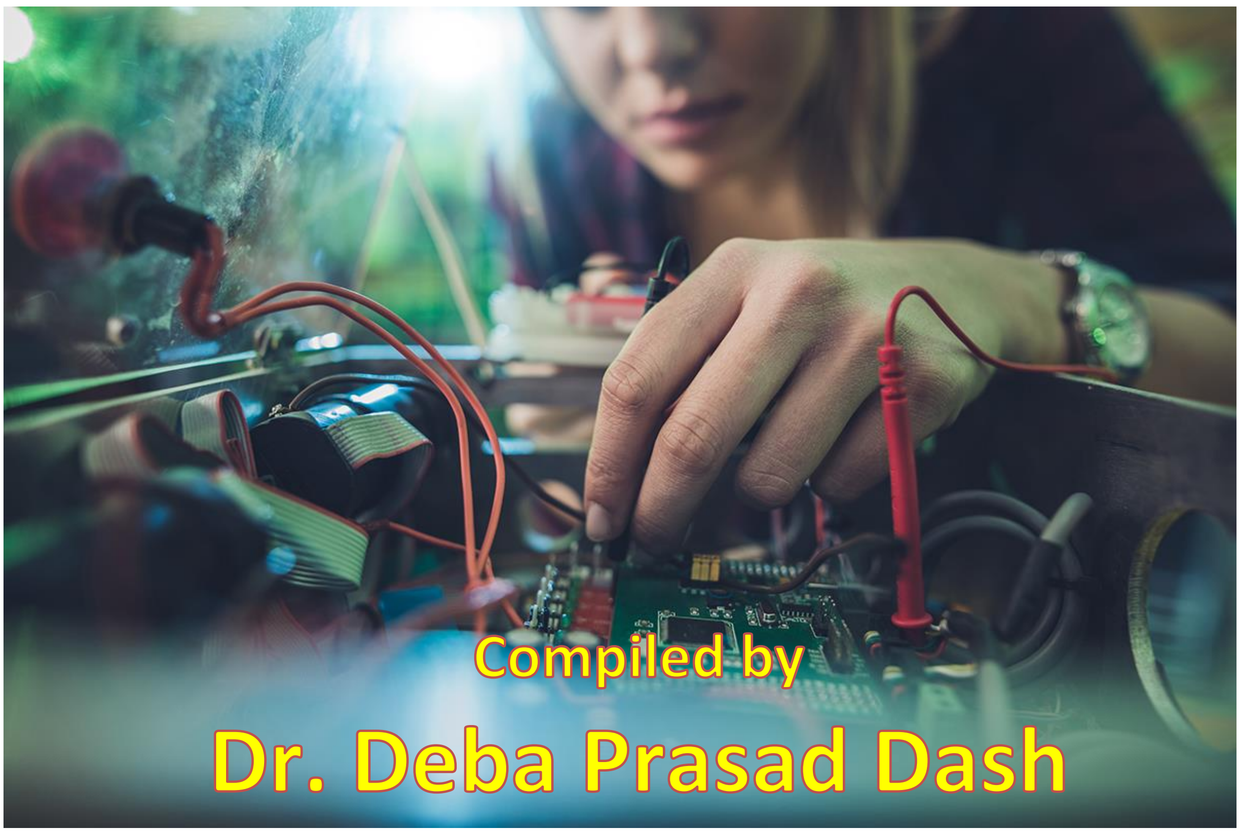
GOVERNMENT COLLEGE OF ENGINEERING, KALAHANDI

Lecture notes

on

BASIC ELECTRICAL ENGINEERING
(Module I)



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MODULE-I

1. CIRCUIT ANALYSIS:

1.1 Kirchhoff's Laws: -

1.1.1 Kirchhoff's current law or point law (KCL):

Statement: - *In any electrical network, the algebraic sum of the currents meeting at a point is zero.*

$\Sigma I = 0$ ……at a junction or node

Assumption: - Incoming current = positive

Outgoing current = negative

1.1.2. Kirchhoff's voltage law or mesh law (KVL):

Statement: - *The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the emfs in that path is zero.*

$\Sigma IR + \Sigma emf = 0$ …………………………...round the mesh

Assumption: - i) Rise in voltage (If we go from negative terminal to positive terminal) = positive

ii) Fall in voltage (If we go from positive terminal to negative terminal) = negative

iii) If we go through the resistor in the same direction as current then there is a fall in potential. Hence this voltage is taken as negative.

iv) If we go through the resistor against the direction of current then there is a rise in potential. Hence this voltage drop is taken as positive.

