PROPERTY OF FLUID

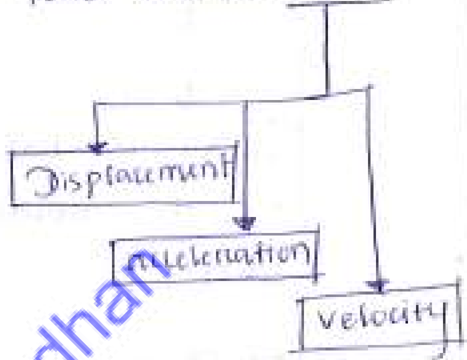
MODULE : 1

Introduction -

Fluid Mechanics

Fluid → Any substance which can flow is called fluid

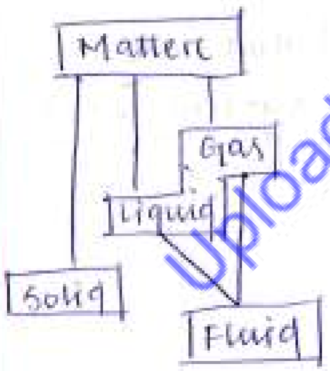
Mechanics → we study about force and its effect



substance : which have

- Finite mass
- Occupay space
- Tangiable (which we can feel).

Flow : It is relative change of position of particles w.r.t time.



→ Finite mass

→ Fluid does not have any certain shape.

→ It occupies shape of vessel.

→ which can flow under its own wt.

Fluid Mechanics -

Fluid mechanics means study of effect of force on fluid.

2 Types –

1. Fluid at rest — Fluid statics.
2. Fluid at motion – Fluid dynamics.

Microscopic and macroscopic Approch –

Study is based on the molecular level.

Avg. on the overall behaviour of the fluid is studied.

Fluid as a continuum –

In a fluid, the entermolecular spacing between the fluid particles is treated as negligible and the entire fluid mass system is assumed as continious distribution of mass, which is known as continuum.

Fluid properties –

- **Intensive properties:**

Intensive properties are those that are independent of mass of the fluid system.

Example: Temperature
Pressure
Density.

- **Extensive properties:**

Extensive properties are those whose values depend on the size or extent of the system.

Example: Total mass
Total volume
Total momentum

Mass Density :

Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density.

→ It is denoted by the symbol ρ (rho).

→ The unit of mass density in SI unit is Kg/m^3.

→ The density of liquid may be considered as constant while that of gases changes with the variation of pressure and temperature.

$$\rho = \frac{\text{mass of fluid}}{\text{volume of fluid}}.$$

→ The value of density of water is $1 gm/cm^3$ or $1000 Kg/m^3$.

Specific Gravity :

Specific gravity is defined as the ratio of the weight density of a fluid to the weight density of a standard fluid. For liquid, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density.

→ It is dimensionless quantity.

→ It is denoted by the symbol S.

$$\boxed{S = \frac{\text{weight density of fluid}}{\text{weight density of a standard fluid}}.}$$

→ The value of specific gravity of water is $9810 N/m^3$.

Relative Density :

Relative density is the ratio of density of one substance with respect to other substance.

$$\boxed{\rho_{1/2} = \frac{\rho_1}{\rho_2}}$$

Specific weight :

Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density.

→ It is denoted by w.

$$w = \frac{\text{weight of fluid}}{\text{volume of fluid}}$$

$$= \frac{\text{mass of fluid} \times \text{acceleration due to gravity}}{\text{volume of fluid}}$$

$$= \frac{\text{mass of fluid} \times g}{\text{volume of fluid}}$$

$$= \rho \times g$$

$$\boxed{w = \rho g}$$

→ Both depends upon temperature and pressure.

→ Its SI unit is N/m^3.

→ $W_{water} = 9.81 \times 1000$ Newton/m^3.

Specific volume :

Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

→ Specific volume is the reciprocal of mass density.

$$\text{Specific volume} = \frac{\text{volume of fluid}}{\text{mass of fluid}}$$

$$= \frac{1}{\dfrac{\text{mass of fluid}}{\text{volume of fluid}}}$$

$$= \frac{1}{\rho}$$

A good friend is like a four-leaf clover; hard to find and lucky to have

→ Its SI unit is m^3/kg.

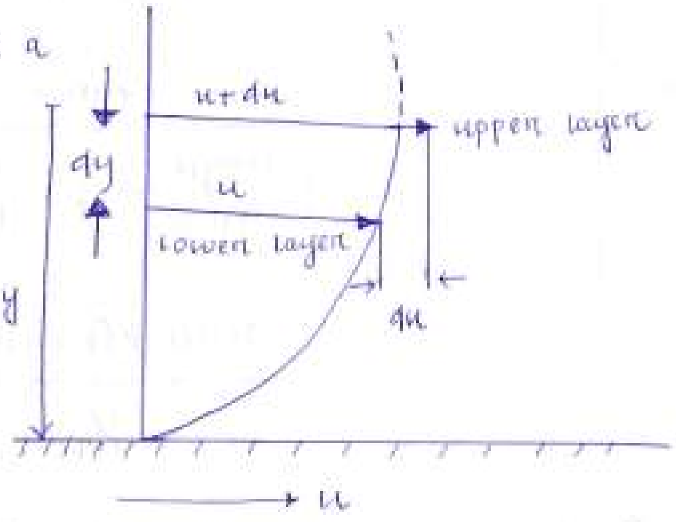
Compressibility :

It is measure of change in volume of fluid in response to a change in pressure.

VISCOSITY :

It is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

It is a measure of the internal fluid friction which causes resistance to flow. It is primarily due to cohesion and molecular momentum exchange between fluid layer and as flow occured.



When two fluid layer, at a distance "dy" apart, move one over the other at different velocities u and u+du, viscosity together with relative velocity causes a shear stress, acting between the fluid layer. The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to y.

It is denoted by symbol τ (Tow).

$$\therefore \tau \propto \frac{du}{dy} \quad \text{or} \quad \tau = \mu \frac{du}{dy}$$

where μ is the constant of proportionality & is known as co-efficient of dynamic viscosity or only viscosity. $\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

$$\therefore \mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

~~Where μ is the ...~~

$$\mu = \frac{\text{Shear stress}}{\frac{\text{change of velocity}}{\text{change of distance}}}$$

$$= \frac{\text{Force} \times \text{area}}{\left(\frac{\text{length}}{\text{time}}\right) \times \frac{1}{\text{length}}}$$

$$= \frac{\text{Force} \times (\text{length})^2}{1/\text{Time}}$$

* Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

Unit of Viscosity –

In SI system $= \frac{N\text{-}s}{m^2} = \text{pas}\cdot s$ $\left[1 \text{ pascal} = \frac{N}{m^2}\right]$

In MKS system $= \frac{Kg\text{-}sec}{m^2}$

In CGS system $= \frac{\text{dyne-sec}}{cm^2}$

* one $\frac{Kgf\text{-}sec}{m^2} = \frac{9.81\,N}{m^2} = 9.81$ poise

Kinematic viscosity –

It is defined as the ratio between dynamic velocity and density of fluid.

$$\nu = \frac{\text{viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

The units of Kinematic viscosity

$$\nu = \frac{\text{units of } \mu}{\text{units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{length})^2 \times \frac{\text{mass}}{(\text{length})^3}}$$

$$= \frac{\text{force} \times \text{time}}{\text{mass}/\text{length}}$$

$$= \frac{\text{mass} \times \frac{\text{length}}{(\text{time})^2} \times \text{time}}{\frac{\text{mass}}{\text{length}}}$$

$$= \frac{(\text{length})^2}{\text{time}}.$$

In MKS system = $metre^2/sec$

In SI system = m^2/sec.

In CGS system = cm^2/s.

→ 1 stoke = $10^{-4}\ m^2/s$

→ centistoke = $\frac{1}{100}$ stoke.

Newtonian Fluids:

→ Fluids which obey Newton's law of viscosity are known as Newtonian fluids.

→ According to Newton's law of viscosity, shear stress is directly proportional to the rate of deformation or velocity gradient across the flow.

$$\tau \propto \frac{du}{dy}$$

$$\Rightarrow \tau = \mu \frac{du}{dy}$$

Dynamic viscosity:-

- Dimension of $\mu = [ML^{-1}T^{-1}]$
- unit of μ = $N\text{-}s/m^2$ or pa.s.
- In cgs unit μ is expressed as 'poise'

$$1 \text{ poise} = 0.1\ N\text{-}s/m^2$$

- (μ) water = $10^{-3}\ N\text{-}s/m^2$
- (μ) air = $1.81 \times 10^{-5}\ N\text{-}s/m^2$.

Kinematic viscosity—

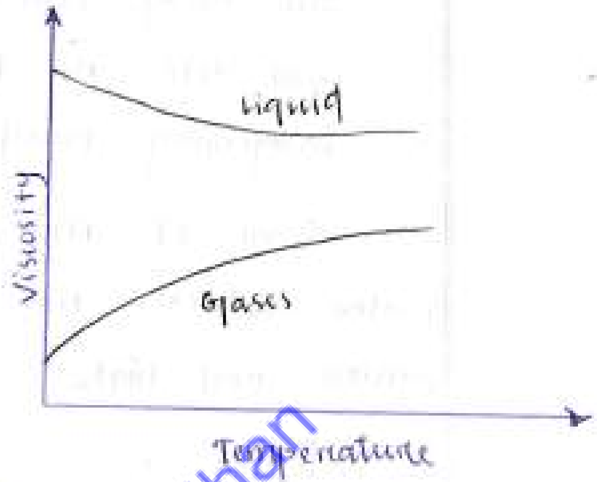
- Dimension of ν = $[L^2T^{-1}]$
- unit = m^2/s or cm^2/s.
- 1 stoke = $10^{-4}\ m^2/s$

$V_{water} = 1 \times 10^{-6}\ m^2/s$

$V_{air} = 15 \times 10^{-6}\ m^2/s$

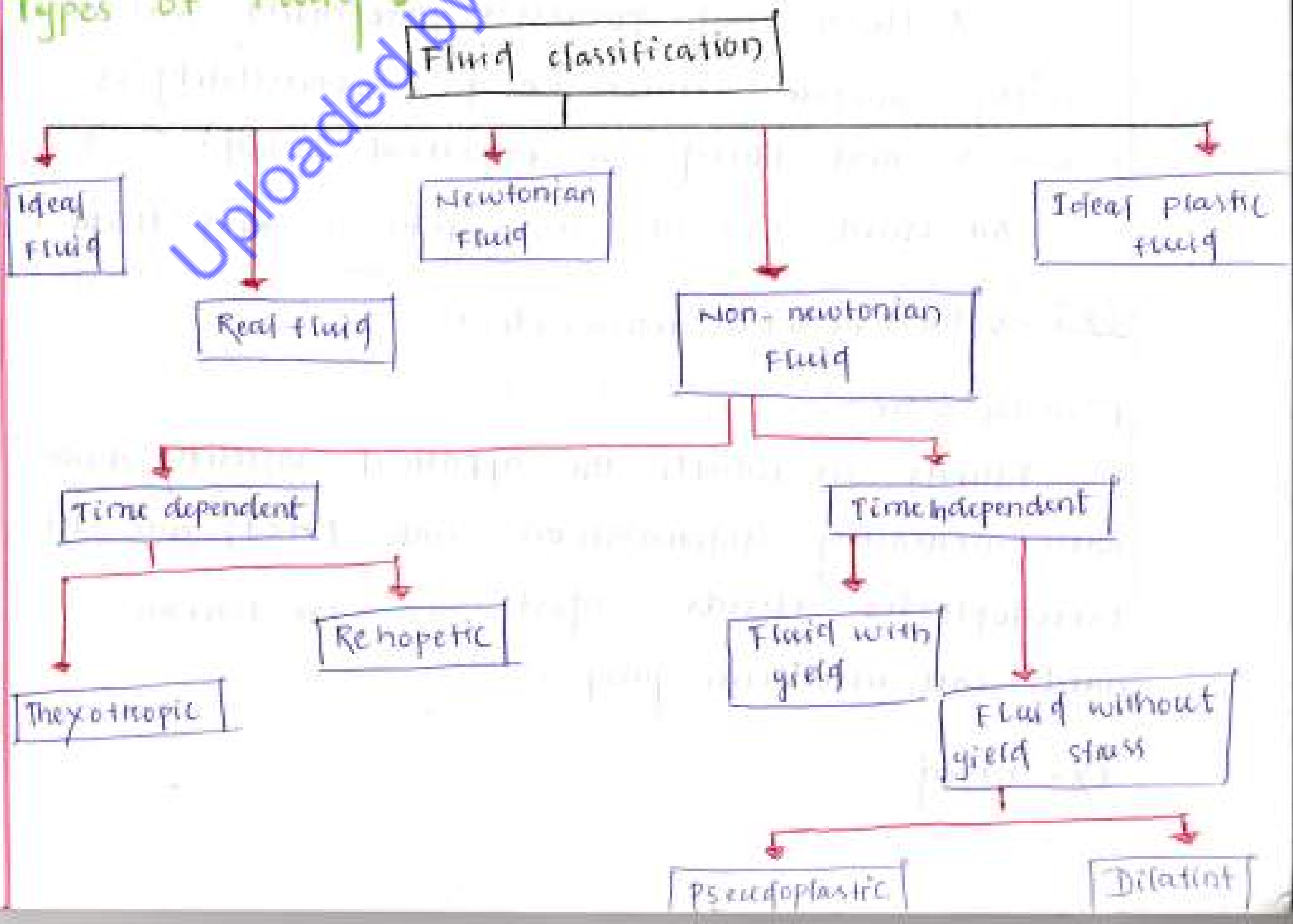
Variation of viscosity with temperature —

→ Increase in temperature causes a decrease in the viscosity of a liquid. whereas viscosity of gases increases with temperature growth.



