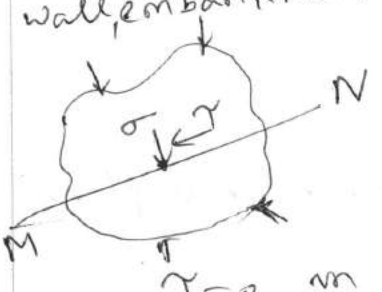
Shear strength of soil — Module IV

Shear strength of soil is the maximum resistance to shear stress just before the failure. Soils are very less subjected to direct shear but the shear stress is develop within the soil mass is due to direct compression.

- The failure of the soil mass occurs when the shear stress induced due to application of compressive loads which exceeds the shear strength of soil mass, the failure of the soil is due to the relative moment of soil particle over each other.
- The shear strength is develop within the body due to following reasons.
- Due to frictional resistance beween the particle in the soil mass (It is the combination of sliding friction & Rolling friction)
- Cohesion force between the particle or attraction between the particle.
- The structural resistance for displacement due to interlocking between soil particles

* Shear strength of a soil is the engineering property like permeability & compressibility of soil. Shear strength gives an idea about the stability of soil which further helps to design the foundation of a structure, retaining wall, embankment made using earth.



σ or σ_n = Normal stress

τ = Shear stress

3 principal planes ⊥lar to eachother

3 principal stresses ⊥lar to eachother

$\tau = 0$ in principal plane

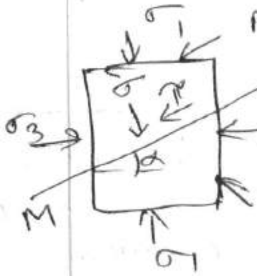
$\sigma_1, \sigma_2, \sigma_3 \rightarrow$ Principal stresses

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

σ_1 = Major principal stress

σ_2 = Intermediate principal stress

σ_3 = Minor principal stress



Analytical

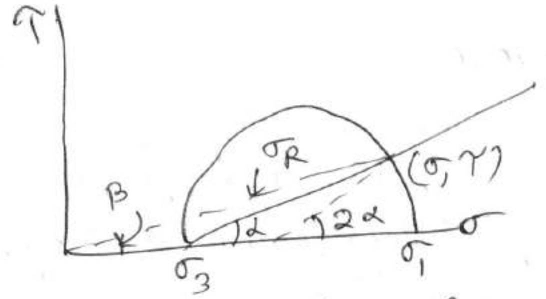
$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\alpha$$

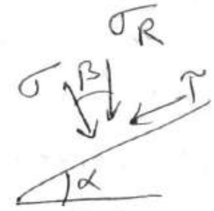
$$\tau = \frac{\sigma_1 - \sigma_3}{2}\sin 2\alpha$$

Graphical (Mohr Circle)

σ_R = Resultant stress $= \sqrt{\sigma^2 + \tau^2}$

β = angle of obliquity (angle between σ & σ_R)





Pole (origin of planes)

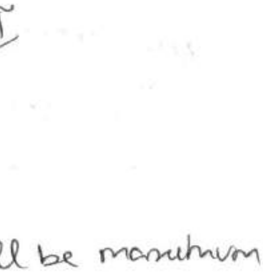
For failure plane angle of obliquity will be maximum

$S \rightarrow$ shear strength

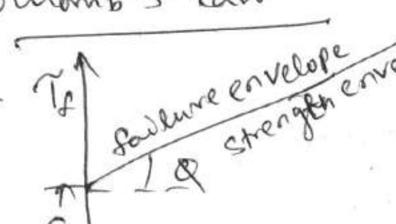
$\tau \rightarrow$ shear stress

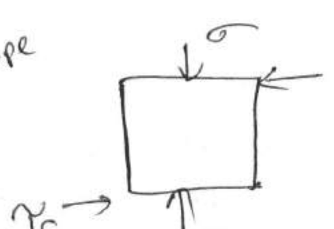
$\tau_f \rightarrow$ failure shear stress

If $\tau < S$ no shear failure $\boxed{\tau_f = S}$



Coulomb's law





Coulomb's eqn $\boxed{S = C + \sigma \tan\phi}$

σ = Normal stress on the plane

c = cohesion

ϕ = angle of internal friction

Angle of Repose :- Natural slope of a soil heap.

Angle of repose & angle of shearing resistance are not equal to each other. However for a loose sand, these two are nearly equal.

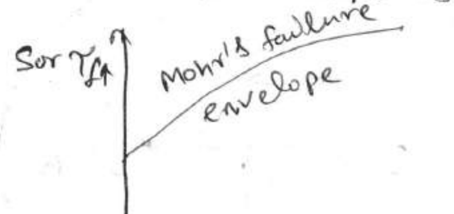
Angle of shearing resistance :- Angle of inclination of failure envelope.

Mohr's theory :- Soil fails essentially due to shear

- Failure shear stress depends on σ
- σ_2 has no effect on the behaviour of soil

$$S = f(\sigma)$$

- As per Mohr's theory Mohr's failure envelope is Curvilinear



$S = c + \sigma \tan\phi$
- general equation not used nowadays

$S = c' + \sigma' \tan\phi'$
- in terms of effective stress
- c' & ϕ' are called effective shear strength parameters
- used for drained conditions of soil

$S = C_u + \sigma \tan\phi_u$
- in terms of total stress
- ϕ_u & C_u are called total shear strength parameters
- Used for undrained conditions of soil.

Mohr circle :- He has developed a graphical method for the determination of stresses acting over a principal plane.

Sign convention for compressive force (+ve)

- It is represented right side of the x-axis or positive x-axis
- Shear stress → +ve
- It represents at the upper side of y-axis or +ve y direction
- Horizontal axis represents the normal stress
- Vertical axis represents shear stress.

