

Module III CONSOLIDATION

When the soil is subjected to any compressive force like all other materials its volume decreases.

The property of the soil due to which a decrease in volume occurs under compressive force is known as the compressibility of soil. The compression of soils can occur due to one or more of the following causes.

1. Compression of solid particles and water in voids
2. Compression (decrease in volume of soil mass under stress)
3. Compression and expulsion of air in the voids
3. Expulsion of water in the voids.

- The two most important requirements for stability & safety of the structure are

a. Deformation especially vertical deformation called settlement of the soil should not be excessive & must be within permissible limits.

b. The shear strength of the foundation soil should be adequate to withstand the stresses induced

(a) The compression of solid particle is very small which is also considered as negligible.

- The compression of air is very easy as the air is highly compressible in nature. Hence the volume of air is rapidly reduced after the application of load.

- This expulsion of air content is only possible in case of partially saturated soil and the process is known as compaction.

- In case of fully saturated soil the reduction in the volume is possible only when the water content is expelled out from the void space of soil mass.

- This process is known as consolidation. (gradual process)

According to Terzaghi every process involving a decrease in the water content of a saturated soil without replacement of the water by air is called process of consolidation.

Swelling is a opposite process which involves an increase in the water content due to increase in the volume of voids.

- In sands consolidation may be generally considered to keep pace with construction while in clays the process of consolidation proceeds long after the construction has been completed.

Note: The reduction in the volume is only because of the rearrangement of solid particle as the water expelled out from the void space and creat vacuum spaces which is filled of by the soil.

- A very negligible amount of volume reduction is possible because of distortion and bending, fracture of solid particle.

Comparison between Compaction and Consolidation

- | | |
|---|--|
| 1. Expulsion of pore air | Expulsion of porewater |
| 2. Soil involved is partially saturated soil. | Fully saturated soil. |
| 3. Applies to cohesive as well as cohesionless soils. | Applies to Cohesive Soils only |
| 4. Brought about by artificial or human agency | Brought about by application of load or by natural agencies |
| 5. Dynamic loading is commonly applied | Static loading is commonly applied |
| 6. Improves bearing capacity and settlement characteristic | - Same |
| 7. Relatively quick process | relatively slow process |
| 8. Relatively complex phenomenon involving expulsion, compression & dissolution of pore air in water. | Relatively Simple Phenomenon |
| 9. Useful in primarily in embankments & earth dams | Useful as a means of improving the Properties of foundation soil |
| 10. Almost an instantaneous phenomenon | It is a time dependent phenomenon |

Stages of consolidation

There are 3 stages of consolidation

1. Initial consolidation
2. Primary consolidation
3. Secondary consolidation

Initial consolidation :-

- When the load is applied to the partially saturated soil a decrease in the volume of soil mass occurs which is due to the expulsion and compression of air from the void space of soil mass. A very small reduction in the volume of soil particle is also possible. This consolidation stage is called initial consolidation

* In case of fully saturated soil the reduction in volume is possible due to compression of solid particles only.

Primary consolidation :-

⇒ It is the most important stage of consolidation

- The further reduction in the volume is due to the expulsion of water from the void space of soil mass
- When the pressure is applied on the saturated soil initially all the applied pressure is taken by the water, but the water is incompressible in nature
- After the application of load it creates a head difference due to the compression of solid particle, after the initial consolidation stage so the water starts flowing.
- But the flow of water depends upon the permeability of soil, hence in case of fine grained soil the primary consolidation process is very slow.
- But in coarse grained soil the primary consolidation process is very high

Secondary consolidation :-

- After the completion of primary consolidation the secondary consolidation process occurs.
- In this stage of consolidation the reduction in volume is possible at a very slow rate.

Means

The reduction in volume continues at a very slow rate even after the excess hydrostatic pressure developed by the applied pressure is fully dissipated and the primary consolidation is complete. This additional reductive in volume is called Secondary consolidation.

- The process is just the plastic readjustment of solid particle.

$$\text{Total Consolidation} = S_i + S_c + S_{sc}$$

(St)

S_i = initial consolidation

S_c = settlement due to primary consolidation

S_{sc} = Secondary " "

For stiff clay $S_t = S_i + S_c$
 S_{sc} is minor

Terzaghi's model :-

- This model is suggested by Terzaghi which contains a cylinder, piston, valve, tube, spring, water

- Initially a 10kN load is applied on the piston and the valve is closed so there is no expulsion of water is possible. Hence, the load which is applied is taken by the spring to maintain the equilibrium.

- When the valve is opened the water starts flowing through the tube as the load, which is applied is taken by the water but not by the spring.

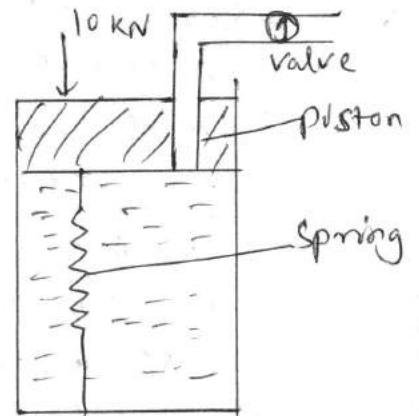
- As the water expelled out gradually the piston is moving downwards after reaching certain point the applied load is carried out by both water and the spring.

- After all the water is expelled out from the cylinder the entire load is carried by the spring only.

- Considering the spring as soil mass, the applied load is considered as the static load, which is applied on the soil and consider the model as a saturated

Soul system under a permanent load.

