FLUID KINEMATICS -

MODULE - 2

Fluid Kinematics is the study of Fluid motion regardless of the cause of motion.

Here we don't discuss about the force for which the fluid is in motion but we discuss about

- velocity
- acceleration
- displacement

We study the Kinematic behaviour of the fluid

↓

It means we study the velocity, acceleration displacement of the fluid in different coordinate and trace points.

Methods of Describing Fluid Motion -

The fluid motion is described by two method. They are

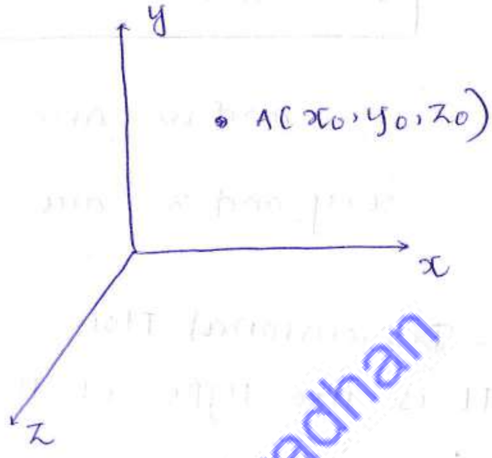
(a) Lagrangian method (b) Eulerian method.

- In the Lagrangian method, a single fluid particle is followed during its motion and its velocity, acceleration, density, etc. are described.
- In the Eulerian method, the velocity, acceleration, pressure, density etc are described at a point in flow field. The Eulerian method is commonly used in Fluid mechanics.

Lagrangian Approach —

- In this approach each fluid particle is observed w.r.t time.
- The Kinematic behaviour of the fluid particle will be the fraction of its identity.

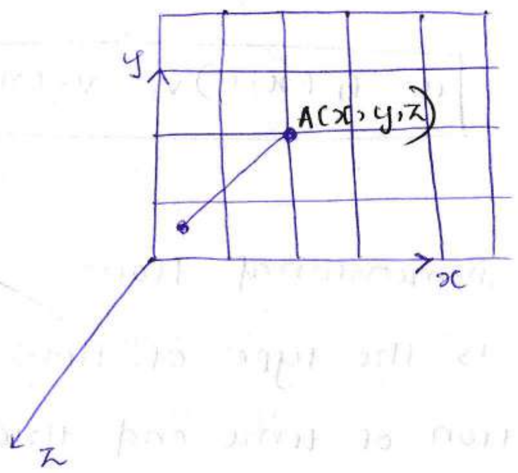
$$\overline{S} = f(x, y, z, t)$$



Eulerian Approach —

- Defines a frame of reference and entire flow field is described with space co-ordinate.
- The Kinematic behaviour will be the function of its space coordinate and time.

$$\overline{S} = f(x, y, z, t)$$



Dimension of Flow —

- One - Dimension of flow :

It is the type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. For a steady one- dimensional flow, the velocity is a function of one-space-co-ordinate only. The variation of velocities in other two mutually perpendicular direction is assumed negligible. Hence, mathematically,

$$\boxed{u = f(x),\ v = 0 \text{ and } w = 0}$$

where, u, v and w are velocity components.

x, y and z are directions respectively.

- Two-Dimensional Flow –

It is the type of flow in which the velocity is a function of time and two rectangular space co-ordinates say x and y. For a steady two-dimensional flow the velocity is a function of two space co-ordinates only. The variation of velocity in the third direction is negligible. Thus, mathematically,

$$\boxed{u = f_1(x, y)\ v = v_2(x, y) \text{ and } w = 0}$$

- Three-Dimensional flow –

It is the type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-Dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only. Thus, mathematically

$$\boxed{u = f_1(x, y, z), v = f_2(x, y, z) \text{ and } w = f_3 = x, y, z}$$

Types of Flow —

- Steady flow and unsteady flow —

* R is the set of fluid properties.

* v is the velocity of the fluid particle.

→ If R and v is invariant with respect to time, then it is called steady flow.

$$\boxed{\left.\frac{\partial R}{\partial t}\right|_{space} = 0} \qquad \boxed{\left.\frac{\partial v}{\partial t}\right|_{space} = 0}$$

→ If R and v varies with respect to time it is called unsteady flow.

$$\boxed{\left.\frac{\partial R}{\partial t}\right|_{space} \neq 0} \qquad \boxed{\left.\frac{\partial v}{\partial t}\right|_{space} \neq 0}$$

- Uniform and Non-uniform flow —

→ If R and v is invariant with respect to space (direction) for a particular interval of time is called uniform flow.

$$\boxed{\left.\frac{\partial R}{\partial space}\right|_{time} = 0} \qquad \boxed{\left.\frac{\partial v}{\partial space}\right|_{time} = 0}$$

→ If R and v varies with respect to space for a particular interval of time called non-uniform flow -

$$\boxed{\left.\frac{\partial R}{\partial space}\right|_{time} \neq 0}$$

$$\boxed{\left.\frac{\partial v}{\partial space}\right|_{time} \neq 0}$$

- Steady and uniform flow –

→ If R and v are invariant with respect to space and time.

$$\boxed{\frac{\partial R}{\partial t}, \frac{\partial R}{\partial space} = 0}$$

$$\boxed{\frac{\partial v}{\partial t}, \frac{\partial R}{\partial space} = 0}$$

- Non-uniform and unsteady flow –

→ If R and v are varies with respect to space and time.

$$\boxed{\frac{\partial R}{\partial t}, \frac{\partial R}{\partial space} \neq 0}$$

$$\boxed{\frac{\partial v}{\partial t}, \frac{\partial R}{\partial space} \neq 0}$$

- compressible and incompressible flow –

→ If the density of fluid does not changes with respect to pressure. It is called as compressible flow.

$$\boxed{\frac{\partial \rho}{\partial p} = 0}$$

where ρ is constant and invariant wrt. change in pressure.

