KINEMATICS

MODULE-1

It deals with the relative motion of a mechanism without taking considension of forces producing the motion.

- It deals with displacement, velocity and acceleration of a exting mechanism.

Dynamics –

It involves the calculation of forces impresed often different Part of a mechanism.

Mechanism and Machine –

If a number of bodies are all in such a way that the motion of one causes constant and creaditable motion to the other it is Known as mechanism. A mechanism modifies a motion.

Machine –

A machine is a mechanism or a combination of mechanism which gives a definite motion to the parts also modifies and transmites the availeved mechanical energy into some kind of desirable work.

Analysis –

It is the study of motion and forces concerning different part of an exiting mechanism.

Syntesis –

It involves disign of its different parts.

Link –

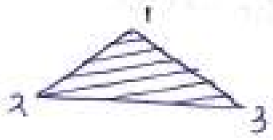
A mechanism made of number of resistance bodies of which some may have motion relative to other.

- A resistance bodies or a group of resistance bodies with rigid connection preventing their relative motion is known as a link.

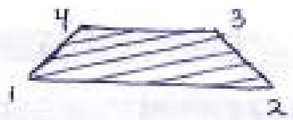
Link is classified into 3 parts –

(a) Binary link

(b) Ternary link



(c) Quantenary link



Kinematic pair –

A Kinematic pair or simple a pair is a joint of two links having the relative motion between them.

Types of Kinematic pair –

- Nature of contact.
- Nature of mechanical constraint.
- Nature of relative motion.

★ Kinematic pair according to nature of contact –

- Higher pair –

When a pair has a point or line contact between the links it is higher pair.

EX – wheel rooling on a surface.
Cam and follower.
Tooth gear
Ball and roller bearing.

- Lower pair -

A pair of links having surface or area of contact between the member is known as lower pair.

EX - Noot tuning in a screw.
shaft rotating on the bearing.
All pairs of sliding crank mehanism.
Universal joint.

★ Kinematic pair according to nature of mechanical constraint -

- closed pair -

when the element of a pair are heabl together mechanically it is known as closed pair.

- One is fully and closed to another and the contact between this two can be broken only be destraction of atleast one member.

EX - All lower pairs and some higher pair as shown.

- Unclosed pair -

When two links of a pair are contact either due to force of gravity and some spring action consitute and unclosed pair.

★ Kinematic pair according to nature of relative motion -

- sliding pair -

If two links are sliding motion relative to each other they form a sliding pair.

Ex- Rectangular road in a rectangular hole prisime forms a sliding pair.

- Turning / Revoalving pair —

When two links have turning or revolving motion relative to each other they constitute turning or revolving pair.

Ex- In sliding crank mechanism all the pair except the slider and guide.

- Rolling pair —

When the links of a pair have rolling motion relative to each other they forms the rolling pair.

Ex- Ball and roller bearing.

Rolling of a wheel on the surface.

- Screw pair —

If two matching links have a turning as well as sliding motion between them they form a screw pair.

- Spherical pair —

When one links in form of a spher inside a fixed linked it is spherical pair.

Ex- Ball and socket joint in a spherical pair.

Joints —

A joint is a movable connection between links and allows relative motion between the links.

$\boxed{\text{No. of pairs} = n-1}$

Types of joint -

- Binary joint - 1 pair
- Ternary joint - 2 pair
- Quantenary ~~pair~~ joint - 3 pair.

Binary joint -

If two links are joint at the same connection it is called binary joint.

Ternary joint -

If three links are joint at the same connection it is called as ternary joint.

- It is consider equivalent to two binary pair.

Quantenary joint -

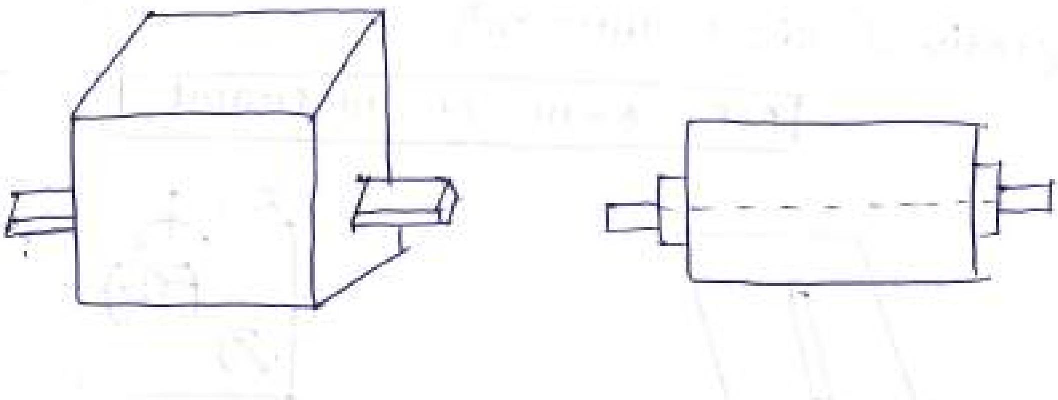
If four links are joint at the same connection it is known as quantennary joint.

- It is consider equivalent to three binary pair.

Types of constrained motion -

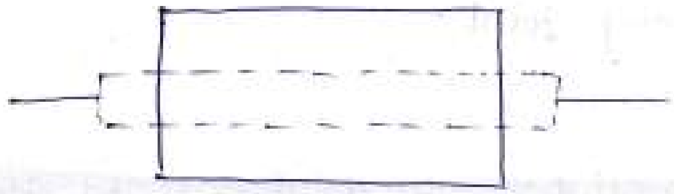
- completely constrained motion -

when a motion between two elements of a pair is in a definite direction irrespective of the direction of force applied it is known as completely constrained motion.



- Incompletely constraint motion –

When the motion between two elements of upon the direction and depends upon the direction of forces applied it is known as incompletely constraint motion.



- Successfully constraint motion –

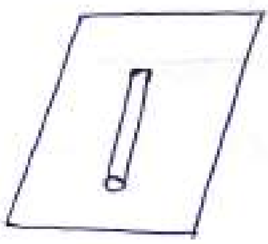
When the motion between two elements of a pair is possible in more than one direction but it is made to have motion only in one direction by using some external mean, it is successfully constraint motion.

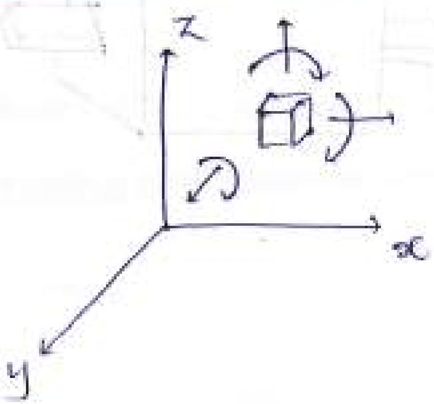
Degree of freedom (DOF) –

Degree of freedom of a pair defined as the number of independent relative motions both translational and rotational a pair can have.

- Translational motion along any three mutually ⊥r axis.
- Rotational about these axis

$$\boxed{DOF = 6 - \text{no. of restraint}}$$



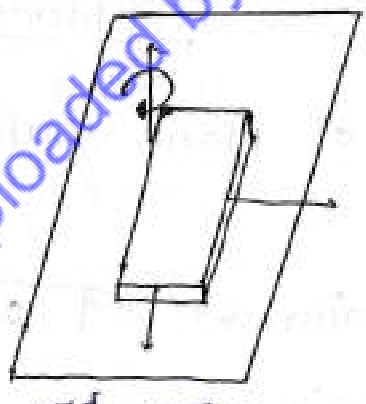


Linkage mechanism and structure —

- A linkage is obtained if one of the link of a Kinematic chain is fixed to the ground.
- If motion of any of movable links results in definite motion to other then linkage called a mechanism.
- If one of the link of a redundant chain is fixed it is Known as a structure or locked system. So DOF of a structure = 0.
- A structure with -ve DOF is Known as super structure.

Mobility of mechanism —

According to the number of these general or common restraint a mechanism can be classified into different order. A zero order mechanism will have no such general restraints. A 1st order mechanism has one general restraint.



3rd order.

N = Total no. of links

F = Total no. of freedom.

P_1 = No. of pair having 1° of freedom of 5th order.

P_2 = No. of pair having 2° of freedom of 4th order.

$$\boxed{F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5}$$

No. of movable links = N-1

so, total degree of freedom of the links will be 6(N-1).

1° freedom imposes 5 restrient on a mechanism. Reducing it degree of freedom by $5P_1$. Similarly 2° of freedom imposes 4 restrient on a mechanism. Reducing it degree of freedom by $4P_2$. The other pair having 3°, 4° and 5° of freedom, reducing the degree of freedom of the mechanism as the above equation.

- In most of the mechanism any two dimensions ~~are~~ such as four links in a slider crank mechanism, in which the displacement is possible along two axis and rotation about one axis therefore for a plane mechanism.

$$\boxed{F = 3(N-1) - 2P_1 - P_2}$$

This equation is also known as Grubler's criterion.

- For degree of freedom of plane mechanism in which each.

P_1 = No. of lower pairs or 2nd order

P_2 = Mostly higher pair

$$\boxed{F = 3(N-1) - 2P_1}$$

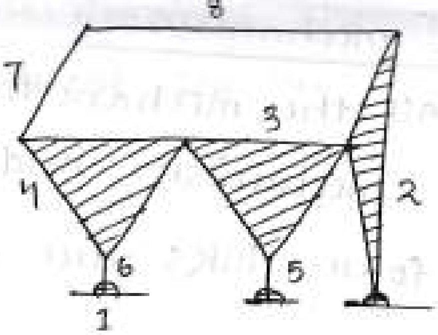
This is known as Kutzback's Criteria.

Q. No. of binary links :

" " total links

" " joints or pair

degree of freedom ?



⇒ No. of binary links = 4

No. of ternary links = 4

No. of Quertanary links = 0

Total no. of links = 8

No. of joints / pair = 10

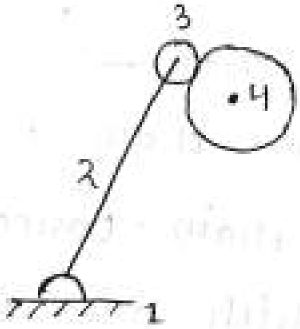
$F = 3(N-1) - 2P_1$

$= 3(8-1) - 2 \times 10$

$= 21 - 20$

$= 1$

Q.