- As the model behaves at different intervals of time likewise practically the saturated soul is also behave in the same way



Behaviour of saturated soul under pressure :-

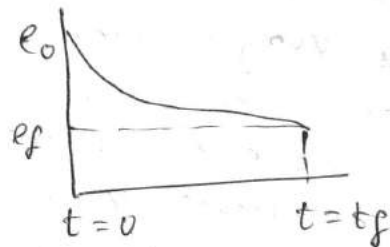
- The pressure developed on the water after the application of compressive load is called hydrodynamic pressure or is called excess hydrostatic pressure
- After that the water starts flowing and the pore water pressure inside the soul mass is reduced hence the value of effective stress will increase

$$\sigma' = \sigma - u(\downarrow)$$

$$e_0 > e_f$$

t_0 - initial time

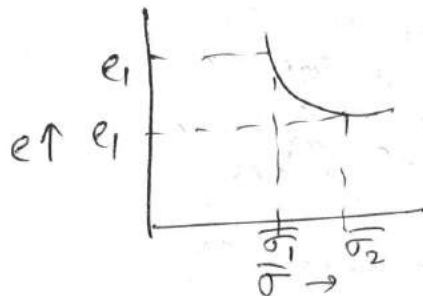
t_f final time



Relation between effective stress & void ratio

$$e_1 > e_2$$

$$\bar{\sigma}_2 > \bar{\sigma}_1$$

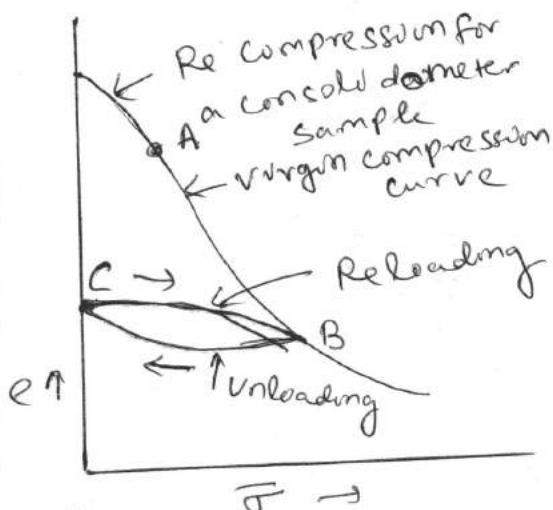


Role of stress History / Consolidation of laterally confined soil

- Soil tend to retain the effects of stress changes that have taken place in their geological history in the form of their structure. A soil which is subjected to a certain effective stress for the first time in its geological history will be more compressible than when it has been subjected to a larger effective stress in its earlier history but is now relieved of that effective stress due to some reason.

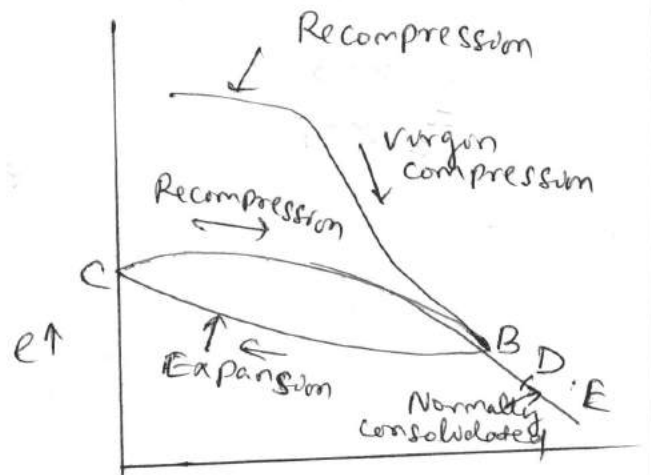
- When a soil is stressed to a level greater than the maximum stress to which it was ever subjected in the past, perhaps some kind of breakdown in the soil structure occurs resulting in a much higher compressibility indicated by a steep void ratio-effective stress ($e - \bar{\sigma}$) curve

Figure 1 The initial flatter portion of the $e - \bar{\sigma}$ curve is called re compression curve & the steeper portion after break in the curve (attributed to a breakdown in structure) is called virgin compression curve because the soil is experiencing 1st time stresses in this part. Somewhere between these two parts of curve lies the point A corresponding to the maximum value of stress the soil has ever experienced called preconsolidation stress $\bar{\sigma}_c$ or $(\bar{\sigma}_p')$



Fig(1)

$e - \bar{\sigma}$ curve on arithmetic & logarithmic plot



Fig(2)

$\log \bar{\sigma}$

Normally Consolidated: - When the existing effective stress $\bar{\sigma}$ is the maximum that it has ever experienced in its stress history. $\bar{\sigma} = \bar{\sigma}_c$ (or σ_p')

OR it has the type of consolidation when the soil has not subjected to previous loading in other words the applied load over a type of soil which has not experienced any kind of loading previously so after the loading the soil shows some kind of settlement in it which is considered as normally consolidated soil.
Ex: - AB - loading curve

Over consolidated soil: -

- If the existing effective stress is than preconsolidation stress that is $\bar{\sigma} < \bar{\sigma}_c$ (σ_p')

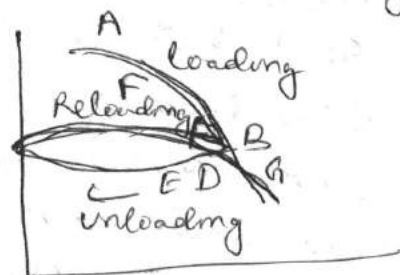
- A soil is said to be over consolidated soil if it had been subjected to any previous loading in the past which is more than the present one

Ex CD = reloading curve

Under consolidated soil: -

- If the soil deposit has not reached equilibrium under the applied overburden load it is said to be under consolidated soil. This normally occurs in an area which is recently backfilled.