→ If the density of fluid change wrt pressure, it is called as incompressible flow.

$$\boxed{\frac{\partial \rho}{\partial p} \neq 0}$$

ρ is the function of pressure.

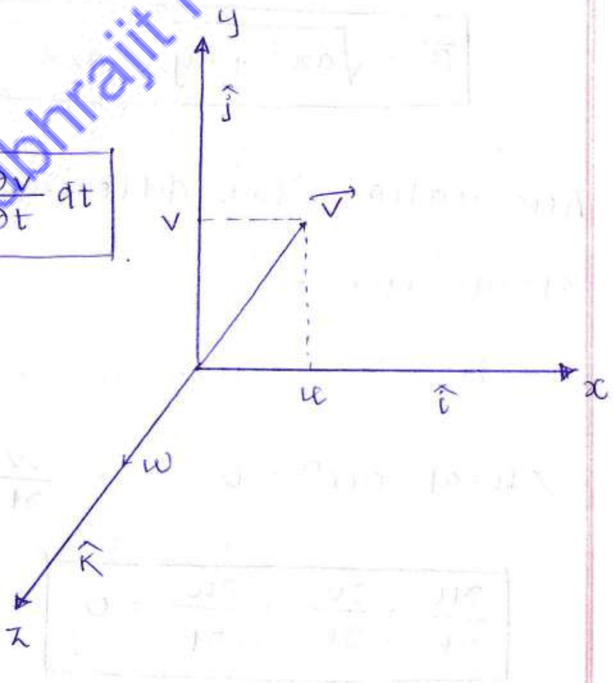
Acceleration of fluid flow —

Let V is the resultant velocity at any point in a fluid flow. Let u, v and w are its component in x, y and z direction. The velocity components are functions of space-co-ordinates and time. Mathematically, the velocity components are given as.

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t)$$

Resultant velocity, $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$$V = \sqrt{u^2 + v^2 + w^2}$$



change in velocity —

$$\boxed{d\vec{V} = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz + \frac{\partial V}{\partial t}dt}$$

acceleration —

$$\vec{a} = \frac{dV}{dt}$$

$$\vec{a} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt} + \frac{\partial V}{\partial z}\frac{dz}{dt} + \frac{\partial V}{\partial t}\cancel{\frac{dt}{dt}}$$

$$= \underbrace{u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}}_{} + \underbrace{\frac{\partial V}{\partial t}}_{}$$

change in velocity wrt space (direction).
↓
(convective accleration)

Temporal acceleration
|
change in velocity wrt time
↓
(Local acceleration)

$$\boxed{\vec{a} = \text{convective acceleration} + \text{local acceleration}}$$

acceleration along x- direction –

$$\vec{a}_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

y-direction –

$$\vec{a}_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

z-direction –

$$\vec{a}_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Total acceleration –

$$\boxed{\vec{a} = \sqrt{a_x^2 + a_y^2 + a_z^2}}$$

Acceleration for different types of flow –

- Steady flow –

$$\vec{a} = \text{convective acc}^n + \text{local acc}^n.$$

$\Rightarrow$ local $acc^n = 0$, $\frac{\partial v}{\partial t} = 0$

$$\boxed{\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t} = 0}$$

- Uniform flow –

$\Rightarrow$ convective $acc^n = 0$

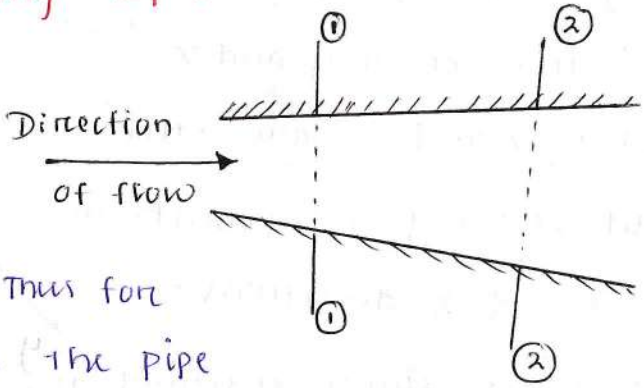
$$\underbrace{\boxed{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}}_{\text{becomes zero.}} + \frac{\partial w}{\partial t}$$

$$\boxed{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = 0}$$

- steady and uniform flow –

$$\boxed{\vec{a} = 0}$$

General form of continuity Equations –



The equation based on the principle of ~~conv~~ conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section. The quantity of fluid per second is constant.

Let V_1 = average velocity at cross-section (1-1)

ρ_1 = Density at section (1-1)

A_1 = Area of pipe at section (1-1)

V_2, ρ_2, A_2 are corresponding values at section (2-2)

Then rate of flow at section (1-1) $= \rho_1 A_1 V_1$

Rate of flow at section (2-2) $= \rho_2 A_2 V_2$.

