac{17.70}{0.0267}\ cm^3 = 662.9\ cm^3$

Void ratio, $e_{min} = \frac{337.1}{662.9} = 0.508$

Void ratio in the natural sate, $e = 0.63$

Density Index, $I_D = \frac{(e_{max} - e)}{(e_{max} - e_{min})}$

$$= \frac{0.810 - 0.630}{0.810 - 0.508} = \frac{0.180}{0.302} = 0.596$$

$\therefore$ $I_D$ = **59.6%.**

**Example 2.23** The dry unit weight of a sand sample in the loosest state is 13.34 $kN/m^3$ and in the densest state, it is 21.19 $kN/m^3$. Determine the density index of this sand when it has a porosity of 33%. Assume the grain specific gravity as 2.68.

Solution:

$\gamma_{min}$(loosest state) = 13.34 kN/m$^3$

$\gamma_{max}$(densest state) = 21.19 kN/m$^3$

Porosity, $n$ = 33%

Void ratio, $e_0 = \dfrac{n}{(1-n)} = 33/67 = 0.49$

$$\gamma_0 = \frac{G \cdot \gamma_w}{(1-e_0)} = \frac{2.68 \times 9.81}{(1+0.49)} \text{ kN/m}^3 = 17.64 \text{ kN/m}^3$$

Density Index, $I_D$

$$= \frac{(\gamma_{max})}{(\gamma_0)} \left( \frac{\gamma_0 - \gamma_{min}}{\gamma_{max} - \gamma_{min}} \right)$$

$$= \frac{21.19}{17.64} \times \frac{(17.64 - 13.34)}{(21.19 - 13.34)} = \frac{21.19}{17.64} \times \frac{4.30}{7.85} = 0.658 = \mathbf{65.8\%}$$

Alternatively:

$$\gamma_{min} = \frac{G \cdot \gamma_w}{(1-e_{max})} \quad \text{or} \quad 13.34 = \frac{2.68 \times 9.81}{(1+e_{max})}$$

$\therefore$ $e_{max} = 0.971$

$$\gamma_{max} = \frac{G \cdot \gamma_w}{(1-e_{min})} \quad \text{or} \quad 21.19 = \frac{2.68 \times 9.81}{(1+e_{min})}$$

$\therefore$ $e_{min} = 0.241$

$\therefore$ $I_D = \dfrac{(e_{max} - e_0)}{(e_{max} - e_{min})}$, (by Eq. 3.8)

$$= \frac{(0.971 - 0.49)}{(0.971 - 0.241)} = \frac{0.48}{0.73} = \mathbf{56.8\%.}$$

# INDEX PROPERTIES OF SOILS

## 3. **ILLUSTRATIVE EXAMPLES**

**Example 3.1** During a test for water content determination on a soil sample by pycnometer, the following observations were taken

(1) Mass of wet soil sample = 1000 gm

(2) Mass of pycnometer with soil and filled with water = 2000 gm

(3) Mass of pycnometer filled with water only = 1480 gm

(4) Specific gravity of solids = 2.67

Determine the water content.

Solution:

We know that $w = [(\frac{M2-M1}{M3-M4})(\frac{G-1}{G}) - 1] \times 100$

$=[\frac{1000}{(2000-1480)} \times \left(\frac{2.67-1}{2.67}\right) - 1] \times 100$

$= 20.28\%$

Hence water content of the sample is 20.28%.

**Example 3.2** The mass of an empty gas jar was 0.498 Kg. When completely filled with water its mass was 1.528 Kg. An oven dried sample of soil mass 0.198 Kg was placed in the jar and water was added to fill the jar and its mass was found to be 1.653 Kg. Determine the specific gravity of particle.

Solution:

$$G = \frac{M_{2-M_1}}{(M_{2-M_1})-(M_{3-M_4})}$$

$$= \frac{0.198}{0.198-(1.653-1.528)}$$

= 2.71

**Example 3.3** A soil sample consisting of particles of size ranging from 0.5 mm to 0.01 mm, is put on the surface of still water tank 5 metres deep. Calculate the time of settlement of the coarsest and the finest particle of the sample, to the bottom of the tank. Assume average specific gravity of soil particles as 2.66 and viscosity of water as 0.01 poise.