Figure in back side of this page

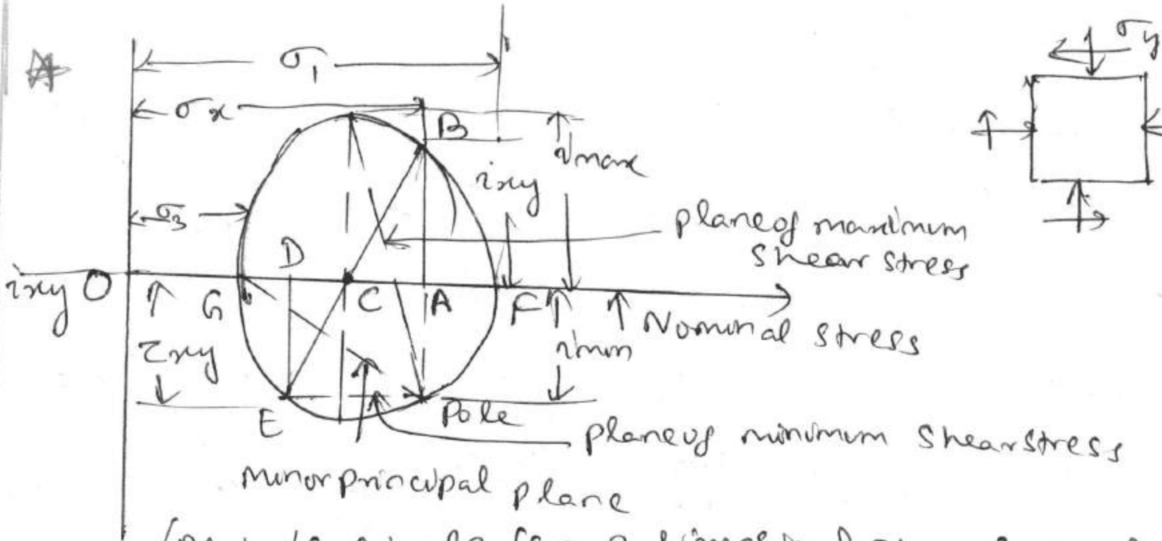
Terzaghi's effective Stress Principle

According to this theory the shear strength of soil in case of saturated soil is the function of the effective stress which is responsible for the failure. So, he rectify the failure as

$$S = c' + \bar{\sigma} \tan \phi'$$

c' & ϕ' refers to the cohesion & angle of internal friction. Corresponding to effective stress value or effective shear strength parameter.

c & ϕ refers to total strength parameter.



(Mohr's circle for 2 dimensional stress system)

Mohr-Coulomb failure theory :-

This theory was first proposed by coulomb (1776) and later generated by Mohr.

Mohr (1900) failure theory is based on the hypotheses that material fail

$$\tau_f = F(\sigma) \quad \text{or} \quad S = f(\sigma)$$

Coulomb assumed the relation between τ_f & σ to be linear and give

$$\tau_f = C + \sigma \tan\phi$$

Coulomb observed that one component of the Shearing strength, called the intrinsic Cohesion (apparent cohesion) is constant for a given soil and is independent of the applied stress. The other component, namely the frictional resistance some what similar to sliding friction in solids, varies directly as the magnitude of the normal stress on the plane of rupture.

where τ_f = shear strength of soil.

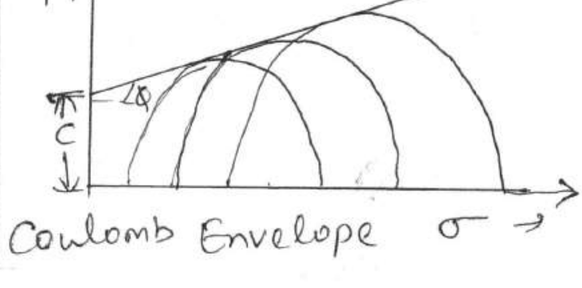
C = apparent cohesion

σ = Normal stress on the plane of rupture

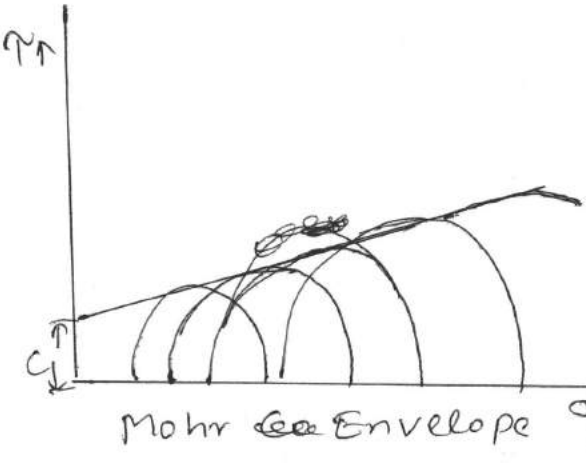
ϕ = Angle of internal friction.

where C & ϕ are referred as shear strength parameters

Mohr generalized the shear envelope also known as failure envelope, as a curve which becomes f latter with increasing normal stress as shown in figure

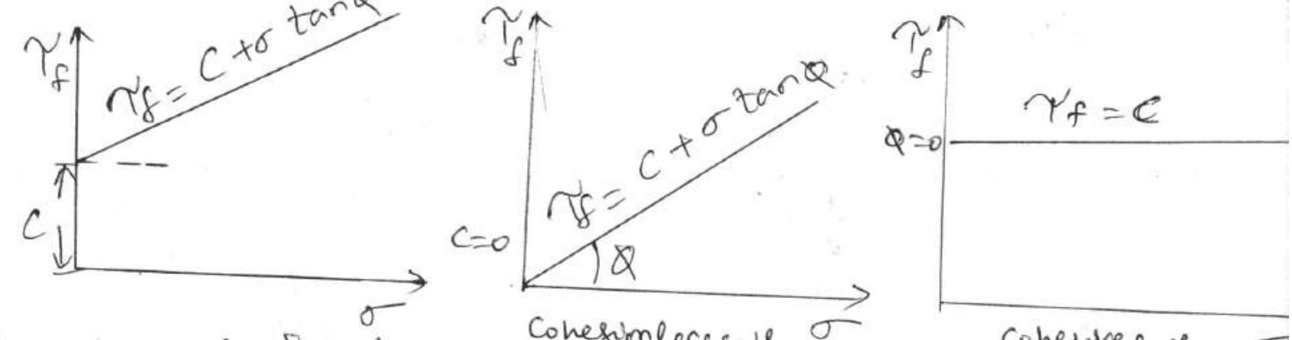


Coulomb Envelope $\sigma \rightarrow$



Mohr Envelope $\sigma \rightarrow$

Based on the values of shear strength parameters soils can be described as (i) cohesive soil (C − Φ)
(ii) Cohesionless soil (C = 0)
(iii) Purely cohesive soil (Φ = 0)

