STRESS DISTRIBUTION IN SOIL

10.0 Illustrative Examples

Example 10.1: A concentrated load of 22.5 kN acts on the surface of a homogeneous soil mass of large extent. Find the stress intensity at a depth of 15 meters and (*i*) directly under the load, and (*ii*) at a horizontal distance of 7.5 metres. Use Boussinesq's equations.

Solution:

According to Boussinesq's theory,

$$\sigma_z = \frac{Q}{z^2} \cdot \frac{(3/2\pi)}{[1+(r/z)^2]^{5/2}}$$

(*i*) Directly under the load:

$$r = 0. \therefore r/z = 0$$

$$z = 15 \text{ m } Q = 22.5 \text{ kN}$$

$$\therefore \quad \sigma_z = \frac{22.5}{15 \times 15} \cdot \frac{(3/2\pi)}{(1+0)^{5/2}}$$

$$= \mathbf{47.75\ kN/m^2.}$$

(*ii*) At a horizontal distance of 7.5 metres:

$$r = 7.5 \text{ m } z = 15 \text{ m}$$

$$r/z = 7.5/15 = 0.5$$

$$\therefore \quad \sigma_z = \frac{22.50}{15 \times 15} \cdot \frac{(3/2\pi)}{[(1+(0.5)^2]^{5/2}}$$

$$= \mathbf{27.33\ N/m^2.}$$

Example 10.2: A load 1000 kN acts as a point load at the surface of a soil mass. Estimate the stress at a point 3 m below and 4 m away from the point of action of the load by Boussinesq's formula. Compare the value with the result from Westergaard's theory.

Solution:

Boussinesq's theory:

$$\sigma_z = \frac{Q}{z^2} \cdot \frac{(3/2\pi)}{[1+(r/z)^2]^{5/2}}$$

Here $r = 4$ m, $z = 3$ m and $Q = 1000$ kN

$$\therefore \quad \sigma_z = \frac{1000}{3 \times 3} \cdot \frac{(3/2\pi)}{[1+(4/3)^2]^{5/2}} = \mathbf{4.125\ kN/m^2}$$

Westergaard's Theory:

$$\sigma_z = \frac{Q}{z^2} \cdot \frac{(1/\pi)}{[1+2(r/z)^2]^{3/2}}$$

$$\therefore \quad \sigma_z = \frac{1000}{3 \times 3} \cdot \frac{(1/\pi)}{[1+2(4/3)^2]^{3/2}} = \mathbf{3.637\ kN/m^2.}$$

Example 10.3: A line load of 100 kN/metre run extends to a long distance. Determine the intensity of vertical stress at a point, 2 m below the surface and (i) directly under the line load, and (ii) at a distance of 2 m perpendicular to the line. Use Boussinesq's theory.

Solution:

$$q' = 100 \text{ kN/m}$$

$$z = 2 \text{ m}$$

$$\sigma_z = (q'/z).\frac{(2/\pi)}{[1+(x/z)^2]^2}$$

(*i*) Referring to Fig. at point A_1,

$$x = 0$$

$$\therefore \quad \sigma_z = (q/z)\times(2/\pi) = \frac{100}{2}\times\frac{2}{\pi} \text{ kN/m}^2 = \mathbf{31.83 \text{ kN/m}^2}$$

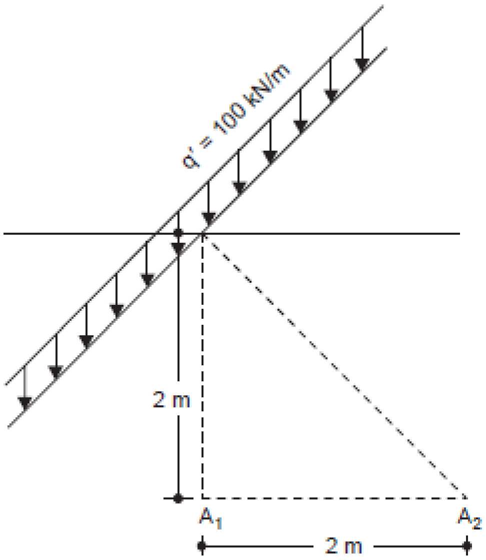

Fig. Line load

(*ii*) $x = 2$ m at point A_2,

$$x/z = 1$$

$$\therefore \quad \sigma_z = \frac{q'}{z}.\frac{(2/\pi)}{4}$$

$$= \frac{q'}{z}.\frac{1}{2\pi}$$

$$= \frac{100}{2}.\frac{1}{2\pi}$$

$= 25/\pi$ kN/m^2

$=$ **7.96 kN/m^2.**

Example 10.4: The load from a continuous footing of 1.8 metres width, which may be considered to be a strip load of considerable length, is 180 kN/m^2. Determine the maximum principal stress at 1.2 metres depth below the footing, if the point lies (i) directly below the centre of the footing, (ii) directly below the edge of the footing, and (iii) 0.6 m away from the edge of the footing. What is the maximum shear stress at each of these points? What is the absolute maximum shear stress and at what depth will it occur directly below the middle of the footing?

Solution:

$$B = 2b = 1.8 \text{ m}$$
$$q = 180 \text{ kN/m}^2$$
$$z = 1.2 \text{ m}$$

Referring to Fig.