$F = 3(N-1) - 2P_1 - P_2 - 1$

$= 3(4-1) - 2 \times 3 - 1$

$= 12 - 3 - 4$

$= 3(4-1) - 2 \times 3 - 2 - 1$

$= 12 - 3 - 6 - 2 - 1$

$= 12 - 9 - 3$

$= 12 - 12$

$= 0$

$F = 3(N-1) - 2P_1 - P_2 - 1$

$= 3(4-1) - 2 \times$

Simple mechanism –

All the mechanism having four links are simple mechanism and the mechanism having more than four links are compound mechanism.

Example of SM –

- Four bar mechanism –

  It consists of four links and four turning pair.

- Slider crank mechanism –

  It consists of four link three turning pair and one sliding pair.

- Double slider crank mechanism –

  It consist of four link two turning pair and two sliding pair.

Four Bar chain –

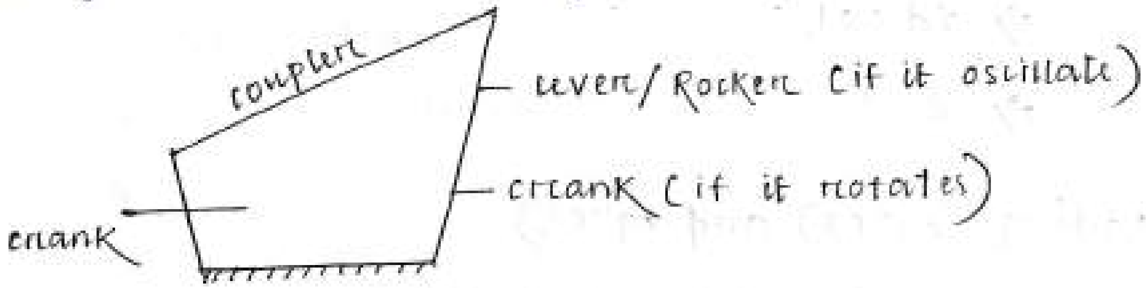
A four bar chain is the most fundamental of plain kinematic chain. Basically it consist of four rigid links which are connected in form of quadrilateral by four pin joints.

– A link that makes a complet revolution is called the crank.

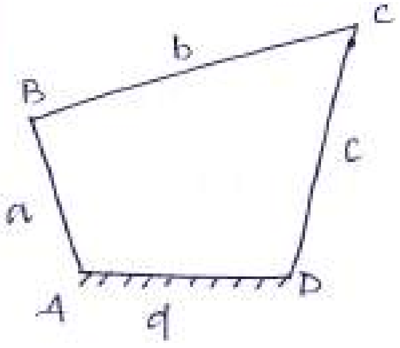
– This link opposite to fixed linked is called as capler and the four the link is called as a leaver or rocker.

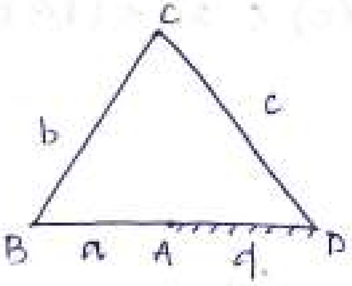
– If it oscillate or another crank if it rotates.

— The linkage can exit if the sum of the three links is greater than the length of one link.



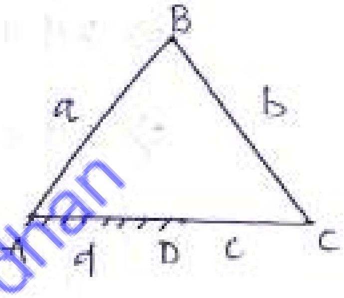
Q.





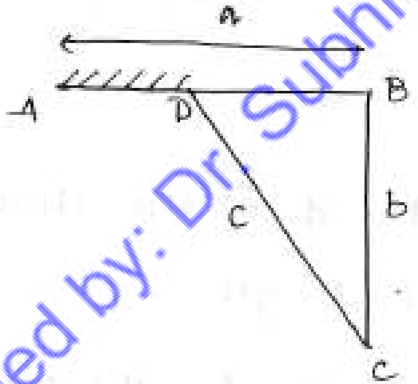
(fig-1)

$a+d < b+c$



(fig-2)

$d+c < a+b$



(fig-3)

$b < c + (a-d)$

from fig-1 = $a+d < b+c$ —— (i)

fig-2 = $c+d < a+b$ —— (ii)

fig-3 = $b < c+(a-d)$

$= d+b \leq c+a$ —— (iii)

Add eqn(i) and eqn(ii)

$a+d < b+c$

$c+d < a+b$.

$\Rightarrow 2d + (\cancel{a+c}) < 2b + (\cancel{a+c})$

$\Rightarrow 2d < 2b$

$\Rightarrow d < b$.

Adding eq^n(2) and eq^n(3)

$c + d + d + b < a + b + c + a$

$\Rightarrow 2d + (\cancel{b+c}) < 2a + (\cancel{b+c})$

$\Rightarrow 2d < 2a$

$\Rightarrow d < a$.

Adding eq^n(1) and eq^n(3)

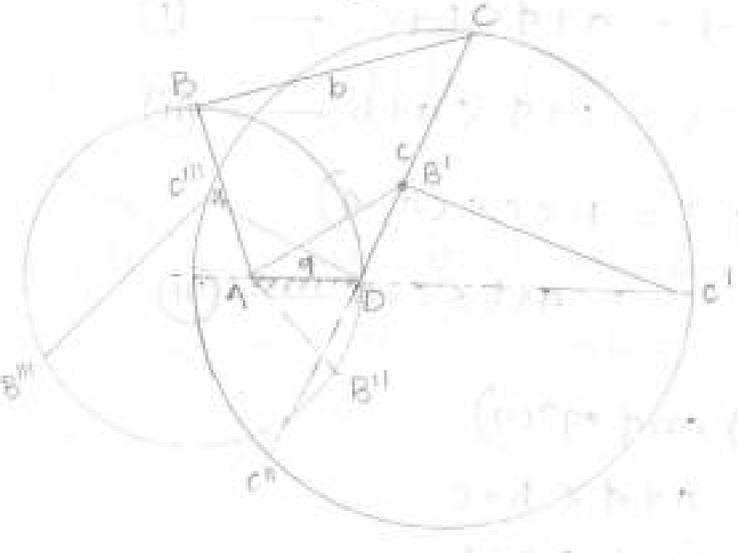
$2d + (\cancel{a+b}) < 2c + (\cancel{a+b})$

$\Rightarrow 2d < 2c$

$\Rightarrow d < c$.

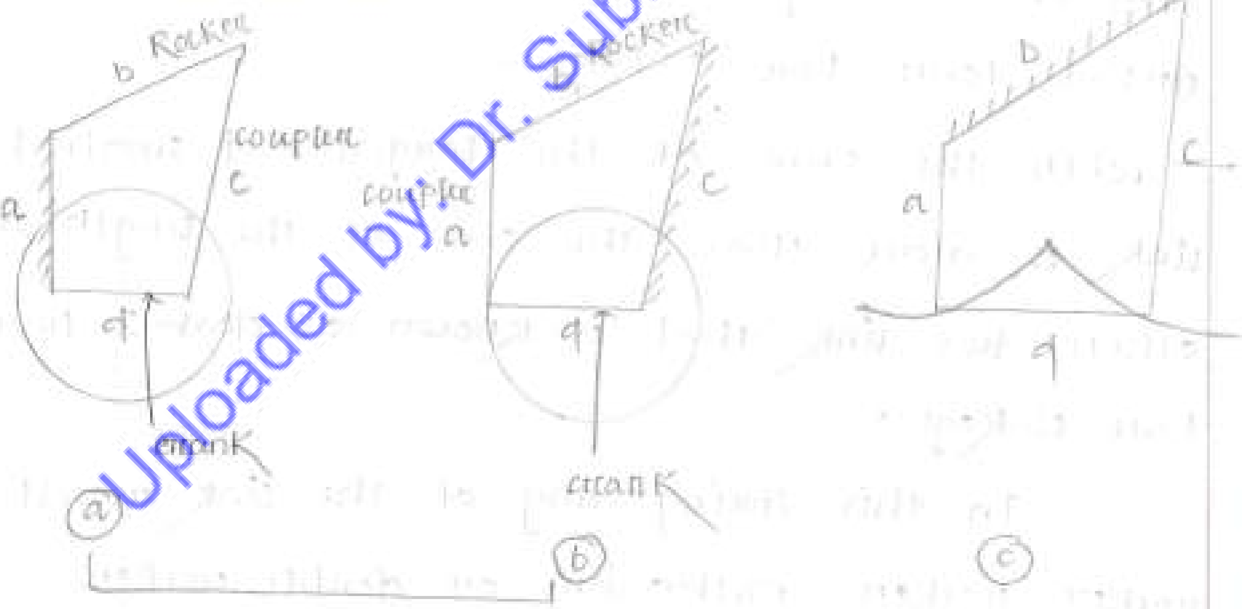
This justifies that the d is the shorted link is less than a, b and c length.

— The shortest link is fixed, the sum of the shortest and longest linked is less than the sum of the other two links.



If 'c' is rotate through a full circle is if it to be a crank then the condition to be realised are the same as above. Also it can be shown that if both the links A and B rotates through full circle then the link 'b' also makes one complete rotation relative to the fixed linked 'd' the mechanism is known as crank - crank or doubled crank or darag crank, mechanism or rotery rotary converter.

The relative motion between two adjacent link remains the same irrespective of which link it is fixed to the frame. The different mechanism (inversions) obtained by fixing different links of this kind of chain will be as followed.



crank and rocker mechanism

— If any of the adjacent link to 'd' i.e. 'a' or 'c' is fixed 'd' can have a full revolution (crank) and the link oposite to it where as in figure (b) 'c' is fixed and the 'd' is the crank and 'b' oscillates this mechanism is known as crank rocker or crank lever mechanism or rotary oscillating converter.

— If the link opposite to shortest link that is link 'b' is fixed and the shortest link is made a coupler, the other two link 'a' and 'c' would oscillate the mechanism is known as Rocker - Rocker or double rocker, or double lever mechanism or oscillating oscillatory converter.

$c+d > a+b$ (link can't be rotate case - II)

$c+d < a+b$ (one link is rotating case - I)

— In a linkage in which the sum of the lengths of longest and shortest length is less than the sum of the length of other two link is known as class - 1 four bar linkage.

— when the sum of the largest and smallest link is more than the sum of the length of other two link that is known as class - 2 four bar linkage.

In this fixing any of the link result is rocker - rocker mechanism or double rocker mechanism.

So this mechanism is also known as Grashof's law.