According to law of conservation of mass

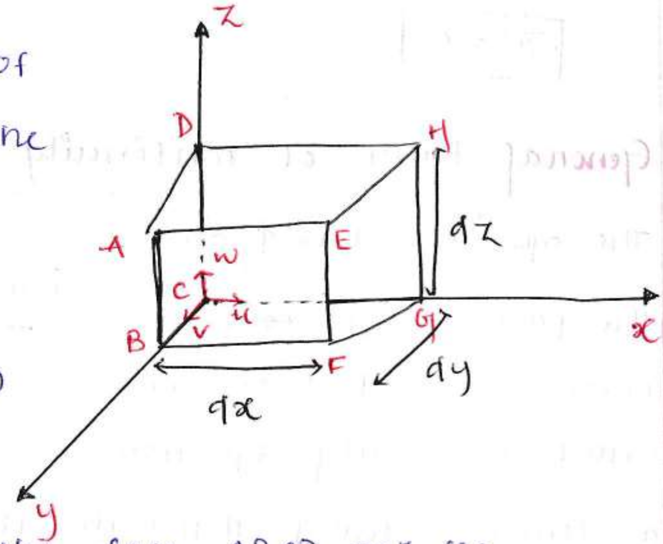
Rate of flow at section(1-1) = Rate of flow at section (2-2)

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2} \quad \text{——— (1)}$$

From eq^n(1) is applicable to the compressible as well as incompressible fluids and is called continuity equation. If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation reduces to

$$\boxed{A_1 V_1 = A_2 V_2}$$

Continuity Equation in three-dimensions –



Consider a fluid element of lengths dx, dy and dz in the direction of x, y and z.

Let u, v and w are the inlet velocity components in x, y and z directions.

Mass of fluid entering the face ABCD per sec.

$$= \rho \times \text{velocity in } x\text{-direction} \times \text{Area of } ABCD$$

$$= \rho \times u \times (dy \times dz)$$

The mass of fluid leaving the face EFGH per second $= \rho u \cdot dy dz + \frac{\partial}{\partial x}(\rho u dy dz) dx$

Gain of mass in x-direction

$$= \text{mass through ABCD} - \text{mass through EFGH}$$

$$= \rho u dy dz - \rho u dy dz - \frac{\partial}{\partial x}(\rho u dy dz) dx$$

$$= -\frac{\partial}{\partial x}(\rho u dy dz) dx$$

$$= -\frac{\partial}{\partial x}(\rho u) dx dy dz.$$

Similarly, the net gain of mass in y-direction

$$= -\frac{\partial}{\partial y}(\rho v) dx dy dz$$

In z-direction $= -\frac{\partial}{\partial z}(\rho w) dx dy dz.$

$\therefore$ Net gain of masses $= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx dy dz.$

since the mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element. But mass of fluid in the element is ρ, dx, dy, dz and its rate of increase with time is $\frac{\partial}{\partial t}(\rho . dx, dy, dz)$ or $\frac{\partial \rho}{\partial t}, dx, dy, dz$.

Equating two expression.

$$-\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]dx\,dy\,dz = \frac{\partial \rho}{\partial t}.dx\,dy\,dz.$$

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0}$$

General for continuity equation.

- For steady flow —

$$\frac{\partial \rho}{\partial t}=0 \qquad \rho \neq f(t)$$
$$\rho = f(x,y,z)$$

$$\boxed{\frac{\partial}{\partial x}\rho u+\frac{\partial}{\partial y}\rho v+\frac{\partial}{\partial z}\rho w=0}$$

- For steady and incompressible flow.

$\rho = \text{constant}$

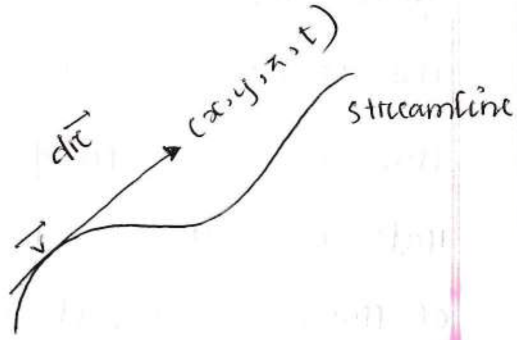
$\rho \neq f(x,y,z)$.

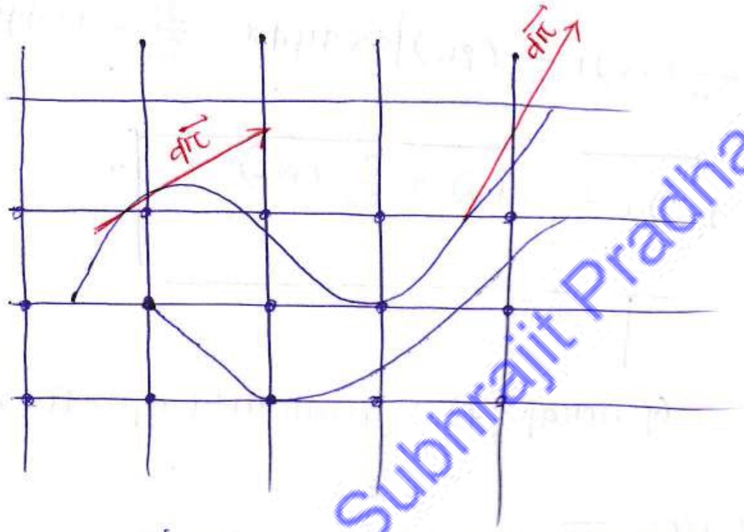
$$\boxed{\frac{\partial}{\partial x}u+\frac{\partial}{\partial y}v+\frac{\partial}{\partial z}w=0}$$

Flow Visualization Technique –

→ Streamline

It is an imaginary line drawn in the flow fluid such that the tangent drawn at any point on this line represent the direction of the velocity vector at a particular point.





at an instantaneous point of time
↳ Eularian approach

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$\vec{V}$ and $d\vec{r}$ are co-linear.

$$\theta = 0^\circ$$

$$\vec{V} \times d\vec{r} = V\,dr\sin\theta$$

$\boxed{\vec{V} \times d\vec{r} = 0}$ zero vector $= 0\hat{i} + 0\hat{j} + 0\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & v & w \\ dx & dy & dz \end{vmatrix} = 0$$

$\Rightarrow \hat{i}(v\,dz - w\,dy) - \hat{j}(u\,dz - w\,dx) + \hat{k}(u\,dy - v\,dx) = 0\hat{i} + 0\hat{j} + 0\hat{k}$.