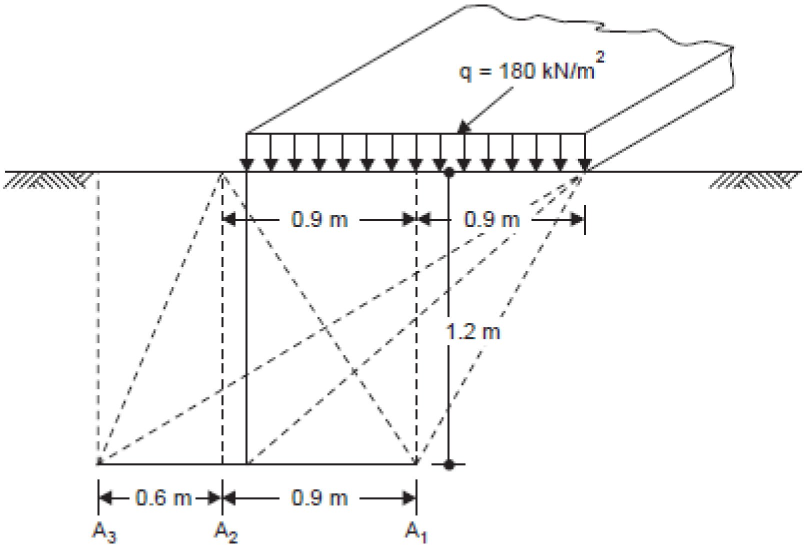

Fig. Strip load

(*i*) For point A_1,

$$\frac{\theta_0}{2} = \tan^{-1}\frac{0.9}{1.2} = 36°.87 = 0.6435 \text{ rad.}$$
$$\theta_0 = 1.287 \text{ rad.}$$

Maximum principal stress

$$\sigma_1 = (q/\pi)(\theta_0 + \sin\theta_0)$$
$$= \frac{180}{\pi}(1.287 + 0.960) = \mathbf{128.74\ kN/m^2}$$

Maximum shear stress,

$$\tau_{max} = \frac{q}{\pi}.\sin\theta_0 = \frac{180}{\pi} \times 0.960 = \mathbf{55.00\ kN/m^2}$$

(ii) for point A_2,

$$\theta_0 = \tan^{-1}\frac{1.8}{1.2} = 56^\circ.31 = 0.9828 \text{ rad.}$$

$$\sigma_1 = \frac{180}{\pi}(0.9828 + 0.8321) = \mathbf{104\ kN/m^2}$$

$$\tau_{max} = \frac{180}{\pi}.(0.8321) = \mathbf{47.68\ kN/m^2}$$

(iii) for point A_3,

$$\theta_1 = \tan^{-1}\frac{0.6}{1.2} = 26^\circ.565 = 0.464 \text{ rad.}$$

$$\theta_2 = \tan^{-1}\frac{2.4}{1.2} = 63^\circ.435 = 1.107 \text{ rad.}$$

$$\theta_0 = (\theta_2 - \theta_1) = (1.107 - 0.464) \text{ rad.} = 0.643 \text{ rad.}$$

$$\sigma_1 = (180/\pi)\ (0.643 + 0.600) = \mathbf{71.22\ kN/m^2}$$

$$\tau_{max} = \frac{180}{\pi} \times 0.6 = \mathbf{34.38\ kN/m^2}$$

Absolute maximum shear stress = q/π = $180/\pi$ = **57.3 kN/m²**

This occurs at a depth $B/2$ or 0.9 m below the centre of the footing.

Example 10.5: A circular area on the surface of an elastic mass of great extent carries a uniformly distributed load of 120 kN/m^2. The radius of the circle is 3 m. Compute the intensity of vertical pressure at a point 5 metres beneath the centre of the circle using Boussinesq's method.

Solution:

Radius 'a' of the loaded area = 3 m

$$q = 120 \text{ kN/m}^2$$

$$z = 5 \text{ m}$$

$$z = q\left[1 - \frac{1}{\{1 + (a/z)^2\}^{3/2}}\right]$$

$$= 120\left[1 - \frac{1}{\{1 + (3/5)^2\}^{3/2}}\right]$$

$$= 120\left[1 - \frac{1}{(34/25)^{3/2}}\right]$$

$$= \mathbf{44.3\ kN/m^2.}$$

Example 10.6: A ring foundation is of 3.60 m external diameter and 2.40 m internal diameter. It transmits a uniform pressure of 135 kN/m^2. Calculate the vertical stress at a depth of 1.80 m directly beneath the centre of the loaded area.

Solution:

$$a_i = 2.40/2 = 1.20 \text{ m}$$
$$a_o = 3.60/2 = 1.80 \text{ m}$$
$$z = 1.80 \text{ m}$$
$$q = 135 \text{ kN/m}^2$$
$$\sigma_z = q \cdot K_{B_C}$$

where
$$K_{B_c} = \left[\frac{1}{\left\{1+\left(\frac{a_i}{z}\right)^2\right\}^{3/2}} - \frac{1}{\left\{1+\left(\frac{a_o}{z}\right)^2\right\}^{3/2}}\right]$$
$$= \left[\frac{1}{\left\{1+\left(\frac{120}{180}\right)^2\right\}^{3/2}} - \frac{1}{\left\{1+\left(\frac{180}{180}\right)^2\right\}^{3/2}}\right]$$
$$= 0.222$$

$\therefore$ $$\sigma_z = 135 \times 0.222 \approx 30 \text{ kN/m}^2.$$

Example 10.7: A raft of size 4 m-square carries a load of 200 kN/m^2. Determine the vertical stress increment at a point 4 m below the centre of the loaded area using Boussinesq's theory. Compare the result with that obtained by the equivalent point load method and with that obtained by dividing the area into four equal parts the load from each of which is assumed to act through its centre.

Solution:

(*i*) Square Area:

Imagine, as in Fig. the area to be divided into four equal squares. The stress at *A* will be four times the stress produced under the corner of the small square.