Grashof's law —

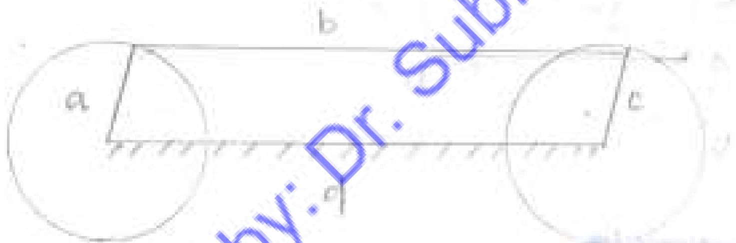
It states that a four bar mechanism has atleast one revolving link if the sum of the largest and smallest link is less than the sum of the length of other two links.

— If the sum of the length of largest and shortest link is equal to the length of the sum of other two links, the four bar inversion result in mechanism similar to those a given by Grashof's law. In this links become collinear and may have to be guided in proper directions.

EX - parallel crank four bar linkage

Deltoid linkage

Parallel crank four bar linkage —

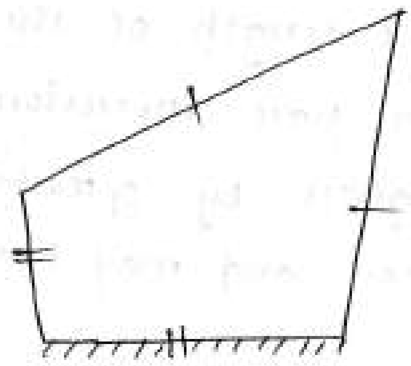


— If in a four bar linkage two opposite links are parallel and equal in length then any of the link can be made fixed.

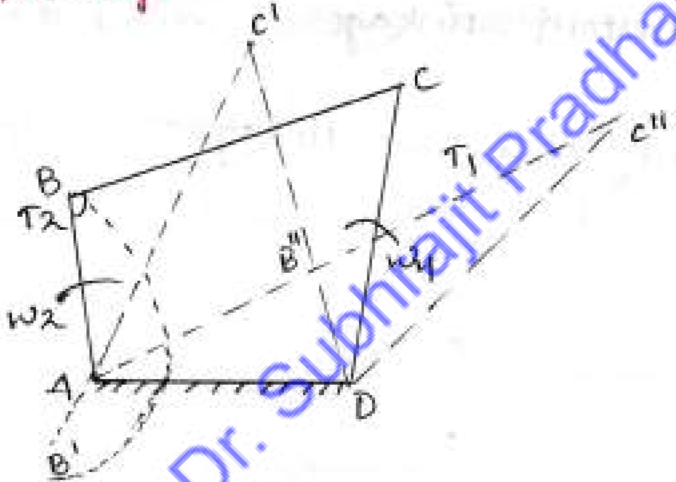
— The two link adjasent to the fixed linked will always act as two cranks, the four links form a parallel to gram in all position of the crank provided the crank rotates.

Deltoid linkage —

In this the equal linkes are adjouent to each other when any of the shorter link is fixed a double crank mechanism is obtained in which one revolution of longer link causes two revolution of shorter link.



Mechanical Advantage –



$$\text{Power} = 2\pi N$$

$$w = \frac{2\pi N}{60}$$

The mechanical advantage of a mechanism is the ratio of the output force or torque to input force on torque at any instant. If the friction and inertian force are ignore and the input torque is applied to the link to drive the output link with a resisting torque then

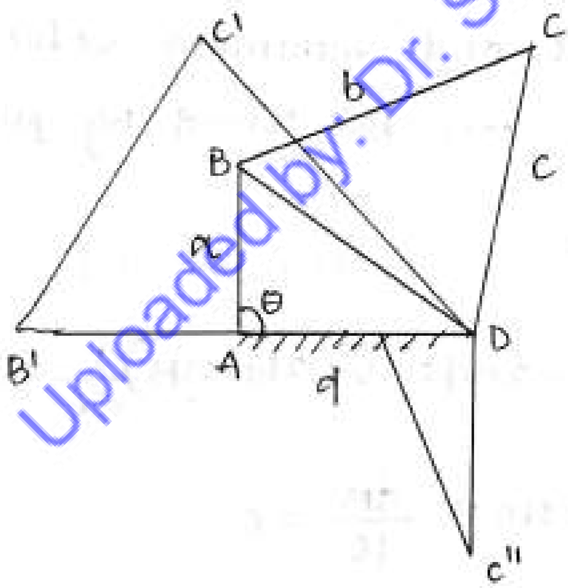
power input = power output

$\Rightarrow T_2 \omega_2 = T_4 \omega_4$

$\Rightarrow \dfrac{T_4}{T_2} = \dfrac{\omega_4}{\omega_2}$ (mechanical advantage)

Incase of crank and rocker mechanism the velocity ω_4 of the output link DC (rocker) becomes zero at extrem position AB'C'D and AB"C"D that is when the input link AB is in line with the coupler BC the angle 'ν' between them is either zero or 180°. It makes the mechanical advantages to be infinite at such position a small input torque can over come a large input load, the extrem position of the linkage are known as <u>toggle position</u>.

Transmission Angle—



The angle is between the output link and the coupler is known as transmission angle.

If the link AB is the input link is the forced applied to the output link DC. transmission through the couple BC the particular value of the force in the coupler rod the torque transmitted to the output link about 'd' is maximum when the angle μ is 90° and when the angle between BC and DC is zero the mechanism is blocked or jammed. when 'μ' deviates from 90° the torque output decreases so μ is kept more than 45°.

Applying cosine law to triangle ABD and ΔBCD.

$$a^2+d^2-2ad\cos\theta = K^2 \quad \text{—— (I)}$$

$$b^2+c^2-2bc\cos\mu = K^2 \quad \text{—— (II)}$$

Equation are equating the two eqⁿ(I) & eqⁿ(II)

$$a^2+d^2-2ad\cos\theta = b^2+c^2-2bc\cos\mu = K^2$$

$$\Rightarrow a^2+d^2-b^2-c^2-2ad\cos\theta+2bc\cos\mu=0$$

For the maximum value and minimum value of the transmission angle can be found by putting

$$\frac{d\mu}{d\theta}=0$$

$$\Rightarrow (a^2+d^2-b^2-c^2-2ad\cos\theta+2bc\cos\mu)\frac{1}{d\theta}=0$$

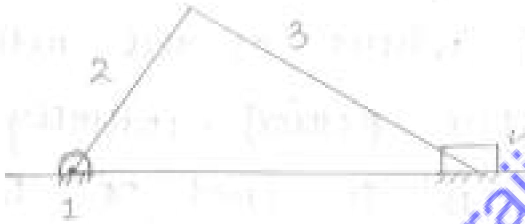
$$\Rightarrow 2ad\sin\theta - 2bc\sin\mu\frac{d\mu}{d\theta}=0$$

$$\Rightarrow \boxed{\frac{d\mu}{d\theta} = \frac{ad\sin\theta}{bc\sin\mu}}$$

Inversion of slider crank mechanism –

When one of the turning pair of a four bar chain is replaced by a sliding pair. It's given a single slider crank chain or simple slider crank chain. It is possible to replace two sliding pair of a four bar chain to gate a double slider crank chain. Different mechanism often by fixing different link of a kinematic chain are known as inversion.

1st Inversion –

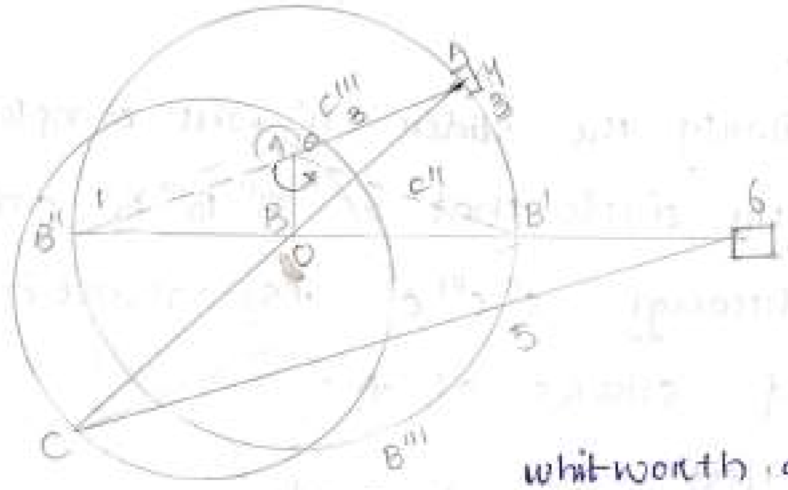


This inversion is obtain, when ~~#~~ link '1' is fixed and link '2' and '4' made the crank and slider respectively.

Application –

- Reciprocating engine
- Reciprocating pump

2nd Inversion –



Whitworth quick return mechanism.

fixing of the link 1 of the slider crank chain results in the 2nd inversion. Here 'AB' become the crank, this makes the links 1 to rotate about 'O'.

Application-

- whitworth quick return mechanism.
- Rotary engine.

Whitworth quick return mechanism—

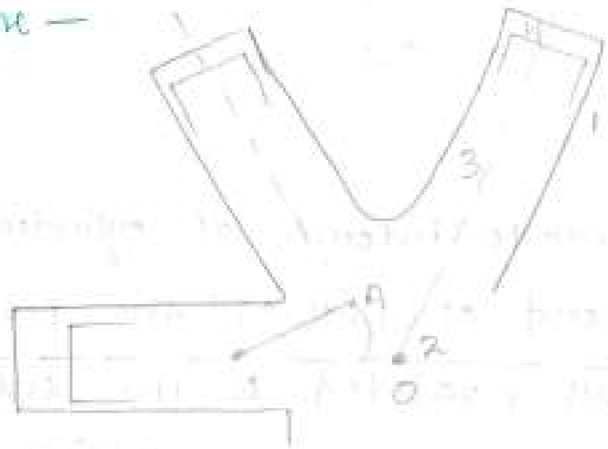
It is a mechanism used in the workshop to cut metal. The forward stroke take little longer and cut the metal where as the return stroke is idle takes shorter period. initially late the slider 4 be B at B'' so that 'C' be at C'. This cutting tool 'b' will be of the extreme less position with the moment of the crank the slider transverse the path B' B B'' where as pt. 'C'. moves from C'C'' position. The cutting tool 'b' will have the forward stroke. Finally the slider 'B' assumes the position B' and the cutting tool 'b' is in the extreme right position.

Similarly the slider '4' will complete the rest of the circle that is B'' B''' B' and C passes through C'' C''' C'. So there is a backward stroke at A.

Here.
θ = output angle
β = acute angle from B''AB'.

$$\text{Quick return ratio} = \frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\alpha}{\beta}$$

Rotary Engine –



It can be observed that will the rotation of link '3' the link '1' rotate about the centre 'O' and the slider '4' respocate on it. This also implise that if the slider is made to reciprocate to link '1' the crank '3' will rotate about 'A' and the link about 'O'. The slider's replaced by piston and the link one by cylinder privated at 'O'. Moreover instead of one cylinder, seven cylinder or nine cylinder symmetrical placed at regular same plain or in parallel plane.