→ ~~[illegible]~~ The reason for the above phenomena is that, in liquids ; viscosity is primarily due to molecular cohension which decreases with increase in volume due to temperature increment, while in gases, viscosity is due to molecular momentum transfer which increases with increase in number of collision between gas molecules.

Types of Fluid :



Ideal Fluid :

The ideal fluid is an incompressible fluid. This type of fluid does not have any viscosity and surface tension.

The ideal fluid is also known as a perfect fluid. In real life, no fluid has such properties so it is an imaginary fluid.

Some of the liquid can be considered ideal like water which has very low viscosity, low surface tension and high resistance to compression.

Thus water can be considered an ideal fluid for all practical purposes without incurring much appreciable error in arriving at the result.

Real Fluid :

The real fluid is a compressible fluid. Every real fluid present possess some viscosity and surface tension. A real fluid is also known as practical fluid.

A fluid that possessess properties such as viscosity, surface tension, and compressibility is known as real fluid or practical fluid.

All fluid present in nature is real fluid.

Ex: water, petrol, Kerosene, etc.

Pseudoplastic :

Fluids in which the apparent viscosity decreases with increasing deformation rate $(n<1)$ are called pseudoplastic fluids. Most non-newtonian fluids fall into this group.

Ex- Blood

Dilatant :

If the apparent viscosity increases with increasing deformation rate (n>1), the fluid is termed as dilant (or shear thickening).

Ex - Starch suspensions.

Bingham plastic :

Fluids that behave as a solid until a minimum yield stress, τ_y and flow after crossing this limit are known as ideal plastic or Bingham plastic.

The corresponding shear stress model is

$$\tau = \tau_y + \mu \frac{du}{dy}$$

Ex - Toothpaste, creams.

Thixotropic :

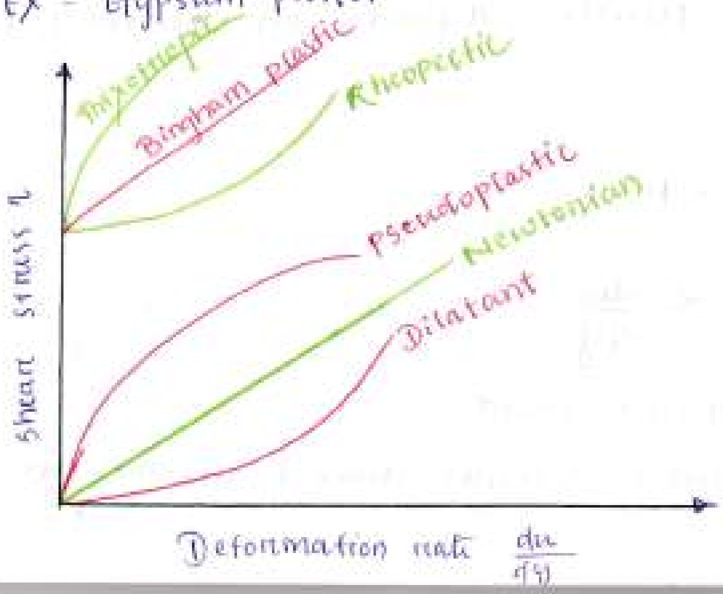
Apparent viscosity (η) for thixotropic fluids decreases with time under a constant applied shear stress.

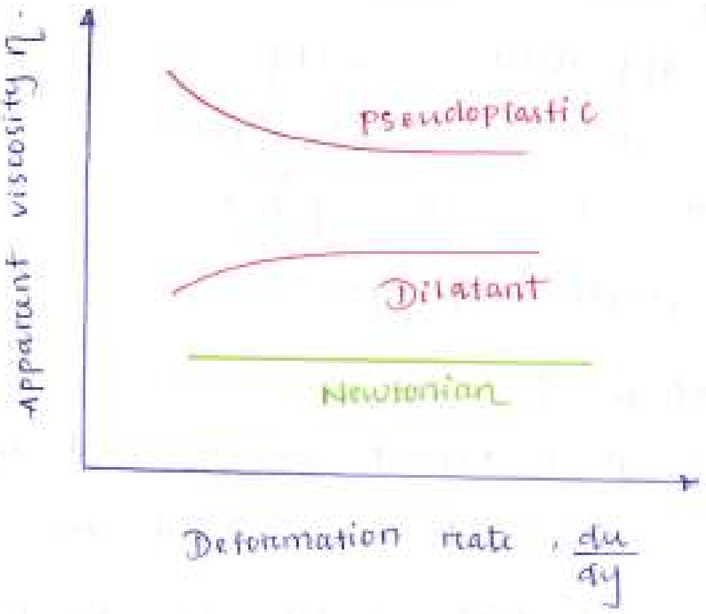
Ex - paints, printer inks.

Rheopetic :

Apparent viscosity (η) for rheopetic fluids increases with time under constant shear stress.

Ex - Gypsum pastes.





RHEOLOGY:

Rheology is the branch of science which deals with the studies of different types of fluid behaviours.

Ideal fluid -

- An ideal fluid is a fluid that is incompressible and no internal resistance to flow.
- Invicide (no viscosity)
- Shear stress will be zero
- Constant normal stress.

Newtonian fluids —

Newtonian fluids depends on newton's law of viscosity.

$$\tau = \mu \frac{du}{dy}$$

$$\tau \propto \frac{du}{dy}$$

Non-newtonian fluids —

- Non-newtonian fluids doesn't depends on power law

$$\boxed{\tau = m\left(\frac{du}{dy}\right)^n}$$

where :

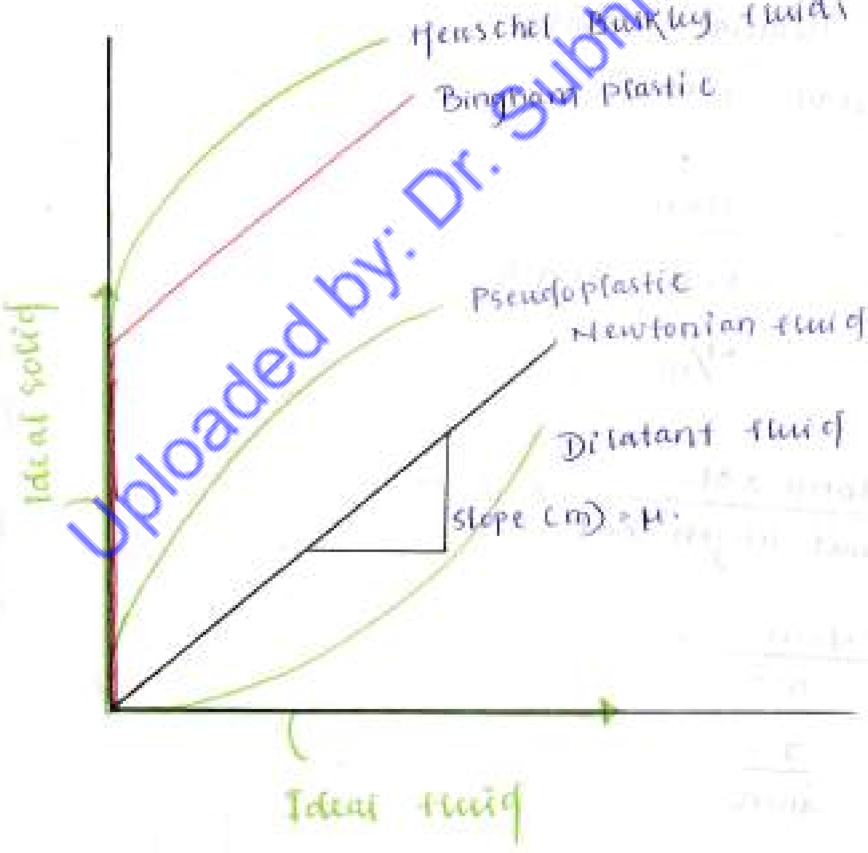
m = consistancy index

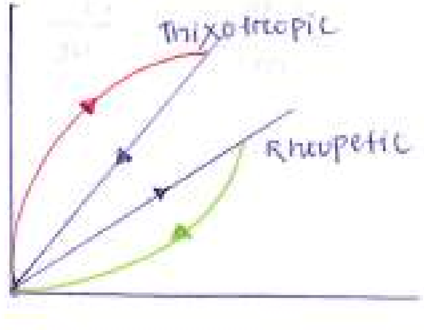
n = flow index

Henschel Bulkley fluids —

Henschel Bulkley fluids is a generalized model of a non-newtonian fluid, in which the strain experienced by the fluid is related to the stress in a complicated, non-linear way.

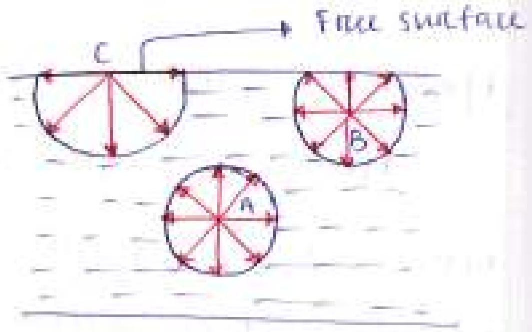
→ when we apply yield stress and the fluid start to flow then it will behave a non-newtonian fluid.





Surface Tension —

Free surface of liquid behaves like a stretched membrane and it tries to minimise its area upto maximum possible extent. This is known as surface tension.



→ Surface tension is defined as the tensile force acting on the surface of a liquid in contact with gas or on the surface between two immiscible liquid.

→ In MKS unit kgf/m.

→ In SI unit is N/m.

Free surface —

→ constant normal stress

→ Zero shear stress

$$\sigma = \frac{\text{Force}}{\text{unit length}}$$

$$= N/m$$

or

$$\sigma = \frac{\text{Force} \times \Delta L}{\text{unit length} \times \Delta L}$$

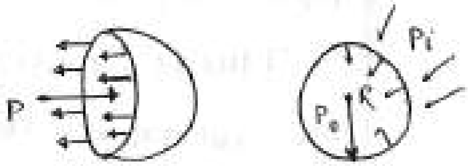
$$= \frac{N\text{-}m}{m^2}$$

$$= \frac{J}{\text{Area}}$$

$$\sigma = \frac{\text{surface energy}}{\text{Area}} = \frac{N\text{-}m}{m^2} = \frac{N}{m}$$

- **Water Droplet :**

— water droplet has only 1 free surface.



P_i = inside pressure

P_o = outer pressure.

$$\boxed{P = P_i - P_o}$$

This is called excess pressure.

For equilibrium

$\Sigma F = 0$

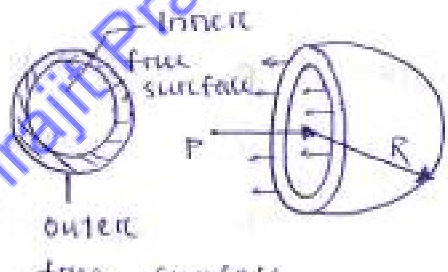
$\left(\xrightarrow{P} \quad \xleftarrow{\sigma} \right)$ should be same

$$P \times \pi R^2 = \sigma \times 2\pi R$$

$$\Rightarrow \boxed{P = \frac{2\sigma}{R}}$$

- **Soap Bubble :**

— Soap bubble has 2 free surface.