Solution:

$$v = \frac{D^2\,\gamma_w\,(G-1)}{18\times10^6\eta}$$

$$= \frac{D^2(G-1)}{1.835\times10^6\times\eta}$$

Here G = 2.66 and $\eta = 0.01\times10^{-4} = 10^{-6}$ KN-s/m$^2$

$$v = \frac{D^2}{1.835}\times\frac{2.66-1}{10^6(10^{-6})}$$

$= 0.905\ D^2$

Where v is in m/sec and D is in mm

For coarsest particle D = 0.5 mm

$v = 0.905(0.5)^2 = 0.2263$ m/sec

t = h/v = 5/0.2263 = 22.1 seconds

for the finest particle, D = 0.01 mm

$v = 0.905(0.01)^2 = 9.05\times10^{-5}$ m/sec

$t = \frac{5}{9.05\times10^{-5}} = 55249$ sec = 15 hours 20 min 49 seconds.

**Example 3.4** A soil has a liquid limit of 25 % and plastic limit is 15 %. Determine the plasticity index. If the water content of the soil in its natural condition in the field is 20 %, find the liquidity index and relative consistency.

Solution:

$w_l = 25\ \%$ $\quad$ $w_p = 15\ \%$ $\quad$ $w = 20\ \%$

plasticity Index $I_p = w_l - w_p$

$= 25 - 15 = 10\ \%$

Liquidity index = $I_l = \frac{w - w_p}{I_p}\times 100$

$$= \frac{0.2 - 0.15}{0.1} \times 100 = 50\ \%$$

Relative consistency = $I_c = \frac{w_l - w}{I_p} \times 100$

$$= \frac{0.25 - 0.2}{0.1} \times 100 = 50\ \%$$

**Example 3.5** 50 grams of oven dried soil sample is taken for sedimentation analysis. The hydrometer reading in a 100 ml soil suspension 30 minutes after the commencement of sedimentation test is 24.5. The effective depth for Rh = 25, found from the calibration curve is 10.7 cm. The meniscus correction is found to be +0.5 and the composite correction as – 2.50 at the test temperature of 30°C. Taking the specific gravity of particles as 2.75 and viscosity of water as 0.008 poise, calculate the smallest particle size which would have settled during this interval of 30 minutes and the percentage of particles finer than this size.

Solution:

$R_h' = 24.5$ $\quad\quad$ $R_h = 24.5+0.5 = 25$ $\quad\quad$ $R = 24.5 - 2.5 = 22$

$$D = \sqrt{\frac{3000\eta}{(G-1)\gamma_w}}\sqrt{\frac{H_e}{t}}$$

Where D is in mm, $H_e$ is in cm and t is in minute.

Here $\eta = 0.008\times10^{-4}$ KN-s/m$^2$

$H_e = 10.7$ cm

$G = 2.75$ and $\gamma_w = 9.81$ KN/m$^3$

$t = 30$ min.

$$D = \sqrt{\frac{3000\times0.008\times10^{-4}}{(2.75-1)9.81}} \times \sqrt{\frac{10.7}{30}}$$

$= 7.06\times10^{-3}$ mm

$= 0.00706$ mm

The percentage finer is given by

$$N' = \frac{100GR}{Md(G-1)}$$

Where $M_d$ = mass of dry soil = 50 gm

$$N' = \frac{100\times2.75}{50(2.75-1)} \times 22$$

$= 69.1$ %.

**Example 3.6** In a specific gravity test with pycnometer, the following observed readings are available:

Weight of the empty pycnometer = 7.50 N

Weight of pycnometer + dry soil = 17.30 N

Weight of pycnometer + dry soil + water filling the remaining volume = 22.45 N

Weight of pycnometer + water = 16.30 N

Determine the specific gravity of the soil solids, ignoring the effect of temperature.

Solution:

The given weights are designated $W_1$ to $W_4$ respectively.