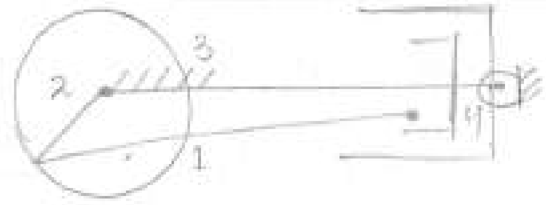
3rd inversion –

By fixing the link '3' of the slider crank chain the 3rd inversion is obtain, the link '2' is the crank and the link '4' oscillates.

Application –

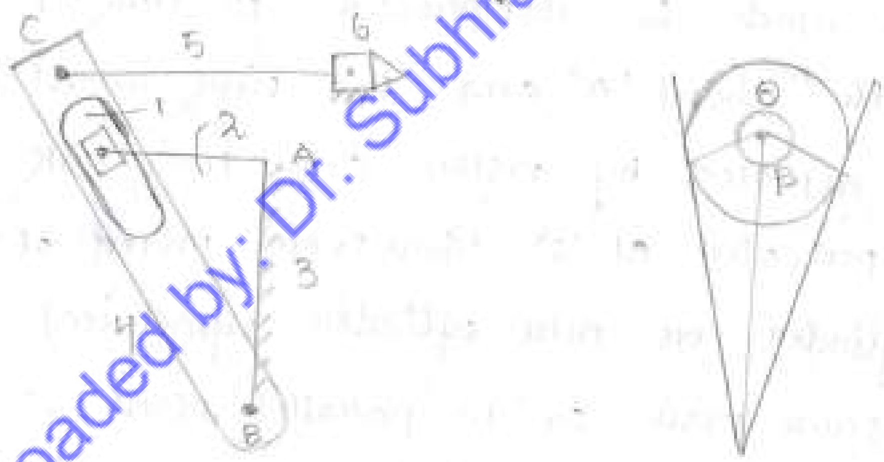
- oscillatory cylinder engine.
- crank and slotted leaver mechanism.

Oscillatory cylinder Engine –



The link '4' is made inform of cylinder and piston is fixed to the end of link '1' the piston reciproca inside the cylinder pivatated to the fixed link '3' the arrangement is known as oscillating cylinder engine. In which the piston reciprocates in the oscillating cylinder and the crank rotates.

- Crank and slotted mechanism –

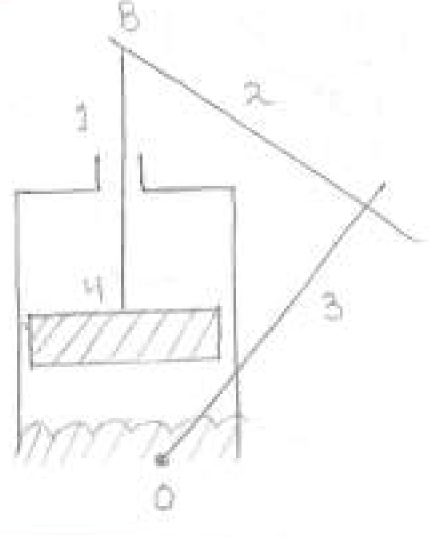


If the cylinder of an oscillatory engine is made inform of guide and the piston in form of slider the arrangement is often as crank and slotted lever mechanism the time of forward stroke is proposnal to angle 'θ' where as for the return stoke proposal to β provided the crank rotate in clockwise.

The comparission of wheat-whorth mechanism to crank and slotted lever mechanism :

- The crank 3 of wheat whorth mechanism is longer than its fixed link "2" where as the crank '2' is of slotted lever mechanism is shorter than its the fixed link '3'.
- The coupler of link one of wheat whorth mechanism makes a complete rotation where as the coupler of link '4' in slotted lever mechanism oscillate about the pivot point 'B'.
- The coupler link holding the tool can be pivated to the main coupler link at any convinate point 'c' in both cases. However for the same displacement of the tool it is more convinate if the pt 'c' is taken ~~extension~~ external of the main coupler link in case of wheat ~~woor~~ whorth mechanism and beyond the extrem position of the slider in the slotted lever mechanism.

Fourth Inversion –



If the link '4' of a slider crank mechanism is fixed the fourth inversion is obtained. The link '3' can be oscillate about the fixed pivot pt 'B' and end 'O' to reciprocated along the axis of the fixed link '4'.

Application –

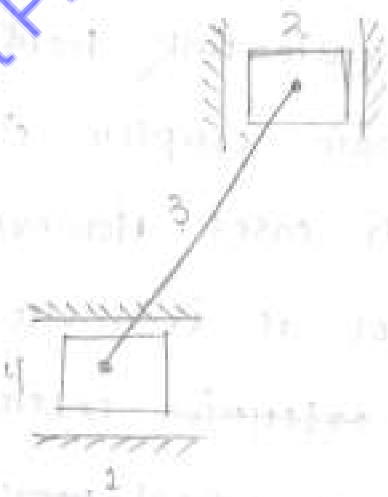
Hand pump –

The link '4' is made inform of cylinder and plunger is fixed to the link reciprocate in it.

Double slider crank chain –

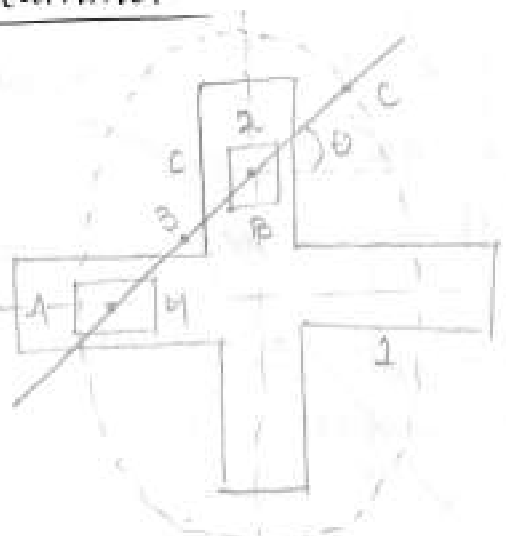
1. First Inversion –

This inversion is obtained when link one is fixed and two adjacent pair 2,3 and 3,4 are turning points and the other two pair 1,2 and 4,1 are sliding pair or prisimatic pair.



Application –

• Elliptical Trammel –



The link '1' is in the form in which the fixed link are is in the form of gas for slider '2' and '4'. with the movement of the slider any point 'c' on the link '3' except the midpoint of AB will an ellipse on a fixed plain. The mid point of AB will a circle.

Let at any instant the link '3' makes angle with x-axis. considering the displacement of the slider from the centre of the triangle.

$$x = BC\cos\theta$$

$$\Rightarrow \cos\theta = \frac{x}{BC}$$

$$y = AC\sin\theta$$

$$\Rightarrow \sin\theta = \frac{y}{AC}$$

$$\frac{x^2}{BC^2} + \frac{y^2}{AC^2} = \cos^2\theta + \sin^2\theta = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The equation suggest the above equation is a equation of a ellipse.

If 'c' is the midpoint in such condition

$$AC = BC$$

$$\frac{x^2}{BC^2} + \frac{y^2}{AC^2} = 1$$

$$\Rightarrow \frac{x^2}{AC^2} + \frac{y^2}{AC^2} = 1$$

$$\Rightarrow x^2 + y^2 = AC^2.$$

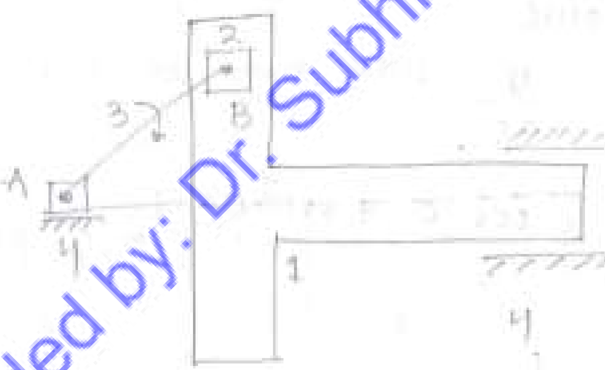
The equation suggest the above equation is a equation of a circle .

2. Second inversion –

If any of the slide blocks of 1st inversion is fixed . The 2nd inversion of double slider crank chain is obtained . when the link '4' is fixed the end 'B' of the crank '3' rotates about 'A' and the link 1 reciprocates in horizontal direction its application is scotch - yoke

Application –

- Scotch - yoke mechanism –



It is used to convert the rotary motion into sliding motion as the link '3' rotates . The horizontal portion of link '1' slides or reciprocate in the fixed link '4'.

Application –

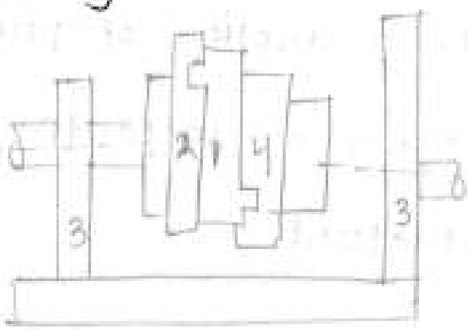
- Valve actuates in high pressure oil and gas pipe line .

3. Third Inversion -

This inversion is obtained when the link '3' of the 1st inversion is fixed and the link '1' is free to move.

Application

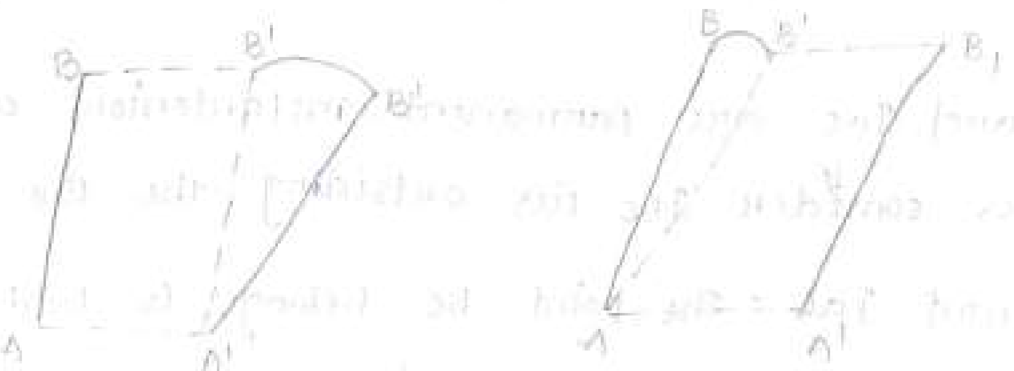
- Oldham's coupling -



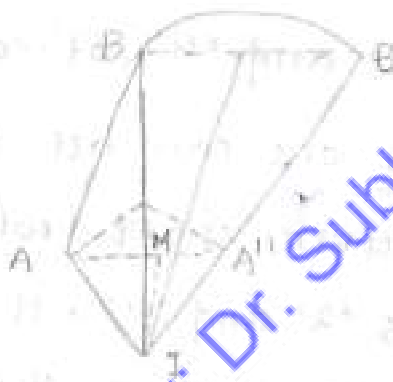
If a rotating link '2' and '4' of a mechanism are replaced by two shafts one can act as the driver and the other as driven shaft with their axes at the pivot the links '2' and '4'. It is used to connect 2 parallel shafts. when the distance between them is very small.