$\left(\xrightarrow{P} \quad \xleftarrow{\sigma} \right)$

$\Sigma F = 0$

$$P \times \pi R^2 = \sigma(2 \times 2\pi R)$$

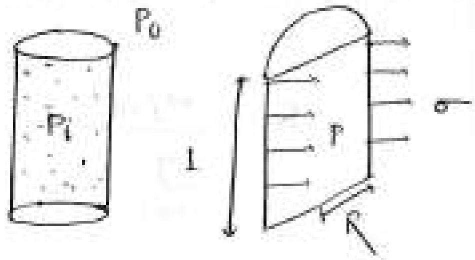
$$\Rightarrow P \times \pi R^2 = \sigma(4\pi R)$$

$$\Rightarrow \boxed{P = \frac{4\sigma}{R}}$$

$$\boxed{P_{soap\ bubble} = 2 \times P_{water\ droplet}}$$

- **Water Jet :**

— water jet has only 1 free surface.



$$P \times 2RL = \sigma 2L$$

$$\Rightarrow \boxed{P = \frac{\sigma}{R}}$$

Thermodynamic Properties:

Fluids consist of liquids or gases. But gases are compressible fluids and hence thermodynamic properties play an important role. With the change of pressure and temperature, the gases undergo large variation in density. The relationship between pressure (absolute), specific volume and temperature of a gas is given by the equation of state as

$$PV = RT$$

$$\text{or} \quad \frac{P}{\rho} = RT$$

where, P = absolute pressure of a gas in N/m^2

V = specific volume $= \frac{1}{\rho}$

R = Gas constant

T = Absolute temperature in °K.

ρ = Density of a gas.

- In MKS units $= \dfrac{Kgf\text{-}m}{Kg°K}$

- SI unit P is Newton/m^2 or N/m^2.

$$R = \frac{N/m^2}{\frac{Kg}{m^3} \times K} = \frac{Nm}{Kg\text{-}K} = \frac{Joule}{Kg \cdot K}$$

$$= \frac{J}{Kg \cdot K}$$

For air, R in MKS $= 29.3 \dfrac{Kgf \cdot m}{Kg° K}$

$$R \text{ in SI} = 29.3 \times 9.81 \frac{Nm}{Kg°K}$$

$$= 287 \frac{1}{Kg \cdot K}$$

Isothermal process :

If the change in density occurs at constant temperature, then the process is called isothermal and relationship between pressure (P) and density (ρ).

$$\boxed{\frac{P}{\rho} = \text{constant}}$$

Adiabatic process :

If the change in density occurs with no heat exchange to and from the gas, the process is called adiabatic. And if no heat is generated within the gas due to friction, the relationship between pressure and density.

$$\boxed{\frac{P}{\rho^K} = \text{constant}}$$

where,

K = Ratio of specific heat of a gas at constant pressure and constant volume.

$$\boxed{K_{air} = 1.4}$$

$$\boxed{PV = mRT}.$$

$$\Rightarrow \boxed{PV = n \times m \times RT}$$

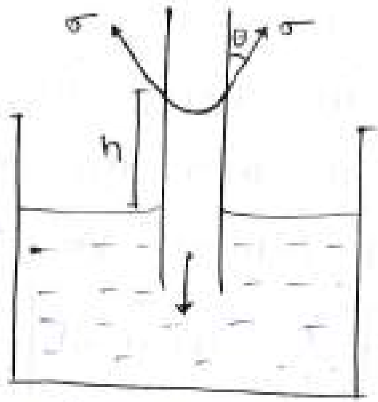
where,

m = mass

n = mole

R = universal gas constant = 8314 J/kg mole K

Capillary:



→ Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.

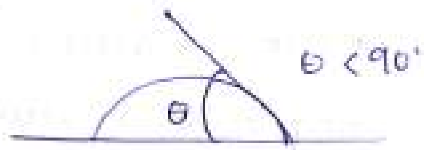
→ The rise of liquid surface is known a capillary rise while the fall of the liquid surface is known as capillary depression.

→ The phenomena of rise ~~of~~ fall of of a liquid in a small diameter tube is known as capillary. This is due to surface tension.

- Wetting liquid –

  * More adhesive force.

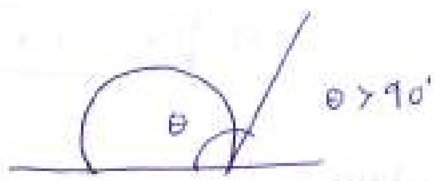
  Eq: water



- Non-wetting liquid –

  * More cohesive force (Interaction Force)

  Eq: Mercury



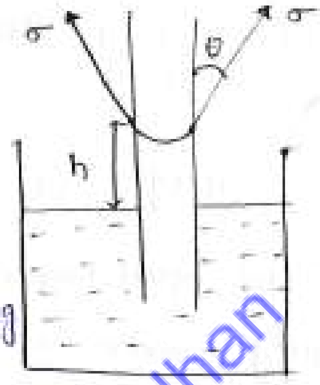
Expression for capillary ~~for~~ Rise:

Consider a glass tube of small diameter 'd' opened at both ends and is inserted in a liquid. The liquid will rise in the tube above the level of the liquid.

Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

Let σ = surface tension of liquid.

θ = Angle of contact between liquid and glass tube.



The weight of liquid of height h in the tube = (Area of tube × h) × ρ × g

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g \quad \text{——— (1)}$$

where,

ρ = density of liquid.

Vertical component of the surface tensile force

$$= (\sigma \times \text{circumference}) \times \cos\theta$$

$$= \sigma \times \pi d \times \cos\theta \quad \text{——— (2)}$$

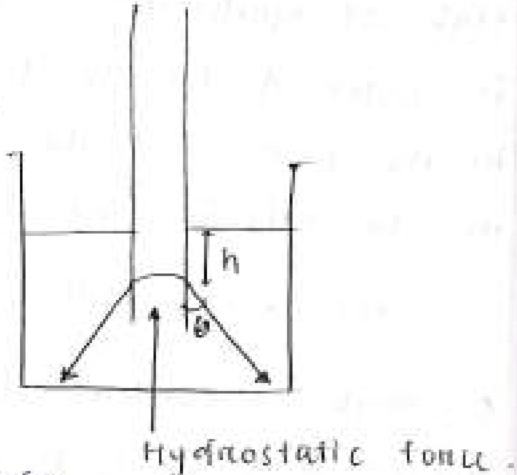
For equilibrium, eqn (1) and eqn (2) -

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

$$\Rightarrow h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g}$$

$$\boxed{h = \frac{4\sigma\cos\theta}{\rho \times g \times d}}$$

Expression for capillary fall -



If the glass tube is dipped in mercury, the level of mercury in the tube will be lower that the general level of the outside liquid.

let h = Height of depression tube.

Then in equilibrium, two forces are acting on the mercury inside the tube. First one is due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos\theta$.

Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth h × Area.

$$= p \times \frac{\pi}{4} d^2$$

$$= \rho g \times h \times \frac{\pi}{4} d^2 \quad (\because p = \rho g h)$$

Equating the two sides.

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

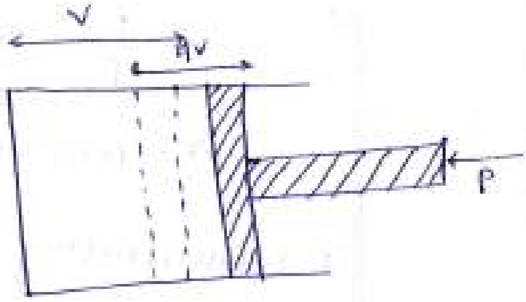
$$\Rightarrow \boxed{h = \frac{4\sigma \cos\theta}{\rho g d}}$$

* Value of θ for mercury and glass tube is 130°.

- Compressibility and Bulk Modulus —

Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain

consider a cylinder fitted with a piston -



Let V = Volume of a gas enclosed in the cylinder.

P = Pressure of gas when volume is V.

Let the pressure is increased to $P+dP$, the volume of gas decreases from V to $V-dV$.

Then increase in pressure = dp Kgf/m²

Decrease in volume = dv

$\therefore$ volumetric strain $= -\dfrac{dv}{v}$

-ve sign means the volume decreases with increase of pressure.

$\therefore$ Bulk modulus, $K = \dfrac{\text{Increase of pressure}}{\text{Volumetric strain}}$

$$= \frac{dP}{-dv/v} = \frac{-dP}{dv}v \qquad \text{(i)}$$

$$\text{compressibility} = \frac{1}{K} \qquad \text{(ii)}$$

Relationship between Bulk modulus (K) and pressure (P) for a Gas -

The relationship between bulk modulus of elasticity (K) and pressure for a gas for two different processes of compression are as:

- For Isothermal process -

The relationship between pressure (P) and density (ρ) of a gas as:

$$\frac{p}{\rho} = \text{constant}$$

$$pv = \text{constant} \qquad \left(\because v = \frac{1}{\rho}\right)$$

Differentiating this equations, we get (p and v both are variables).

$$p\,dv + v\,dp = 0$$

$$\Rightarrow p\,dv = -v\,dp$$

$$\Rightarrow p = -\frac{v\,dp}{dv}$$

Substituting this value in eq^n(1)

$$\boxed{K = p}$$

- For Adiabatic process —

$$\frac{p}{\rho^K} = \text{constant}$$

$$pv^K = \text{constant}$$

Differentiating, we get $p\,d(v^K) + v^K(dp) = 0$

$$\Rightarrow p \times K \times v^{K-1}\,dv + v^K\,dp = 0$$

$$\Rightarrow pK\,dv + v\,dp = 0$$

$$\Rightarrow pK\,dv = -v\,dp$$

$$pK = -\frac{v\,dp}{dv}$$

Hence from eq^n(1) we have,

$$\boxed{K = pK}$$

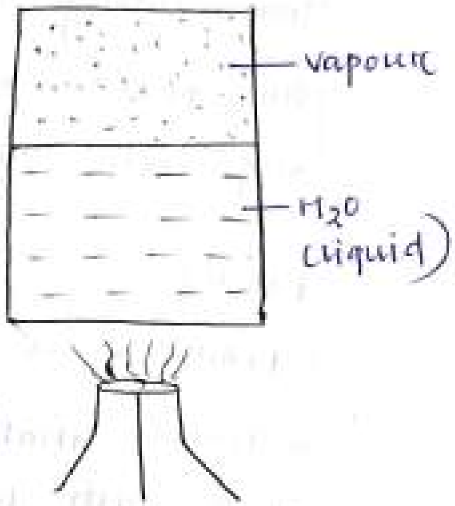
where,

K = Bulk modulus

K = Ratio of specific heats.

Vapour pressure –

- A change from the liquid state to the gaseous state is known as vaporization. The vaporization (which depends upon the prevailing pressure and temperature condition) occurs because of continuous escaping of the molecules through the free liquid surface.



- Vapour pressure is the pressure exerted by the vapour on a liquid. Vapour pressure of the liquid increases with temperature. For that reason the liquid at higher pressure boils at higher temperature.

At boiling point

$$\boxed{VP = P_{atm}}$$

Saturation Temperature –

At a given pressure the temperature at which any pure liquid changes phase is called saturation temperature. Similarly, at a given temperature the pressure at which any liquid changed fluid is called saturation temperature.

Cavitation –

It is a phenomenon when liquid flows into a region where its pressure is reduced to vapour pressure and its start vapourizing and the vapour pockets or bubbles are form in the liquid. When these bubbles are carried along the liquid until high pressure region, they collaps. When the vapour pressure bubbles collaps a cavity is formed and the surrounding liquid rushes to fill the cavity. This process of formation of vapour bubbles and their collapsing is called cavitation.

Pitting —

Pitting is a type of corrosion that occurs in materials that have protective films. It is an attack with localized holes on the metal's surface. The attack can penetrate the metal very rapidly, while some parts of the metal surface remains free from corrosion.

Pitting.

Compressibility (B)

$$B = \frac{1}{K}$$

$$= -\frac{dv/v}{dP}$$

$$\boxed{B = -\frac{dv/v}{dP}}$$

It is the inverse of Bulk modulus of elasticity.