$v\,dz - w\,dy = 0$

$\Rightarrow v\,dz = w\,dy$

$\Rightarrow \frac{dz}{w} = \frac{dy}{v}$ —— (I)

$u\,dz - w\,dx = 0$

$\Rightarrow u\,dz = w\,dx$

$\Rightarrow \frac{dz}{w} = \frac{dx}{u}$ —— (II)

From eqⁿ (I) and eqⁿ (II) we get.

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$ Relation for streamline.

Q. A 2D incompressible flow field is given by

$$\vec{V} = 3x\hat{i} - 3y\hat{j}$$

Find the equation of steamline passing through (1,1)

$\Rightarrow u = 3x$, $v = 3y$.

we know that,

$$\frac{dx}{u} = \frac{dy}{v}.$$

$$\Rightarrow \frac{dx}{3x} = \frac{dy}{-3y}$$

$$\Rightarrow \frac{dx}{x} = -\frac{dy}{y}$$

Integrating bothsides

$\ln x = -\ln y + \ln c$

$\Rightarrow \ln x + \ln y = \ln c$

$\Rightarrow \ln(x \cdot y) = \ln c$.

Antilog take.

$\Rightarrow \boxed{xy = c}$

when $x = 1$, $y = 1$

$\boxed{c = 1}$

So, the eqⁿ will be

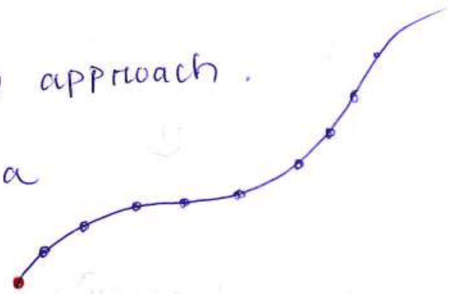
$xy = 1$

$\Rightarrow \boxed{y = \frac{1}{x}}$

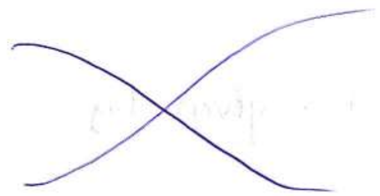
★ $\boxed{\text{Two streamline never intersect with each other.}}$

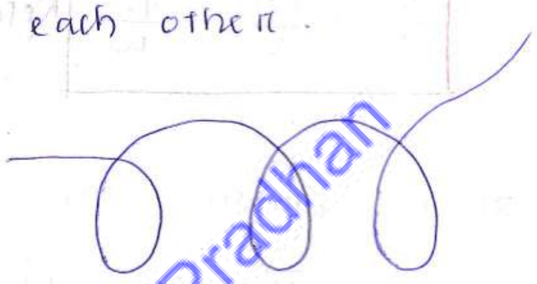
Path line —

- Pathline based on lagrangian approach.



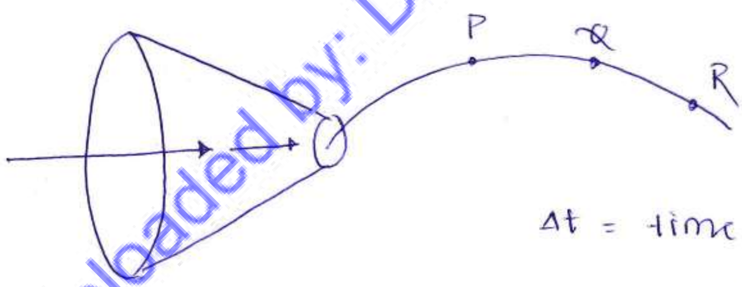
- A locus of fluid particle in a flow field is called pathline.
- 2 pathline intersect with each other.





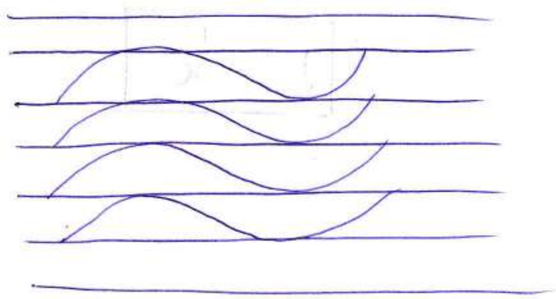
- It can intersect itself or can from a loop.

Streak line —



Δt = time interval for observation.

- Streakline is an immaginary line which is passing through the point which have pased through a same point after a given interval of time.