AB = loading curve
BEC = unloading curve
CFD = reloading curve
DG = extension of 1st curve



Over Consolidation Ratio: -

$$OCR = \frac{\bar{\sigma}_c}{\bar{\sigma}} = \frac{\sigma_p'}{\sigma}$$

Max^m effective stress to which the soil has been subjected in its stress history

Existing effective stress in the soil (present)

For normally consolidated soil $OCR = 1$

For pre/over consolidated soil $OCR > 1$

Under consolidated soil

$$OCR < 1$$

Fig(1) & Fig(2) are obtained by testing a clay sample in a consolidometer. Branch AB of the plot represents a soil that has never been subjected to compressive loads on the semilogarithmic plot in Fig(2) thus appears as a straight line. After the load has reached a value represented by point B the soil sample is unloaded in stages. Corresponding to this release of load an expansion curve BC is obtained. It can be seen that the unloading has not restored the soil to the original state. There is a permanent deformation due to an irreversible alteration in the soil structure. The part of deformation that is recovered is attributed to the elastic rebound of the soil skeleton. The specimen is loaded again after the unloading.

The reloading curve lies slightly above the rebound curve & the two meet just before point B. It is evident that the reloaded soil is not compressed as much as the soil that was not loaded before.

On the semilogarithmic plot this portion of the reloading curve for the now preconsolidated soil has a convex curvature upwards.

If the loading is now resumed beyond point B the curve merges smoothly into the straight line DE that would have been obtained had the loading not been interrupted at B.

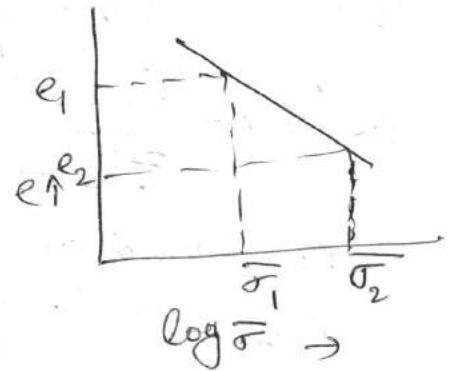
An empirical observation that can be made that $e_v \text{ vs } \log \bar{\sigma}$ is always a st. line for normally consolidated clay. For over consolidated clay is always a convex curvature upwards.

Determination of (C_c) Compression index

For normally consolidated clay slope of the straight line portion of e vs $\log \bar{\sigma}$

This parameter is known as Compression index

$$C_c = \frac{e_1 - e_2}{\log_{10} \bar{\sigma}_2 - \log_{10} \bar{\sigma}_1} = \frac{\Delta e}{\log_{10} \left(\frac{\bar{\sigma}_2}{\bar{\sigma}_1} \right)}$$



C_c is a constant for a given soil & it is not function of the effective stress.

Higher the compression index value larger the resulting vertical deformation in clays

e_1 = initial void ratio corresponding to the initial pressure $\bar{\sigma}_1$

e_2 = void ratio at increased pressure $\bar{\sigma}_2$

Skempton (1944) has given equation for estimating C_c

For remoulded clay sample $C_c = 0.007 (W_L - 10)$

For undisturbed clay of medium to low sensitivity

$$C_c = 0.009 (W_L - 10)$$

Hough (1957) For precompressed soil

$$C_c = 0.3 (e_0 - 0.27)$$

e_0 = in situ void ratio

The slope of the expansion curve (branch BC) in Fig 2 is called swelling index C_s

can be given by $e_2 = e_1 + C_s \log \left(\frac{\bar{\sigma}_1}{\bar{\sigma}_2} \right)$

C_s is a measure of the volume increase in a soil consequent to release of stress. (Swelling index) Expansion Index

e_2 = void ratio at a stress $\bar{\sigma}_2$ after a release in stress from $\bar{\sigma}_1$

Coefficient of Compressibility (a_v) :-

It is defined as the decrease in void ratio per unit increase of pressure

$$a_v = \frac{-\Delta e}{\Delta \sigma} = \frac{e_1 - e_2}{\sigma_2 - \sigma_1}$$

OR
$$a_v = \frac{e_0 - e}{\sigma' - \sigma_0'} = \frac{-\Delta e}{\Delta \sigma'}$$
 (-ve sign indicates decrease in void ratio)

e_0 = void ratio under pressure σ_0'

e = void ratio under pressure σ'

Coefficient of volume change (m_v) :-

It is defined as the change in volume of a soil per unit of initial volume due to a given unit increase in the pressure.

$$m_v = \frac{-\Delta V}{V_0} \cdot \frac{1}{\Delta \sigma'} = \frac{a_v}{1+e_0}$$

We know $e = \frac{V_v}{V_s} \Rightarrow 1+e = \frac{V_v}{V_s} + 1 = \frac{V_v + V_s}{V_s} = \frac{V}{V_s}$

$$V = (1+e) V_s$$

$$V_0 = (1+e_0) V_s$$

$$V_1 = (1+e_1) V_s$$

$$\Delta V = (V_0 - V_1) = (1+e_0) V_s - (1+e_1) V_s = (e_0 - e_1) V_s$$

$$\Delta V = \Delta e V_s$$

Substitute V_0 & ΔV

$$m_v = \frac{-\Delta e}{(1+e_0)} \cdot \frac{1}{\Delta \sigma'} = \frac{1}{(1+e_0)} \cdot \frac{(-\Delta e)}{\Delta \sigma'}$$

$$m_v = \frac{a_v}{1+e_0}$$

When the soil is laterally confined the change in volume is proportional to change in the thickness ΔH & initial volume is proportional to the initial thickness H_0

$$m_v = \frac{-\Delta H}{H_0} \cdot \frac{1}{\Delta \sigma'}$$

$$\Delta H = -m_v H_0 \Delta \sigma'$$

-ve sign void ratio or thickness decrease with the increase in the pressure

ΔH = change in thickness due to pressure increment

Consolidation settlement :-

When the soil stratum of thickness H has fully consolidated under pressure increment $\Delta\sigma'$

$$p_f = m_v H \Delta\sigma'$$

Using void ratio (e)

$$p_f = \frac{H}{1+e_0} C_c \log_{10} \frac{(\sigma'_0 + \Delta\sigma')}{\sigma'_0} \quad \text{For normally consolidated clay}$$