Elasticity (K)

$$K = \frac{\text{stress}}{\text{strain}}$$

$$\boxed{K = \left(-\frac{dP}{dv/v}\right)}$$

$$\boxed{K_{(gas)} \downarrow \quad \text{Temp} \uparrow}$$

$$\boxed{\begin{array}{ll} K_{liq} \downarrow & \text{Temp} \uparrow \\ K_{gas} \uparrow & \text{Temp} \uparrow \end{array}}$$

K is not constant for liquid.

FLUID STATICS-

Consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure or simply pressure and this ratio is represented by P. Hence mathematically the pressure at a point in a fluid at rest is,

$$P = \frac{dF}{dA}$$

If the force (F) is uniformly distributed over the area (A), then pressure at any point is given by,

$$P = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}.$$

∴ Force or pressure force, $F = P \times A$.

The units of pressure are –

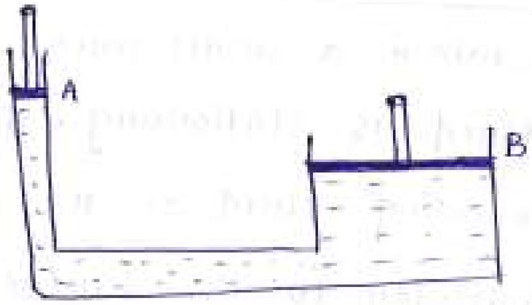
- Kgf/m^2 and Kgf/cm^2 in MKS units.
- $Newton/m^2$ or N/m^2 and N/mm^2 in SI units. N/m^2 is known as pascal and is represented by Pa. ~~unit~~

$$KPa = \text{Kilo pascal} = 1000\,N/m^2$$

$$\text{bar} = 100\,KPa = 10^5\,N/m^2.$$

- Study of fluid at rest.
- Study of fluid in Rigid body motion.

Pascal's law -

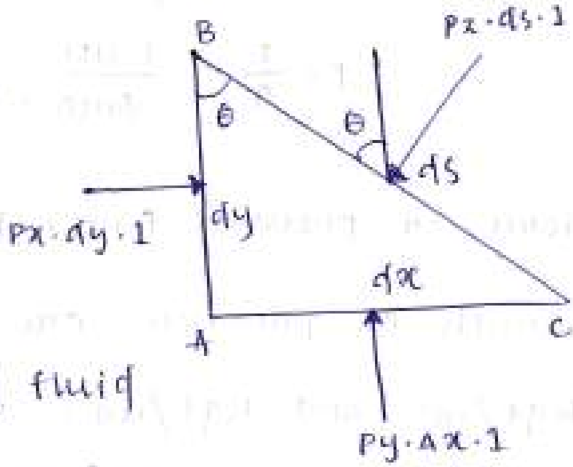


consider the column is filled with water, and a piston has blocked the ends of each column A and B. If piston A is pressed, ~~both~~ piston B will rise. We have just applied pascal's law to our fluid pressure.

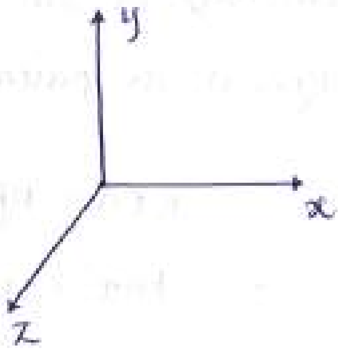
According to pascal's law,

" The external static pressure applied on a confined liquid is distributed or transmitted evenly throughout the liquid in all directions.

$$\boxed{F = PA}$$



The fluid element is of very small dimensions i.e. dx, dy and ds.



consider an arbitrary fluid element of wedge shape in a fluid mass at rest. Let the width of the element perpendicular to the plane of paper is unity and P_x, P_y and P_z are the pressure or intensity of pressure acting on the face AB, AC and BC ~~resp~~ respectively.

Let $\angle ABC = \theta$.

Force on the face AB = $P_x \times$ Area of face AB

$$= P_x \times dy \times 1$$

Similarly, force on the face AC = $P_y \times dx \times 1$

Force on the face BC = $P_z \times ds \times 1$.

weight of element = Mass of element $\times g$.

$$= (\text{volume} \times \rho) \times g$$

$$= \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g$$

where, ρ = density of fluid.

Resolving the forces in x-direction,

$$P_x \times dy \times 1 - P(ds \times 1) \sin(90° - \theta) = 0$$

$$P_x \times dy \times 1 - P_z ds \times 1 \cos\theta = 0$$

$$ds \cos\theta = AB = dy$$

$$P_x \times dy \times 1 - P_z \times dy \times 1 = 0$$

$$P_x = P_z \quad \text{--- (1)}$$

Similarly, resolving the forces in y-direction.

$$P_y \times dx \times 1 - P_z \times ds \times 1 \cos(90° - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

$$P_y \times dx - P_z\, ds \sin\theta - \frac{dx\,dy}{2} \times \rho \times g = 0$$

But $ds \sin\theta = dx$.

$$P_y\, dx - P_z \times dx = 0 \quad \text{--- (2)}$$

$$P_y = P_z$$

from eqⁿ(I) and eqⁿ(II)

$$\boxed{P_x = P_y = P_z}$$

The above equation shows that the pressure at any point in x, y and z directions is equal.

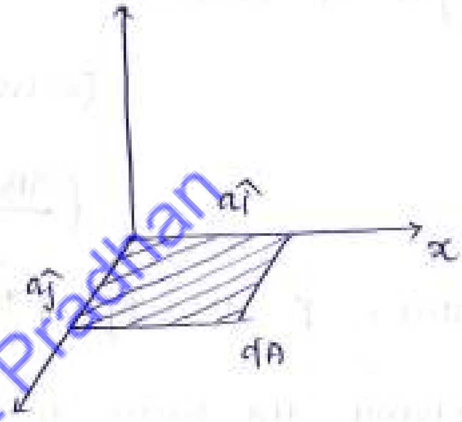
Vector product –

$d\vec{A} = a\hat{i} \times a\hat{j}$

$= a^2 \sin\theta$

$\Rightarrow dA = a^2$

$dA \neq 0$ (vector) (small area)



Unit –

$P = \frac{\text{Newton}}{m^2}$ or Pascal (SI unit)

$P = \frac{\text{Dyne}}{cm^2}$ (CGS unit)

Non-SI unit –

$1 \text{ bar} = 10^5 \text{ pa}$

$1 \text{ Kpa} = 1000 \text{pa}$

$1 \text{ bar} = 100 \text{ KPa}$

1 Torque = 1 mm of Hg

1 atm = 760 mm of Hg

1 Torque $= \frac{1}{760}$ atm

1 atm = 1.013 bar

OR

1 atm = 101.3 KPa

2nd law of Motion –

It states that the time rate of change of the momentum of a body is equal in both magnitude and direction to the force imposed on it.

$$\boxed{F = ma}$$

Pressure Measurement –

The Pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure.

- **Absolute pressure –**

  It is defined as the pressure which is measured with reference to absolute vacuum pressure.

- **Gauge pressure –**

  It is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure.

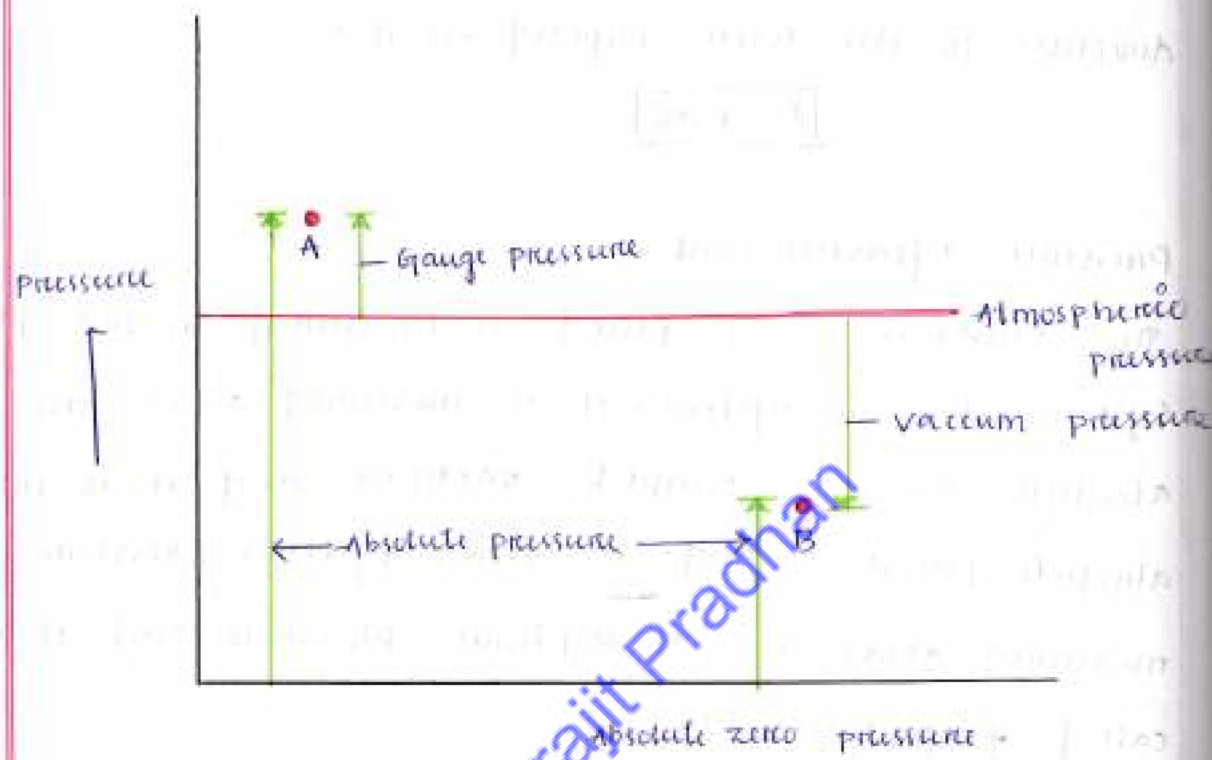
- **Vaccum pressure –**

  It is defined as the pressure which the liquid is less than Patm and it is measured Patm.

Absolute pressure = atmospheric pressure + Gauge pressure

$$\boxed{P_{ab} = P_{atm} + P_{gauge}}$$

vaccum pressure = Atmospheric pressure — Absolute pressure.



(Relationship between pressure)

Mechanical Advantage of pascal's law —

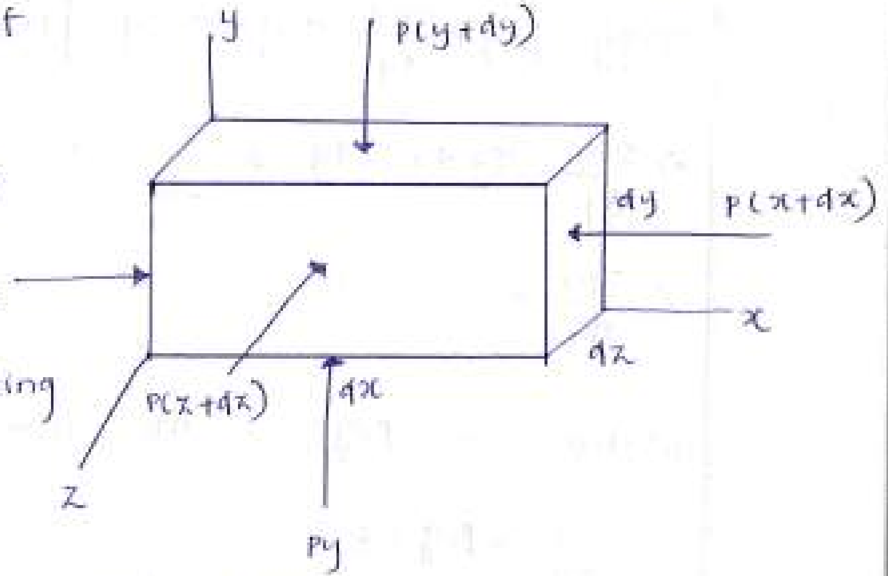
We use pascal's law in hydrolics. we apply pressure on the small cross-section area and it transfer to a area where the cross-section area is more. so, the force is amplify.