Then,

the weight of dry soil solids, $W_s = W_2 - W_1$

$= (17.30 - 7.50)\text{ N} = 9.80\text{ N}$

The specific gravity of soil solids is given by Eq. 3.1:

$$G = \frac{W_s}{W_s - (W_3 - W_4)} \text{ (ignoring the effect of temperature)}$$

$$= \frac{9.80}{9.80 - (22.45 - 16.30)}$$

$$= \frac{9.80}{(9.80 - 16.15)} = 2.685$$

∴ Specific gravity of the soil solids = **2.685.**

**Example 3.7** In a specific gravity test in which a density bottle and kerosene were used, the following observations were made:

Weight of empty density bottle = 0.6025 N

Weight of bottle + clay sample = 0.8160 N

Weight of bottle + clay + kerosene filling the remaining volume = 2.5734 N

Weight of bottle + kerosene = 2.4217 N

Temperature of the test = 27°C

Specific gravity of kerosene at 27°C = 0.773

Determine the specific gravity of the soil solids.

What will be the value if it has to be reported at 4°C?

Solution:

Assume the specific gravity of water at 27°C as 0.99654.

Let the weight be designated as $W_1$ through $W_4$ in that order.

Wt of dry clay sample, $W_s = (W_2 - W_1) = (0.816 - 0.6025)\text{ N} = 0.2135\text{ N}$

By Eq. 3.2,

$$G = \frac{W_s \cdot G_k}{W_s - (W_3 - W_4)}$$

$G_k$ here is given as 0.773.

$$\therefore \quad G = \frac{0.2135 \times 0.773}{0.2135 - (2.5734 - 2.4217)} \approx 2.67$$

If the value has to be reported at 4°C, by Eq. 3.3,

$$G_{T_2} = G_{T_1} \cdot \frac{G_{w_{T_2}}}{G_{w_{T_1}}}$$

$$\therefore \quad G_{4^\circ} = G_{27^\circ} \cdot \frac{1}{0.99654} = \frac{2.67 \times 1}{0.99654} = \mathbf{2.68.}$$

**Example 3.8** A pycnometer was used to determine the water content of a sandy soil. The following observation were obtained:

Weight of empty pycnometer = 8 N

Weight of pycnometer + wet soil sample = 11.60 N

Weight of pycnometer + wet soil + water filling remaining volume = 20 N

Weight of pycnometer + water = 18 N

Specific gravity of soil solids (determined by a separate test) = 2.66

Compute the water content of the soil sample.

Solution:

The weights may be designated $W_1$ through $W_4$ in that order,

$$w = \left[\frac{(W_2 - W_1)}{(W_3 - W_4)}\left(\frac{G-1}{G}\right) - 1\right] \times 100$$

Substituting the values,

$$w = \left[\frac{(11.60 - 8.00)}{(20 - 18)} \times \frac{(2.66 - 1)}{2.66} - 1\right] \times 100$$

$$= \left[\frac{3.6}{2.0} \times \frac{1.66}{2.66} - 1\right] \times 100$$

$$= (1.1233 - 1) \times 100 = \mathbf{12.33}$$

∴ Water content of the soil sample = **12.33%.**

**Example 3.9** The following data were obtained during an in-situ unit wt determination of an embankment by the sand-replacement method:

Volume of calibrating can = 1000 ml

Weight of empty can = 9 N

Weight of can + sand = 25 N

Weight of sand filling the conical portion of the sand-pouring cylinder = 4.5 N

Initial weight of sand-pouring cylinder + sand = 54 N

Weight of cylinder + sand, after filling the excavated hole = 41.4 N

Wet weight of excavated soil = 9.36 N

In-situ water content = 9%

Determine the in-situ unit weight and in-situ dry unit weight.

Solution:

Sand-replacement method of *in-situ* unit weight determination:

Weight of sand filling the calibrating can of volume 1000 ml = (25 – 9) N = 16 N

Unit weight of sand = 16/1000 N/cm$^3$ = 0.016 N/cm$^3$

Weight of sand filling the excavated hole and conical portion of the sand pouring cylinder = (54 – 41.4) = 12.60 N

Weight of sand filling the excavated hole = (12.6 – 4.5) = 8.10 N

Volume of the excavated hole $= \frac{8.10}{0.016}$ cm$^3$ = 506.25 cm$^3$

Weight of excavated soil = 9.36 N

*In-situ* unit weight, $\gamma$ = 9.36/506.25 N/cm$^3$ = **18.15 kN/m³**

Water content, $w$ = 9%

*In-situ* dry unit weight, $\gamma_d = \frac{\gamma}{(1+w)} = \frac{18.15}{(1+0.09)}$ kN/m$^3$ = **16.67 kN/m³.**

**Example 3.10** A field density test was conducted by core-cutter method and the following data was obtained:

Weight of empty core-cutter = 22.80 N

Weight of soil and core-cutter = 50.05 N

Inside diameter of the core-cutter = 90.0 mm

Height of core-cutter = 180.0 mm

Weight or wet sample for moisture determination = 0.5405 N

Weight of oven-dry sample = 0.5112 N

Specific gravity so soil grains = 2.72

Determine (a) dry density, (b) void-ratio, and (c) degree of saturation.