H = initial thickness of consolidating layer

e_0 = initial void ratio of the layer

C_c = Compression index

σ'_0 = initial effective pressure at the middle of the layer

$\Delta\sigma'$ = effective pressure increment at the middle of the layer

Over consolidated soil

$$p_f = \frac{C_s}{1+e_0} H \log_{10} \frac{\sigma'}{\sigma'_0}$$

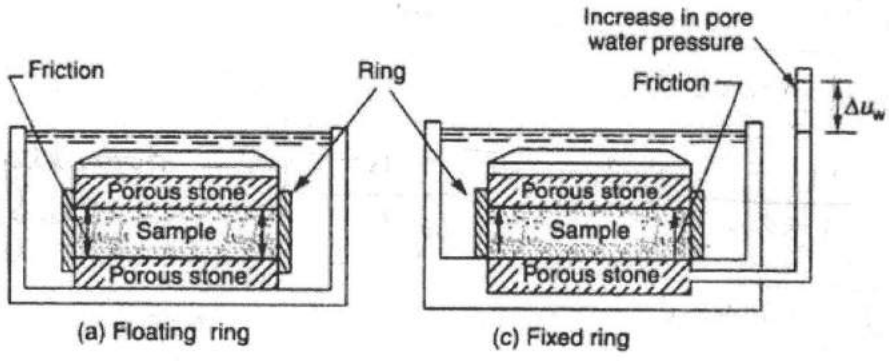
$$\sigma' = \sigma'_0 + \Delta\sigma'$$

Recompression Index :-

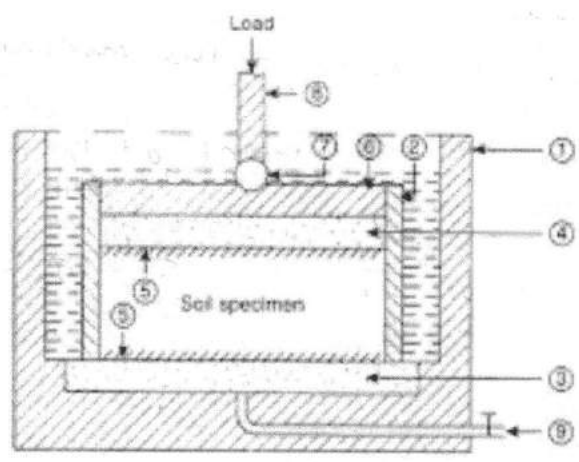
$$C_r = \frac{-\Delta e}{\log \left(\frac{\bar{\sigma} + \Delta\bar{\sigma}}{\bar{\sigma}} \right)}$$

The load is applied at the time of reloading which is less than the previous loading which is applied at the very beginning over the soil

Consolidation Test :-

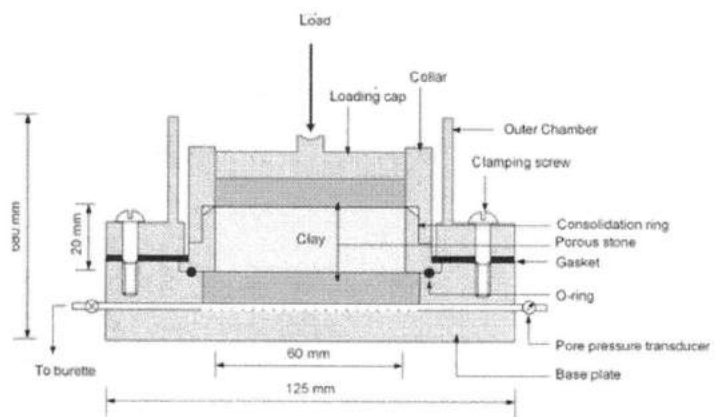


Types of consolidometers



- (1) Consolidation cell
- (2) Specimen ring
- (3) Bottom porous stone
- (4) Top porous stone
- (5) Filter paper
- (6) Pressure pad
- (7) Pressure ball
- (8) Loading yoke
- (9) Outlet valve

Sectional elevation of a fixed-ring consolidometer.



Consolidation Test :-

Aim:- To Study the Compressibility & Characteristics of

soil
- It is a laboratory method and this test is performed by using Consolidometer or by oedometer apparatus

Oedometer apparatus :-

- It consists of a consolidation cell (which is cylindrical in shape), two porous stones are provided at the top and bottom of soil specimen.

- 2 types of cylindrical container

(i) Free ring cell (both top and bottom porous stone are free to move)

(ii) Fixed ring cell (bottom porous stone is not moving)

In free ring cell compression occurs from both top & bottom while in fixed ring cell the soil sample moves only downward relative to the soil sample

(i.e. only the top porous stone is permitted to move downwards as the specimen compresses)

In free ring cell the friction between the ring & the soil is somewhat less than in the fixed ring cell.

- The inside surface of the ring should be frictionless

- Thickness of soil specimen

(i) 20mm thick

(ii) Ratio between diameter of ring and thickness of the soil specimen = 3

(iii) The thickness of soil specimen should not be less than 10 times of the maximum size of soil particle

- Internal dia. of cell varying from 60mm to 100mm

- The soil sample is loaded in increments of vertical stress under each stress increment the sample is allowed to consolidate till there is little or no further compression, with all the excess pore water pressure being completely dissipated.