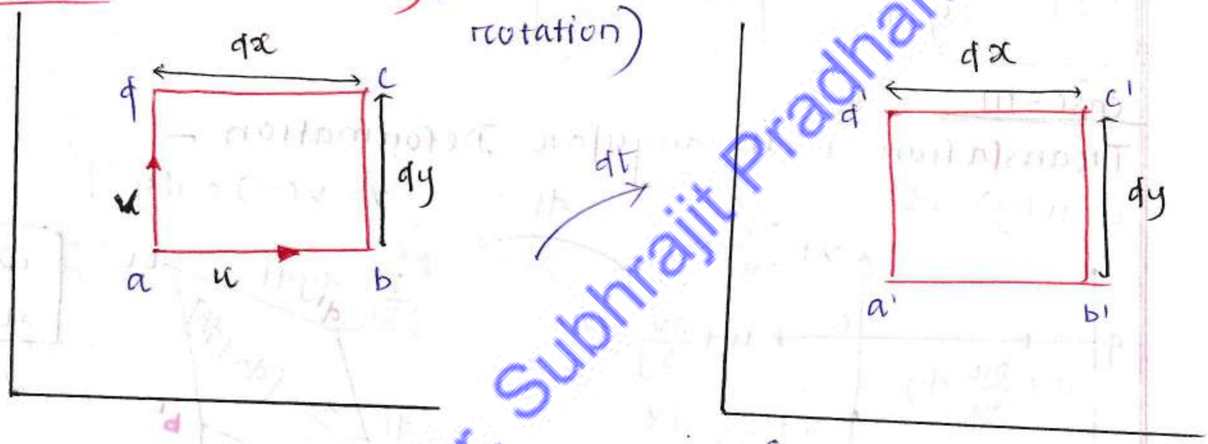
- If the flow is steady, then.

$$\left.\begin{array}{l}\text{Streamline}\\ \text{Pathline}\\ \text{Streak line}\end{array}\right\}\text{ coincide.}$$

Fluid element Motion –

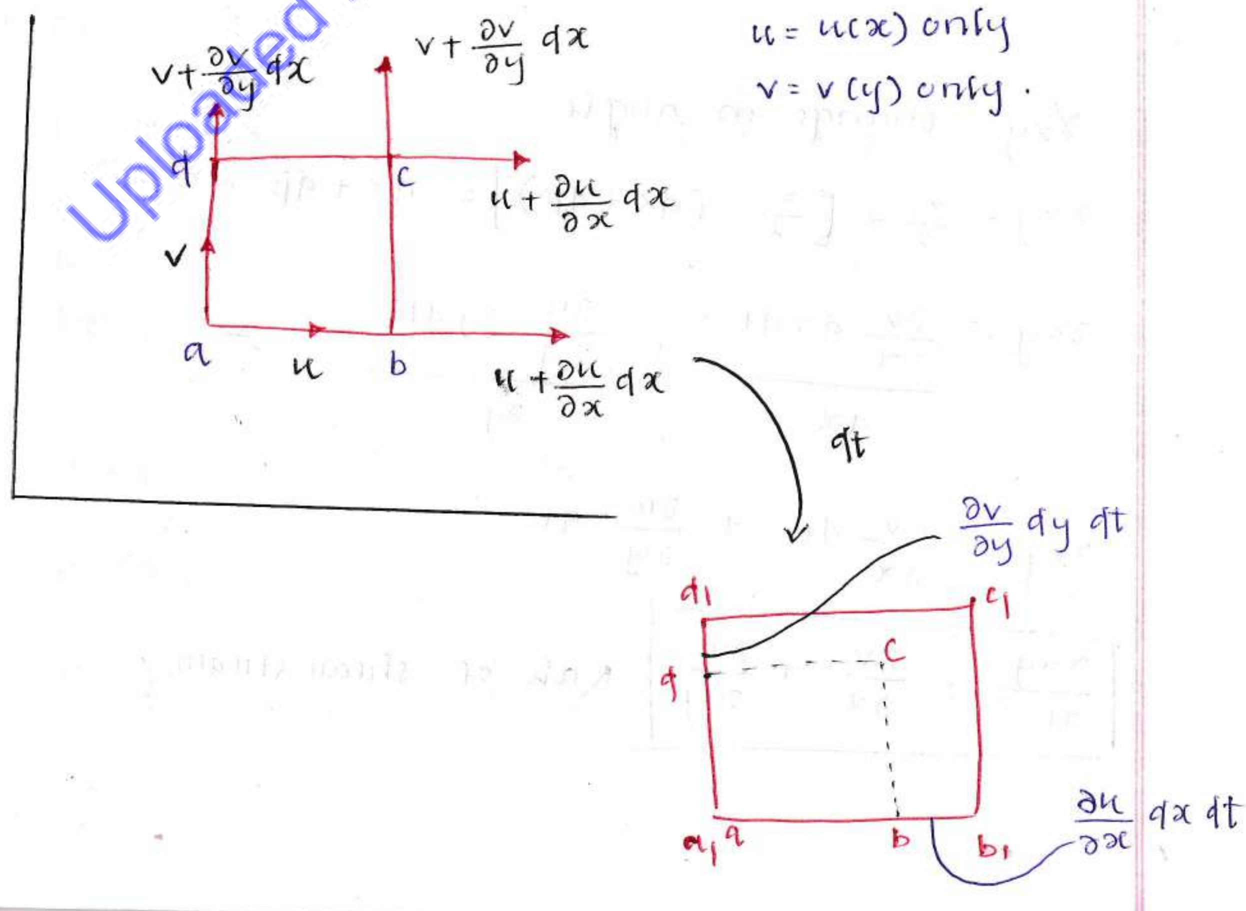
- Translation without deformation.
- Translation with linear deformation.
- Translation with angular deformation.

case-I (Uniform Flow) (Translation without deformation and rotation)



(Pure translation)

case-II Translation with linear deformation.