Solution:

Weight of soil in the core-cutter ($W$) = (50.05 – 22.80) = 27.25 N

Volume of core-cutter ($V$) $= (\pi/4) \times 9^2 \times 18\ cm^3 = 1145.11\ cm^3$

Wet unit weight of soil ($\gamma$) $= W/V = \dfrac{27.25}{1145.11}\ N/cm^3 = 23.34\ kN/m^3$

Weight of oven-dry sample = 0.5112 N

Weight of moisture = (0.5405 – 0.5112) = 0.0293 N

Moisture content, $w$ $= \dfrac{0.0293}{0.5112} \times 100\% = 5.73\%$

Dry unit weight, $\gamma_d$ $= \dfrac{\gamma}{(1+w)} = \dfrac{23.34}{(1+0.0573)}\ kN/m^3$

$= \mathbf{22.075\ kN/m^3}$

Grain specific gravity, $G$ = 2.72

$$\gamma_d = \frac{G \cdot \gamma_w}{(1+e)} \quad \text{or} \quad 22.075 = \frac{2.72 \times 9.81}{(1+e)}$$

whence, the void-ratio, $e$ $\approx$ **0.21**

Degree of saturation, $S = wG/e$ $= \dfrac{0.0573 \times 2.72}{0.21} = \mathbf{74.2\%}$.

**Example 3.11** A soil sample consists of particles ranging in size from 0.6 mm to 0.02 mm. The average specific gravity of the particles is 2.66. Determine the time of settlement of the coarsest and finest of these particles through a depth of 1 metre. Assume the viscosity of water as 0.001 N-sec/m2 and the unit weight as 9.8 kN/m$^3$.

Solution:

By Stokes' law

$$v = (1/180) \cdot (\gamma_w/\mu_w)(G-1)D^2$$

where $v$ = terminal velocity in cm/sec,

$\gamma_w$ = unit weight of water in kN/m$^3$,

$\mu_w$ = viscosity of water in N-sec/m$^2$,

$G$ = grain specific gravity, and

$D$ = size of particle in mm.

$$\therefore \quad v = \frac{1}{180} \times \frac{9.80}{0.001}(2.66-1)D^2 \approx 90D^2$$

For the coarsest particle, $D$ = 0.6 mm

$v = 90 \times (0.6)^2$ cm/sec. = 32.4 cm/sec.

$t = h/v$ = 100.0/32.4 sec. = 3.086 sec.

For the finest particle, $D$ = 0.02 mm.

$v = 90(0.02)^2$ cm/sec. = 0.036 cm/sec.

$t = h/v = \dfrac{100.000}{0.036}$ sec. = 2777.78 sec. = 46 min. 17.78 sec.

This time of settlement of the coarsest and finest particles through one metre are nearly **4 sec.** and **46 min. 18 sec.** respectively.

**Example 3.12** The liquid limit of a clay soil is 56% and its plasticity index is 15%. (a) In what state of consistency is this material at a water content of 45%? (b) What is the plastic limit of the soil? (c) The void ratio of this soil at the minimum volume reached on shrinkage, is 0.88. What is the shrinkage limit, if its grain specific gravity is 2.71?

Solution:

Liquid limit, $W_L = 56\%$

Plasticity index, $I_p = 15\%$

$I_p = w_L - w_p$, by Eq. 3.37.

$\therefore$ $15 = 56 - w_p$

Whence the plastic limit, $w_p = (56 - 15) = \mathbf{41\%}$

$\therefore$ At a water content of 45%, the soil is in the *plastic state* of consistency.

Void ratio at minimum volume, $e = 0.88$

Grain specific gravity, $G = 2.71$

Since at shrinkage limit, the volume is minimum and the soil is still saturated,

$$e = w_s G$$

or $$w_s = e/G = 0.88/2.71 = 32.5\%$$

$\therefore$ Shrinkage limit of the soil = **32.5%.**

**Example 3.13** A soil has a plastic limit of 25% and a plasticity index of 30. If the natural water content of the soil is 34%, what is the liquidity index and what is the consistency index? How do you describe the consistency?