- Usually a load is kept for 24 hrs. The stresses commonly used are 25, 50, 100, 200, 400 & 800 kN/m².
- Vertical deformation of the specimen is measured by means of a dial gauge.

- To determine the rate of compression under each stress increment dial gauge readings are taken at different elapsed times after a load is placed.

One schedule of elapsed time - 0.25, 1, 2.25, 4.0, 6.25, 9.0, 12.25, 16.0, 20.25, 25, 36, 49, 60, 120, 240 mins etc till last reading is taken at 24 hrs.

- Another schedule which is in common practice 0.25, 0.5, 1, 2, 4, 8, 15, 30, 60, 120 mins etc

- The 24 hr reading gives the final compression under each stress increment

- To obtain void ratio vs pressure relationship the equilibrium void ratio is computed with the help of 24 hr reading.

- After the consolidation under last stress increment is over, the specimen is unloaded in two or three stages & the soil allowed to swell only the final swell readings are taken at each unloading stage and after the completion of swelling the consolidation ring with the soil specimen is taken out, dried in an oven to determine the wt. of solids & final water content.

To determine pre consolidation stress :-

pre consolidation pressure is the greatest effective stress to which the soil has been subjected to in the past & under which it has undergone full consolidation.

The most popular method suggested by A Casagrande (1938) Procedure:

1. By judgement of eye point A, the point of maximum curvature on the consolidation curve is located.

2. At A a horizontal line is drawn

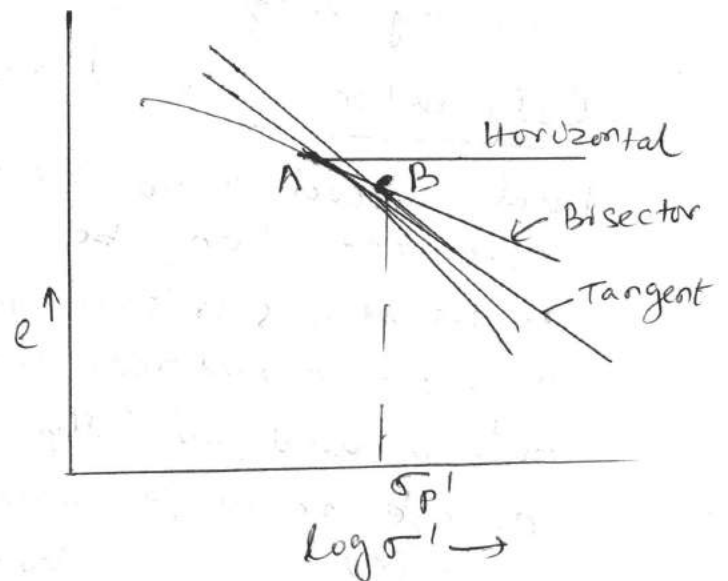
3. A tangent is drawn to the curve at A

4. The angle obtained by steps (2) & (3) is bisected.

5. The straight line part of the curve is extended

back to meet the bisector line obtained in step 4

The point of intersection of these lines (Point B) gives the pre consolidation pressure or stress σ_p' ($\bar{\sigma}_c$)



The Consolidation test data are used to determine the following things

- (i) Void ratio and Coefficient of volume change
- (ii) Coefficient of consolidation
- (iii) Coefficient of permeability

- The Consolidation test is carried out to determine the volume compressibility characteristic of a soil, the relation between void ratio & effective stress of soil corresponding to load increment as well as load decrement and the permeability characteristic of soil and its value.

- It will also help to identify the settlement characteristic of soil indirectly.

- The test result can be determined by 2 methods

- (i) By change in height method
- (ii) By change in void ratio method

Calculation of void ratio and Coefficient of Volume Change

Final void ratio at the end of each pressure increment can be calculated by two methods

1. Height of solids method → for both saturated and unsaturated samples
2. Change in void ratio method

↓
this is used for fully saturated specimen.

Change in height method:-

$$H_s = \frac{M_d}{G A \rho_w} = \frac{W_d}{G A}$$

H_s = Height of solids of specimen (cm)

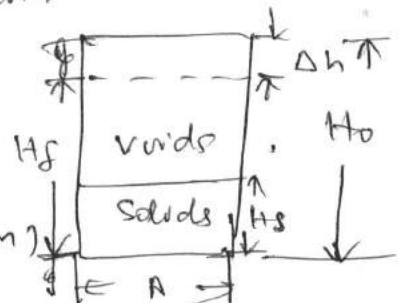
M_d = Mass of dried specimen

W_d = wt of dried specimen

A = cross-sectional area of specimen (cm²)

G = sp. gr. of soil

$$e = \frac{H - H_s}{H_s}$$



H = Specimen height (cm) at equilibrium under various applied pressures = $H_1 + \Delta H$
 $= H_0 + \sum \Delta H$

H_0 = initial ht. of specimen

ΔH = change in the specimen thickness under any pressure increment.

H_1 = ht. of specimen at the beginning of the load increment

S & w are calculated knowing e values at the beginning & at the end

Change in void ratio method :-

$$e_f = W_f G$$

$$e = \frac{V - V_s}{V_s} = \frac{V}{V_s} - 1 \Rightarrow V = V_s(1+e)$$

$$A \times H = V_s(1+e) \quad \text{By partial differentiation} \quad A dh = V_s de$$

W_f = final water content at the end of the test

e_f = final void ratio at the end of the test

$$\frac{\Delta e}{1+e} = \frac{\Delta H}{H} \quad \Delta e = \text{Change of void ratio under each pressure increment}$$

$$\Delta e = \frac{1+e_f}{H_f} \Delta H$$

H_f = final ht. of specimen at the end of the test

$$m_v = - \frac{\Delta e}{1+e_0} \frac{1}{\Delta \sigma}$$

void ratio method

$$m_v = - \frac{\Delta H}{H_0} \frac{1}{\Delta \sigma}$$

change in thickness method

(m^3/kN)