1.2. MAXWELL'S LOOP CURRENT METHOD (MESH ANALYSIS):

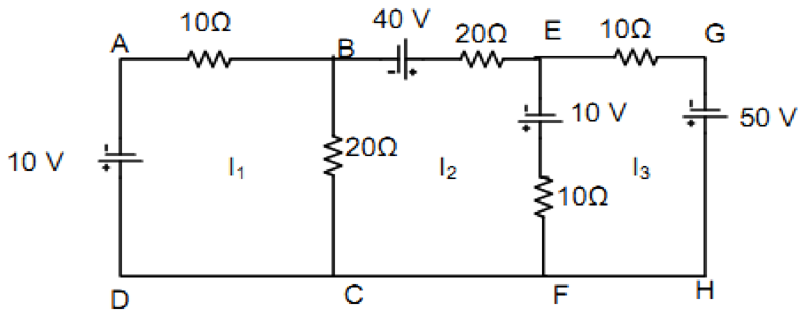
Statement: - *This method determines branch currents and voltages across the elements of a network.* The following process is followed in this method: -

• Here, instead of taking branch currents (as in Kirchoff's law) loop currents are taken which are assumed to flow in the clockwise direction.

• Branch currents can be found in terms of loop currents

• Sign conventions for the IR drops and battery emfs are the same as for Kirchoff's law.

Example: Find I_1, I_2 and I_3 in the network shown in Fig below using loop current method.



$$-I_1 \times 10 - (I_1 - I_2) \times 20 - 10 = 0$$
$$\Rightarrow \; 3I_1 - 2I_2 = -1 \qquad (1)$$

For mesh BEFCB,

$$40 - I_2 \times 20 + 10 - (I_2 - I_3) \times 10 - (I_2 - I_1) \times 20 = 0$$
$$\Rightarrow \; 2I_1 - 5I_2 + I_3 = -5 \qquad (2)$$

For mesh EGHFE,

$$-10I_3 + 50 - (I_3 - I_2) \times 10 - 10 = 0$$
$$\Rightarrow \; I_2 - 2I_3 = -4 \qquad (3)$$

Equation (2) x 2 + Equation (3)

$$4I_1 - 9I_2 = -14 \qquad (4)$$

Solving eq^n (1) & eq^n (4)

$I_1 = 1$ A, $I_2 = 2$ A, $I_3 = 3$ A

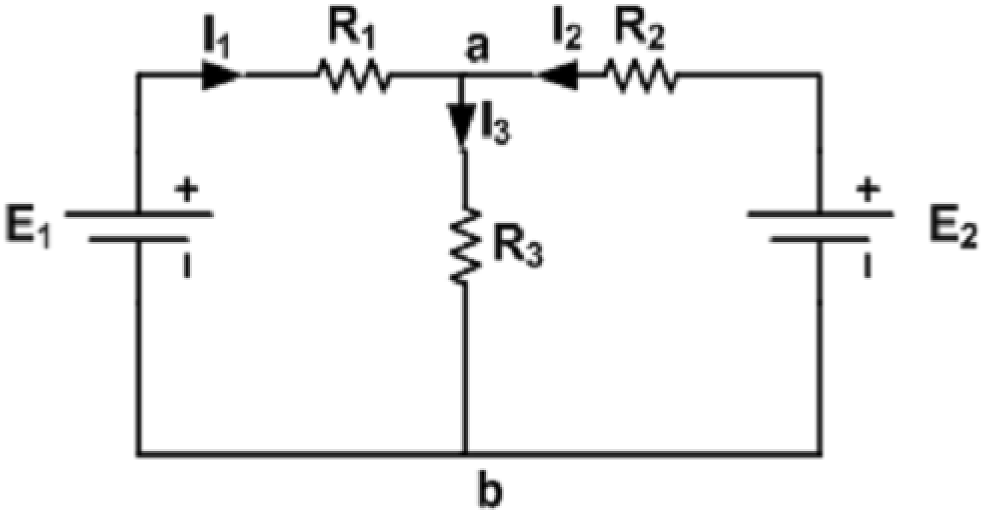
1.3. NODAL ANALYSIS:

Statement: - This method determines branch currents in the circuit and also Voltages at individual nodes.