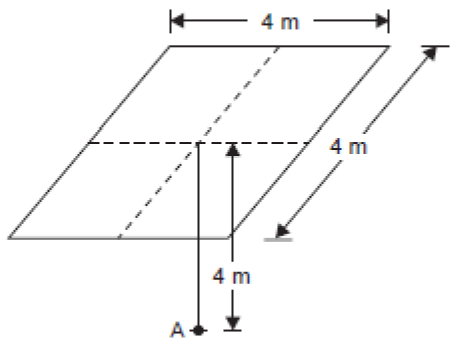

Fig. Uniform load on square area

$m = 2/4 = 0.5$, $n = 2/4 = 0.5$

$$I_\sigma = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\cdot\frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1}\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1-m^2n^2}\right]$$

$$= \frac{1}{4\pi}\left[\frac{2\times0.5\times0.5\sqrt{0.25+0.25+1}}{1.50+0.25\times0.25}\cdot\frac{0.25+0.25+2}{0.25+0.25+1} + \tan^{-1}\frac{2\times0.5\times0.5\sqrt{0.25+0.25+1}}{1.50-0.25\times0.25}\right]$$

$$= \frac{1}{4\pi}\left[\frac{0.5\sqrt{1.5}}{1.5625}\times\frac{2.50}{1.50} + \tan^{-1}\frac{0.5\sqrt{1.5}}{1.4375}\right] = 0.0840$$

(The value may be obtained from Tables or Charts also.)

$\therefore \qquad \sigma_z = 4 \times 200 \times 0.084 = \mathbf{67.2\ kN/m^2}$

(*ii*) Equivalent point load method:

$$Q = 200 \times 16 = 3200 \text{ kN}$$

$$\sigma_z = \frac{Q}{z^2}\cdot\frac{(3/2\pi)}{[1+(r/z)^2]^{5/2}} = \frac{3200}{16}\times\frac{(3/2\pi)}{1^{5/2}} = \mathbf{95.5\ kN/m^2}$$

(*iii*) Four equivalent point loads:

From Example 10.3, $\sigma_z = \mathbf{71.14\ kN/m^2}$

Thus, percentage error in the equivalent point load method

$$= \frac{(95.5-67.2)}{67.2}\times100 = \mathbf{42.11}$$

Percentage error in four equivalent point loads approach

$$= \frac{(71.14-67.20)}{67.20}\times100 = \mathbf{5.86.}$$

Example 10.8: A rectangular foundation, 2 m × 4 m, transmits a uniform pressure of 450 kN/m^2 to the underlying soil. Determine the vertical stress at a depth of 1 metre below the foundation at a point within the loaded area, 1 metre away from a short edge and 0.5 metre away from a long edge. Use Boussinesq's theory.

Solution:

Depth $z = 1$ m. $q = 450$ kN/m^2

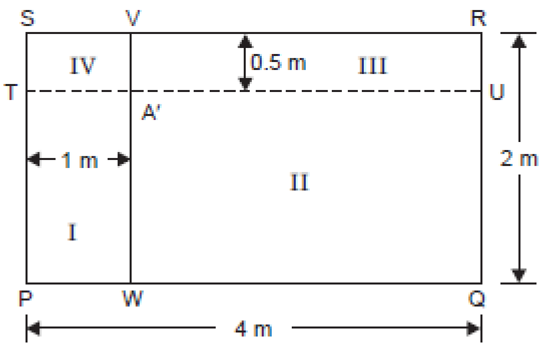

Fig. Stress at a point inside a loaded area

The loaded area and the plan position of the point A′ at which the vertical stress is required are shown in Fig. The area is divided into four parts as shown, such that A′ forms a corner of each.

$$\sigma_z = q\left[I_{\sigma_I} + I_{\sigma_{II}} + I_{\sigma_{III}} + I_{\sigma_{IV}}\right]$$

Area I: $m = 1/1 = 1$; $n = 1.5/1 = 1.5$

$$I_{\sigma_I} = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\cdot\frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1}\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1-m^2n^2}\right]$$

$$= \frac{1}{4\pi}\left[\frac{2\times1\times1.5\sqrt{1^2+1.5^2+1}}{1^2+1.5^2+1+1^2\times1.5^2}\cdot\frac{1^2+1.5^2+2}{1^2+1.5^2+1} + \tan^{-1}\frac{2\times1\times1.5\sqrt{1^2+1.5^2+1}}{1^2+1.5^2+1-1^2\times1.5^2}\right]$$

$$= 0.1936$$

Area II: $m = 1.5/1 = 1.5$; $n = 3/1 = 3$

$$I_{\sigma_{II}} = \frac{1}{4\pi}\left[\frac{2\times1.5\times3\sqrt{1.5^2+3^2+1}}{1.5^2+3^2+1+1.5^2\times3^2}\cdot\frac{1.5^2+3^2+2}{1.5^2+3^2+1} + \tan^{-1}\frac{2\times1.5\times3\sqrt{1.5^2+3^2+1}}{1.5^2+3^2+1-1.5^2\times3^2}\right]$$

$$= 0.2290$$

Area III: $m = 0.5/1 = 0.5$; $N = 3/1 = 3$

$$I_{\sigma_{III}} = \frac{1}{4\pi}\left[\frac{2\times0.5\times3\sqrt{0.5^2+3^2+1^2}}{0.5^2+3^2+1+0.5^2\times3^2}\cdot\frac{0.5^2+3^2+2}{0.5^2+3^2+1} + \tan^{-1}\frac{2\times0.5\times3\sqrt{0.5^2+3^2+1}}{0.5^2+3^2+1-0.5^2\times3^2}\right]$$