VELOCITY MECHANISM (Instanteneous centre method) -

Some times a body has simultaneously a motion of rotation as well as translation such as wheel of a car, a sphere.

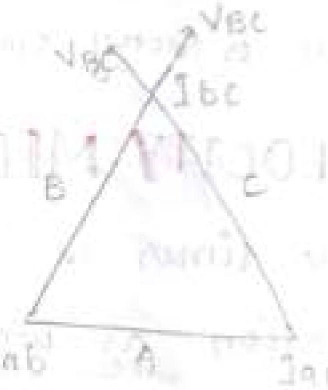


rolling on the ground without sleeping such motion will have combined effect of relation and translession the motion of the link AB. So, gradual that it is difficult to see the two separate motions but 'B' moves faster than 'A' thus combined motion of rotation's translation of the link 'AB' may be assume to be a motion of pure rotation about the centre 'I' known as instantanious centre of rotation or centro/virtualcent.



Kennedy Theorem -

It states that if 3 links move relative to each other they have '3' instanteeial centre lies on a straight line.

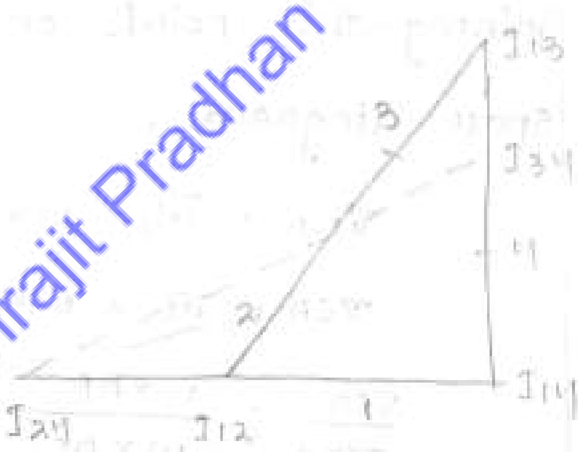


$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

Iab and Iac are parmanent instantenious centre. Let us consider Ibc lies outsiding the line joining Iab and Iac. The point bc belongs to both link b and c.

If we consider I_{bc} on the link b its velocity V_{bc} must be perpendicular to the line joining I_{ab} and I_{bc}. Now consider I_{bc} on the link 'c'. If velocity V_{bc} must be perpendicular to the line joining I_{ac} and I_{bc}. Therefore the velocity of point I_{bc} ~~am~~ can't be ~~In~~ perpendicular to both I_{ab}, I_{bc} and I_{ac}, I_{bc} unless the point I_{bc} lies on the line joining the point I_{ab} and I_{ac}.

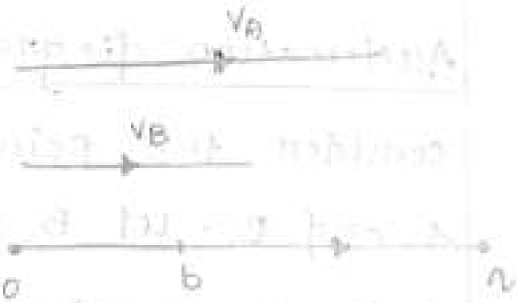
123	234	143	214
12	23	14	12
13	24	13	14
23	34	34	24



Graphical Analysis of velocity and Acceleration –

Relative velocity method –

V_{AB}



V_A magnitude represented by $\overline{oa}$

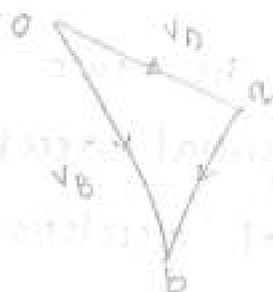
V_B magnitude represented by $\overline{ob}$

V_{AB} – velocity of 'A' w.r.t B = $V_A - V_B$

$\overline{ba} = \overline{oa} - \overline{ob}$

V_{BA} – velocity of 'B' w.r.t A = $V_B - V_A$

$\overline{ob} = \overline{ob} - \overline{oa}$



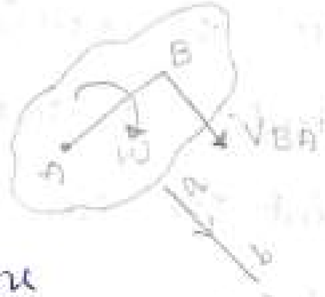
$V_{BA} = V_B - V_A$

$V_{AB} = V_A - V_B$

$V_{BA} = - V_{AB}$

Motion of a line–

The velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining. This points on the configuration on space diagram.



$$V_{BA} = \overline{ab} = \omega \times AB$$

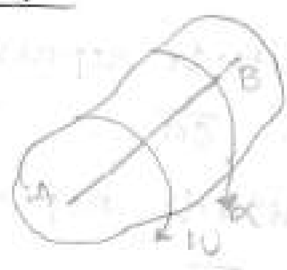
$$V_{CA} = \overline{ac} = \omega \times AC$$

$$\frac{\overline{V_{CA}}}{V_{BA}} = \frac{\omega \times AB}{\omega \times AC} = \frac{AC}{AB}$$

$$\Rightarrow \frac{\overline{ac}}{\overline{ab}} = \frac{AC}{AB}$$

Acceleration diagram for a link–

Consider two point of a link A and B. Let B moves w.r.t A with an angular velocity ω rad/sec and α rad/sec² be the angular acceleration of the link A and B. So is there will be two acceleration present.



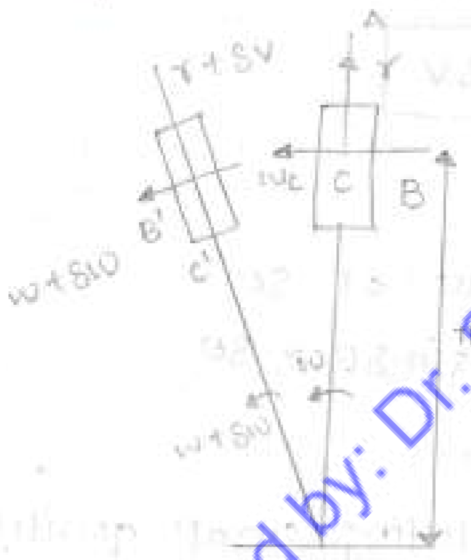
- tangential acceleration
- Radial acceleration.

$a^r ba$ - Radial acceleration of B w.r.t A.

$a^t ba$ - Tangential acceleration of B w.r.t A.

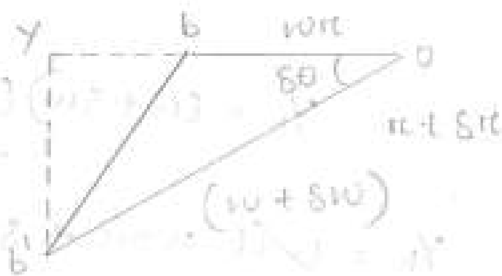
Accl^n	Magnitude	Direction	sense
$a^r ba$	$\omega^2 \times AB = \frac{vba^2}{AB}$	Parallel to AB	B to A
$a^t ba$	$\alpha \times AB = ba$	cw	⊥ to AB.

Coriolis component of acceleration -



Acceleration

Velocity



When the point on one link is sliding ~~of~~ along another rotating link such as the in quick-return motion mechanism then the Coriolis component acceleration must be calculated.

ω - Angular velocity OA

v - Velocity of the slider B along the link OA at the time t sec.

ωr - Velocity of the slider B w.r.t O at the time t sec.

$bx = (v + \delta v)\cos\delta\theta - v$

For small value of $\delta\theta$

$\cos\delta\theta \simeq 1$

$$\boxed{bx = \cancel{v} + \delta v - \cancel{v} = \delta v}$$

$xb_1 = (r + \delta v)\sin\delta\theta$

For small value of $\delta\theta$

$\sin\delta\theta \simeq \delta\theta$

$= (r + \delta v)\delta\theta$

$xb_1 = r\delta\theta + \delta v\,\delta\theta$ (Neglecting small quantity $\delta v\,\delta\theta \approx 0$)

$$\boxed{\overleftarrow{xb_1} = rv\,\delta\theta}$$

Left slide $\rightarrow$ $y_b = (\omega + \delta\omega)(r + \delta r)\cos\delta\theta - \omega r$

$(\cos\delta\theta \approx 1)$

$y_b = \cancel{\omega r} + \omega\delta r + \delta\omega r + \delta\omega\delta r - \cancel{\omega r}$

$$\boxed{\overrightarrow{yb} = \omega\delta r + \delta\omega r}$$

$$yb' = (\omega + \delta\omega)(r + \delta r)\sin\delta\theta \quad (\sin\delta\theta \approx \delta\theta)$$

$$= (\omega r + r\delta\omega + \omega\,\delta r + \delta r - \delta\omega)\,\delta\theta$$

$$= \omega r\,\delta\theta + r\,\delta\omega\,\delta\theta + \omega\,\delta r\,\delta\theta + \delta r\,\delta\omega\,\delta\theta$$

(Neglecting small quantity)

$$\boxed{\downarrow yb' = \omega r\,\delta\theta}$$

<u>Change the velocity in radial direction –</u>

$$\boxed{\uparrow bx - b'y = \delta v - \omega r\,\delta\theta}$$

<u>Change in velocity in tangential direction –</u>

$$\boxed{\overleftarrow{b_1x + by} = v\,\delta\theta + r\,\delta\omega + \omega\,\delta r}$$

Radial component of acceleration of the slider B w.r.t O on the link or acting radially outward.

$$a^r_{BO} = \lim_{\delta t \to 0} \frac{\delta v - \omega r\,\delta\theta}{\delta t}$$

$$= \frac{dv}{dt} - \omega r \frac{d\theta}{dt} \rightarrow \omega$$

$$\boxed{a^r_{BO} = \frac{dv}{dt} - \omega^2 r}$$

<u>Tangential acceleration –</u>

$$a^t_{BO} = \lim_{\delta t \to 0} \frac{v\,\delta\theta + r\,\delta\omega + \omega\,\delta r}{\delta t}$$

$$= v\frac{d\theta}{dt} + r\frac{d\omega}{dt} + \omega\frac{dr}{dt}$$

$$= v\omega + r\alpha + \omega v$$

$$\boxed{\overleftarrow{a^t_{BO}} = 2v\omega + \alpha r}$$

Radial component of acceleration $a^r_{c_0} = \downarrow \omega^2 r$

Tangential component of acceleration

$$a^t_{co} = \overleftarrow{\alpha r}$$

Radial component of slider B w.r.t C.

$$\uparrow a^r_{BC} = a_{BO} - a_{CO}$$

$$= \frac{dv}{dt} - \omega^2 r - (-\omega^2 r)$$

$$= \uparrow \frac{dv}{dt}$$

Tangential component of the slider B w.r.t C.