The following steps are adopted in this method: -

- Identify all the nodes in the network.
- One of these nodes is taken as reference node in at zero potential
- The node voltages are measured w.r.t the reference node
- KCL to find current expression for each node
- This method is easier if all the current sources are present. If any voltage source is present, convert it to current source.
- The number of simultaneous equations to be solved becomes (n-1) where 'n' is the number of independent nodes.

Explanation: -



At node 'a' $I_1 + I_2 = I_3$

By ohms law, $I_1 = \frac{E_1 - V_a}{R_1}, I_2 = \frac{E_2 - V_a}{R_2}, I_3 = \frac{V_a}{R_3}$

Therefore, $$V_a\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0$$

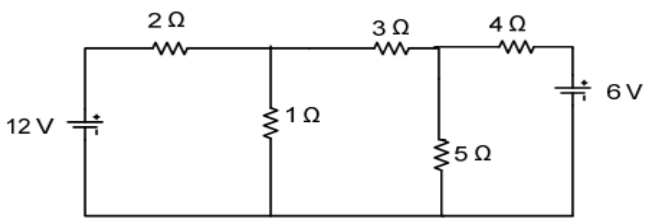
or, $$V_a\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0$$

or, $$V_a\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0$$

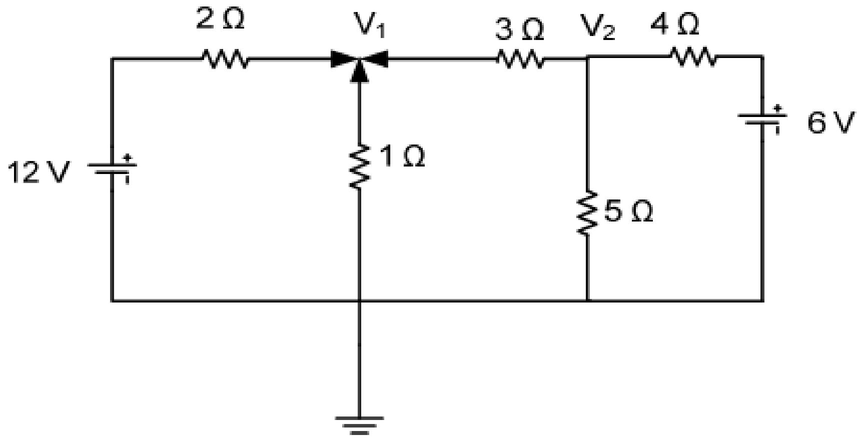
Hence,

- Node voltage multiplied by sum of the entire conductance connected to this node. This term is positive.
- The node voltage at the other end of each branch (connected to this node multiplied by conductance of this branch). This term is negative.

Example: - Use nodal analysis to find currents in the different branches of the circuit shown below.



Solution: - Let V_1 and V_2 are the voltages of two nodes as shown in Fig. below



Applying KCL to node-1, we get

$$\frac{12-V_1}{2}+\frac{0-V_1}{1}+\frac{V_2-V_1}{3}=0$$

$$\Rightarrow 36-3V_1-6V_1+2V_2-2V_1=0$$

$$\Rightarrow -11V_1+2V_2=36\ldots\ldots\ldots\ldots\ldots(1)$$

Again applying KCL to node-2, we get:-

$$\frac{V_1 - V_2}{3} + \frac{0 - V_2}{5} + \frac{6 - V_2}{4} = 0$$

$$\Rightarrow 20V_1 - 47V_2 + 90 = 0$$

$$\Rightarrow 20V_1 - 47V_2 = -90 \ldots\ldots\ldots\ldots\ldots(2)$$

Solving Eq (1) and (2) we get V_1 = 3.924 Volt and V_2 = 3.584 volt

Current through 2 Ω resistance = $\frac{12\text{-}V_1}{2} = \frac{12\text{-}3.924}{2} = 4.038$ A