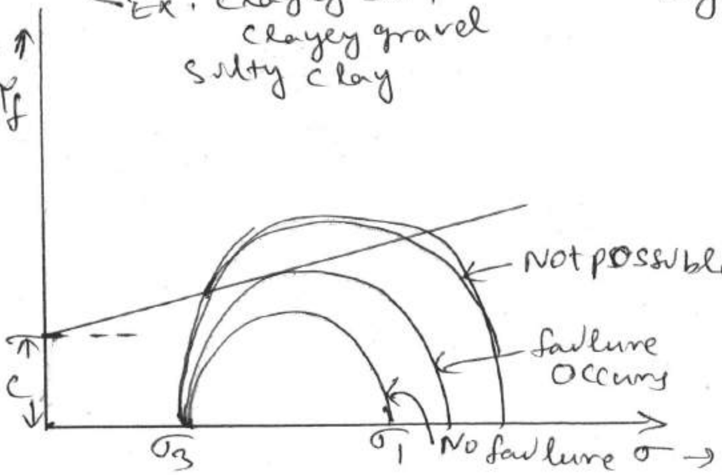


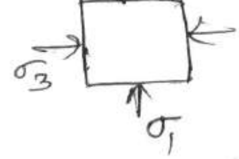
Case (a) C − Φ soil — Ex. clayey sand, clayey gravel, silty clay

Cohesionless soil, Case (b) C = 0 — exa. Dry sand

Cohesive soil, Case (c) Φ = 0 — Ex plastic clay

Strength envelope for the three types of soils.





α_f = failure plane inclination with major principal plane

$\alpha_f = 45° + \Phi/2$

$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$; $\tau_f \le \tau_{max}$

Theory of failure in soil :

- There are many theories develop on the failure of soil mass.
- Mohr's , Coulomb theory has well defined the failure of soil mass hence it is accepted by the soil engineer.
- This theory contains following things :-
1. The failure due to the failure of material in shear
2. The ultimate shear stress depends on the normal stress acting on the failure plane & properties of material
3. In case of 3dimensional system the criteria for failure is independent of σ_2

Drainage Condition :-

1. Unconsolidated Undrained test (UU)
2. Consolidated undrained test (CU)
3. Consolidated drained test (CD)

UU Test :-

- It is a quick test
- 1st stage of the test soil is in unconsolidated test which is called confining test.
- 2nd stage of the test is known as shear stage where the soil is undrained .
- This kind of drainage condition is provided for saturated clay

CU Test :-

- It is suitable for investigating the earth dam case which may fail due to the sudden draw down in it.
- In this test the soil sample is consolidated initially by the application of stress on it.
- Then this sample is taken for the direct shear stress.
- In case of triaxial test the specimen is allowed to consolidated fully under the application of load over it.

CD Test

- The drainage valves are opended through out the experiment this test is very time taking test.
- This type of test is normally conducted over clay & very fine sand.

Note: Consolidation stage 

Stages of shear test :-

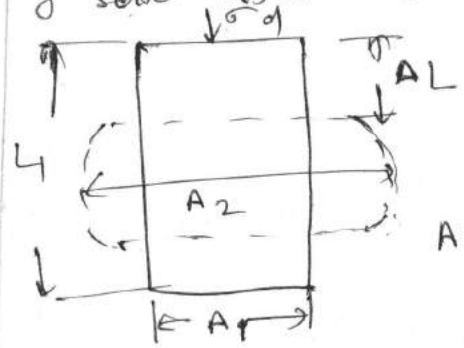
- 1st stage or consolidation stage (application of σ_c)
- 2nd stage or shearing stage (application of σ_d)

When the valve of drain pipe is opened, when σ_c is applied (1st stage), pore water escapes and soil gets consolidated otherwise it remains unconsolidated.

When the valve is open during application of σ_d, drained condition is obtained

* UU Test - Drain valve is always kept closed (quick test)
* CU Test - Valve is open in 1st stage & closed in 2nd stage
* CD test - valve is always kept open (slow test)

- c & φ are shear strength parameters & not the property of soil & it varies with drainage conditions & test performed

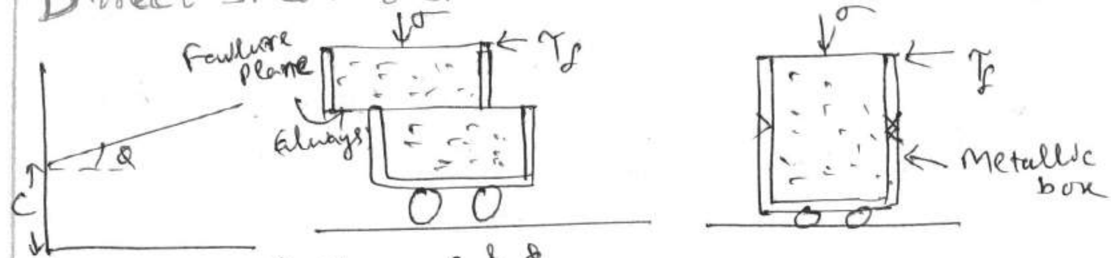


$$\sigma_d = \frac{\text{axial load}}{\text{area at failure}} = \frac{Q_a}{A_2}$$

$$A_2 = \frac{V_1 \pm \Delta V}{L_1 - \Delta L}$$

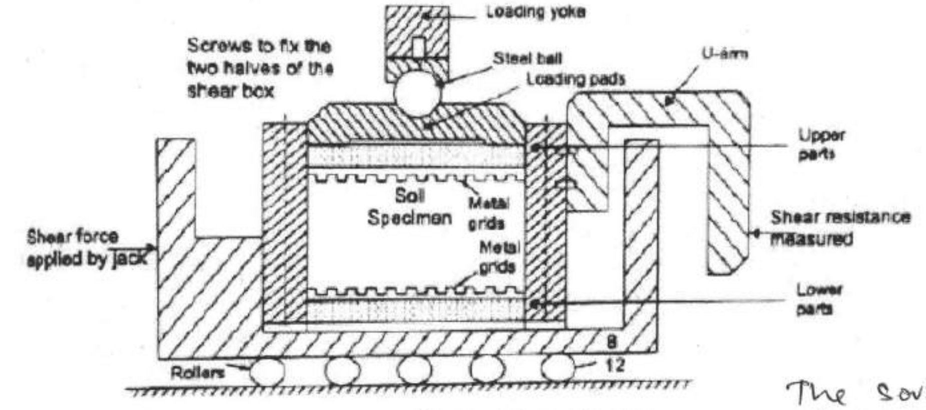
$\Delta V = 0$, undrained test

Direct shear test:-



To find analytically c & φ

$\tau_f = c + \sigma \tan\phi$



Parts of direct shear box

This test is generally used for cohesionless soils.

The soil specimen is taken of dimension 60 x 60x25 mm

Strain controlled direct shear machine consists of shear box, soil container, loading unit, proving ring, dial gauge to measure shear deformation and volume changes. A two piece square shear box is one type of soil container used.