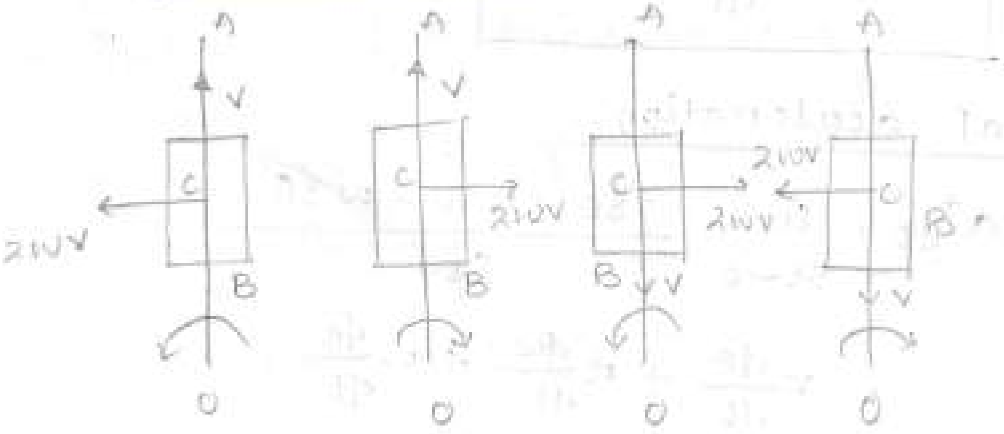
$$a^t_{BC} = a_{BO} - a_{CO}$$

$$= 2v\omega + \cancel{\alpha r} - \cancel{\alpha r}$$

$$\boxed{a^t_{BC} = 2v\omega}$$

The tangential component acceleration of slider B wrt to C on the link is known as coriolis component of acceleration and always $\perp$ar to the link

$$\boxed{a^c_{BC} = a^t_{BC} = 2\omega v}$$



Mechanism Synthesis -

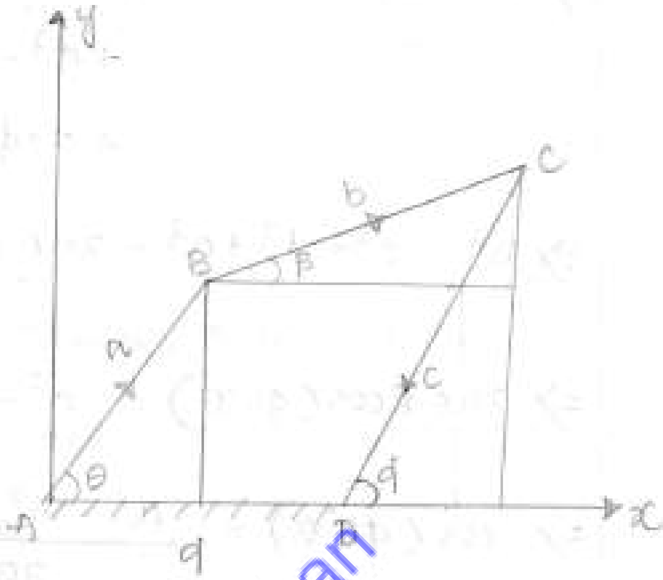
Freudenstein's Equation-

$AB = a$

$BC = b$

$CD = c$

$AD = d$



For the equilibrium of mechanism the sum of component along x-axis and y-axis must be zero.

Displacement Analysis -

Taking summesation along x-axis

$$\Sigma x = 0$$

$$a\cos\theta + b\cos\beta = c\cos\phi + d$$

$$\Rightarrow b\cos\beta = c\cos\phi + d - a\cos\theta$$

squaring both the side

$$\Rightarrow b^2\cos^2\beta = c^2\cos^2\phi + d^2 + a^2\cos^2\theta + 2cd\cos\phi - 2ad\cos\theta - 2ac\cos\theta\cos\theta \quad \text{--- (i)}$$

Taking summesation along y-axis -

$$\Sigma y = 0$$

$$a\sin\theta + b\sin\beta = c\sin\theta$$

$$b\sin\beta = c\sin\theta - a\sin\theta$$

squaring both the side

$$b^2\sin^2\beta = c^2\sin^2\theta + a^2\sin^2\theta - 2ac\sin\theta\sin\theta \quad \text{--- (ii)}$$

so adding eq^n (I) and eq^n (II) we get

$$\Rightarrow b^2(\cos^2\beta + \sin^2\beta) = c^2(\sin^2\phi + \cos^2\phi) + a^2(\sin^2\theta + \cos^2\theta)$$
$$+ d^2 - 2ac(\cos\phi \cdot \cos + \sin\phi \sin\theta)$$
$$+ 2cd\cos\phi - 2ad\cos\theta$$

$$\Rightarrow b^2 = c^2 + d^2 + a^2 - 2ac\cos(\theta - \phi) + 2cd\cos\phi - 2ad\cos\theta$$

$$\Rightarrow 2ac\cos(\phi - \theta) = a^2 - b^2 + c^2 + d^2 + 2cd\cos\phi - 2ad\cos\theta$$

$$\Rightarrow \cos(\phi - \theta) = \frac{a^2 - b^2 + c^2 + d^2}{2ac} + \frac{d}{a}\cos\phi - \frac{d}{c}\cos\theta$$

$$\frac{d}{a} = K, \quad \frac{d}{c} = K_2, \quad \frac{a^2 - b^2 + c^2 + d^2}{2ac} = K_3.$$

$$\Rightarrow \boxed{\cos(\phi - \theta) = K_1\cos\phi - K_2\cos\theta + K_3} \longrightarrow \text{(III)}$$

Equation (3) is called freuden stein eq^n. since it is difficult to find the value of ϕ.

so,
$$\sin\phi = \frac{2\tan\phi/2}{1 + \tan^2\phi/2}, \quad \cos\phi = \frac{1 - \tan^2\phi/2}{1 + \tan^2\phi/2}.$$

$$\cos\phi \cdot \cos\theta + \sin\phi\sin\theta = K_1\cos\phi - K_2\cos\theta + K_3.$$

$$\Rightarrow \frac{1 - \tan^2\theta/2}{1 + \tan^2\theta/2} \cdot \cos\theta + \frac{2\tan\theta/2}{1 + \tan^2\theta/2}\sin\theta =$$
$$K_1\left(\frac{1 - \tan^2\theta/2}{1 + \tan^2\theta/2}\right) - K_2\cos\theta + K_3.$$

$$\Rightarrow \left(1-\tan^2 \theta/_2\right)\cos\theta + 2\tan\theta/_2 \sin\theta = K_1\left(1-\tan^2 \phi/_2\right) - K_2\cos\theta\left(1+\tan^2\phi/_2\right) + K_3\left(1+\tan^2\phi/_2\right)$$

<u>According to all this term -</u>

$$-\tan^2\phi/_2 \cos\theta + K_1\tan^2\phi/_2 + K_2\cos\theta + \tan^2\phi/_2 - K_3\tan^2\phi/_2 + 2\sin\theta\tan\phi/_2 = -\cos\theta + K_1 - K_2\cos\theta + K_3$$

$$\left[(1-K_2)\cos\theta + K_3 - K_1\right]\tan^2\phi/_2 - 2\sin\tan\phi/_2 + \left[K_1+K_3-(1+K_3)\cos\theta\right] = 0$$

$$\Rightarrow A\tan^2\phi/_2 + B\tan^2\phi/_2 + C = 0$$

where,

$$A = (1-K_2)\cos\theta + K_3 - K_1$$

$$B = -2\sin\theta$$

$$C = K_1 + K_3 - (1+K_3)\cos\theta$$

$$\boxed{\therefore \tan\phi/_2 = \frac{-B \pm \sqrt{B^2-4AC}}{2A}}$$

To determine angle 'β'. we have to resolving in x-axis and y-axis and squaring.

<u>displacement along x-axis -</u>

$$a\cos\theta + b\cos\beta = c\cos\phi + d$$

$$c\cos\phi = a\cos\theta + b\cos\beta - d$$

Squaring both sides.

$$c^2\cos^2\phi = a^2\cos^2\theta + b^2\cos^2\beta + d^2 + 2ab\cos\theta\cos\beta - 2bd\cos\beta - 2ad\cos\theta \quad \text{——} \quad (1)$$

displacement along y-axis —

$$a\sin\theta + b\sin\beta = c\sin\phi$$

squaring both side.

$$c^2\sin^2\phi = a^2\sin^2\theta + b^2\sin^2\beta + 2ab\sin\theta - \sin\beta \quad \text{—— (2)}$$