Current through 1 Ω resistance = $\frac{0\text{-}V_1}{1} = -3.924$ A

Current through 3 Ω resistance = $\frac{V_1\text{-}V_2}{3} = 0.1133$ A

Current through 5 Ω resistance = $\frac{0\text{-}V_2}{5} = -0.7168$ A

Current through 4 Ω resistance = $\frac{6\text{-}V_2}{4} = 0.604$ A

As currents through 1Ω and 5Ω are negative, so actually their directions are opposite to the assumptions.

Series Circuit Resistors:

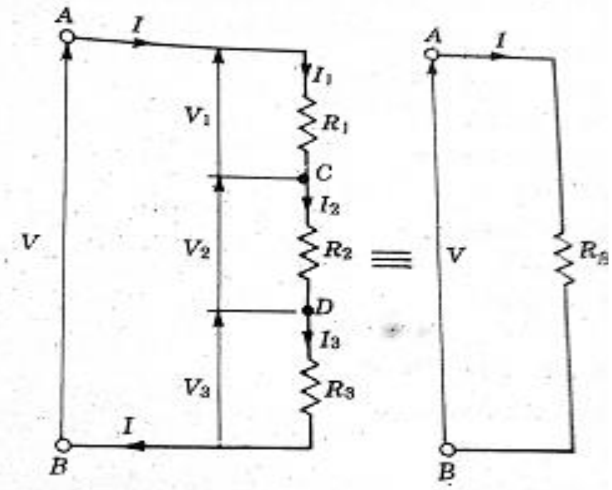


Fig. 4.13. Series circuit and its equivalent.

By KVL in mesh A – C – D – B – A

$$V_{AC} + V_{CD} + V_{DB} + V_{BA} = 0 \;\; ; \;\; -V_1 - V_2 - V_3 + V = 0$$

$$V = V_1 + V_2 + V_3$$

By Ohm's law

$$V_1 = R_1 I$$

$$V_2 = R_2 I$$

$$V_3 = R_3 I$$

From the equivalent circuit shown in Fig. 4.13 (b),

$$V = R_S I$$

Combining the Eqs. (4.7.1) to (4.7.5) we get

$$R_S I = R_1 I + R_2 I + R_3 I$$

$$R_S = R_1 + R_2 + R_3$$

Parallel Ckt. Resistors:

$V_1 = V_2 = V_3 = V$

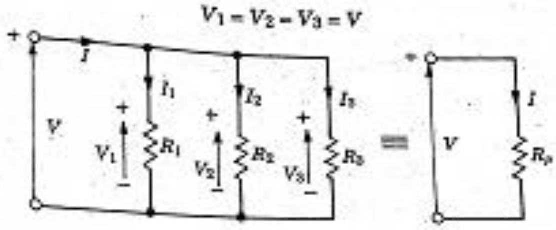


Fig. 4.17. Resistors in parallel and their equivalent.

We are interested in finding a single resistance R_p which has the same volt-ampere relation combination.

By KCL,

$$I = I_1 + I_2 + I_3$$

By Ohm's law

$$I_1 = \frac{V_1}{R_1} = \frac{V}{R_1}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V}{R_2}$$

$$I_3 = \frac{V_3}{R_3} = \frac{V}{R_3}$$

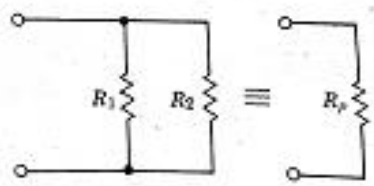
$$I = \frac{V}{R_p}$$

Substituting these values in Eq. (4.11.1)