A proving ring is used to indicate the shear load taken by the soil initiated in the shearing plane.

Procedure:

The soil specimen used in the test is usually of size 60mmx60mm

1. This is a simple and very commonly used test which is performed in a shear box apparatus.
2. A shear box apparatus which consists of brass box square or circular in cross section.
3. The box is split horizontally into two parts which are held together with the help of two screws.
4. The joint between the two parts of the box is at the level of the centre of soil sample.

5. The lower part of the box is rigidly held in position in a container which rests over slides or rollers and can be pushed forward at a constant rate by a general jack driven by electric motor by hand

6. The upper part of the box butts against a calibrated steel proving ring for measuring the shear force.

7. The soil sample is compacted in the shear box by clamping both the parts together with the help of two screws and is held between the metal grids and porous plates placed above the top and below the bottom of the soil sample.

8. For drainage tests perforated metal grids are used and for undrained tests solid metal grids are used.

9. The metal grids have linear slots or serrations to have proper grip with the soil sample and are so oriented that the serrations are perpendicular to the direction of the shearing force applied to the soil sample.

10. Normal load is applied on the soil sample from a loading yoke through a steel ball, bearing upon a metal pressure pad. The pressure pad fits into the shear box over the upper porous plate.

11. The screws clamping the two parts of the shear box are removed and a shear force is applied to the lower part of the box through the geared jack and the movement of the lower part of the box is transmitted through the soil sample to the upper part of the box and hence to the proving ring.

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12. The deformation of the proving ring indicates the magnitude of the applied shear force which is gradually increased until the soil sample fails.

13. A number of identical soil samples are tested under increasing normal loads and the required maximum shear force is recorded.

14. From the load and cross sectional area of the sample the corresponding normal and shear stresses are calculated.

15. A graph is plotted between the normal stress as abscissa and the shear stress as ordinate. Such a plot gives failure envelope for the soil under the given test conditions.

Advantages :-

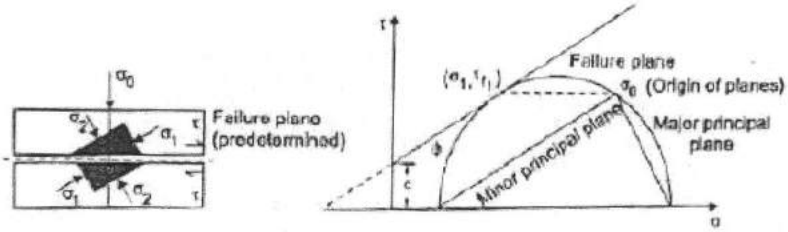
1. It is easy to test sands and gravels.
2. Large samples can be tested in large shear boxes, as small samples can give misleading results due to imperfections such as fractures and fissures, or may not be truely representative.
3. Samples can be sheared along predetermined planes, when the shear strength along fissures or other selected planes are needed.

Disadvantages:-

1. The failure plane is always horizontal in the test and thus may not be the weakest plane in the sample. Failure of the soil occurs progressively from the edges towards the centre of the sample.
2. There is no provision for measuring pore water pressure in the shear box and so it is not possible to determine effective stresses from undrained tests.
3. The shear box apparatus cannot give reliable undrained strengths because it is impossible to prevent localized drainage away from the shear plane.

Mohr's circle representation of stress conditions:-

In the direct shear test the failure plane is predetermined and it is horizontal. The figure shows the stress conditions during failure.



Conditions of stress in the shear box

Fig. 13.7 (b) Mohr's circle for direct shear test

In order to find the direction of principal planes at failure Mohr's circle is drawn such that the failure envelope is tangential to the Mohr's circle. which represents the failure condition for a particular normal stress.

Limitations:-

1. The stress conditions are complex primarily because
of the nonuniform distribution of normal and shear
stresses on the plane.
2. There is virtually no control of the drainage
of the soil specimen as the water content of a saturated
soil changes rapidly with stress.
3. The area of the sliding surface at failure will be
less than the original area of soil specimen and strictly
speaking this should be accounted for.
4. The ridges of the metal gratings embedded on the
top & bottom of the specimen causes distortion of the
specimen to some degree.
5. The effect of lateral restraint by the side walls of
the shear box is likely to affect the results.
6. The failure plane is predetermined and this may not
be the weakest plane. In fact this is the most
important limitation of the direct shear test.

Advantages

1. The direct shear test is a simple test compared to the triaxial compression test.
2. since the thickness of the sample is small quick drainage.

Disadvantages of Direct Shear test

1. ~~The~~ shear stress is not uniformly distributed being more at the edge than at the center.
2. The failure plane is predetermined
3. shear displacement causes reduction in area under shear

⇒ ~~The~~

Advantages of Triaxial compression test :

1. The specimen is free to fail along the weakest plane unlike in the direct shear test
2. The stress distribution on the failure plane is uniform.
3. There is a complete control of drainage conditions.
4. precise measurement of pore pressure and volume change are possible during the test
5. The stress induced on any plane within the specimen at any stage of the test can be determined.
6. Failure occurs near the middle of sample.

Modified failure envelope :-

The following relation between shear strength parameter and principal stress at failure.

$$\frac{\sigma_1' - \sigma_3'}{2} = \frac{\sigma_1' + \sigma_3'}{2} \sin\phi + c' \cos\phi$$

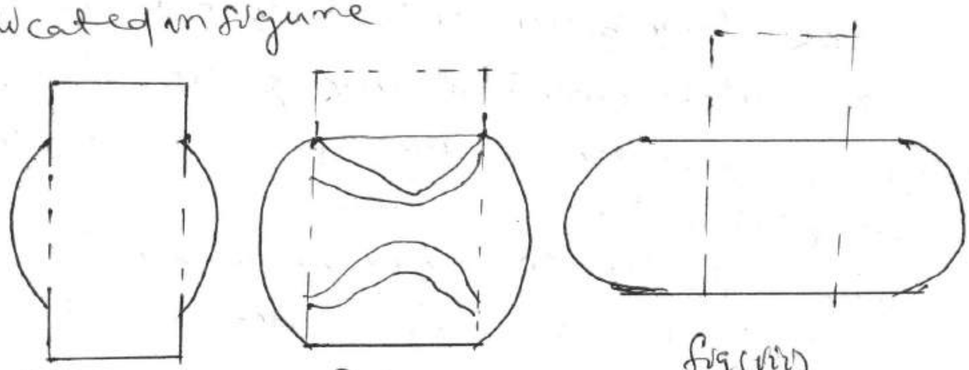
$$\Rightarrow \frac{\sigma_1' - \sigma_3'}{2} = \frac{\sigma_1' + \sigma_3'}{2} \tan\psi + a'$$

$$a' = c' \cos\phi \quad \& \quad \tan\psi = \sin\phi'$$

$$\phi = \sin^{-1}(\tan\psi) \qquad c' = \frac{a'}{\cos\phi'}$$

Type of failure of soil specimen in the triaxial test :-

Depending on the soil type & its physical properties a soil specimen can exhibit one of the three failure patterns indicated in figure



fig(v) fig(vi) fig(vii)