Solution:

Plastic limit, $w_p = 25\%$

Plasticity index, $I_p = 30$

By Eq. 3.37, $I_p = w_L - w_p$

$\therefore$ Liquid limit, $w_L = I_p + w_p = 30 + 25 = 55\%.$

Liquidity index, $$I_L = \frac{(w - w_p)}{I_p}$$

where $w$ is the natural moisture content.

$\therefore$ Liquidity index, $$I_L = \frac{(34 - 25)}{30} = \mathbf{0.30}$$

Consistency index, $$I_c = \frac{(w_L - w)}{I_p}$$

$\therefore$ Consistency index, $$I_c = \frac{(55 - 34)}{30} = \mathbf{0.70}$$

The consistency of the soil may be described as '*medium soft*' or '*medium stiff*'.

**Example 3.14** A fine grained soil is found to have a liquid limit of 90% and a plasticity index of 50. The natural water content is 28%. Determine the liquidity index and indicate the probable consistency of the natural soil.

Solution:

Liquid limit, $w_L = 90\%$

Plasticity index, $I_p = 50$

$I_p = w_L - w_p$

$\therefore$ Plastic limit, $w_p = w_L - I_p = 90 - 50 = 40\%$

The natural water content, $w = 28\%$

Liquidity index, $I_L$, by Eq. 3.40, is given by

$$I_L = \frac{w - w_p}{I_p} = \frac{28 - 40}{50} = -\frac{12}{50} = -0.24 \text{ (negative)}$$

Since the liquidity index is negative, the soil is in the semi-solid state of consistency and is stiff; this fact can be inferred directly from the observation that the natural moisture content is less than the plastic limit of the soil.

**Example 3.15** A clay sample has void ratio of 0.50 in the dry condition. The grain specific gravity has been determined as 2.72. What will be the shrinkage limit of this clay?

Solution:

The void ratio in the dry condition also will be the void ratio of the soil even at the shrinkage limit: but the soil has to be saturated at this limit.

For a saturated soil,

$$e = wG$$

or $$w = e/G$$

$\therefore$ $$w_s = e/G = 0.50/2.72 = 18.4\%,$$

Hence the shrinkage limit for this soil is **18.4%**.

**Example 3.16** The following are the data obtained in a shrinkage limit test:

Initial weight of saturated soil = 0.956 N

Initial volume of the saturated soil = 68.5 cm$^3$

Final dry volume = 24.1 cm$^3$

Final dry weight = 0.435 N

Determine the shrinkage limit, the specific gravity of grains, the initial and final dry unit weight, bulk unit weight, and void ratio.

Solution:

From the data,

Initial water content, $w_i = \frac{(0.956 - 0.435)}{0.435} \times 100 = 119.77\%$

the shrinkage limit is given by

$$w_s = \left[w_i - \frac{(V_i - V_d)}{W_d} \cdot \gamma_w\right] \times 100$$

$$= \left[1.1977 - \frac{(68.5 - 24.1)}{0.435} \times 0.01\right] \times 100 = \mathbf{17.70\%}$$

Final dry unit weight $= \frac{0.435}{24.1}$ N/cm$^3$ = **17.71 kN/m$^3$**

Initial bulk unit weight $= \frac{0.956}{68.5}$ N/cm$^3$ = **13.70 kN/m$^3$**

Grain specific gravity $= \dfrac{1}{\left(\dfrac{\gamma_w}{\gamma_d} - \dfrac{w_s}{100}\right)} = \dfrac{1}{\left(\dfrac{9.81}{17.71}\right) - \left(\dfrac{17.70}{100}\right)} = \mathbf{2.65}$

Initial dry unit weight $= \dfrac{\gamma_{d_i}}{(1 + w_i)} = \dfrac{13.70}{(1 + 1.1977)}$ = **6.23 kN/m$^3$**

Initial void ratio = $w_i G = 1.1977 \times 2.65 = \mathbf{3.17}$

Final void ratio = $w_s G = 0.1770 \times 2.65 = \mathbf{0.47}$.