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

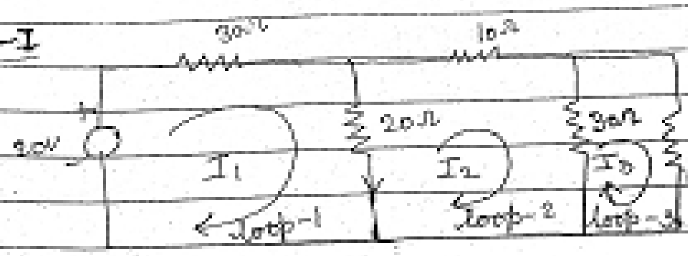
When only *two* resistors are in parallel (Fig. 4.18)



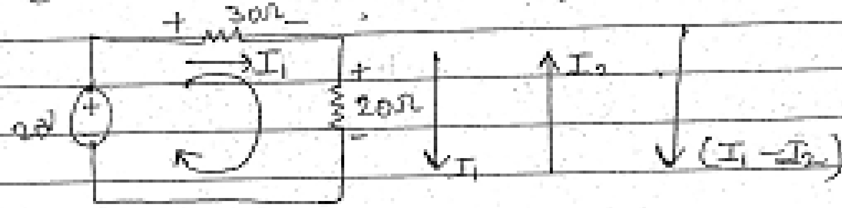
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{\text{product}}{\text{sum}}$$

Step-I



step-2 (Consider loop-1 and apply KVL)



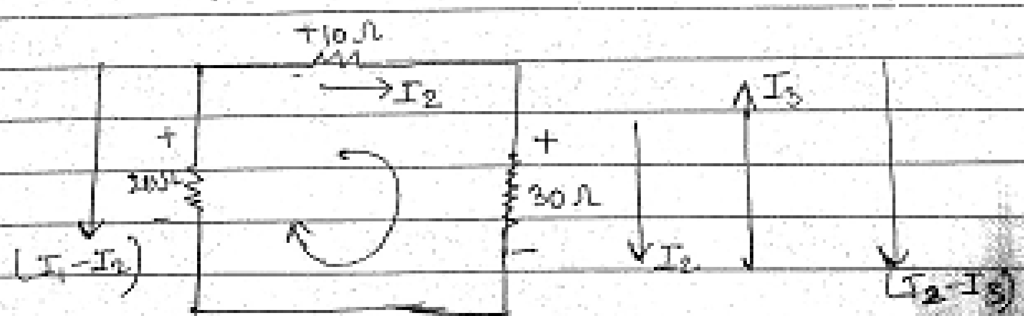
$$20 - 30I_1 - 20(I_1 - I_2) = 0$$

$$\Rightarrow 20 - 30I_1 - 20I_1 + 20I_2 = 0$$

$$\Rightarrow 50I_1 - 20I_2 = 20$$

$$\Rightarrow 50I_1 - 20I_2 + 0I_3 = 20 \quad \text{—— (1)}$$

step-3 (Consider loop-2 and apply KVL)



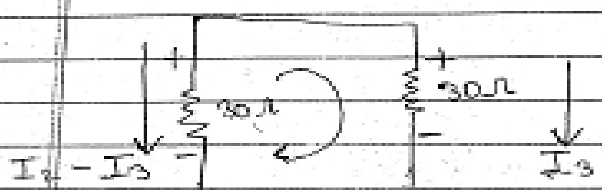
Solve the above ckt. By using KVL. And also Loop analysis method (mesh)

$20(I_1 - I_2) - 10I_2 - 30(I_2 - I_3) = 0$

$\Rightarrow 20I_1 - 20I_2 - 10I_2 - 30I_2 + 30I_3 = 0$

$\Rightarrow 20I_1 - 60I_2 + 30I_3 = 0$ —— (2)

Step-4 (Considered, loop-3 and apply KVL



$30(I_2 - I_3) - 30I_3 = 0$

$\Rightarrow 30I_2 - 60I_3 = 0$

$\Rightarrow 0I_1 + 30I_2 - 60I_3 = 0$ —— (3)

from eq (1), (2) & (3) can be written in matrix form

$50I_1 - 20I_2 + 0I_3 = 20$ —— (1)

$20I_1 - 60I_2 + 30I_3 = 0$ —— (2)