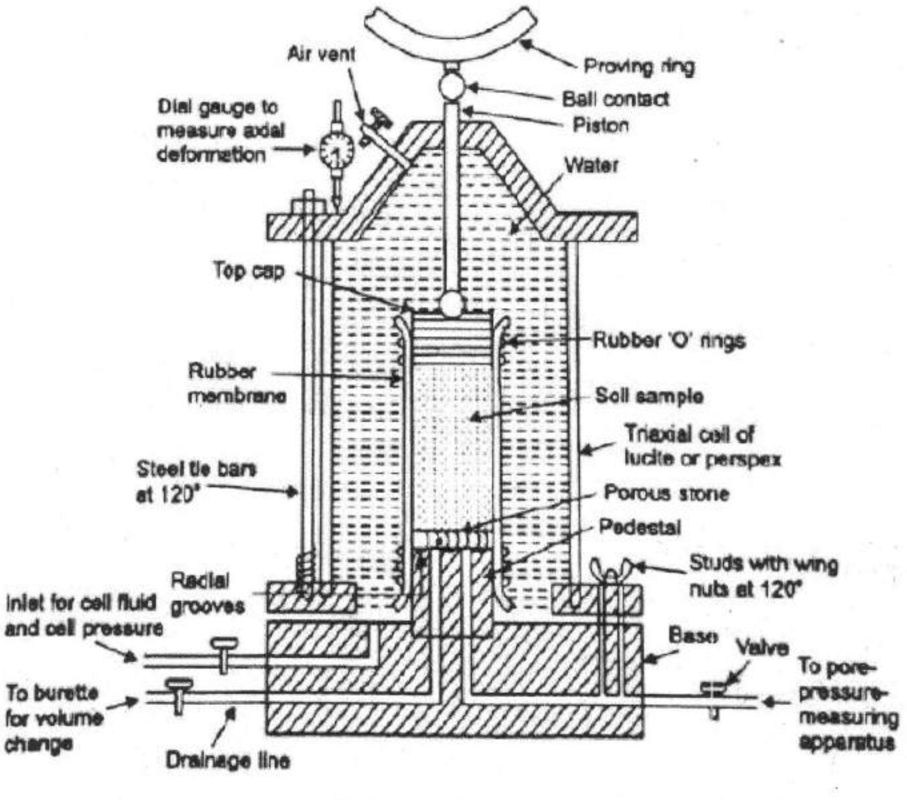
Fig(v) Brittle failure with a well defined failure

In fig(vi) is shown a semi plastic failure with shear cones and some lateral bulging

In fig(vii) is a typical plastic failure with excessive lateral bulging and absence of failure plane

Triaxial Compression Test :-

Triaxial test is more reliable because measuring shearstrength. The test is called triaxial because three principal stresses are assumed to be known and are controlled.



Triaxial cell with accessories

Three principal stresses are applied to the soil sample out of which two are applied water pressure inside the confining cell and are equal. The third principal stress is applied by a loading ram through the top of the cell and is different to the other two principalstresses.

The triaxial compression test consists of two stage.

First stage:- In this a soil sample is set in the triaxial cell and confining pressure is then applied.

Second Stage. In this additional axial stress (deviator stress) is applied which induces shear stresses in the sample. The axial stress is continuously increased Until the sample fails.

During both the stages, the applied stages, axial strain, and pore water pressure or change in sample volume can be measured.

Apparatus required:

1. A constant rate of strain compression machine of which the following is a brief description of one is in common use.

(a) A loading frame in which the load is applied by yoke acting through an elastic dynamometer, more commonly called a proving ring which used to measure the load. The frame is operated at a constant rate by a small electric motor.

(b) A hydraulic pressure apparatus including an air compresser and water reservoir in which air under pressure acting on the water raises it to the required pressure, together with the necessary control valves and pressure dials.

2. A triaxial cell to take 3.8 cm dia and 7.6 cm long samples. in which the sample can be subjected to an all round hydrostatic pressure, together with a vertical compression load acting through a piston. The vertical load from the piston acts on a pressure cap. The cell is usually designed with a nonferrous metal top and base connected by tension rods and with walls formed of perspex. These connections are generally provided through the base for the cell fluid inlet, pore water outlet from the bottom of the soil sample and the drainage outlet from the top of the soil sample and sometimes this drainage outlet may be omitted in smaller cells.

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In the top cap there is an air release valve is kept open during filling of the cell with water.

(b) General

(i) 3.8 cm internal diameter 12.5 cm long sample tubes. The soil specimen used in the test is cylindrical shape with length 2 to 2.5 times the diameter.

(ii) Rubber rings.

(iii) An open ended cylindrical section former 3.8 cm inside diameter fitted with a small rubber tube in its side.

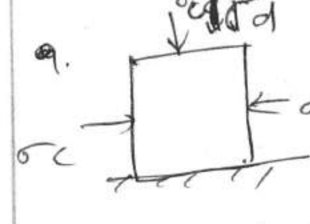
Procedure:-

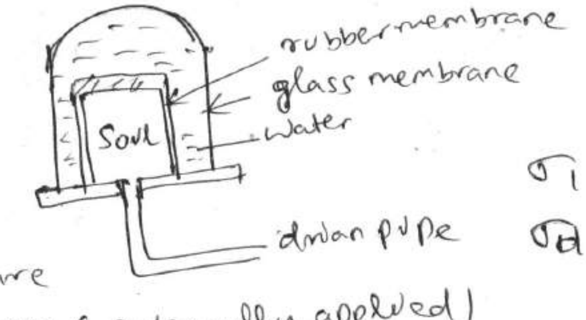
1. Generally 38 mm dia. & 76 mm height specimen is used.
2. Specimen is encased by a thin rubber membrane and set into a plastic cylindrical chamber.
3. Depending upon the drainage conditions of the test porous or non porous discs are placed on the top and bottom ends of the soil sample and the rubber membrane is sealed onto these ends by rubber rings.
4. The triaxial cell is filled with water and pressure is applied to the water, which is turn is transmitted to the soil sample all round and at the top. The pressure is called "cell pressure" "confining pressure" or "chamber pressure".
5. The vertical axial compressive load called deviator stress is applied on the soil sample by a stainless steel piston passing through the centre of the top cap.
6. Additional axial stress is applied while keeping the cell pressure constant. This introduces shear stresses on all planes except the horizontal and the vertical planes.