**Example 3.17** The Atterberg limits of a clay soil are: Liquid limit = 75%; Plastic limit = 45%; and Shrinkage limit = 25%. If a sample of this soil has a volume of 30 $cm^3$ at the liquid limit and a volume 16.6 $cm^3$ at the shrinkage limit, determine the specific gravity of solids, shrinkage ratio, and volumetric shrinkage.

Solution:

The phase diagrams at liquid limit, shrinkage limit, and in the dry state are shown in Fig.

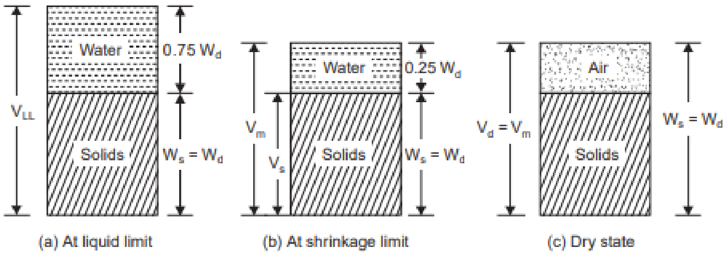


**Fig.** . Phase diagrams of the clay soil

Difference in the volume of water at LL and SL = (30 – 16.6) $cm^3$ = 13.4 $cm^3$

Corresponding difference in weight of water = 0.134 N

But this is (0.75 – 0.25) $W_d$ or 0.50 $W_d$ from Fig.

$\therefore \quad 0.50\,W_d = 0.134$

$W_d = 0.268$ N

Weight of water at SL = 0.25 $W_d$ = 0.25 × 0.268 = 0.067 N

$\therefore$ Volume of water at SL = 6.7 $cm^3$

Volume of solids, $V_s$ = Total volume at SL – volume of water at SL.

= (16.6 – 6.7) $cm^3$ = 9.9 $cm^3$.

Weight of solids, $W_d = 0.268$ N

$$\therefore \quad \gamma_s = \frac{0.268}{9.9} \text{ N/cm}^3 = 0.027 \text{ N/cm}^3 = 27 \text{ kN/m}^3$$

$$\therefore \text{Specific gravity of solids} = \frac{\gamma_s}{\gamma_w} = \frac{27}{9.81} = 2.71$$

$$\text{Shrinkage ratio,} \quad R = \frac{W_d}{V_d} = \frac{26.8}{16.6} = \mathbf{1.61}$$

Volumetric shrinkage, $V_s = R(w_i - w_s) = R(w_L - w_s)$, here

$= 1.61\,(75 - 25) = \mathbf{80.5\%}$

**Example 3.18** The mass specific gravity of a saturated specimen of clay is 1.84 when the water content is 38%. On oven drying the mass specific gravity falls to 1.70. Determine the specific gravity of solids and shrinkage limit of the clay.

Solution:

For a saturated soil,

$$e = w.G$$

$$\therefore \quad e = 0.38\,G$$

Mass specific gravity in the saturated condition

$$= \frac{\gamma_{sat}}{\gamma_w} = \frac{(G+e)}{(1+e)} = \frac{(G+0.38\,G)}{(1+0.38\,G)}$$

$$\therefore \quad 1.84 = \frac{1.38\,G}{1+0.38\,G}$$

whence $G = 2.71$

$\therefore$ Specific gravity of the solids = **2.71**

the shrinkage limit is given by

$$w_s = \left(\frac{\gamma_w}{\gamma_d} - \frac{1}{G}\right) \times 100$$

where $\gamma_d$ = dry unit weight in dry state.

But, $\gamma_d$ = (mass specific gravity in the dry state) $\gamma_w$

$= 1.70\,\gamma_w$

$$\therefore \quad w_s = \left(\frac{\gamma_w}{1.70\,w} - \frac{1}{G}\right)100 = \left(\frac{1}{1.70} - \frac{1}{2.71}\right)100$$

$= 21.9\%$

$\therefore$ Shrinkage limit of the clay = **21.9%.**

**Example 3.19** A saturated soil sample has a volume of 23 cm$^3$ at liquid limit. The shrinkage limit and liquid limit are 18% and 45%, respectively. The specific gravity of solids is 2.73. Determine the minimum volume which can be attained by the soil.

Solution:

The minimum volume which can be attained by the soil occurs at the shrinkage limit. The phase diagrams of the soil at shrinkage limit and at liquid limit are shown in Fig.