Adding eq^n(I) and eq^n(II)

$$c^2(\cos^2\phi + \sin^2\phi) = a^2(\cos^2\phi + \sin^2\theta) + b^2(\sin^2\beta + \cos^2\beta)$$
$$+ 2ab(\cos\theta\cos\beta + \sin\theta\sin\beta) - 2ab\cos\beta$$
$$- 2ad\cos\theta + d^2$$

$$\Rightarrow c^2 - a^2 - b^2 - d^2 + 2bd\cos\beta + 2ad\cos\theta = 2abc(\cos\{\theta - 1\})$$

$$\sin\beta = \frac{2\tan\beta/2}{1+\tan^2\beta/2}, \quad \cos\beta = \frac{1-\tan^2\beta/2}{1+\tan^2\beta/2}.$$

substituting these value in above eq^n

$$\frac{c^2 - a^2 - b^2 - d^2 + 2ab\cos\beta + 2ad\cos\theta}{2ab} = \cos(\theta - \beta)$$

$$\frac{c^2 - a^2 - b^2 - d^2}{2ab} = K_5, \quad \frac{d}{a} = K_1, \quad \frac{d}{b} = K_4$$

$$\cos(\theta - \beta) = K_5 + K_1\cos\beta + K_4\cos\theta$$

To determine the value of β,

$$\cos\theta\left(\frac{1-\tan^2\beta/2}{1+\tan^2\beta/2}\right) + \sin\theta\left(\frac{2\tan\beta/2}{1+\tan^2\beta/2}\right) =$$

$$K_1\left(\frac{1-\tan^2\beta/2}{1+\tan^2\beta/2}\right) + K_4\cos\theta + K_5.$$

$$= \frac{\cos\theta - \cos\theta \tan^2 \beta/2}{1+\tan^2\beta/2} + \frac{\sin\theta\, 2\tan\beta/2}{1+\tan^2\beta/2} = \frac{K_1 - K_1\tan^2\beta/2}{1+\tan^2\beta/2} + K_4\cos\theta + K_5$$

$$\Rightarrow \frac{\cos\theta - \cos\theta\tan^2\beta/2 + 2\sin\theta\tan\beta/2}{1+\tan^2\beta/2} = K_1(K_1\tan^2\beta/2) + K_4\cos\theta(1+\tan^2\beta/2) + K_5(1+\tan^2\beta/2)$$

$$\Rightarrow \tan^2\beta/2\,(K_1 - K_5 - K_4\cos\theta - \cos\theta) + 2\sin\theta\cdot\tan\beta/2 + \cos\theta - K_1A + K_5 - K_4\cos\theta = 0$$

$$\Rightarrow [K_1 - K_5 - \cos\theta(1+K_4)]\tan^2\beta/2 + 2\sin\theta\cdot\tan\beta/2 + \cos\theta - K_1 + K_5 - K_4\cos\theta = 0$$

$$\Rightarrow \tan^2\beta/2\,[K_1 - K_5 - \cos\theta(1+K_4)] + 2\sin\theta\cdot\tan\beta/2 + \cos\theta(1-K_4) - K_1 + K_5 = 0$$

$$\Rightarrow \tan^2\beta/2\,[\underbrace{-K_1 + K_5 + \cos\theta(1+K_4)}_{D}] - \underbrace{2\sin\theta}_{E}\cdot\tan\beta/2 - \underbrace{\cos\theta(1-K_4)}_{F} + \underbrace{K_1 - K_5}_{F} = 0$$

$$\tan\beta/2 = \frac{-E \pm \sqrt{E^2 - 4DF}}{2D}$$

where,

$D = (K_4+1)\cos\theta - K_1 + K_5$

$E = -2\sin\theta$

$F = (K_4-1)\cos\theta + K_1 + K_5$

Velocity Analysis –

ω_1 = Angular velocity of link AB $= \frac{d\theta}{dt}$

ω_2 = Angular velocity of link BC $= \frac{d\beta}{dt}$

ω_3 = Angular velocity of link CD $= \frac{d\phi}{dt}$

Displacement along x-axis –

$$a\cos\theta + b\cos\beta = c\cos\phi + d$$

Differentiating w.r.t time 't'

$$-a\sin\theta\frac{d\theta}{dt} - b\sin\beta\frac{d\beta}{dt} + c\sin\phi\frac{d\phi}{dt} = 0$$

$$\Rightarrow -a\sin\theta\,\omega_1 - b\sin\beta\,\omega_2 + c\sin\phi\,\omega_3 = 0 \quad \text{—— (i)}$$

Again differentiating w.r.t time 't'.

$$-a\cos\theta\frac{d^2\theta}{dt^2} - b\cos\beta\frac{d^2\beta}{dt^2} + c\sin\phi\frac{d^2\phi}{dt} = 0 \quad \text{—— (a)}$$

Displacement along y-axis –

$$a\sin\theta + b\sin\beta = c\sin\phi$$

Differentiating w.r.to time 't'.

$$a\cos\theta\frac{d\theta}{dt} + b\cos\beta\frac{d\beta}{dt} = c\sin\phi\frac{d\phi}{dt} = c$$

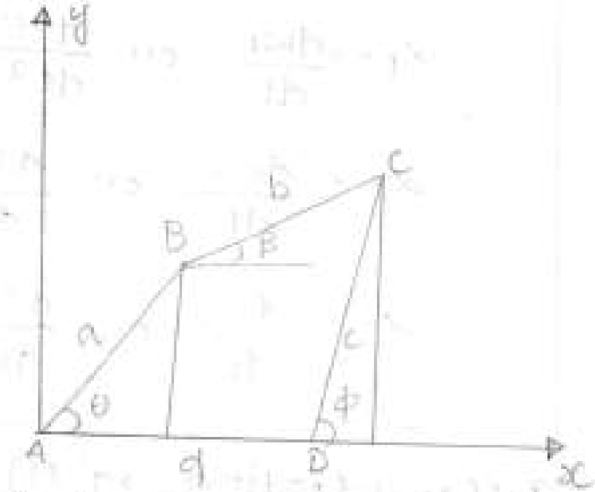
$$a\cos\theta\,\omega_1 + b\cos\beta\,\omega_2 = c\cos\phi\,\omega_3 \quad \text{—— (ii)}$$

Again differentiating w.r.to time 't'.

$$-a\sin\theta\frac{d^2\theta}{dt^2} - b\sin\beta\frac{d^2\beta}{dt} + c\sin\phi\frac{d^2\phi}{dt} = 0 \quad \text{—— (b)}$$

multiplying by $\cos\beta$ and $\sin\beta$
by both eqn (I) and (II)

~~$-a\omega_1 \sin\theta$~~



$$-a\omega_1 \sin\theta \cdot \cos\beta - b\omega \sin\beta \cos\beta + c\omega_3 \sin\phi \cos\beta = 0$$

$$a\omega_1 \cos\theta \cdot \sin\beta + b\omega_2 \cos\beta \sin\beta - c\omega_3 \cos\phi \sin\beta = 0$$

$$a\omega_1 (\cos\theta \sin\beta - \cos\beta \sin\theta) + c\omega_3 (\sin\phi \cos\beta - \cos\phi \sin\beta) = 0$$

$$c\omega_3 \sin(\phi - \beta) = -a\omega_1 \sin(\beta - \theta)$$

$$\boxed{\omega_3 = \frac{-a\omega_1 \sin(\beta - \theta)}{c \sin(\phi - \beta)}}$$

multiplying $\cos\phi$ to eqn (I) $\sin\phi$ to eqn (II)

$$-a\omega_1 \sin\theta \cdot \cos\phi - b\omega_2 \sin\beta \cos\phi + c\omega_3 \sin\phi \cos\phi = 0$$

$$-a\omega_1 \cos\theta \cdot \sin\phi + b\omega_2 \cos\beta \sin\phi - c\omega_3 \sin\phi \cos\phi = 0$$

$$a\omega_1 (\cos\theta \sin\phi - \sin\theta \cos\phi) + b\omega_2 (\cos\beta \sin\phi - \sin\beta \cos\phi) = 0$$

$$a\omega_1 [\sin(\phi - \theta)] + b\omega_2 [\sin(\phi - \beta)] = 0$$

$$b\omega_2 \sin(\phi - \beta) = -a\omega_1 \sin(\phi - \theta)$$

$$\boxed{\omega_2 = \frac{-a\omega_1 \sin(\phi - \theta)}{b \sin(\phi - \beta)}}$$

Acceleration Analysis –

$$\alpha_1 = \frac{d\omega_1}{dt} \text{ or } \frac{d^2\theta}{dt^2}$$

$$\alpha_2 = \frac{d\omega_2}{dt} \text{ or } \frac{d^2\beta}{dt^2}$$

$$\alpha_3 = \frac{d\omega_3}{dt} \text{ or } \frac{d^2\phi}{dt^2}$$

Differentiating eq^n(I) w.r.t time (t).

$$-a\left[\omega_1 \cos\theta \frac{d\phi}{dt} + \sin\theta \frac{d\omega_1}{dt}\right] - b\left[\omega_2 \cos\beta \frac{d\beta}{dt} + \sin\beta \frac{d\omega_2}{dt}\right]$$
$$+ c\left[\omega_3 \cos\phi \frac{d\phi}{dt} + \sin\phi \frac{d\omega_3}{dt}\right] = 0$$

$$\Rightarrow -a\left[\omega_1 \cos\theta\, \omega_1 + \sin\theta\, \alpha_1\right] - b\left[\omega_1 \cos\beta\, \omega_2 + \sin\beta \cdot \alpha_2\right] +$$
$$c\left[\omega_3 \cos\phi\, \omega_3 + \sin\phi\, \alpha_3\right] = 0$$

$$\Rightarrow -a\omega_1^2 \cos\theta - a\sin\theta\,\alpha_1 - b\omega_2 \cos\beta - b\sin\beta\,\alpha_2 +$$
$$c\omega_3^2 \cos\phi + c_3 \sin\phi\,\alpha_3 = 0 \quad \text{—— (III)}$$

Differentiating eq^n(II) w.r.to time.

$$a\left[\omega_1(-\sin\theta)\frac{d\theta}{dt} + \cos\theta \frac{d\omega_1}{dt}\right] + b\left[\omega_1(-\sin\beta)\frac{d\beta}{dt} + \cos\beta \frac{d\omega_2}{dt}\right] - c\left[\omega_3(-\sin\phi)\cdot\frac{d\phi}{dt} + \cos\phi \frac{d\phi\,\omega_3}{dt}\right]$$

$$\Rightarrow a\left[\omega_1(-\sin\theta)\,\omega_1 + \cos\theta\,\alpha_1\right] + b\left[\omega_2(-\sin\beta)\,\omega_2 + \cos\beta\,\alpha_2\right] +$$
$$c\left[\omega_3(-\sin\phi)\,\omega_3 + \cos\phi\,\alpha_3\right] = 0$$

$$\Rightarrow -a\omega_1^2 \sin\theta + a\cos\theta\,\alpha_1 - b\omega_2^2 \sin\beta + b\cos\beta\,\alpha_2 + c\omega_3^2 \sin\phi -$$
$$c\cos\phi\,\alpha_3 = 0 \quad \text{—— (IV)}$$

multiplying eqn (III) by $\cos\phi$ and eqn (IV) by $\sin\phi$, we get.

$$-a\omega_1^2\cos\phi\cos\theta - a\sin\theta\alpha_1\cos\phi - b\omega_2^2\cos\beta\cos\phi - b\sin\beta\alpha_2\cos\phi +$$
$$c\omega_3^2\cos^2\phi + c\alpha_3\sin\phi\cos\phi = 0$$

$$-a\omega_1^2\sin\theta\sin\phi + a\cos\theta\alpha_1\sin\phi - b\omega_2^2\sin\beta\sin\phi + b\alpha_2\cos\beta\sin\phi +$$
$$c\omega_3^2\sin^2\phi - c\alpha_3\sin\phi\cos\phi = 0$$