$$= 0.1368$$

Area IV: $m = 0.5/1 = 0.5$; $n = 1/1 = 1$

$$I_{\sigma_{IV}} = \frac{1}{4\pi}\left[\frac{2\times0.5\times1\sqrt{0.5^2+1^2+1}}{0.5^2+1^2+1+0.5^2\times1^2}\cdot\frac{0.5^2+1^2+2}{0.5^2+1^2+1} + \tan^{-1}\frac{2\times0.5\times1\sqrt{0.5^2+1^2+1}}{0.5^2+1^2+1-0.5^2\times1^2}\right]$$

$$= 0.1202$$

$$\therefore \quad \sigma_z = 450(0.1936 + 0.2290 + 0.1368 + 0.1202)$$

$$= \mathbf{305.8\ kN/m^2}.$$

Example 10.9: A rectangular foundation 2 m × 3 m, transmits a pressure of 360 kN/m^2 to the underlying soil. Determine the vertical stress at a point 1 metre vertically below a point lying outside the loaded area, 1 metre away from a short edge and 0.5 metre away from a long edge. Use Boussinesq's theory.

Solution:

$$z = 1 \text{ m}; \; q = 360 \text{ kN/m}^2$$

since the point at which the stress is required is outside the loaded area, rectangles are imagined as shown in Fig. 10.27, so as to make A' a corner of all the concerned rectangle. With the notation of Fig. 10.27,

$$\sigma_z = q\left(I_{\sigma_{I}} - I_{\sigma_{II}} - I_{\sigma_{III}} - I_{\sigma_{IV}}\right)$$

Area I: $m = 2.5/1 = 2.5$; $n = 4/1 = 4$

$$I_{\sigma_I} = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\cdot\frac{m^2+n^2+2}{m^2+n^2+1}+\tan^{-1}\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1-m^2n^2}\right]$$

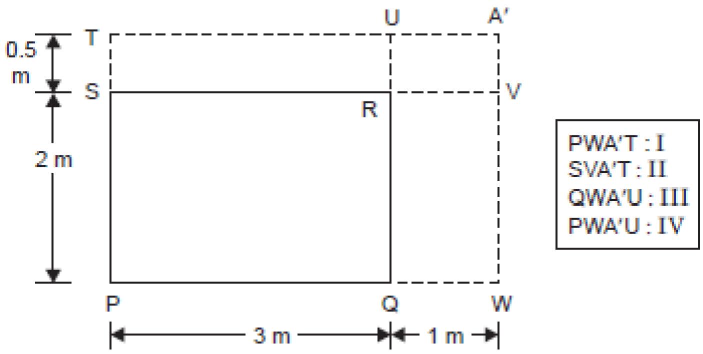

Fig. Stress at a point outside loaded area

$$= \frac{1}{4\pi}\left[\frac{2\times2.5\times4\sqrt{2.5^2+4^2+1}}{2.5^2+4^2+1+2.5^2\times4^2}\cdot\frac{2.5^2+4^2+2}{2.5^2+4^2+1}+\tan^{-1}\frac{2\times2.5\times4\sqrt{2.5^2+4^2+1}}{2.5^2+4^2+1-2.5^2\times4^2}\right]$$

$= 0.2434$

Area II: $m = 0.5/1 = 0.5$; $n = 4/1 = 4$

$$I_{\sigma_{II}} = \frac{1}{4\pi}\left[\frac{2\times0.5\times4\sqrt{0.5^2+4^2+1}}{0.5^2+4^2+1+0.5^2\times4^2}\cdot\frac{0.5^2+4^2+2}{0.5^2+4^2+1}+\tan^{-1}\frac{2\times0.5\times4\sqrt{0.5^2+4^2+1}}{0.5^2+4^2+1-0.5^2\times4^2}\right]$$

$= 0.1372$

Area III: $m = 1/1 = 1$; $n = 2.5/1 = 2.5$

$$I_{\sigma_{III}} = \frac{1}{4\pi}\left[\frac{2\times1\times2.5\sqrt{1^2+2.5^2+1}}{1^2+2.5^2+1+1^2\times2.5^2}\cdot\frac{1^2+2.5^2+2}{1^2+2.5^2+1}+\tan^{-1}\frac{2\times1\times2.5\sqrt{1^2+2.5^2+1}}{1^2+2.5^2+1-1^2\times2.5^2}\right]$$

$= 0.2024$

Area IV: $m = 0.5/1 = 0.5$; $n = 1/1 = 1$

$$I_{\sigma_{IV}} = \frac{1}{4\pi}\left[\frac{2\times0.5\times1\sqrt{0.5^2+1^2+1}}{0.5^2+1^2+1+0.5^2\times1^2}\cdot\frac{0.5^2+1^2+2}{0.5^2+1^2+1}+\tan^{-1}\frac{2\times0.5\times1\sqrt{0.5^2+1^2+1}}{0.5^2+1^2+1-0.5^2\times1^2}\right]$$

$= 0.1202$

$\therefore \quad \sigma_z = 360(0.2434 - 0.1372 - 0.2024 + 0.1202)$

$= \mathbf{8.64 \text{ kN/m}^2}$.

SHEAR STRENGTH

11.0 ILLUSTRATIVE EXAMPLES

Example 11.1: The stresses at failure on the failure plane in a cohesionless soil mass were: Shear stress = 4 kN/m^2; normal stress = 10 kN/m^2. Determine the resultant stress on the failure plane, the angle of internal friction of the soil and the angle of inclination of the failure plane to the major principal plane.