Hydrostatics law —

A hydrostatics law state that rate of increase of pressure in a vertically downward direction in fluid/liquid is equal to weight density of the liquid.

consider a three dimensional fluid element at rest along with the forces acting on it.

weight of the element
$= w(dx \times dy \times dz)$



P_x = pressure acting on the left face along x-direction.

$P_x + \frac{\partial p}{\partial x} dx$ = pressure acting on the right face.

For equilibrium of the element -

$$\Sigma F_x = 0$$

$$P_x(dy \times dz) = \left[P_x + \frac{\partial P_x}{\partial x} \cdot dx\right] dy \cdot dz \quad [\text{Divide } (dy \times dz)]$$

$$P_x = P_x + \frac{\partial P_x}{\partial x} dx$$

$$\therefore \frac{\partial P_x}{\partial x} \cdot dx = 0$$

But $dx \neq 0$

$$\therefore \frac{\partial P_x}{\partial x} = 0 \quad \text{—— (i)}$$

Similarly for 'z' direction -

$$\therefore \frac{\partial P_z}{\partial z} = 0 \quad \text{—— (ii)}$$

From eqn (i) and eqn (ii) can be concluded that there is no change in pressure in x and z direction at the same level or height. Hence, pressure at the same level in the fluid at rest remains constant.

For equilibrium in y-direction.

$$\Sigma F_y = 0$$

$$P_y \cdot (dx \cdot dz) = \left[P_y + \frac{\partial P_y}{\partial y} dy\right] dx \cdot dz + w(dx \times dy \times dz)$$

$\Rightarrow P_y = \left[P_y + \frac{\partial P_y}{\partial y}\cdot dy\right] + w\cdot dy$ [Divide $dx \times dz$]

$\Rightarrow \frac{\partial P_y}{\partial y}\cdot dy + w\cdot dy = 0$

$\Rightarrow \frac{\partial P_y}{\partial y} + w = 0$

substitute $w = \rho\cdot g$ in the above equation.

$$\frac{\partial P_y}{\partial y} + \rho\cdot g = 0$$

$\Rightarrow \frac{\partial P_y}{\partial y} = -\rho g$ —— (iii)

since $P_y = f(y)$ hence using perfect differentiation in relation (3)

separating variables we get,

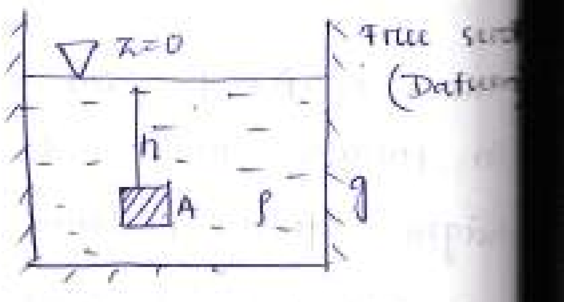
$$dP = -\rho\cdot g\, dy.$$

Integrating both sides.

$$\int dP = \int -\rho g\, dy$$

$$\Rightarrow \boxed{P = -\rho g\cdot y + C}$$

How using Hydrostatic law we can find pressure at a point –



$$\frac{dP}{dz} = \gamma$$

$$dP = \gamma\cdot dz$$

$$\int_{P_{atm}}^{P_A} dP = \int_0^h \gamma\, dz$$

$$P_A - P_{atm} = \gamma h - 0$$

$$\Rightarrow P_A = P_{atm} + \gamma h.$$

$\boxed{P_A = P_{atm} + \rho g h}$ Absolute pressure at A or total pressure.

pressure at free surface $= P_{atm}$

$= (P_{atm})_{gauge} = 0$

$\boxed{P_A = \rho g h}$

Pressure Measurement

- **Manometers**
  - **Simple Manometer**
    - Pizometer
    - U- tube manometer
    - single column or well type or micromanometer
    - Inclined U- tube manometer
  - **Differential Manometer**
    - Differential U- tube manometer
    - Inverted U- tube manometer
    - Inclined U- tube Differential manometer
- **Mechanical Gauges**
  - Bourdan tube
  - pressure transducer
  - Blow pressure gauge
  - Diaphragam pressure

Manometers-

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid.

Mechanical Gauges-

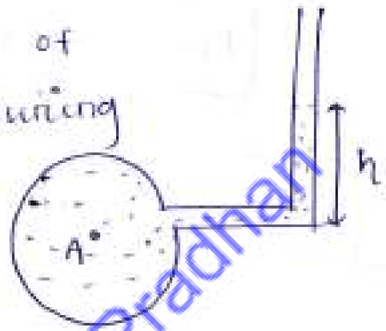
Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight.

Simple Manometers —

A simple manometers consist of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere.

- Piezometer —

It is the simplest form of manometer used for measuring gauge pressure. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere. The rise of liquid gives the pressure head at the point. If at a point A, the height of liquid 'h' in piezometer tube, then pressure at A



$$\boxed{P_A = \rho \times g \times h \ \frac{N}{m^2}}$$

Drawback —

- It can measure +ve pressure only it ~~convert~~ convert measure vaccume pressure.
- It cannot measure high pressure.

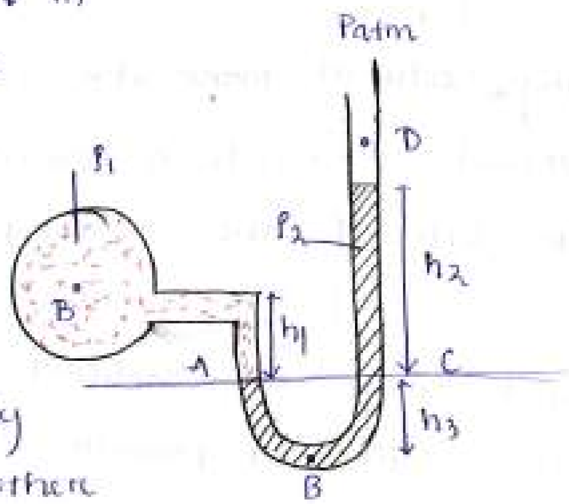
$P_A = \rho g h$.

$h \propto P_A$

$h \propto \frac{1}{\rho g}$ or $\frac{1}{\gamma}$ less the density more the height

- It cannot measure gauge pressure.
- It can only moderate on low pressure.
- Pressure of lighter liquid cannot be measured.

U-tube Manometer —



It consists of glass tube bent in U-shape. one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

Let B is the point at which pressure is to be measured whose value is P.

h_1 = height of ~~liquid~~ light ~~liquid~~ above the datum line.

h_2 = height of heavy liquid above the datum line.

S_1 = specific gravity of liquid.

ρ_1 = Density of light liquid $= 1000 \times S_1$

S_2 = specific gravity of heavy liquid

ρ_2 = Density of heavy liquid $= 1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line in the left column and in the right column of U-tube manometer should be same.

Pressure above left column $= P \times \rho_1 \times g \times h_1$

" " right " $= \rho_2 \times g \times h_2$.

Equating the two pressure $= P \times \rho_1 g h_1 = \rho_2 g h_2$

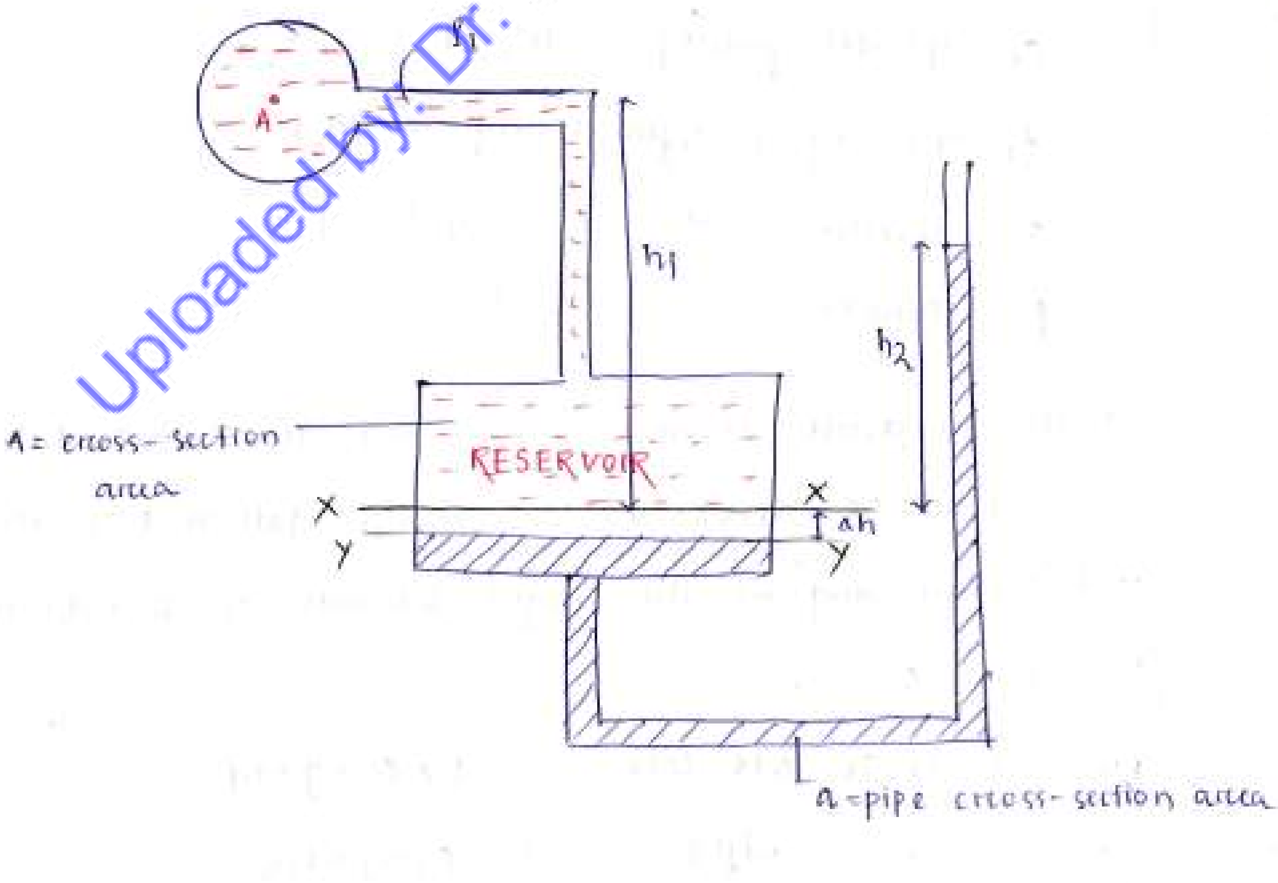
$$\Rightarrow \boxed{P = \rho_2 g h_2 - \rho_1 \times g \times h_1}$$

• Single column or Micromanometer or well type Manometer —

Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area as compared to the area of the tube is connected to one of the limbs of the manometer.

Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limbs. The other limb may be vertical or inclined.

(a) Vertical single column Manometer —



The vertical single column manometer. Let X-X be the datum line in the reservoir and in the right limb of the manometer. When it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Δh = Fall of heavy liquid in right limb

h_1 = Height of centre of pipe above X-X.

P_A = pressure at A.

A = cross-sectional area of the reservoir

a = cross-sectional area of the right limb

S_1 = sp.gr. of liquid in pipe.

ρ_1 = Density of liquid in pipe

ρ_2 = Density of liquid in reservoir.

Displacement volume of left limb = Rise of volume in right limb

$$A \times \Delta h = a \times h_2$$

$$\Rightarrow \boxed{\Delta h = \frac{a \times h_2}{A}}$$

Pressure in the right limb above Y-Y = $\rho_2 \times g \times (\Delta h \times h_2)$

Pressure in the left limb above Y-Y = $\rho_1 \times g \times (\Delta h + h_1) + P_A$.