$u = u(x)$ only

$v = v(y)$ only.

$$\text{Normal strain} = \varepsilon_{xx} = \frac{l_f - l_i}{l_i}$$

$$= \frac{dx + \frac{\partial u}{\partial x} dx\,dt - dx}{dx}$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} dt$$

$$\boxed{\frac{\varepsilon_{xx}}{dt}} = \frac{\partial u}{\partial x}$$

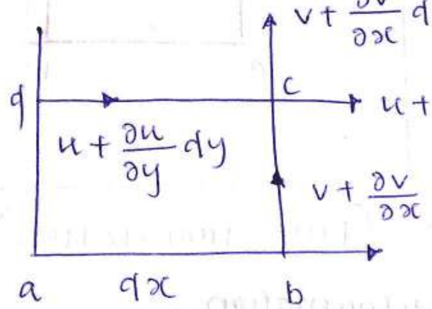
Normal strain rate

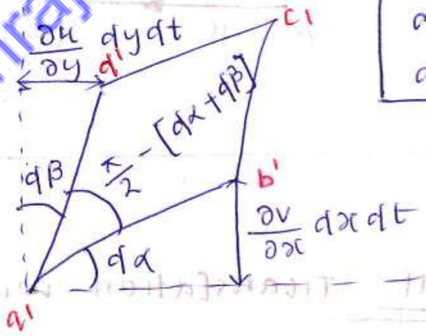
$$\boxed{E_{yy} = \frac{\partial v}{\partial y}} \qquad \boxed{E_{zz} = \frac{\partial w}{\partial z}}$$

case - III

Translation with angular Deformation -

$u = u(y)$ only $\qquad$ dt $\qquad$ $v = v(x)$ only .





$$\boxed{\begin{matrix} ab' = ab \\ ad' = ad \end{matrix}}$$

γ_{xy} = change in angle .

$$\gamma_{xy} = \frac{\pi}{2} - \left[\frac{\pi}{2} - (d\alpha + d\beta)\right] = d\alpha + d\beta .$$

$$\gamma_{xy} = \frac{\frac{\partial v}{\partial x} dx\,dt}{dx} + \frac{\frac{\partial u}{\partial y} dy\,dt}{dy}$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} dt + \frac{\partial u}{\partial y} dt$$

$$\boxed{\frac{\gamma_{xy}}{dt} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}} \quad \text{Rate of shear strain .}$$

Strain Tensor —

Strain at a point in flow field:

$$\epsilon_{ij} = \begin{bmatrix} \epsilon_{xx} & \gamma_{xy}/2 & \gamma_{xz}/2 \\ \gamma_{yx}/2 & \epsilon_{yy} & \gamma_{yz}/2 \\ \gamma_{zx}/2 & \gamma_{zy}/2 & \epsilon_{zz} \end{bmatrix}$$

Shear strain rate

Normal strain rate

2nd order symmetric Tensor

$\gamma_{xy} = \gamma_{yx}$ $\qquad$ $\gamma_{xz} = \gamma_{zx}$ $\qquad$ $\gamma_{yz} = \gamma_{yz}$

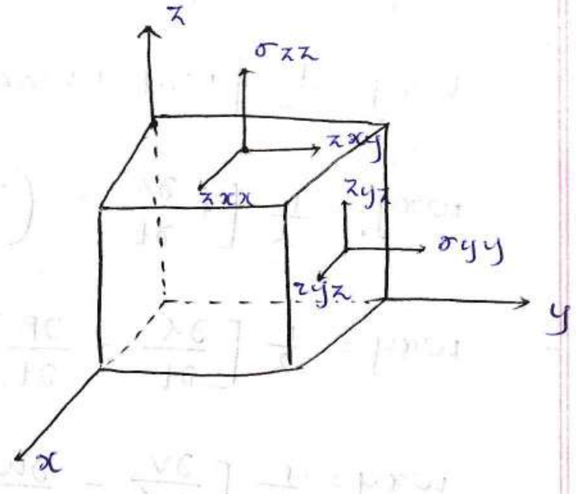
$\epsilon_{xx} = \dfrac{\partial u}{\partial x}$ $\qquad$ $\gamma_{xy} = \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}$

$\epsilon_{yy} = \dfrac{\partial v}{\partial y}$ $\qquad$ $\gamma_{yz} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial z}$

$\epsilon_{zz} = \dfrac{\partial w}{\partial z}$ $\qquad$ $\gamma_{xz} = \dfrac{\partial w}{\partial x} + \dfrac{\partial u}{\partial z}$

Stress Tensor —

Stress at a point



$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$\tau_{xy} = \mu \gamma_{xy}$

$\tau_{xz} = \mu \gamma_{xz}$

$\tau_{yz} = \mu \gamma_{yz}$.

Stokesian Fluid —

$$\left.\begin{aligned}\sigma_{xx} &= -p + 2\mu\frac{\partial u}{\partial x} = -p + 2\mu\epsilon_{xx}\\ \sigma_{yy} &= -p + 2\mu\frac{\partial v}{\partial y} = -p + 2\mu\epsilon_{yy}\\ \sigma_{zz} &= -p + 2\mu\frac{\partial w}{\partial z} = -p + 2\mu\epsilon_{zz}\end{aligned}\right\} \text{For incompressible fluid}$$

$$\epsilon_{ij} = \begin{bmatrix} \epsilon_{xx} & \gamma_{xy}/2 & \gamma_{xz}/2 \\ \gamma_{yx}/2 & \epsilon_{yy} & \gamma_{yz}/2 \\ \gamma_{zx}/2 & \gamma_{zy}/2 & \epsilon_{zz} \end{bmatrix} \Rightarrow \text{strain Tenson}$$