7. The additional axial stress is continuously increased until the failure of the soil sample occurs.

8. The vertical axial load at any stage can be observed from the proving ring.

9.





$\sigma_1 = \sigma_d + \sigma_3$

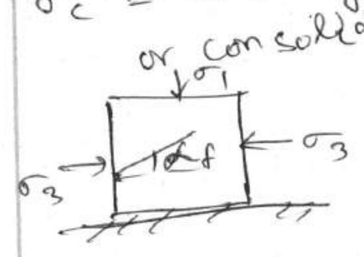
$\sigma_d = \sigma_1 - \sigma_3$

σ_c = Confining pressure

σ_d = deviator stress (externally applied)

water and wind always exerts normal force

σ_c = confining pressure or cell pressure or chamber pressure or consolidation pressure or allround pressure



$\sigma_3 = \sigma_c$

$\sigma_1 = \sigma_c + \sigma_d$

$\sigma_2 = \sigma_c$

σ_3	σ_1	u
10	19	5
20	36	9
30	54	13
40	70	18

Keeping σ_3 constant, determine the deviator stress required to cause shear failure

Using σ_d, determine σ_1

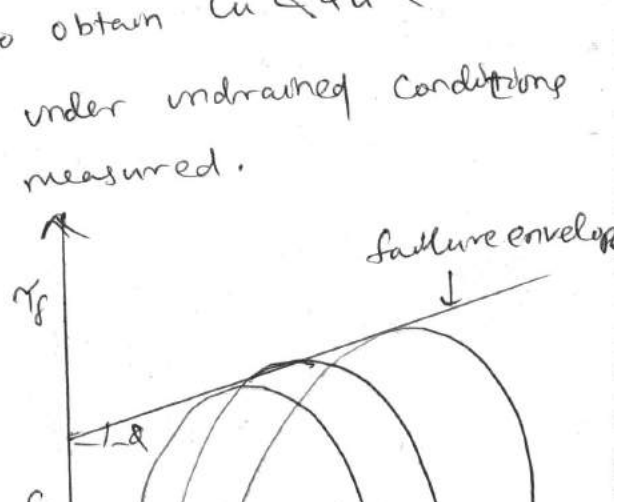
- Open the valve of drain pipe to drain out pore water which gives c' & ϕ'

- Close the drain pipe valve to obtain c_u & ϕ_u (undrained)

when the test is performed under undrained conditions pore water pressure can be measured.

$\sigma_3' = \sigma_3 - u$ & $\sigma_1' = \sigma_1 - u$

- Using σ_3' & σ_1' draw the effective stress circles. A failure envelope tangential to the effective stress circles is drawn & c' & ϕ' are obtained



Total stress circle

- Pore pressure developed in the case of drained test is zero

Plastic equilibrium:- A material is said to be in plastic equilibrium if every point of soil is at verge of failure

Plastic equilibrium equations

$$\sigma_1 = \sigma_3 \tan^2(45 + \frac{\phi_u}{2}) + 2 c_u \tan(45° + \phi_u/2)$$

$$\sigma_1' = \sigma_3' \tan^2(45° + \frac{\phi'}{2}) + 2c' \tan(45 + \frac{\phi'}{2})$$

Direct shear test	Triaxial test	Unconfined Compression test	Vane shear Test
1. Normally drain test is done 2. Sand or fine grain soils are also taken. (not clay) 3. The failure plane is predetermined 4. The Mohr circle cannot be drawn. 5. It is also called box shear test	1. All the 3 types of drainage condition can be maintained in this test 2. It is applicable for all the type of soil 3. The failure plane is determined after the test 4. The Mohr circle can be drawn.	1. Undrained condition is taken only 2. It is also applicable for all type of soil 3. The failure plane is predetermined 4. Mohr circle can be drawn	1. Undrained condition is taken only 2. Clay, very soft clay also 3. Predetermined failure plane 4. Mohr circle can not be drawn

The deviator stress σ_d at any stage of the test is given by

$$\sigma_d = \frac{F}{A_c}$$

where F = deviator force i.e. additional axial force applied through plunger

A_c = Corrected area of C/S of the specimen at that stage

A_i = Initial area of C/S of specimen

L_i = Initial length of specimen

A_c = corrected area of specimen

Initial volume $V_i = A_i L_i$ and

Volume at any stage of compression

$$(V_i + \Delta V) = A_c(L - \Delta L)$$

$$A_c = \frac{V_i + \Delta V}{L - \Delta L}$$

In the case of undrained test on saturated soil sample $\Delta V = 0$ and

$$A_c = \frac{A_i L_i}{L_i - \Delta L} = \frac{A_i}{1 - \frac{\Delta L}{L_i}} = \frac{A_i}{1-\epsilon}$$

Where ϵ is the axial strain at that stage

After finding deviator stress σ_d at failure major principal stress at failure $\sigma_1 = \sigma_d + \sigma_3$ with this set (σ_1, σ_3) value Mohr circle at failure is drawn. The test is conducted on preferably a minimum of three specimen subjected to different values of cell pressure σ_3.

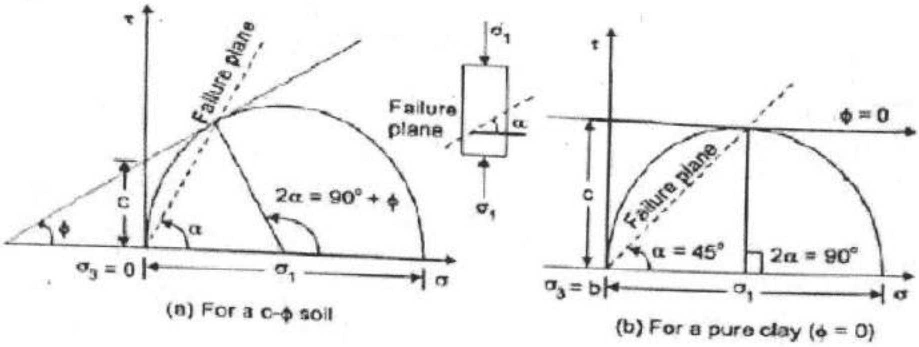
Mohr circle at failure is drawn for each specimen and the common tangent touching all the circle will be failure envelope.

c & ϕ are read out from the plot.