Coefficient of permeability (k) :-

$$k = C_v m_v \gamma_w \left[k = \frac{C_v v \gamma_w}{1+e_0} \right]$$

Determination of C_v :-

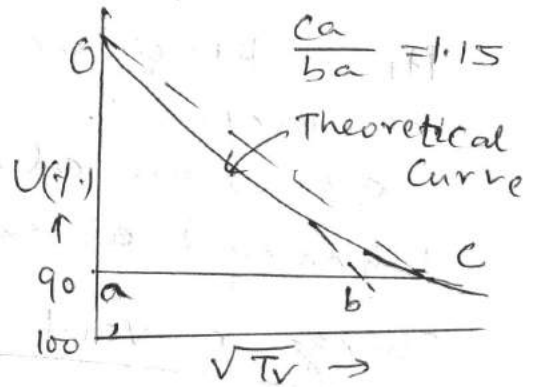
1. Square root of time fitting method
2. Logarithmic time fitting method

This method utilizes the theoretical relationship between U & $\sqrt{T_v}$. The relationship is linear up to the value of U equal to 60%.

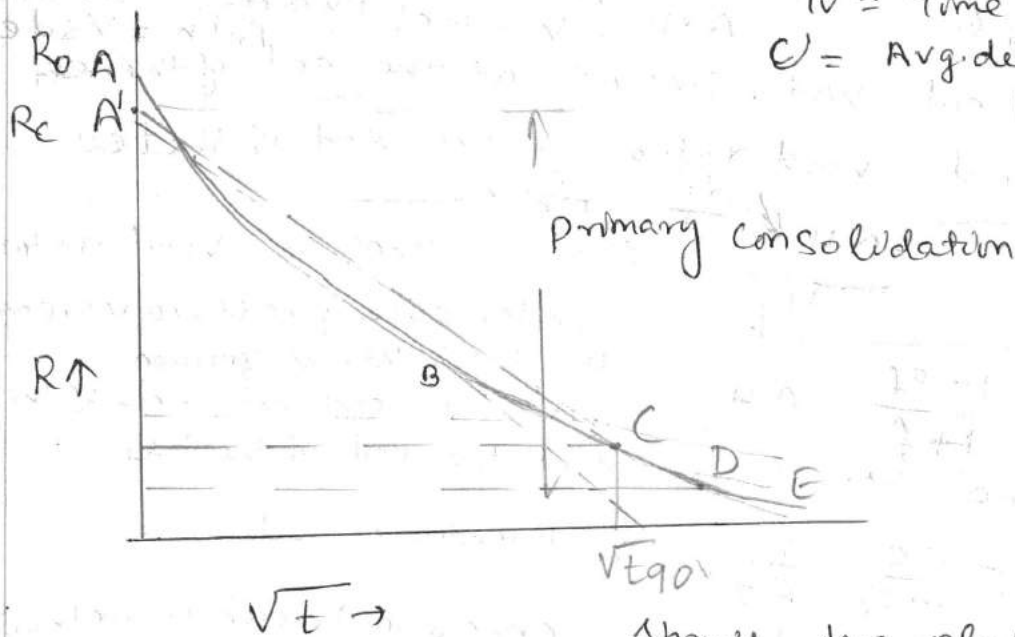
At $U=90\%$ the value of $\sqrt{T_v}$ is 1.15 times the value obtained by the extension of the initial straight line portion.

For a given load increment the dial gauge readings are taken for different time intervals.

A curve is plotted between dial gauge reading (R) & \sqrt{t} .



T_v = Time factor
 C = Avg. degree of consolidation



- The curve ABCDE shows the plot. The curve begins at the dial gauge reading R_0 at time to indicated by point A.

As the load increment is applied there is initial compression. It is obtained by producing back initial linear part of the curve to intersect the dial gauge reading axis at point A'. This corresponds to the corrected zero reading R_c .

- The Consolidation between dial gauge reading R_0 & R_c is the initial compression.

- From the corrected zero reading point A' a line A'C is drawn such that its abscissa is 1.15 times that of the initial linear portion A'B of the curve.

- The intersection of this line with the curve at point C indicates 90% of U. The dial gauge reading corresponding to C as R_{90} & the corresponding abscissa as $\sqrt{t_{90}}$

The point D for 100% primary consolidation can be obtained from R_{90} as

$$R_c - R_{100} = \frac{10}{9} (R_c - R_{90})$$

The consolidation after 100% primary consolidation on the range DE is the secondary consolidation. The value of (C_v) of the soil for that load increment is obtained from the value of $\sqrt{t_{90}}$ obtained from that plot $\sqrt{t_{90}}$

$$C_v = \frac{T_v d^2}{t} = \frac{0.848 d^2}{(\sqrt{t_{90}})^2} = \frac{0.848 d^2}{t_{90}}$$

For $U = 90\%$, $T_v = 0.848$

d = distance of drainage path

H = total thickness

H_0 = initial thickness

H_f = final thickness

For double drainage

$$d = H/2 = \left(\frac{H_0 + H_f}{2} \right)$$

Logarithm of time method:

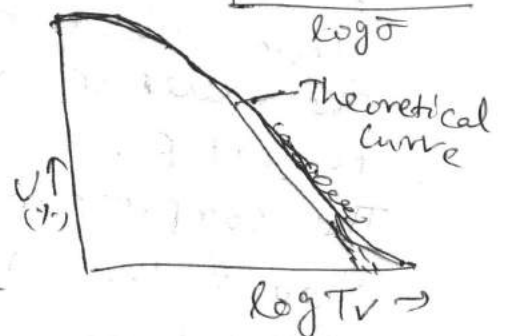
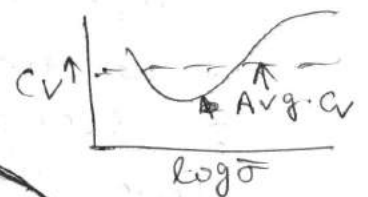
The method given by Casagrande

The curve consists of 3 parts

(i) an initial portion which is parabolic in shape

(ii) a middle portion which is almost linear

(iii) the last portion to which the horizontal axis is an asymptote. It is observed that the point of inflexion on the curve & the lower portion gives the value of 100% consolidation.