Angular velocity (ω_{xy}) —

The rotation (or angular velocity) is defined as the arithmetic mean of the angular velocities of two parallel line segment (i.e. ab and ad) meating at a point.

$$\omega_{xy} = \frac{1}{2}\left[\omega_{ab} + \omega_{ad}\right]$$

$$\omega_{xy} = \frac{1}{2}\left[+\frac{\partial \alpha}{\partial t} + \left(-\frac{\partial \beta}{\partial t}\right)\right]$$

$$\omega_{xy} = \frac{1}{2}\left[\frac{\partial \alpha}{\partial t} - \frac{\partial \beta}{\partial t}\right]$$

$$\omega_{xy} = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

rotation.

similarly —

$$\omega_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_{zx} = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$\vec{w} = \frac{1}{2}(\nabla \times \vec{V})$$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ u & v & w \end{vmatrix}$$

$$= \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \hat{j}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \hat{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\boxed{\vec{w} = w_{yz}\hat{i} + w_{zx}\hat{j} + w_{xy}\hat{k}}$$

Vorticity - (Ω) -

A microscopic measure of rotation at any point in the fluid flow.

It is the actual rotation rate.

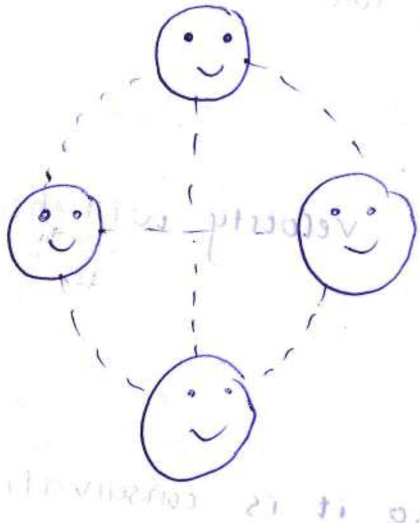
$$\boxed{\Omega = \nabla \times \vec{V}} = \text{curl}\, \vec{V}$$

* It is twice the angular velocity.

• Irrotational Flow -

Fluid particles are not rotating about their mass centres.

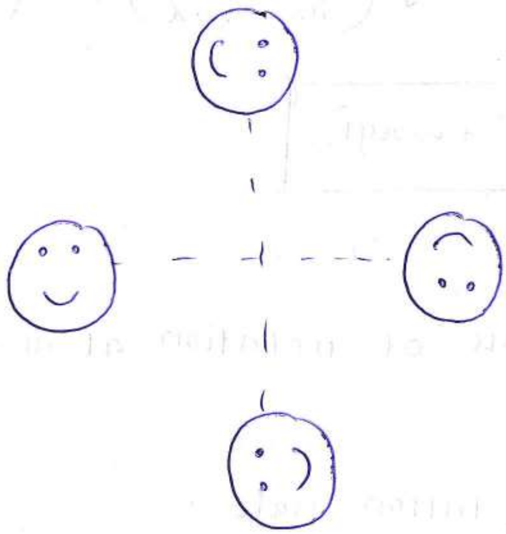
$$\boxed{w_x = w_y = w_z = 0}$$



- Rotational Flow –

Fluid particles are rotating about this mass centres.

$$\boxed{w_x \neq 0}$$



Potential Function –

If a velocity vector field is conservative then the velocity vector will be equal to gradient of a scalar function.

This scalar function is the potential function.

$$\boxed{\vec{V} = \nabla \phi}$$

* potential function exists for a conservative velocity vector field.

Conservative field –

The line integral of the velocity will be independent of path.

$$\boxed{\nabla \times \vec{V} = 0}$$

curl $\vec{V} = 0$ then ~~also~~ also it is conservative field.

$$\Rightarrow \nabla = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right)$$

Hamiltanian operator or Del operator (used for linear vector differential)

$\Rightarrow$ ∇ is not a vector.

$\Rightarrow$ neighter has magnitude or direction.

- $\nabla \times \vec{V} = 0$

  ϕ exists.

- $\phi = f(x, y, z, t)$

$$u = -\frac{\partial \phi}{\partial x}$$

$$v = -\frac{\partial \phi}{\partial y}$$

$$w = -\frac{\partial \phi}{\partial z}$$

Bernoullis Equation —

The Bernoulli's equation was derived on the assumption that fluid is inviscid (non-viscous) and therefore frictionless. But all the real fluid are viscous and hence offer resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoullis equation, these losses have to be taken into consideration. Thus, the Bernoullis equation for real fluids between point 1 and 2 is given as.

$$\boxed{\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L}$$

where, h_L is the frictional loss / energy loss.

Application of Bernoulli's Equation –

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved.

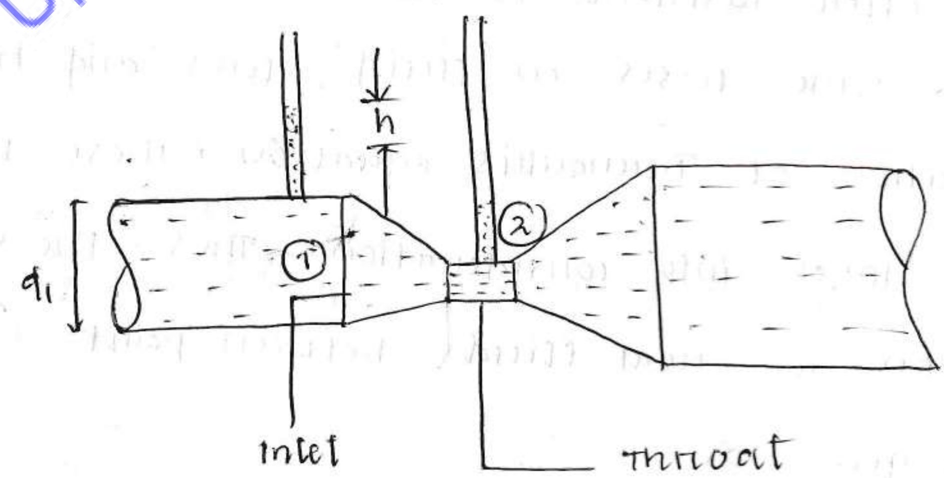
- Venturimeter.
- Orifice meter
- Pitot - tube.