In the triaxial compression test on cylinder soil specimen we have $\sigma_2 = \sigma_3$. Special triaxial cell has been developed to test soil specimen cubical in shape with separate chambers for applying σ_2 & σ_3 separately.

Unconfined Compressive Strength:

The unconfined compressive strength (q_u) is defined as the compressive stress at which an unconfined cylindrical specimen of soil will fail in a simple compression test. The soil sample is not confined laterally and the confining pressure is zero.



$\sigma_2 = \sigma_3 = 0$

Equipment :-

- Compression apparatus with loading frame
- Load & deformation dial gauges
- Sample trimming equipment

Procedure:-

1. Soil sample of size 3.8 cm dia & 7.6 cm long is used.
2. Carefully place the specimen in the compression device between two metal cones attached of two horizontal plates.
3. The upper plate being fixed and the lower one sliding on vertical rods, The sample is placed and center it on the bottom plate
4. Adjust the device so that the upper plate just makes contact with the specimen and set the load and deformation dials to zero.

5. When the test starts the spring extends and lifts up the supporting plate as well as the lower moving plate, thus a compressive load is applied on the soil sample.

6. The compressive load is observed from the calibrated proving ring attached in the loading frame.

7. The vertical axial compression of the soil sample is observed from the dial gauge attached to upper plate which is movable one.

8. The dial gauge reading provides the deformation in the sample and in turn the strains.

9. The proving ring reading provides the corresponding load in turn the axial stress on the sample.

10. Keep applying the load until the load (dial) decreases on the specimen significantly, or the load hole constant for at least four deformation dial readings.

11. Plot graph between axial stress and axial strain. Obtain the peak stress from the graph. This stress is the unconfined compressive strength of the soil.

The unconfined compressive strength is defined as the ratio of axial failure load to cross sectional area of the soil sample when it is not subjected to any lateral pressure.

ΔL = Change in length of sample

L_o = initial length of the sample

$$q_u = \frac{P}{A_c}$$

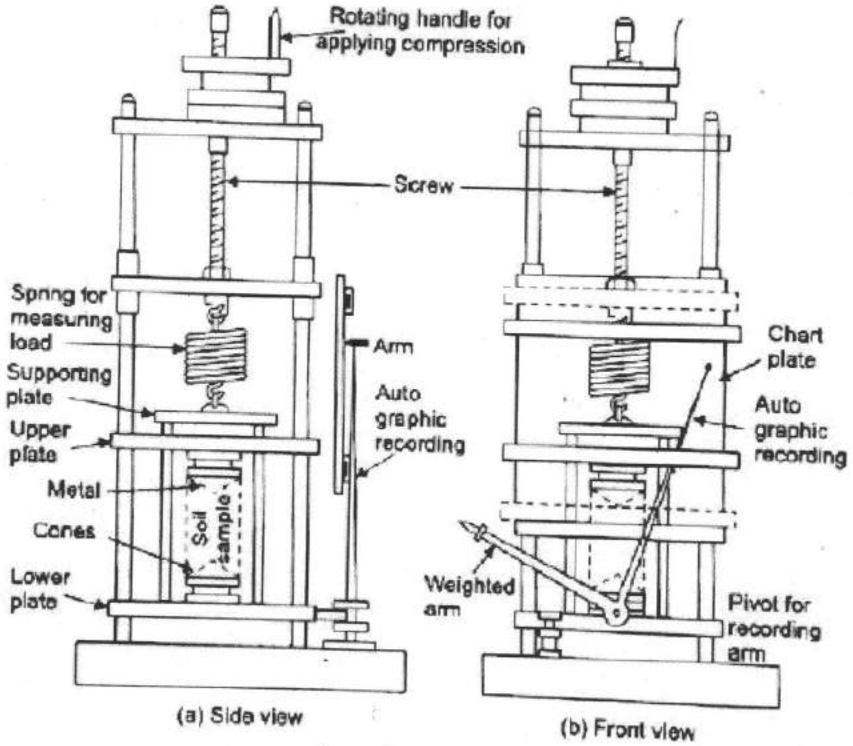
where q_u = unconfined compressive strength.

P = axial load at failure

A_o = initial cross sectional area

A_c = Corrected area at failure $= \frac{A_o}{(1-\varepsilon)}$

ε = axial strain in the sample $= \Delta L / L_o$



- It is a special type of triaxial test with $\sigma_c = 0$
- conducted quickly to have undrained condition
- Suitable for undrained, saturated clay ($\phi u = 0$)

q_u = unconfined compressive strength.

$$\sigma_1 = \sigma_3 \tan^2 (45^\circ + \frac{\phi u}{2}) + 2 C_u \tan (45 + \frac{\phi u}{2})$$

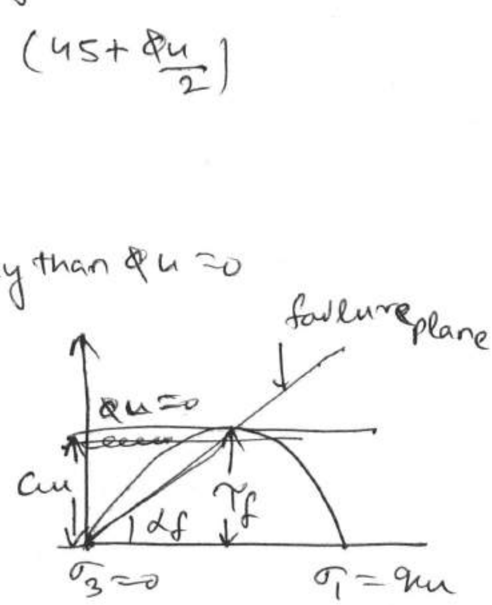
For UCC test $\sigma_3 = 0$ & $\sigma_1 = q_u$