For a given load increment a curve is plotted between the dial gauge reading R & $\log t$.
Let R_0 be the initial dial gauge before the application of load increment.

R_c - corrected dial gauge reading is obtained using the fact that initial portion of the curve is parabolic.

Two points B & C are selected corresponding to some arbitrary time t_1 & $4t_1$ respectively & having vertical intercept a .
Point A' is located such that the vertical intercept between B & A' is also equal to a .
It represents the corrected dial gauge reading R_c

- Corresponding to zero primary consolidation procedure is repeated by selecting 2 other points with time ratio 1:4.

- It should give approximately same location of point A' the consolidation between dial gauge reading R_0 to R_c represented by A & A' is initial compression.

- The final portion of the curve is linear. The point F corresponding to 100% consolidation is obtained from the intersection of two linear parts.
The values of R_{100} & t_{100} are obtained corresponding to point F .

The compression between dial gauge readings R_c & R_{100} is the primary consolidation & between R_{100} & R_f is the secondary consolidation.

Point M corresponding to 50% consolidation

R_{50} is located between R_c & R_{100} & the value of t_{50} is obtained midway.

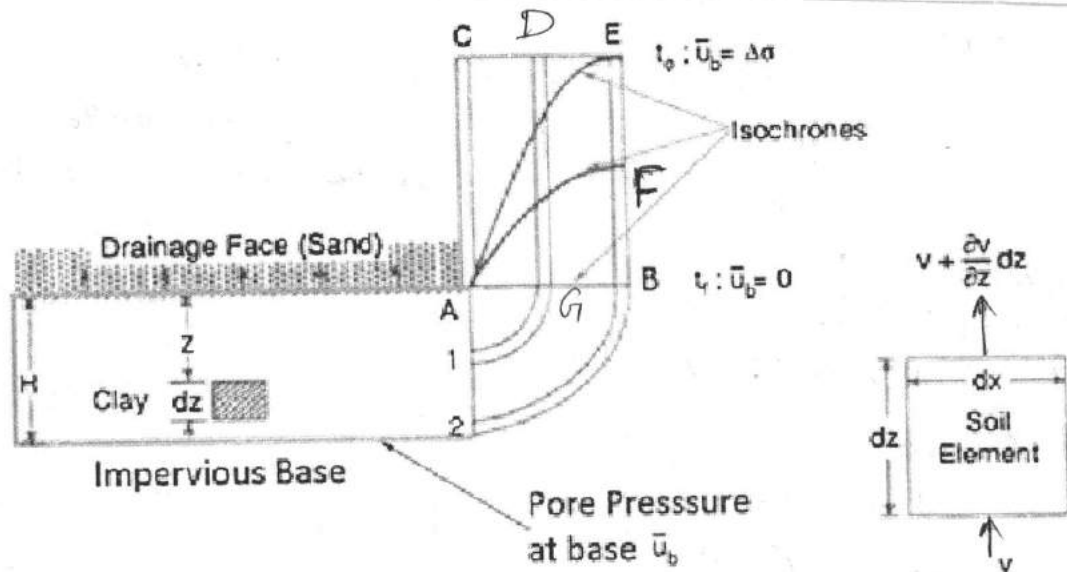
Terzaghi Theory of One dimensional Consolidation:-

Assumption:-

1. The soil is homogeneous & fully saturated
2. Soil particles and water are incompressible
3. The deformation of soil is due to entirely to change in volume
4. Darcy's law for the velocity of flow of water through soil is perfectly valid
5. Coefficient of permeability is constant during consolidation.
6. Load is applied in one direction only & deformation occurs only in the direction of load application.
7. Excess pore water drains out only in the vertical direction.
8. The boundary is a free surface offering no resistance to the flow of water from soil
9. The change in thickness of the layer during consolidation is significant.
10. The time lag in consolidation is due entirely to the permeability of the soil and secondary consolidation is disregarded.

A clay layer of thickness H sandwiched between two layers of sand which serves as drainage faces. When the layer is subjected to a pressure increment $\Delta\sigma$ excess hydrostatic pressure is set up in the clay layer. ~~When the layer is subjected to a pressure increment $\Delta\sigma$ excess hydrostatic pr. is set up in~~

At time $t=0$ the instant of pressure application whole of consolidating pressure $\Delta\sigma$ is carried by the porewater. So that the initial excess hydrostatic pressure u_0 is equal to $\Delta\sigma$ & is represented by a straight line $u = \Delta\sigma$ on the pressure distribution diagram.



The straight line CDE joining the water levels in the piezometric tubes represents this distribution. As the water starts seeping into the sand the excess hydrostatic pressure at the pervious boundaries drops to zero & remains so at all times. After a great time t_p the whole of the excess hydrostatic pressure is dissipated so that $\bar{u} = 0$ represented by line AFB.

At an intermediate time t the consolidating pressure $\Delta\sigma$ is partly carried by water & partly by soil

$$\Delta\sigma = \Delta\sigma' + \bar{u}$$

The distribution of excess hydrostatic pressure \bar{u} at any time t is indicated by the curve AFB joining water levels in the piezometric tubes this curve is known as isochrone. The no. of isochrones can be drawn at various time intervals t_1, t_2, t_3, \dots

-As the consolidation process increased the value of k and m_v is reduced.