Venturimeter –

A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts:

- A short converging part
- Throat
- Diverging part.

Expression for rate of flow through venturimeter –



Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing.

d_1 = diameter at inlet or at section (1)

P_1 = pressure at section (1)

v_1 = velocity of fluid at section (1)

a_1 = area at section (1) $= \frac{\pi}{4} d_1^2$

Similarly, d_2, P_2, v_2 and a_2 are corresponding values at section (2).

Applying Bernoulli's equation at section (1) and (2)

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} \quad (\because z_1 = z_2)$$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\Rightarrow \boxed{h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}} \quad \left(\because h = \frac{P_1 - P_2}{\rho g}\right) \text{ —— (1)}$$

Now applying continuity equation at section (1) and eqn (1)

$$a_1 v_1 = a_2 v_2$$

$$\Rightarrow v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in eqn (1)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g}$$

$$= \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

$$= \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right].$$

$$\Rightarrow v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$\Rightarrow v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}}$$

$$= \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$Q = a_2 v_2$$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$\boxed{Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times 2gh}$$

This equation is called as discharge formula / theoretical discharge.

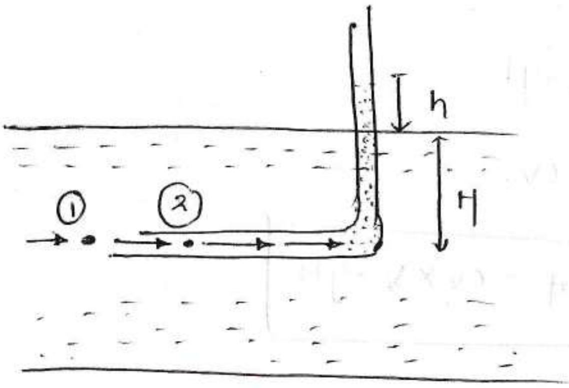
$$Q_{act} = c_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$\boxed{\Rightarrow Q_{act} = c_d \times Q_{theoretical}}$$

∴ c_d = co-efficient of venturimeter and its value is less than 1.

Pitot - tube —

It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy.



P_1 = Intensity of pressure at point ①

v_1 = velocity of flow at (1)

P_2 = pressure at point ②

v_2 = velocity at point ②

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface.

By applying Bernoulli's equation at point (1) and (2),

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$z_1 = z_2$ as point (1) and (2) are on the same line and $v_2 = 0$

$\frac{P_1}{\rho g}$ = pressure head at (1) = H

$\frac{P_2}{\rho g}$ = pressure head at (2) = $(h+H)$

substituting these values, we get

$$H + \frac{v_1^2}{2g} = (h+H) \quad \left(\because h = \frac{v_1^2}{2g}\right)$$

$$\Rightarrow \boxed{v_1 = \sqrt{2gh}}$$

This is theoretical velocity, Actual velocity.

$$(V_1)_{act} = C_v\sqrt{2gh}.$$

For real fluid –

$$v_1 T = \sqrt{2gh}$$

$$\frac{v_1 act}{v_1 T} = cv$$

$$\Rightarrow \boxed{V_{actual} = cv \times \sqrt{2gh}}$$

Flow through pipes or Ducts –

Losses in pipes

Major

- head loss due to friction.

Minor

- Suddern contraction
- Suddern enlargement
- Bend in pipes
- Pipes fittings
- Obstracles.

Major Losses-

- Darcy - weisbach Formula –

$$\boxed{h_F = \frac{4FLV^2}{d \times 2g}}$$

h_f = head loss due to friction

F = co-efficient of the friction which is a functi

of reyonds number (non-dimensional number) -

L = Length of the pipe.

V = mean velocity of flow.

d = diameter of the pipe.

- Chezys Formula –

$$\boxed{h_f = \frac{F'}{\rho g} \times \frac{P}{A} \times L \times v^2}$$

F' = Frictional resistance per unit area of surface per unit velocity.

P = Perimeter of the pipe (wetted perimeter)

P/A = reciprocal $A/P = m$ is the hydraulic mean depth/ hydraulic radius.

$$\Rightarrow \frac{\frac{\pi}{4}d^2}{\pi d} = d/4 = m$$

$$\Rightarrow \boxed{m = \frac{d}{4}}$$

$$h_f = \frac{f'}{\rho g} \times L \times v^2 \times \frac{1}{m}$$

$$\Rightarrow v^2 = \frac{h_f \times \rho g m}{f' L}$$

$$\Rightarrow v = \sqrt{\frac{h_f \times \rho g \times m}{f' L}}$$

where, $\frac{\rho g}{f'} = c$ = chezy's constant

$h_f/L = i$ = head loss per unit length of pipe.

$$v = \sqrt{cmi}$$

$$\Rightarrow \boxed{v = c \times \sqrt{mi}} \quad \because c = \text{constant}$$