Equating these pressure, we have,

$$P_A + \rho_1 g (h_1 + \Delta h) = \rho_2 g \times (h_2 + \Delta h)$$

$$\Rightarrow P_A = \rho_2 g (h_2 + \Delta h) - \rho_1 g (h_1 + \Delta h)$$

$$\Rightarrow P_A = \rho_2 g h_2 + \rho_2 g \Delta h - \rho_1 g h_1 - \rho_1 g \Delta h.$$

$$\Rightarrow P_A = \rho_2 g h_2 - \rho_1 g \Delta h + \rho_2 g h_2 - \rho_1 g h_1$$

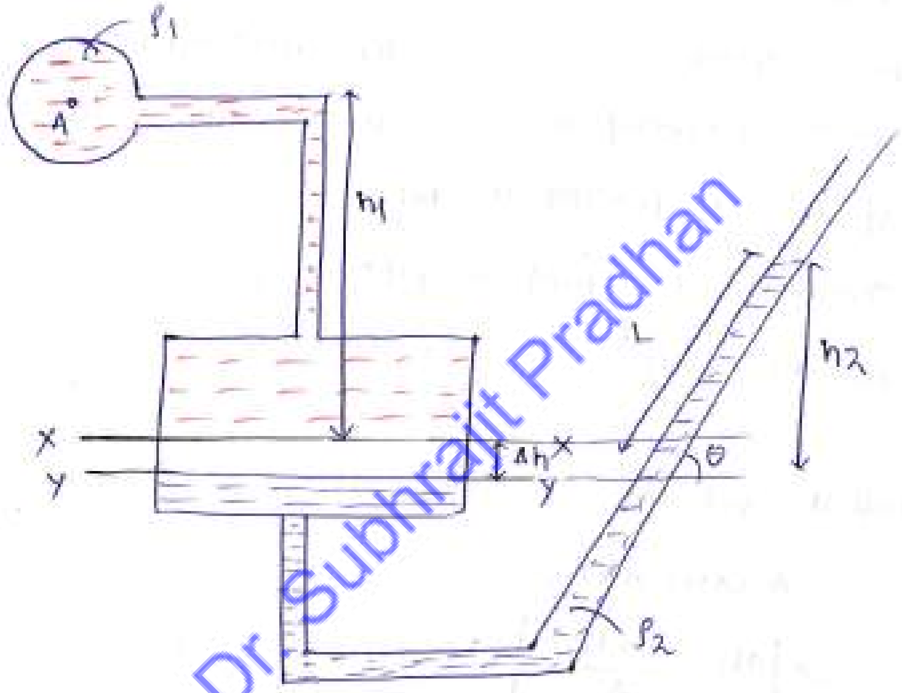
$$\Rightarrow P_A = g \Delta h (\rho_2 - \rho_1) + \rho_2 g h_2 - \rho_1 g h_1$$

$$\Rightarrow \boxed{P_A = g \frac{a \times h_2}{A} (\rho_1 - \rho_2) + \rho_2 g h_2 - \rho_1 g h_1}$$

As the area A is very large as compared to a, hence ratio $\frac{a}{A}$ becomes very small and can be neglected.

$$\boxed{P_A = \rho_2 g h_2 - \rho_1 g h_1}$$

(b) Inclined single column Manometer –



The manometer is more sensitive. Due to inclination the distance move by the heavy liquid in the right limb will be more.

$$\sin\theta = \frac{h_2}{L}$$

$$\Rightarrow \boxed{h_2 = L\sin\theta}$$

Manometric eqn

$$P_A + \rho_1 g(h_1 + \Delta h) = \rho_2 g(h_2 + \Delta h)$$

$$\Rightarrow P_A = \rho_2 g(h_2 + \Delta h) - \rho_1 g(h_1 + \Delta h)$$

$$\Rightarrow P_A = \rho_2 g h_2 + \rho_2 g \Delta h - \rho_1 g h_1 - \rho_1 g \Delta h.$$

$$\Rightarrow P_A = \rho_2 g \Delta h - \rho_1 g \Delta h + \rho_2 g h_2 - \rho_1 g h_1$$

$$\Rightarrow P_A = g\frac{a\times h_2}{A}(\rho_2-\rho_1) + \rho_2 g h_2 - \rho_1 g h_1$$

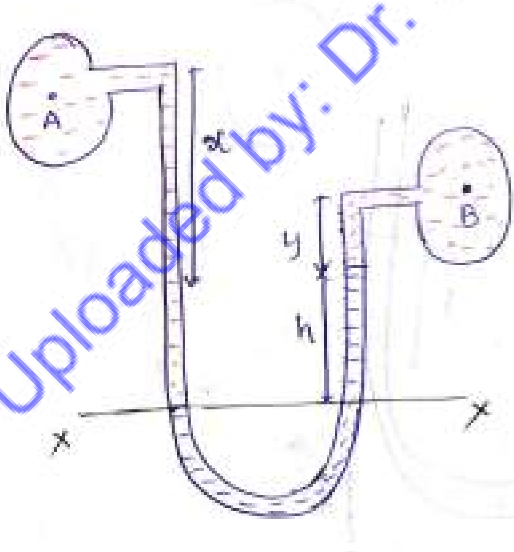
$$\Rightarrow P_A = \frac{ah^2}{A} g(\rho_2-\rho_1) + \rho_2 g h_2 - \rho_1 g h_1$$

$$\Rightarrow \boxed{P_A = \frac{a\, l\sin\theta}{A} g(\rho_2-\rho_1) + \rho_2 g h_2 - \rho_1 g h_1}$$

Differential Manometers —

Differential manometers are the devices used for measuring the difference of pressure between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured.

(A) U-tube differential Manometer —



The two points A and B are at different level and also contains liquids of different sp. gr.. These points are connected to the U-tube differential manometer. Let the pressure at A and B are P_A and P_B.

h = difference of mercury level in the U-tube.

y = Distance of the centre of B, from the mercury level in the right limb x - Distance of the centre of A from the meniscus level in the right limb.

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

Pressure above x-x in the left limb = $\rho_1 g(h+x) + P_A$.

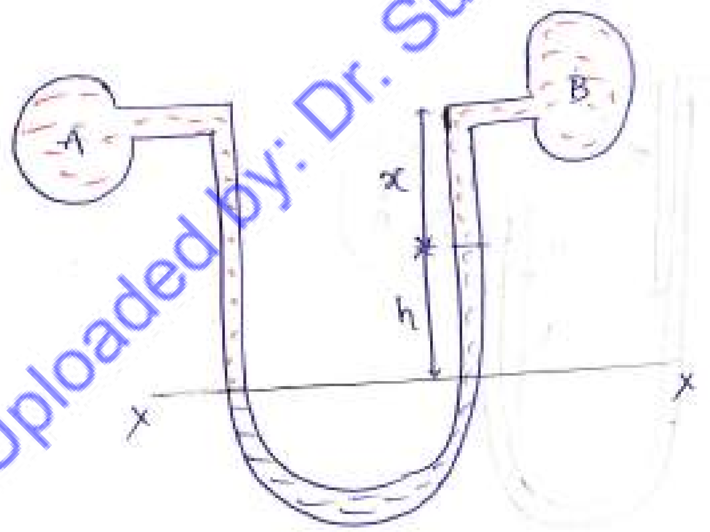
Pressure above x-x in the right limb = $\rho_g \times g \times h + \rho_2 \times g \times y + P_B$

Equating the two pressure

$$\rho_1 g(h+x) + P_A = \rho_g \times g \times h + \rho_2 g y + P_B$$

$$\Rightarrow P_A - P_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h+x)$$

$$\Rightarrow \boxed{P_A - P_B = h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x}$$



The two point A and B are at the same level and contains the same liquid of density ρ_1.

Pressure above x-x in right limb = $\rho_g \times g \times h + \rho_1 \times g \times x + P_B$

Pressure above x-x in left limb = $\rho_1 \times g \times (h+x) + P_A$

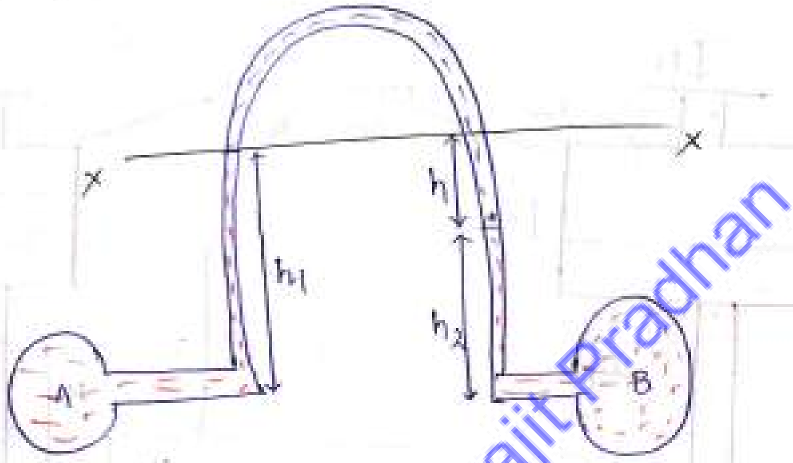
Equating the two pressure.

$$\rho_g \times g \times h + \rho_1 g x + P_B = \rho_1 \times g \times (h+x) + P_A$$

$$\Rightarrow P_A - P_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g (h+x)$$

$$\Rightarrow \boxed{P_A - P_B = g \times h (\rho_g - \rho_1)}$$

(b) Inverted U- tube Differential Manometer -



It consists of an inverted U- tube, containing a ~~liquid~~ light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of two pressure. An inverted U- tube differential manometer connected to the two points A and B. let the pressure at A is more than the pressure at B.

h_1 = Height of liquid in left limb below the datum line x-x.

h_2 = Height of liquid in right limb

h = Difference of light liquid.

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B.

ρ_s = Density of light liquid

P_A = Pressure at A

P_B = Pressure at B

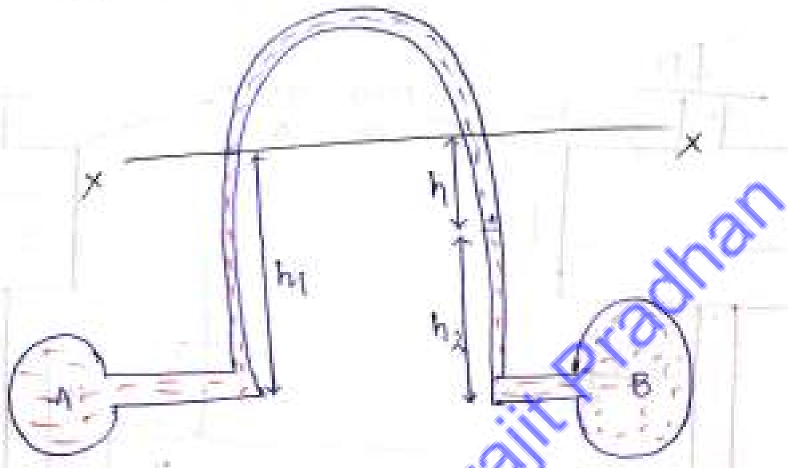
Equating the two pressure.

$$\rho g \times g \times h + \rho_1 g x + P_B = \rho_1 g \times (h+x) + P_A$$

$$\Rightarrow P_A - P_B = \rho g \times g \times h + \rho_1 g x - \rho_1 g (h+x)$$

$$\Rightarrow \boxed{P_A - P_B = g \times h (\rho_g - \rho_1)}$$

(b) Inverted U- tube Differential Manometer -



It consists of an inverted U- tube, containing a ~~liquid~~ light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of two pressure. An inverted U- tube differential manometer connected to the two points A and B. Let the pressure at A is more than the pressure at B.

h_1 = Height of liquid in left limb below the datum line X-X.

h_2 = Height of liquid in right limb

h = Difference of light liquid.

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B.