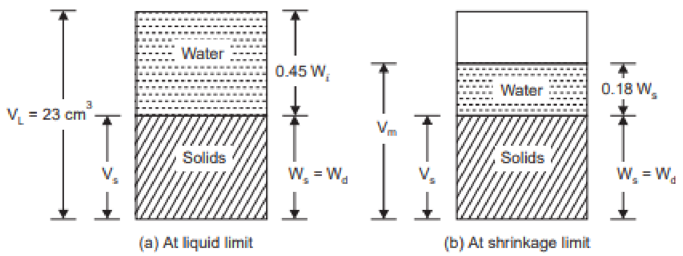


**Fig.** Phase diagrams (Example 3.20)

At liquid limit,

Volume of water = 45 $W_s$ cm$^3$, if $W_s$ is the weight of solids in N.

$$\text{Volume of solids} = \frac{W_s}{G\,\gamma_w} = \frac{W_s}{9.81 \times 10^{-3}} = \frac{1000}{9.81}\,W_s\ \text{cm}^3$$

$$\text{Total volume} = \frac{1000}{9.81}\,W_s + 45\,W_s = 23$$

whence $W_s = 0.2818$ N

At shrinkage limit,

the volume, $V_m = V_s + 0.18\,W_s$

$$= \left(\frac{0.2818}{0.0273} + 18 \times 0.2818\right)\text{cm}^3$$

$= \mathbf{15.4}$ cm$^3$.

**Example 3.20** The liquid limit and plastic limit of a clay are 100% and 25%, respectively. From a hydrometer analysis it has been found that the clay soil consists of 50% of particles smaller than 0.002 mm. Indicate the activity classification of this clay and the probable type of clay mineral

Solution:

Liquid limit, $w_L = 100\%$

Plastic limit, $w_p = 25\%$

Plasticity Index, $I_p = (w_L - w_p)$

$= (100 - 25)\% = 75\%$

Percentage of clay-size particles = 50

Activity, $A = \frac{I_p}{c}$

where $c$ is the percentage of clay-size particles.

$\therefore$ $A = 75/50 = 1.50$

Since the activity is greater than 1.25, the clay may be classified as being **active**.

The probable clay mineral is **montmorillonite**.

**Example 3.21** A clay soil sample has been obtained and tested in its undisturbed condition. The unconfined compression strength has been obtained as 200 kN/m$^2$. It is later remoulded and again tested for its unconfined compression strength, which has been obtained as 40 kN/m$^2$. Classify the soil with regard to its sensitivity and indicate the possible structure of the soil.

Solution:

Unconfined compression strength in the undisturbed state, $q_{uu} = 200$ kN/m$^2$

Unconfined compression strength in the remoulded state, $q_{ur} = 40$ kN/m$^2$

Sensitiviy, $S_t = \frac{q_{uu}}{q_{ur}} = \frac{200}{40} = \mathbf{5}$

Since the sensitivity falls between 4 and 8, the soil may be soil to be "**sensitive**" . The possible structure of the soil may be '**honeycombed**' or '**flocculent**'.

**Example 3.22:** A soil sample has a liquid limit of 20% and plastic limit of 12%. The following data are also available from sieve analysis:

| Sieve size | % Passing |
|---|---|
| 2.032 mm | 100 |
| 0.422 mm | 85 |
| 0.075 mm | 38 |

Classify the soil approximately according to Unified Classification or IS Classification.

Solution:

Since more than 50% of the material is larger than 75-µ size, the soil is a coarse-grained one.

100% material passes 2.032 mm sieve; the material, passing 0.075 mm sieve is also included in this. Since this latter fraction, any way passes this sieve, a 100% of coarse fraction also passes this sieve.

Since more than 50% of coarse fraction is passing this sieve, it is classified as a sand. (This will be the same as the per cent passing 4.75 mm sieve).

Since more than 12% of the material passes the 75-μ sieve, it must be SM or SC.

Now it can be seen that the plasticity index, $I_p$, is (20 – 12) = 8, which is greater than 7.

Also, if the values of $w_L$ and $I_P$ are plotted on the plasticity chart, the point falls above A-line. Hence the soil is to be classified as SC, as per IS classification.

Even according to Unified Classification System, this will be classified as SC, which may be checked easily.