Solution:

$$\text{Resultant stress} = \sqrt{\sigma^2 + \tau^2}$$

$$= \sqrt{10^2 + 4^2} = \mathbf{10.77\ kN/m^2}$$

$$\tan \phi = \tau/\sigma = 4/10 = 0.4$$

$$\phi = \mathbf{21°\ 48'}$$

$$\theta = 45° + \phi/2 = 45° + \frac{21°\ 48'}{2} = \mathbf{55°54'}.$$

Graphical solution

The procedure is first to draw the σ-and τ-axes from an origin *O* and then, to a suitable scale, set-off point *D* with coordinates (10,4), Joining *O* to *D*, the strength envelope is got. The Mohr Circle should be tangential to *OD* to *D*. *DC* is drawn perpendicular to *OD* to cut *OX* in *C*, which is the centre of the circle. With *C* as the centre and *CD* as radius, the circle is completed to cut *OX* in *A* and *B*.

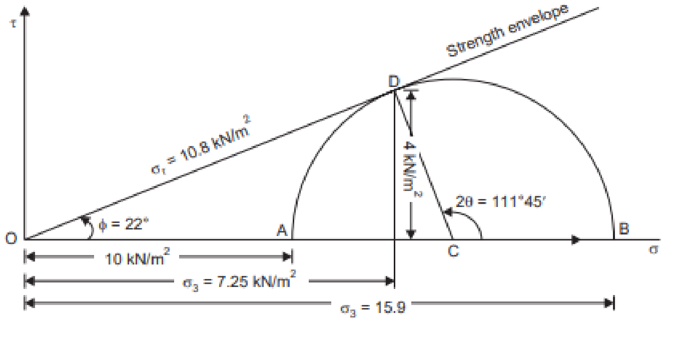

Fig. Mohr's circle

By scaling, the resultant stress = *OD* = 10.8 kN/m^2.

With protractor, ϕ = 22° and θ = 55°53′

We also observe than σ_3 = *OA* = 7.25 kN/m^2 and σ_1 = *OB* = 15.9 kN/m^2.

Example 11.2: Calculate the potential shear strength on a horizontal plane at a depth of 3 m below the surface in a formation of cohesionless soil when the water table is at a depth of 3.5 m. The degree of saturation may be taken as 0.5 on the average. Void ratio = 0.50; grain specific gravity = 2.70; angle of internal friction = 30°. What will be the modified value of shear strength if the water table reaches the ground surface?

Solution:

Effective unit weight $\gamma' = \frac{(G-1)}{(1+e)} \cdot \gamma_w$

$$= \frac{(2.70-1)}{(1+0.5)} \times 10 = 11.33 \text{ kN/m}^3$$

Unit weight, γ, at 50% saturation

$$= \frac{(G+S.e)}{(1+e)} \cdot \gamma_w = \frac{(2.70+0.5\times 0.5)}{(1+0.5)} \times 10 = 19.667 \text{ kN/m}^3$$

(*a*) When the water table is at 3.5 m below the surface:

Normal stress at 3 m depth, $\sigma = 19.67 \times 3 = 59$ kN/m²

Shear strength, $s = \sigma \tan \phi$ for a sand

$= 59 \tan 30° =$ **34 kN/m² (nearly).**

(*b*) When water table reaches the ground surface:

Effective Normal stress at 3 m depth

$$\bar{\sigma} = \gamma' \cdot h = 11.33 \times 3 = 34 \text{ kN/m}^2$$

Shear strength, $s = \bar{\sigma} \tan \phi$

$= 34 \tan 30°$

$=$ **19.6 kN/m² (nearly).**

Example 11.3: The following data were obtained in a direct shear test. Normal pressure = 20 kN/m2, tangential pressure = 16 kN/m^2. Angle of internal friction = 20°, cohesion = 8 kN/m^2. Represent the data by Mohr's Circle and compute the principal stresses and the direction of the principal planes.

Solution:

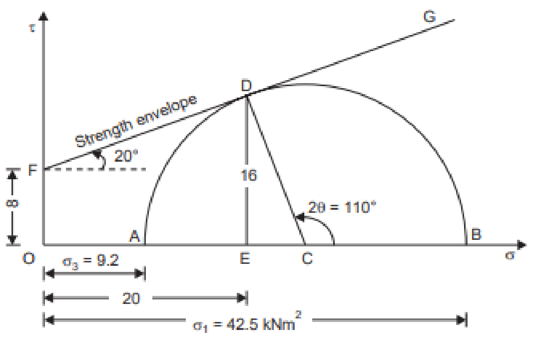

Fig. 8.47 Mohr's circle (Ex. 8.4)

The strength envelope *FG* is located since both *c* and ϕ are given. Point *D* is set-off with co-ordiantes (20, 16) with respect to the origin *O* ; it should fall on the envelope. (In this case, there appears to be slight discrepancy in the data). *DC* is drawn perpendicular to *FD* to meet the σ-axis in *C*. With *C* as centre and *CD* as radius, the Mohr's circle is completed. The principal stresses σ_3 (*OA*) and σ_1 (*OB*) are scaled off and found to be **9.2 kN/m²** and **42.5 kN/m².** Angle *BCD* is measured and found to be 110°. Hence the major principal plane is inclined at **55°** (clockwise) and the minor principal plane at **35°** (counter clockwise) to the plane of shear (horizontal plane, in this case).