The slope of isochrones at any point at a given time indicates the rate of change of \bar{u} with depth.

At any time t hydraulic head $h = \frac{\bar{u}}{\gamma_w}$
Corresponding to excess hydrostatic pr.

$$i = \frac{\partial h}{\partial z} = \frac{1}{\gamma_w} \frac{\partial \bar{u}}{\partial z}$$

The rate of change of \bar{u} along the depth of layer represents the hydraulic gradient

The velocity with which the excess pore water flows at depth z is given by Darcy's law

$$V = Ki = \frac{k}{\gamma_w} \frac{\partial \bar{u}}{\partial z}$$

The rate of change of velocity along the depth of the layer

$$\frac{\partial V}{\partial z} = \frac{k}{\gamma_w} \frac{\partial^2 \bar{u}}{\partial z^2} \quad \text{--- (7)}$$

Consider a small soil element of size dx, dz & of width dy + lar to the xz plane. If v is the velocity of water at the entry into the element the velocity at exit be equal to $v + \frac{\partial v}{\partial z} dz$

The quantity of water entering the soil element
= $v dx dy$

The quantity of water leaving the soil element
= $(v + \frac{\partial v}{\partial z} dz) dx dy$

The net quantity of water dQ squeezed out of the soil element per unit time.

$$\Delta Q = \frac{\partial v}{\partial z} dx dy dz \quad \text{--- (1)}$$

The decrease in volume of soil is equal to the volume of water squeezed out

$$\Delta V = -m_v V_0 \Delta \sigma' \quad \text{--- (2)}$$

V_0 = Vol. of soil element at time $t_0 = dx dy dz$

Change of volume per unit time is given by

$$\frac{\partial (\Delta V)}{\partial t} = -m_v dx dy dz \frac{\partial (\Delta \sigma')}{\partial t} \quad \text{--- (3)}$$

Equating eqn (1) & (3)

$$\frac{\partial v}{\partial z} = -m_v \frac{\partial(\Delta\sigma')}{\partial t} \quad (4)$$

where $\Delta\sigma$ is constant

$$\Delta\sigma = \Delta\sigma' + \bar{u}$$

$$\frac{\partial(\Delta\sigma')}{\partial t} = -\frac{\partial\bar{u}}{\partial t} \quad (5)$$

From Eqn (4) & (5)

$$\frac{\partial v}{\partial z} = m_v \frac{\partial\bar{u}}{\partial t} \quad (6)$$

Combining eqn (6) & (7)

$$\frac{\partial\bar{u}}{\partial t} = \frac{k}{m_v \gamma_w} \frac{\partial^2\bar{u}}{\partial z^2}$$

$$C_v \text{ in } m^2/\text{sec}$$

$$m_v \text{ in } mm^2/kN$$

$$k \text{ in } m/\text{sec}$$

$$\frac{\partial\bar{u}}{\partial t} = C_v \frac{\partial^2\bar{u}}{\partial z^2}$$

$$C_v = \frac{k}{m_v \gamma_w}$$

This is the basic differential eqn of consolidation which relates the rate of dissipation of excess pore pressure with the rate of expulsion of pore water from a unit volume of soil

If f_f = final settlement under pressure increment $\Delta\sigma$
 f = settlement at any intermediate time t

$$\text{Degree of consolidation at time } t = U(\%) = \frac{f}{f_f} \times 100$$

The degree of consolidation is a function of Time factor T_v

$$U(\%) = f(T_v)$$

T_v is a dimensionless parameter

t = time

$$T_v = \frac{C_v t}{d^2}$$

d = drainage path \rightarrow It represents max^m distance a water particle has to travel within the layer to reach a drainage face when a clay layer is bound by two drainage faces double drainage occurs

$$d = H/2$$

For double drainage H = thickness of layer

$$d = H$$

For single "

$$U < 60\% \quad T_v = \frac{\pi}{4} (U/100)^2$$

$$U > 60\% \quad T_v = -0.9332 \log_{10} \left(1 - \frac{U}{100}\right) - 0.0851$$

T_v depends on k, t, d, C_v, m_v

Limitation of Consolidation Theory:-

- The one dimensional consolidation theory is based on number of assumption but the equation has the following situation.

1. The value of coefficient of consolidation has been assume as constant but in practical case it changes with a change in consolidation pressure.
2. The distance (d) of the drainage path cannot be measured accurately in the field but the value which has used is a average value.
3. There is sometimes difficult to ~~locate~~ locate the location of drainage face
4. The equation is based on the one dimensional consolidation process but in actual it is a 3 dimensional process.
5. The initial consolidation and secondary consolidation has neglected but sometimes both the stage of consolidation are required to consider.
6. In the field the time require for loading is neglected, but it has to be considered.
7. In actual practice the pressure distribution may be non linear and not uniform but the theory becomes very complicated when considering the correct distribution of pressure.

Piping and Subsurface Erosion:

Most piping failures are caused by subsurface erosion in or beneath dams. These failures can occur several months or even years after a dam is placed into operation.

In essence, water that comes out of the ground at the toe starts a process of erosion (if the exit gradient is high enough) that culminates in the formation of a tunnel shaped passage (or pipe) beneath the structure. When the passage finally works backward to meet the free water, a mixture of soil & water rushes through the passage, undermining the structure and flooding the channel below the dam. It has been shown that the danger of a piping failure due to subsurface erosion increases with decreasing grain size.

Similar subsurface erosion problems can occur in relieved dry rocks, where water is seeping from a free ~~surface~~ source to a drainage or filter blanket beneath the floor or behind the walls. If the filter fails or is defective and the hydraulic gradients are critical, serious concentration of flow can result in large voids and eroded channels.