$$q_u = 2 C_u \tan (45 + \frac{\phi u}{2})$$

If soil is undrained saturated clay than $\phi u = 0$

$$\Rightarrow C_u = \frac{q_u}{2}$$

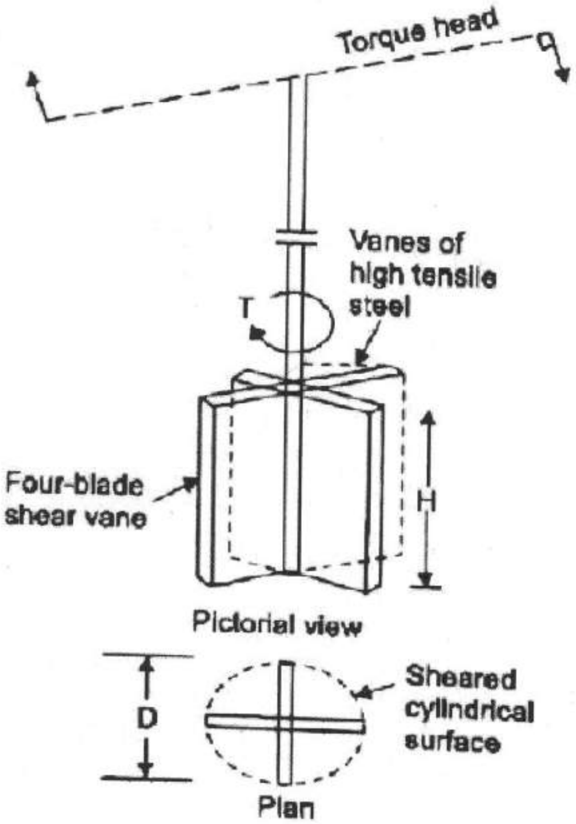
If $\phi = 0 \Rightarrow \alpha = 45^\circ$ $\quad \tau_f = \tau_{max}$



- UCC is the only test in which Mohr circle passes through the origin

UCC is not suitable for fissured clay (cracked) soil will fail along existing crack and does not exhibit the real strength.

Vane Shear Test:



Vane shear apparatus

The vane shear test is quick test used either in laboratory or in the field, to determine the undrained shear strength of cohesive soils.

Apparatus :- The vane shear test apparatus consists of four thin plates called vanes, welded orthogonally to steel rod at its bottom end. A torque measuring device such as a calibrated torsion spring is attached to the rod at its top end which is rotated by a worm gear and worm wheel arrangement

Procedure :- Fill the sampling mould with remoulded soil and level the surface of the sample. Mount the sampling mould under the base of unit and clamp it in position. The vane is pushed gently into the soil sample and it is rotated at a uniform speed by applying a torque or turning moment at the top end of the steel rod.

The rotation of the vane shears the soil along a cylindrical surface. The angle twist of the spring in degrees is indicated by pointer moving on a graduated dial attached to the worm wheel shaft.

The application of the torque is continued till the soil fails in shear.

Calculations :- Torque $T = \theta \times \frac{K}{180}$

where d = diameter of vane (cm)

h = height of vane (cm)

τ = shear strength (kg/cm²)

T = torque applied (kg-cm)

θ = difference of angle (angle of torque)

k = spring factor

Shear strength of soil computed using formula

$$\tau = \frac{T}{\pi D^2 \left(\frac{D}{6} + \frac{H}{2}\right)}$$

- suitable for undrained saturated clay for which $\phi_u = 0$ ($\Rightarrow S = C$)

Derivation of expression for torque

T_f = Torque at failure

H = Ht. of vanes

d = diameter across vanes

τ_f = Shear strength of soils.

Case I The vane is pushed with its top end below the surface of soil so that both top & bottom end partake in shearing.

Note: shearing takes place along cylindrical surface of dia. 'd' & ht. 'h'

Taking moment about the axis of torque rod

$$T_f = (\pi d H \tau_f)\, d/2 + 2\int_0^{d/2} (2\pi r\, dr\, \tau_f)\, r$$

$$= \frac{\pi d^2}{2} H \tau_f + 4\pi \tau_f \int_0^{d/2} r^2 dr$$

$$= \frac{\pi d^2 H \tau_f}{2} + \frac{\pi d^3}{6}\tau_f$$

$$= \pi d^2 \tau_f \left[\frac{H}{2} + \frac{d}{6}\right]$$

Case II The vane is pushed inside the soil with its top end flush with surface of soil so that only bottom end partakes in shearing.

Taking moment about the axis of torque rod we get

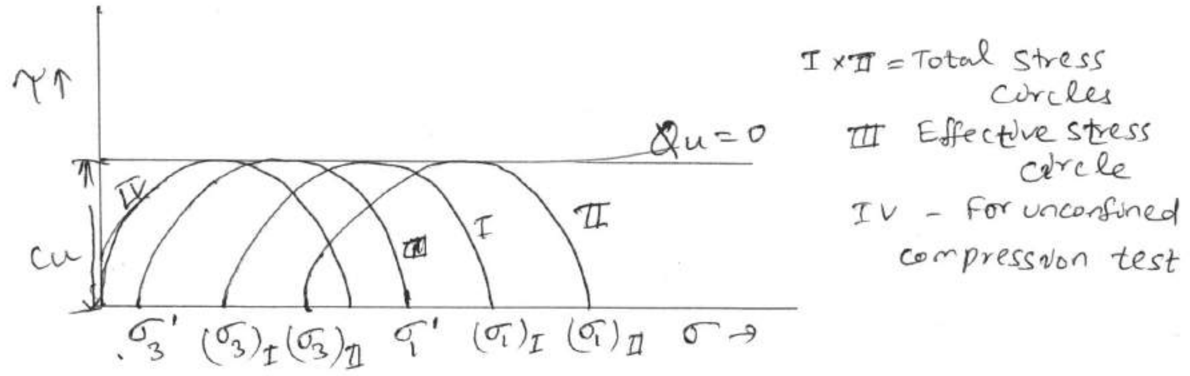
$$T_f = (\pi d H \tau_f)\, d/2 + \int_0^{d/2} (2\pi r\, dr\, \tau_f)\, r$$

$$= \tau_f \pi d^2 \left[\frac{H}{2} + \frac{d}{12}\right]$$

Shear Strength of different soil types :-

I. Shear strength of fully saturated cohesive soils :-

(a) Undrained test :- When undrained tests are conducted on identical specimens of a fully saturated clay with different cell pressures, the Mohr circles obtained will all be of same diameter. The failure envelope is horizontal giving $\phi_u = 0$ and $\tau_f = c_u$. It is clear that $(\sigma_1)_I - (\sigma_3)_{II} = (\sigma_1)_{II} - (\sigma_3)_{II}$



(b) Consolidated undrained test: If consolidated undrained tests are conducted on remoulded fully saturated clay specimens initially consolidated with same cell pressure and subsequently sheared under undrained condition with different cell pressures the Mohr circles obtained will be of same diameter. The failure envelope is horizontal with $\phi_u = 0$ and $\tau_f = c_u$. If the tests are conducted with specimens initially consolidated with same but increased cell pressure and then sheared with different cell pressures the failure envelope obtained is still horizontal with $\phi_u = 0$ but c_u will be greater than in the previous case.