Analytical solution:

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

$$N_\phi = \tan^2 (45° + \phi/2) = \tan^2 55° = 2.04$$

$$\sigma_1 = 2.04\, \sigma_3 + 2 \times 8 \times \tan 55° = 2.04\sigma_3 + 22.88 \quad \text{...(1)}$$

$$\sigma_n = \sigma_1 \cos^2 55° + \sigma_3 \sin^2 55° = 20$$

$$0.33\, \sigma_1 + 0.67\, \sigma_3 = 20 \quad \text{...(2)}$$

Solving, $\sigma_1 = 42.5$ kN/m² and $\sigma_3 = 9.2$ kN/m², as obtained graphically.

Example 11.4: The following results were obtained in a shear box text. Determine the angle of shearing resistance and cohesion intercept:

Normal stress (kN/m^2)	100	200	300
Shear stress (kN/m^2)	130	185	240

Solution:

The normal and shear stresses on the failure plane are plotted as shown:

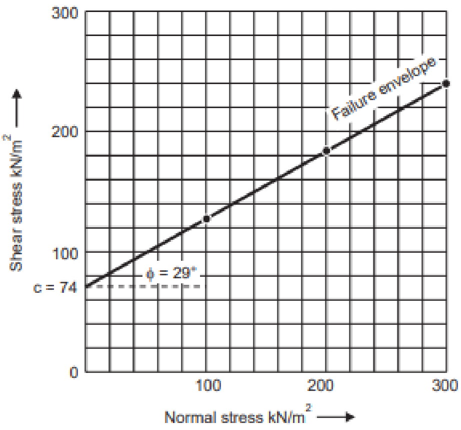

Fig. 8.48 Failure envelope (Ex. 8.5)

The intercept on the shear stress axis is cohesion, c, and the angle of inclination of the failure envelope with the normal stress axis of the angle of shearing resistance, ϕ.

From Fig. 8.48,

$$c = 74 \text{ kN/m}^2$$

$$\phi = \mathbf{29°}.$$

Example 11.5: A series of shear tests were performed on a soil. Each test was carried out until the sample sheared and the principal stresses for each test were:

Test No.	σ_1(kN/m^2)	σ_3(kN/m^2)
1	200	600
2	300	900
3	400	1200

Solution:

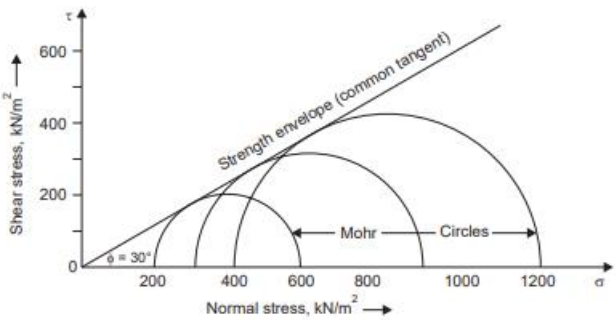

Fig Mohr's circles and strength envelopes

Plot the Mohr's circles and hence determine the strength envelope and angle of internal friction of the soil.

The data indicate that the tests are triaxial compression tests; the Mohr's circles are plotted with $(\sigma_1 - \sigma_3)$ as diameter and the strength envelope is obtained as the common tangent.

Example 11.6: A cylindrical specimen of a saturated soil fails under an axial stress 150 kN/m2 in an unconfined compression test. The failure plane makes an angle of 52° with the horizontal. Calculate the cohesion and angle of internal friction of the soil.

Solution:

Analytical solution:

The angle of the failure plane with respect to the plane on which the major principal (axial) stress acts is:

$$\theta_{cr} = 45° + \phi/2 = 52°$$

$$\therefore \quad \phi/2 = 7° \text{ or } \phi = \mathbf{14°}$$

$$\sigma_1 = 150 \text{ kN/m}^2 \; \sigma_3 = 0$$

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

where $N_\phi = \tan^2(45° + \phi/2) = \tan^2 52°$

$$\sqrt{N_\phi} = \tan 52°$$

$$\therefore \quad 150 = 0 + 2 \times c \tan 52°$$

$$\therefore \quad \text{Cohesion,} \quad c = 75/\tan 52° = \mathbf{58.6 \text{ kN/m}^2}$$

Graphical solution:

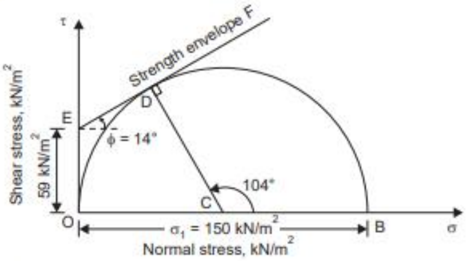

Fig. Mohr's circle and strength envelope

The axial stress is plotted to a suitable scale as *OB*. With *OB* as diameter, the Mohr's circle is established. At the centre *C*, angle *ACD* is set-off as 2 × 52° or 104° to cut the circle in *D*. A tangent to the circle at *D* establishes the strength envelope. The intercept of this on the τ-axis gives the cohesion *c* as 59 kN/m² and the angle of slope of this line with horizontal gives ϕ as 14°. These values compare very well with those from the analytical approach.

Example 11.7: In an unconfined compression test, a sample of sandy clay 8 cm long and 4 cm in diameter fails under a load of 120 N at 10% strain. Compute the shearing resistance taking into account the effect of change in cross-section of the sample.

Solution:

Size of specimen = 4 cm dia. × 8 cm long.

Initial area of cross-section $= (\pi/4) \times 4^2 = 4\pi \text{ cm}^2$.

Area of cross-section at failure $= \dfrac{A_0}{(1-\varepsilon)}$

$$= \frac{4\pi}{(1-0.10)} = 4\pi/0.9 = 40\,\pi/9 \text{ cm}^2$$

Load at failure = 120 N.