+

$$-a\omega_1^2(\cos\theta\cos\phi + \sin\theta\sin\phi) + a\alpha_1(\sin\phi\cos\theta - \cos\phi\sin\theta) -$$
$$b\omega_2^2(\cos\beta\cos\phi + \sin\beta\sin\phi) + c\omega_3^2(\cos^2\phi + \sin^2\phi -$$
$$b\alpha_2(\sin\phi\cos\beta - \cos\phi\sin\beta) = 0$$

$$\Rightarrow -a\omega_1^2\cos(\phi-\theta) + a\alpha_1\sin(\phi-\theta) + b\omega_2^2\cos(\phi-\beta) + b\alpha_2\sin(\theta-\beta)$$
$$+ c\omega_3^2 = 0.$$

$$\Rightarrow \boxed{\alpha = \frac{-a\alpha_1\sin(\phi-\theta) + a\omega_1^2\cos(\phi-\theta) + b\omega_2^2\cos(\theta-\beta) - c\omega_3^2}{b\sin(\phi-\beta)}}$$

multiplying eqn (III) by $\cos\beta$ and eqn (IV) by $\sin\beta$ we get.

$$-a\omega_1^2\cos\theta\cos\beta - a\sin\theta\alpha_1\cos\beta - b\omega_2^2\cos^2\beta - \cancel{b\sin\beta\alpha_2\cos\beta} +$$
$$c\omega_3^2\cos\phi\cos\beta + c_3\sin\phi\cos\beta\alpha_3 = 0$$

$$-a\omega_1^2\sin\theta\sin\beta + a\cos\theta\sin\beta\alpha_1 - b\omega_2^2\sin^2\beta + \cancel{b\cos\beta\sin\beta\alpha_2} +$$
$$c\omega_3^2\sin\phi\sin\beta - c\cos\phi\sin\beta\alpha_3 = 0$$

+

$$-a\omega_1^2\cos(\beta-\theta) + a\alpha_1\sin(\beta-\theta) = b\omega_2^2 + c\omega_3^2\cos(\phi-\beta) +$$
$$c\alpha_3\sin(\phi-\beta) = 0.$$

$$x_3 = \frac{a\omega_1^2 \cos(\beta-\theta) + b\omega_2^2 - a\alpha_1 \sin(\beta-\theta) - c\omega_3^2 \cos(\beta-\phi)}{c \sin(\phi-\beta)}$$

Synthesis of Mechanism –

The synthesis is opposite of analysis the synthesis of mechanism is the design or critaria of mechanism to produce a desire output motion for a given input motion.

In other words, the synthesis of mechanism deals with the determination of proportion of a mechanism for a given i/p or o/p motion.

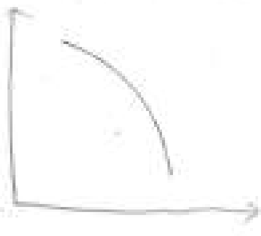
(*) It follower the step –

1. Type of synthesis i.e. the type mechanics to be used.
2. No. of synthesis i.e. the no. of links and no. of joints needed to produce the required motion.
3. Dimensional synthesis i.e. proportion of the length of link necessary.

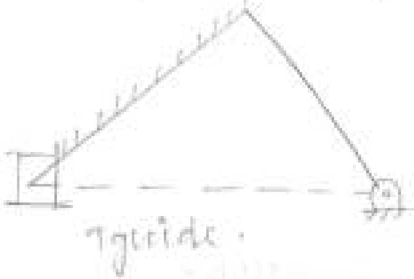
Classification of synthesis –

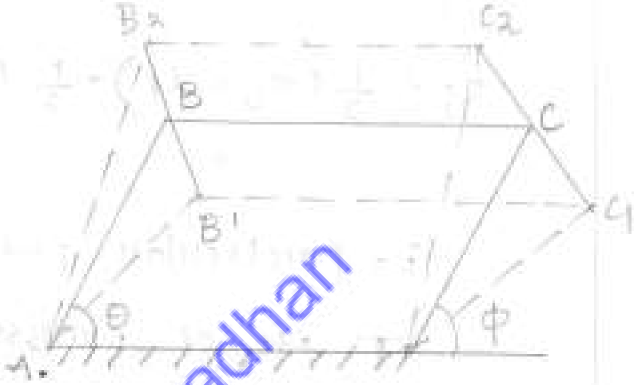
- Function generation –

$y = f(x)$.



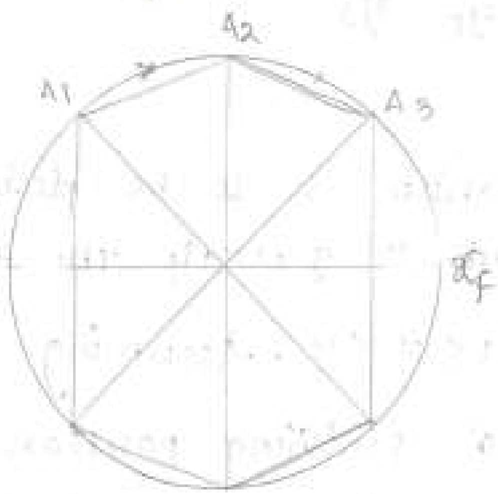
- Path generation –
  - circular motion
  - curvilinear motion –
    combination of ~~transfle~~ translation and rotation.
  - Rectilinear motion.
- Body guidance –





Precision points for function generation –

In design a mechanism to generate a particular function it is usually impossible to accquarately produce the function at more than its points. The points at which the generated and design function agree are known as precision point or acquracy point and must be located. so, as to minimize the error between this points.



The speing between the precision point is called <u>cheby chev specirc</u> - It is for the speing of 'm' points in the range

$x_s \le x \le x_f$ $\quad$ x_s = starting point

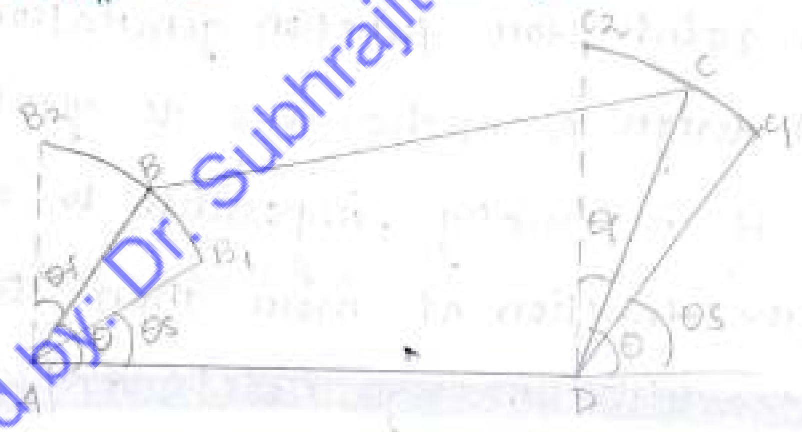
x_f = final point of the preci

$$x_j = \frac{1}{2}(x_s + x_f) - \frac{1}{2}(x_f - x_s)\cos\left[\frac{\pi(2j-1)}{2n}\right]$$

i = particular precision point

N = No. of precission point

Angle relation ship for function generation -



$$\theta = \phi_s + \frac{\theta_f - \theta_s}{x_f - x_s}(x - s_s)$$

$$\phi = \phi_s + \frac{\phi_f - \phi_s}{y_f - y_s}(y - y_s)$$

Question →

A four bar mechanism is to be design by using three precision points to generate the function $y = x^{1.5}$ for the range $1 \le x \le 4$. Assuming 30° starting position and 120°. Finishing position for the input link and 90° starting position and 180° finishing position for the out put link find the value of x, y, θ and ϕ corresponding to three precision point.

$\Rightarrow$ $\cancel{\phi_r = 30^\circ}$ $\theta_s = 30^\circ$ $\phi_s = 90^\circ$

ϕ_s $\theta_f = 120^\circ$ $\phi_f = 180^\circ$

$$\left.\begin{array}{l} y = x^{1.5} \\ 1 \le x \le 4 \end{array}\right\} \text{3point precision} \quad \begin{array}{l} x_f = 4 \\ x_s = 1 \end{array}$$

$$x_1 = \frac{1}{2}(x_s + x_f) - \frac{1}{2}(x_f - x_s)\cos\left[\frac{\pi(2j-1)}{2n}\right]$$

$$= \frac{1}{2}(1+4) - \frac{1}{2}\left(\frac{4-1}{2}\right)\cos\left[\left(\frac{2\times1-1}{2\times4}\right)\times180\right]$$

$$= 1.2$$

$$x_2 = \left(\frac{4+1}{2}\right) - \left(\frac{4-1}{2}\right)\cos\left[\frac{2\times2-1}{2\times3}\times180\right]$$

$$= 2.5$$

$$x_3 = \left(\frac{4+1}{2}\right) - \left(\frac{4-1}{2}\right)\cos\left[\left(\frac{2\times3-1}{2\times3}\times180\right)\right]$$

$$= 3.8$$

$y_1 = x^{1.5}$ $= 1.2^{1.5}$ $= 1.31$

$y_2 = x_2^{1.5}$ $= 2.5^{1.5}$ $= 3.95$

$y_3 = x_3^{1.5}$ $= 3.8^{1.5}$ $= 7.40$

$$\theta_1 = \theta_s + \frac{\theta_f - \theta_s}{x_f - x_s}(x_1 - x_s)$$

$$= 30 + \frac{120-30}{4-1}(1.2-1)$$

$$= 36^\circ$$

$$\theta_2 = 30^\circ + \frac{120-30}{4-1}(2.5-1)$$

$$= 75^\circ$$

$$\theta_3 = 30^\circ + \frac{120-30}{4\cdot 1}(3.8-1)$$

$$= 114^\circ$$

$$\phi_1 = \phi_s + \frac{\phi_f - \phi_s}{y_f - y_s}(y_1 - y_s)$$

$$\boxed{\begin{array}{l} y_s = 1 \\ y_f = 8 \end{array}}$$

$$= 90^\circ + \frac{180^\circ - 90^\circ}{8-1}(1.31-1)$$

$$= 94$$

$$\phi_2 = 90^\circ + \frac{180-90}{8-1}(3.95-1)$$

$$= 128$$

$$\phi_3 = 90^\circ + \frac{180-90}{8-1}(7.40-1)$$

$$= 172.28$$

~~Graphical synthesis of four bar mechanism~~ -