ρ_s = Density of light liquid

P_A = Pressure at A

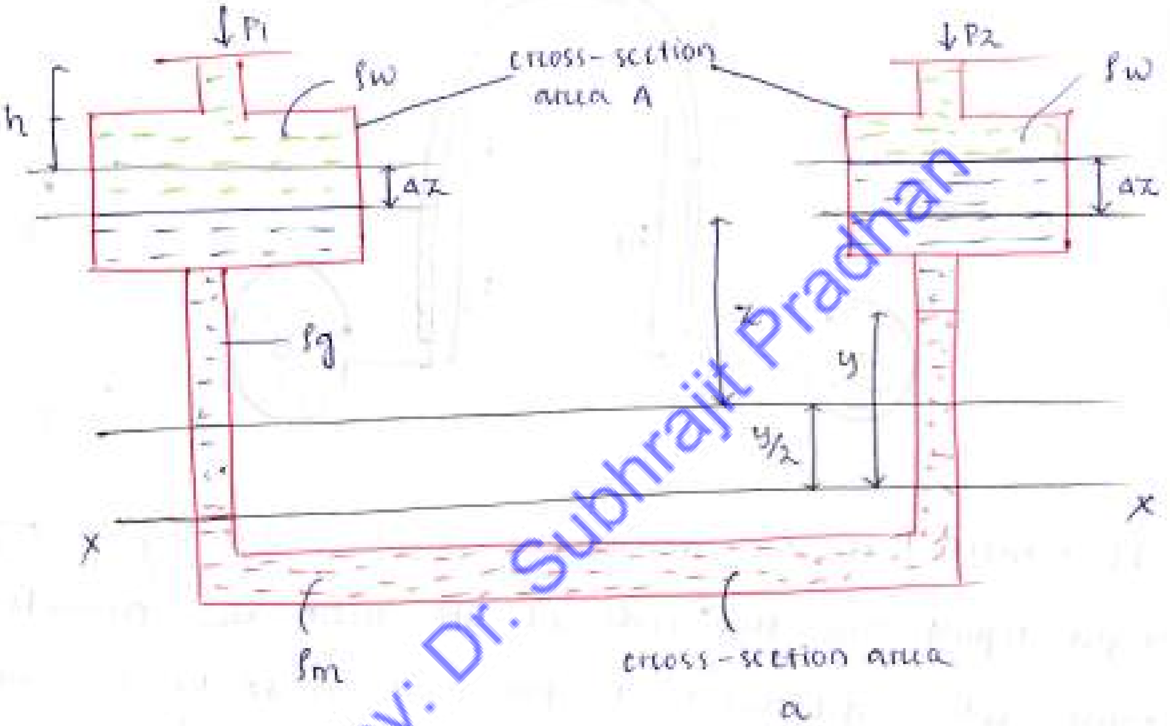
P_B = Pressure at B.

pressure in the left limb = pressure in the right limb

$\Rightarrow P_A - \rho_1 \times g \times h_1 = P_B - \rho_2 \times g \times h_2 - \rho_3 \times g \times h$

$\Rightarrow \boxed{P_A - P_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_3 g h}$

Micro - Manometer (Differential Type) -



If less than 1mm we will notice capillary effect where there will be error in the reading.

- used to measure small pressure difference.

Pascal's law in plane XX -

pressure above XX in left limb = pressure above XX in right limb.

$\Rightarrow P_1 + \rho_w g(h+\Delta z) + \rho g \times g(z - \Delta z + \frac{y}{2}) = P_2 + \rho_w \times g(h - \Delta z) +$

$\rho g \times g \times (z + \Delta z - \frac{y}{2}) + \rho_m \times g \times y$

$\Rightarrow P_1 + \cancel{\rho_w g h} - \rho_w g \Delta z + \cancel{\rho g \times g z} + \rho g g \Delta z + \rho g \times g \times \frac{y}{2}$

$= P_2 + \cancel{\rho_w g h} - \rho_w g \Delta z + \cancel{\rho g \times g z} + \rho g \times g \Delta z - \rho g g \frac{y}{2} + \rho_m g y$

$$\Rightarrow \boxed{P_1 - P_2 = \rho_m g y + 2\rho_g \times g \times \Delta x - 2\rho_g \times g \times \frac{y}{2} - 2\rho_w g \Delta x}$$

According to continuity eqn in gauge liquid the volume displaced from the well = volume of gauge liquid moved in tube.

$$A \times \Delta x = a \times \frac{y}{2}$$

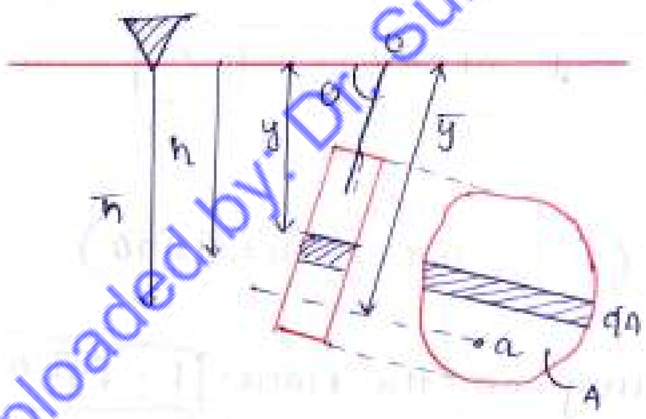
$$\Rightarrow \boxed{\Delta x = \frac{a}{A} \cdot \frac{y}{2}}$$

If a is very very smaller than A, then Δx will be negligible.

$$P_1 - P_2 = \rho_m g y - \rho_g g y$$

$$\Rightarrow \boxed{P_1 - P_2 = (\rho_m - \rho_g) g y}$$

Hydrostatic Force on Surface —



Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid.

ρ = density of fluid

θ = inclined of plane with free surface

y = the surface of element from O

$\bar{y}$ = Distance of centroid of plane from O h = depth of element from free surface.

$\bar{h}$ = depth of centroid of the plane from the surface.

$$\boxed{\frac{h}{y} = \sin\theta} \qquad \boxed{\frac{\bar{h}}{\bar{y}} = \sin\theta}$$

Force on element –

$dF = p\,dA$.

$$\boxed{dF = \rho g h\, dA}$$

$$\boxed{dF = \rho g y \sin\theta\, dA} \quad \left(\because \frac{h}{y} = \sin\theta \Rightarrow h = y\sin\theta\right)$$

Total pressure force on the element –

$F = \int dF$

$F = \int \rho g y \sin\theta\, dA$.

$F = \rho g \sin\theta \int y\, dA$.

$$\boxed{\int y\, dA = \text{First moment of area} = \bar{y} \cdot A}$$

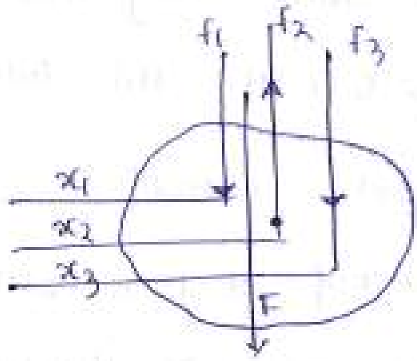
$F = \rho g \sin\theta\, \bar{y} \cdot A$.

$$\Rightarrow \boxed{F = \rho g \bar{h} A} \quad \left(\because \frac{\bar{h}}{\bar{y}} = \sin\theta \Rightarrow \bar{h} = \bar{y}\sin\theta\right)$$

Total force acting on the plane $\boxed{F = \rho g \bar{h} A}$

Varignon Theorem –

The moment of resultant force about point O will be equal to the sum of moments of all forces about O.



$$\boxed{Fx_y = \Sigma F_n x_n} \quad n = 1, 2, 3 \ldots$$

It is used to locate the exact point when the resultant force acts moment of total force (F) about 'O' = moment of all forces about 'O'.

$$F \times y^* = \int dF \cdot y$$

$$F \times y^* = \int (\rho g y \sin\theta \, dA) y$$

$$\rho g \bar{h} A \times y^* = \rho g \sin\theta \int y^2 dA$$

$y^2 dA$ = second moment of area
or
moment of inertia of plane.

$$\boxed{\int y^2 dA = I_0}$$

$$\Rightarrow \rho g \bar{h} A \times y^* = \rho g \sin\theta \, I_0$$

$$\Rightarrow \bar{h} A \times y^* = \sin\theta \, I_0$$

According to parallel axial theorem

$$I_0 = I_G + A\bar{y}^2$$

I_G - moment of inertia of plane about the centroid.

so, $\bar{h} A \times y^* = (I_G + A\bar{y}^2)\sin\theta$.

h^* is the depth of point of application of total pressure force.

$$\boxed{h^* = \text{centre of pressure}}$$

$$\frac{h^*}{y^*} = \sin\theta .$$

$$\bar{h} \cdot A \cdot \frac{h^*}{\sin\theta} = I_G \sin\theta + A\bar{y}^2 \sin\theta .$$

$$\Rightarrow h^* = \frac{I_G \sin^2\theta + A(\bar{y}\sin\theta)^2}{\bar{h} A}.$$

$$\Rightarrow h^* = \frac{I_G \sin^2\theta}{\bar{h} A} + \frac{\cancel{A}\bar{h}^2}{\bar{h}\cancel{A}} \qquad \left(\because \frac{\bar{h}}{\bar{y}} = \sin\theta \quad \bar{h} = \bar{y}\sin\theta\right)$$

$$\Rightarrow \boxed{h^* = \frac{I_G \sin^2\theta}{\bar{h}A} + \bar{h}}$$

For inclined plane.

Centre of pressure is below the centroid of a plane

Case-I

Plane is vertical

$\theta = 90°$

$$h^* = \frac{I_G \sin^2 90°}{\bar{h}A} + \bar{h}$$

$$\Rightarrow \boxed{h^* = \frac{I_G}{\bar{h}A} + \bar{h}} \quad (\because \sin 90° = 1)$$

Case-II

Plane is horizontal

$\theta = 0°$

$$h^* = \frac{I_G \sin^2 0°}{\bar{h}A} + \bar{h}$$

$$\Rightarrow \boxed{h^* = \bar{h}} \quad (\because \sin 0° = 0)$$

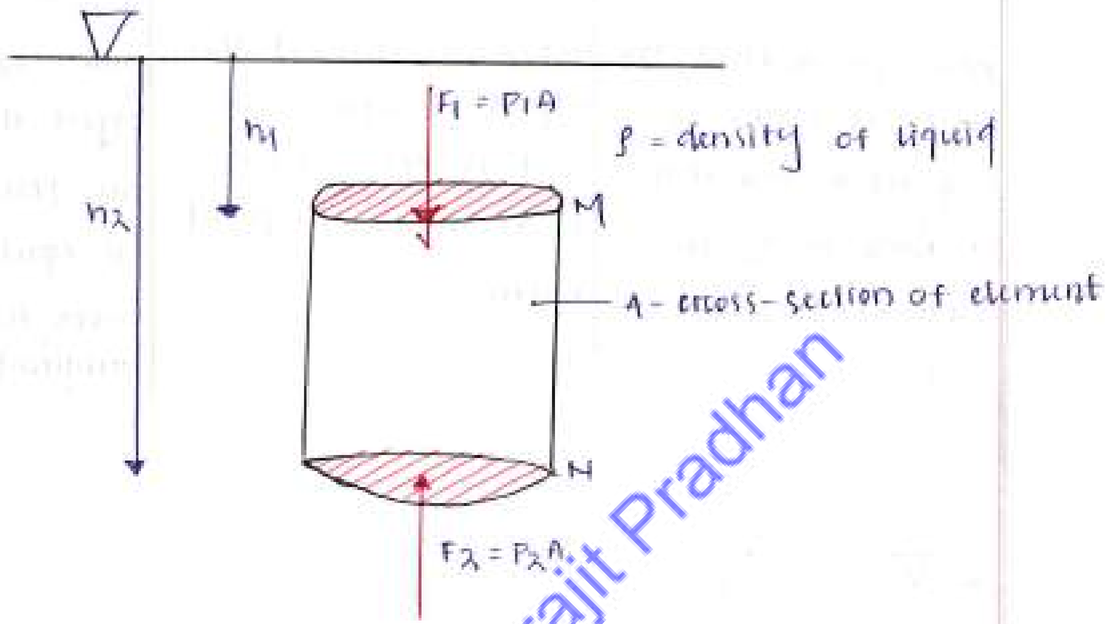
The depth of centroid and center of pressure (COP) is same for a horizontal surface.

Buoyancy—

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

Archimedes' principle -

When a body is submerged either fully or partially then it is acted upon by a force of buoyancy vertically up which is equal to weight of liquid displaced by the body.



Hydrostatic pressure on M

$$P_1 = \rho g h_1$$

Hydrostatic pressure on N

$$P_2 = \rho g h_2$$

$\Sigma F = 0$

Bouyant force

$$F_B = (P_2 - P_1) A$$

$$= (\rho g h_2 - \rho g h_1) A$$

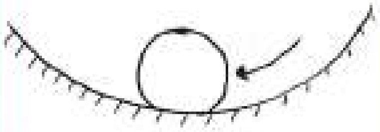
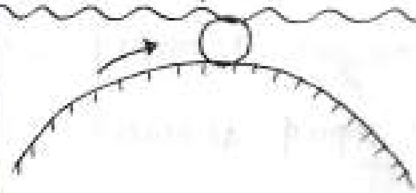
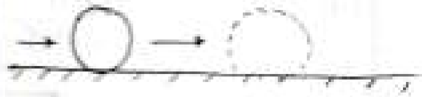
$$= \rho g \underbrace{A (h_2 - h_1)}_{}$$ — volume of cylindrical element.