Axial stress at failure $= \dfrac{120 \times 9}{40\pi} \text{ N/cm}^2$

$$= 2.7/\pi \text{ N/cm}^2$$

$$= 8.6 \text{ N/cm}^2$$

Shear stress at failure $= \dfrac{1}{2} \times 8.6 = 4.3 \text{ N/cm}^2$

The corresponding Mohr's circle is shown in Fig.

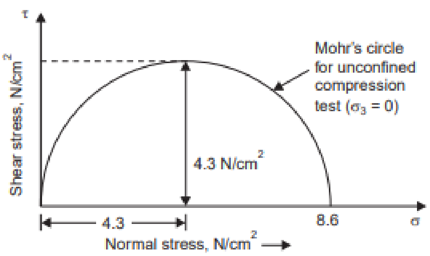

Fig. . Mohr's circle for unconfined compression test

Example 11.8: A cylindrical specimen of a saturated soil fails under an axial stress 150 kN/m^2 in an unconfined compression test. The failure plane makes an angle of 52° with the horizontal. Calculate the cohesion and angle of internal friction of the soil.

Solution:

Analytical solution:

The angle of the failure plane with respect to the plane on which the major principal (axial) stress acts is:

$$\theta_{cr} = 45° + \phi/2 = 52°$$

$$\therefore \quad \phi/2 = 7° \text{ or } \phi = \mathbf{14°}$$

$$\sigma_1 = 150 \text{ kN/m}^2 \; \sigma_3 = 0$$

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

where $N_\phi = \tan^2 (45° + \phi/2) = \tan^2 52°$

$$\sqrt{N_\phi} = \tan 52°$$

$$\therefore \quad 150 = 0 + 2 \times c \tan 52°$$

$\therefore$ Cohesion, $c = 75/\tan 52° = \mathbf{58.6\ kN/m^2}$

Graphical solution:

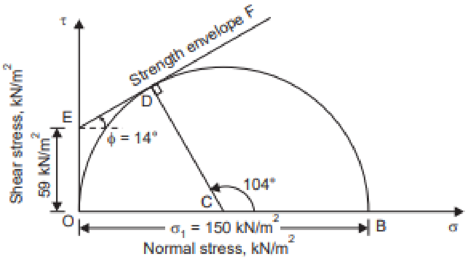

Fig. Mohr's circle and strength envelope

The axial stress is plotted to a suitable scale as *OB*. With *OB* as diameter, the Mohr's circle is established. At the centre *C*, angle *ACD* is set-off as 2 × 52° or 104° to cut the circle in *D*. A tangent to the circle at *D* establishes the strength envelope. The intercept of this on the τ-axis gives the cohesion *c* as 59 kN/m² and the angle of slope of this line with horizontal gives φ as 14°. These values compare very well with those from the analytical approach.

Example 11.9: In a triaxial shear test conducted on a soil sample having a cohesion of 12 kN/ m^2 and angle of shearing resistance of 36°, the cell pressure was 200 kN/m^2. Determine the value of the deviator stress at failure.

Solution:

The strength envelope is drawn through *E* on the τ-axis, *OE* being equal to *C* = 12 kN/m² to a convenient scale, at an angle φ = 36° with the σ-axis. The cell pressure, σ_3 = 200 kN/m² is plotted as *OA*. With centre on the σ-axis, a circle is drawn to pass through *A* and be tangential to the envelope, by trial and error. *AC* is scaled-off, *C* being the centre of the Mohr's circle, which is $(\sigma_1 - \sigma_3)/2$. The deviator stress is double this value. In this case the result is **616 kN/m².** (Fig. 8.54).

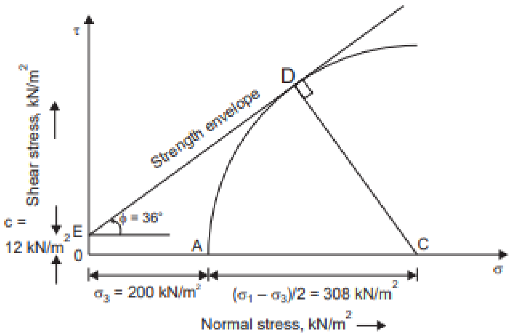

Fig. Mohr's circle for triaxial test

Analytical solution:

$$c = 12 \text{ kN/m}^2$$

$$\phi = 36°$$

$$\sigma_3 = 200 \text{ kN/m}^2$$

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

where $N_\phi = \tan^2 (45° + \phi/2)$.

$$N_\phi = \tan^2 (45° + 18°) = \tan^2 63° = 3.8518$$

$$\sqrt{N_\phi} = \tan 63° = 1.9626$$

$$\therefore \quad \sigma_1 = 200 \times 3.8518 + 2 \times 12 \times 1.9626 = 817.5 \text{ kN/m}^2$$

Deviator stress $= \sigma_1 - \sigma_3 = (817.5 - 200) \text{ kN/m}^2 = 617.5 \text{ kN/m}^2$

Example 11.10: A triaxial compression test on a cohesive sample cylindrical in shape yields the following effective stresses:

Major principal stress ... 8 MN/m^2

Minor principal stress ... 2 MN/m^2

Angle of inclination of rupture plane is 60° to the horizontal. Present the above data, by means of a Mohr's circle of stress diagram. Find the cohesion and angle of internal friction.

Solution:

The minor and major principal stresses are plotted as OA and OB to a convenient scale on the σ-axis. The mid-point of AB is located as C. With C as centre and CA or CB as radius, the Mohr's stress circle is drawn. Angle BCD is plotted as $2\theta_{cr}$ or $2 \times 60° = 120°$ to cut the circle in D. A tangent to the circle drawn at D (perpendicular to CD) gives the strength envelope. The intercept of this envelope, on the τ-axis gives the cohesion, c, and the inclination of the envelope with σ-axis gives the angle of internal friction, φ.

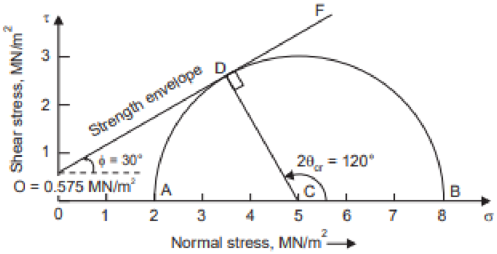

Fig. Mohr's circle and strength envelope

Graphical solution:

The results obtained graphically are: c = **0.575 MN/m²**;

$\phi = 30°$

Analytical method:

$$\sigma_1 = 8 \text{ MN/m}^2 \text{ and } \sigma_3 = 2 \text{ MN/m}^2\ \theta_{cr} = 60°$$

$$\theta_{cr} = 45° + \phi/2 = 60°, \text{ whence } \phi = 30°$$

$$N_\phi = \tan^2(45° + \phi/2) = \tan^2 60° = 3\ ;\ \sqrt{N_\phi} = \sqrt{3}$$

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

$$\therefore \quad 8 = 2 \times 3 + 2 \times c\sqrt{3}, \text{ whence, } c = 1/\sqrt{3} = 0.577 \text{ MN/m}^2$$

The results obtained graphically show excellent agreement with these values.

Example 11.11: A simple of dry sand is subjected to a triaxial test. The angle of internal friction is 37 degrees. If the minor principal stress is 200 kN/m^2, at what value of major principal stress will the soil fail?

Solution:

Analytical method:

$$\phi = 37°$$

$$\sigma_3 = 200 \text{ kN/m}^2$$

For dry sand, $c = 0$.

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

$$= \sigma_3 N_\phi \text{, since } c = 0$$

$$N_\phi = \tan^2(45° + \phi/2) = \tan^2(45° + 18°30') = \tan^2 63°30' = 4.0228$$

$$\therefore \quad \sigma_1 = \sigma_3 N_\phi = 200 \times 4.0228 \text{ kN/m}^2$$

Major principal stress, σ_1 = **804.56 kN/m²**

Graphical method:

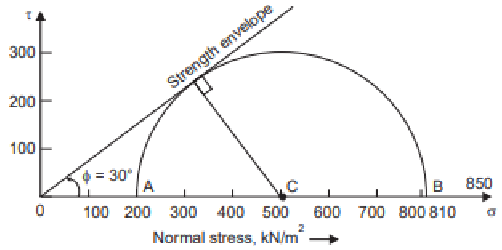

Fig. Mohr's circle and strength envelope

The strength envelope is drawn at 37° to σ-axis, through the origin. The minor principal stress 200 kN/m² is plotted as *OA* on the σ-axis to a convenient scale. With the centre on the σ-axis, draw a circle to pass through *A* and be tangential to the strength envelope by trial and error. If the circle cuts σ-axis at *B* also, *OB* is scaled-off to give the major principal stress, σ_1.

Example 11.12: A thin layer of silt exists at a depth of 18 m below the surface of the ground. The soil above this level has an average dry density of 1.53 Mg/m3 and an average water content of 36%. The water table is almost at the surface. Tests on undisturbed samples of the silt indicate the following values:

$c_u = 45$ kN/m^2; $\varphi_u = 18°$; $c' = 35$ kN/m^2; $\varphi' = 27°$

Estimate the shearing resistance of the silt on a horizontal plane, (a) when the shear stress builds up rapidly and (b) when the shear stress builds up very slowly

Solution:

Bulk unit weight, $\gamma = \gamma_d(1 + w)$

$= 1.53 \times 1.36 = 2.081$ Mg/m^3

Submerged unit weight, $\gamma' = 2.081 - 1.0 = 1.081$ Mg/m^3

Total normal pressure at 18 m depth $= 2.081 \times 9.81 \times 18$

$\sigma = 367.5$ kN/m^2

Effective pressure at 18 m depth $= 1.081 \times 9.81 \times 18$

$\bar{\sigma} = 190.9$ kN/m^2

(*a*) For rapid build-up, the properties for the undrained state and total pressure are to be used:

$s = c_u + \sigma \tan \phi_u$

Shear strength $= 45 + 367.5 \tan 18°$

= 164.4 kN/m²

(*b*) For slow build-up, the effective stress properties and effective pressure are to be used:

$s = c' + \bar{\sigma} \tan \phi'$

Shear strength $= 36 + 190.9 \tan 27°$

= 133.3 kN/m²

Example 11.13: A vane, 10.8 cm long, 7.2 cm in diameter, was pressed into a soft clay at the bottom of a bore hole. Torque was applied and the value at failure was 45 Nm. Find the shear strength of the clay on a horizontal plane.

Solution:

$$T = c\pi\left(\frac{D^2H}{2} + \frac{D^3}{6}\right)$$

for both end of the vane shear device partaking in shear.

$$45/1000 = c\pi\left(\frac{(7.2)^2 \times 10.8}{2} + \frac{7.2^3}{6}\right) \times \frac{1}{100 \times 100 \times 100}$$

$$c = \frac{45 \times 100 \times 100 \times 100}{1000\left(\frac{(7.2)^2 \times 10.8}{2} + \frac{7.2^3}{6}\right)} \text{ kN/m}^2 = 42 \text{ kN/m}^2$$

The shear strength of the clay (cohesion) is **42 kN/m²,** nearly.