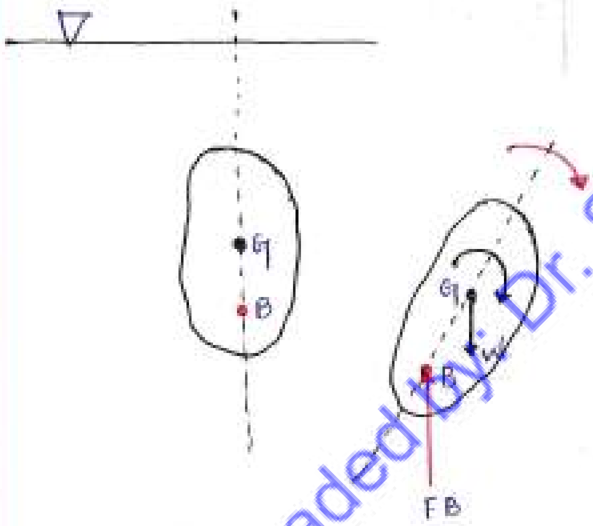
$$= \rho g V \qquad (\because A(h_2 - h_1) = \text{displaced volume of fluid}).$$

$$\boxed{F_B = \rho g V}$$

★ The point at which buoyant force acts is known as centre of Buoyancy. (B).

Stability of Submerged Body –

Stable equilibrium	Unstable equilibrium	Neutral equilibrium
		
When we distrub the object it will move and after some time it come back to original position.	When we distrub the object it will move and it never come back to its original position.	When we distrub the object it will move to a new position and it come to equilibrium but it never come to the original position.

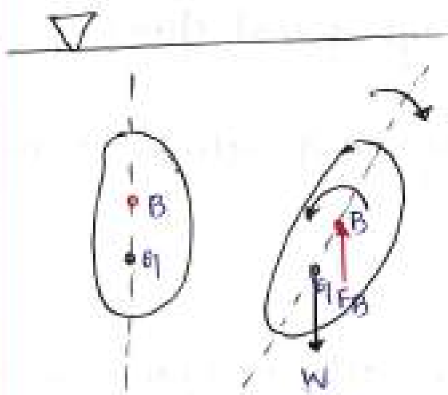


(Unstable equilibrium)

(B is below G)

External moment in c.w direction

Here we see that the line of action of G and B are parallel to each other & form a couple which is in c.w direction and it will not resist the external moment that we applied so it will never come back to the original position.

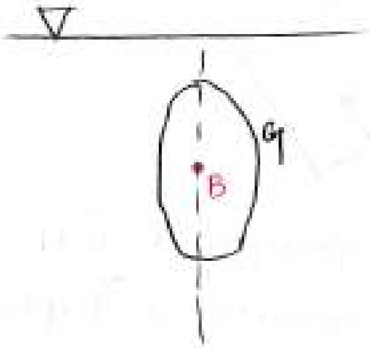


(Stable equilibrium)

(B is above G)

External moment in c.w direction.

Here we see that the line of action of the B and G form a moment anti-clock wise directions that will resist the external moment in c.w direction and due to that it the body will come back to the original position.



(B is at G)
(Neutral Equilibrium)

When body floats —

When the weight of the object is less than the weight of liquid displaced by it.

* When Buoyant force is equal to the weight of the object then it will float.

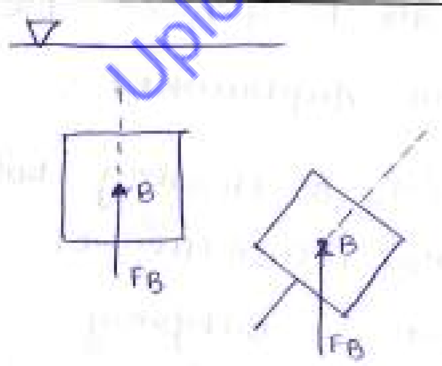
When body sinks —

When the weight of the object is more than the weight of liquid displaced by it.

* When Buoyant force is less than the weight of the object it will sink.

Floatation —

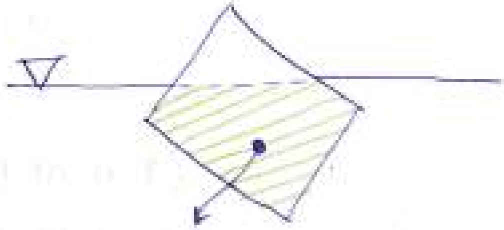
Submerged body



Center of Buoyancy doesnot change when it is disturbed from its mean position.

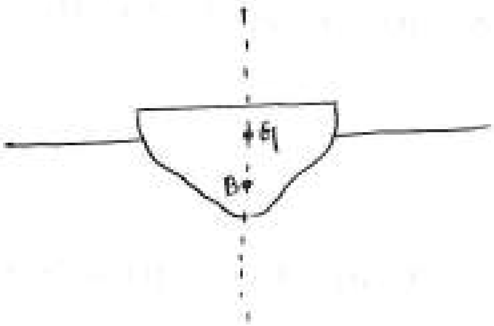
Floating body

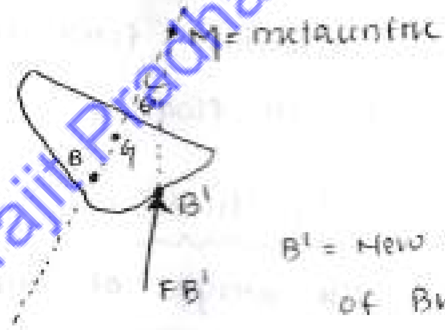
Centre of buoyancy is the centre of gravity of displaced liquid.



Center of buoyancy will shift as the centre of gravity of displaced liquid changes as it shape also changes.

Metacenter and Metacentric height -





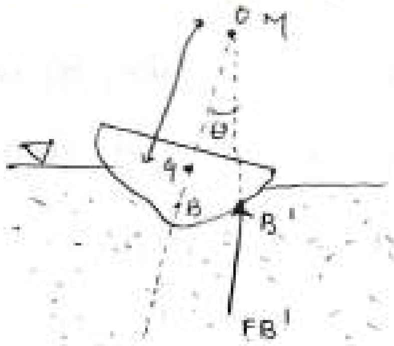
B' = New center of Buoyancy

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

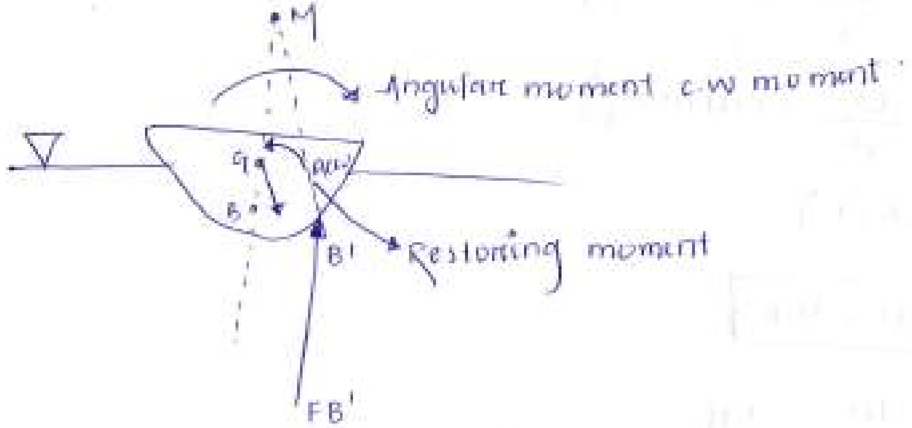
- Metacentre is a point about which a floating body will oscillate, technically it is the interaction of centre of gravity and new center of Buoyancy.

Stability of Floating body -

GM - Metacentric height it is distance between centre of gravity and metacentre.

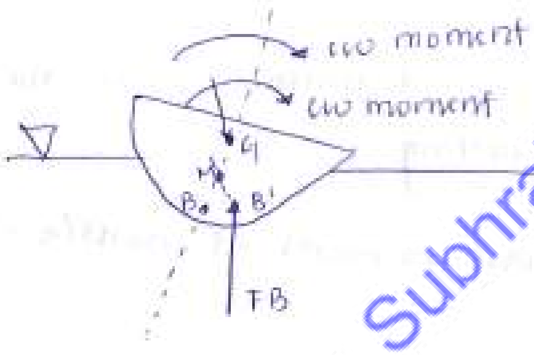


case-I M is above G



$\boxed{\text{stable equilibrium}}$

case-II M is below G



$\boxed{\text{unstable equilibrium}}$

case-III M is at G.

This is condition of $\boxed{\text{Neutral equilibrium}}$.

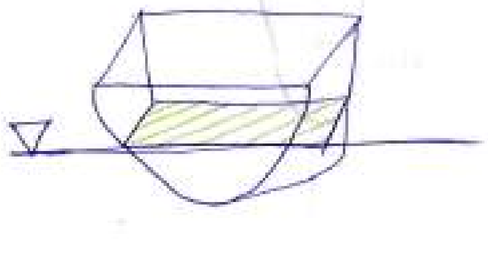
Calculation of Metacentric height -

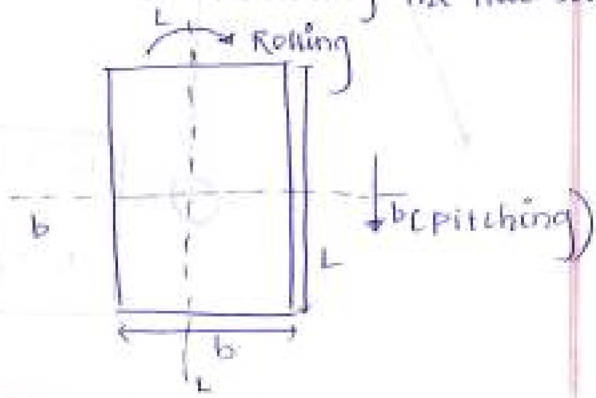
$GM = BM - BG$

$= \frac{I}{\forall} - BG$

where, $\forall$ = displaced volume

I = moment of inertia of surface which is intersecting the free surface.





$$I_{LL} = \frac{Lb^3}{12} \text{ (Rolling)}$$

$$I_{bb} = \frac{bL^3}{12} \text{ (Pitching)}$$

$(b < L)$

so. $\boxed{I_{LL} < I_{bb}}$

$$GM_L = \frac{I_{LL}}{\forall} - BG$$

$$GM_b = \frac{I_{bb}}{\forall} - BG$$

$(GM)_b > (GM)_L$

→ As the I_{LL} is very small. so the boat rolls and some time it turns also

→ chances of unstable is in the direction of rolling

$$\boxed{GM = \frac{I}{\forall} - BG}$$

→ least moment of inertia.

Time period of oscillation –

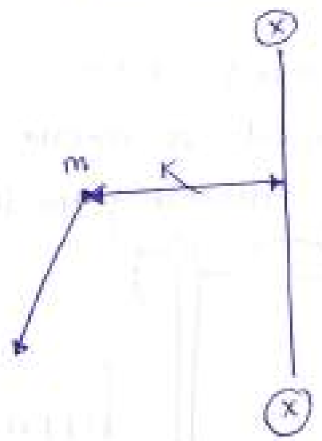
$$\boxed{T = 2\pi\sqrt{\frac{(K)^2}{g(GM)}}}$$

$$\boxed{T \propto \frac{1}{\sqrt{GM}}}$$

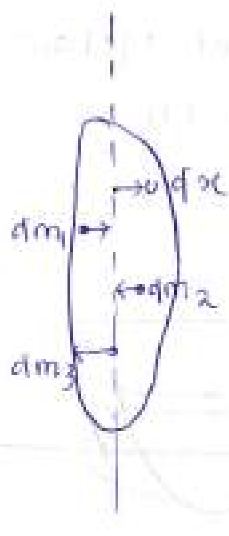
where,

K = radius of gyration

GM = Metacentric height.



$$\boxed{K^2 = \frac{I}{A} \quad K = \sqrt{\frac{I}{A}}}$$



we considere that all the mass are concentrated in a point and that distance is nearly equal to the distance of different masses in the object. That distance is called the radius of gyration.