$0I_1 + 30I_2 - 60I_3 = 0$ —— (3)

$$\begin{bmatrix} 50 & -20 & 0 \\ 20 & -60 & 30 \\ 0 & 30 & -60 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 50 & -20 & 0 \\ 20 & -60 & 30 \\ 0 & 30 & -60 \end{vmatrix} = 111000$$

$$\Delta_1 = \begin{vmatrix} 20 & -20 & 0 \\ 0 & -60 & 30 \\ 0 & 30 & -60 \end{vmatrix} = 54000$$

$$\Delta_2 = \begin{vmatrix} 50 & 20 & 0 \\ 20 & 0 & 30 \\ 0 & 0 & -60 \end{vmatrix} = 24000.$$

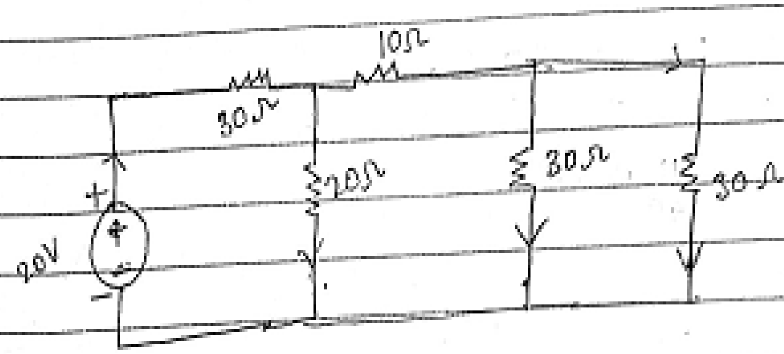
$$\Delta_3 = \begin{vmatrix} 50 & -20 & 20 \\ 20 & -60 & 0 \\ 0 & 30 & 0 \end{vmatrix} = \cancel{10{,}5000}\ 12000$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{54000}{111000} = 0.49\ \text{Amp}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{24000}{111000} = 0.22\ \text{Amp}$$

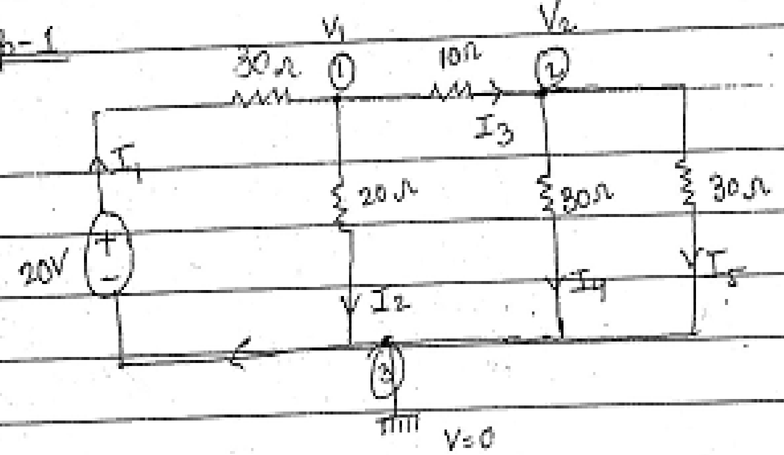
$I_3 = 0.11$ Amp

10/2/18



Find current, voltage and power in 20Ω Resistor using nodal analysis.

Step-1



Junctions → 3

voltage of earthing = 0

When source gives current it is positive when it receives it is negative

Step-2 (Consider junction-1 and apply KCL)

$$I_1 = I_2 + I_3$$

$$I_1 \Rightarrow \frac{0+20-V_1}{30} = \frac{V_1-0}{20} + \frac{V_1-V_2}{10}$$

$$\Rightarrow \frac{20}{30} - \frac{V_1}{30} = \frac{V_1}{20} + \frac{V_1}{10} - \frac{V_2}{10}$$

$$\Rightarrow V_1\left(\frac{1}{30} + \frac{1}{20} + \frac{1}{10}\right) - \frac{V_2}{10} = \frac{2}{3}$$

$$\Rightarrow V_1\left(\frac{2+3+6}{60}\right) - \frac{V_2}{10} = \frac{2}{3}$$

$$\Rightarrow \frac{11}{60}V_1 - \frac{V_2}{10} = \frac{2}{3}$$

$$\Rightarrow 11V_1 - 6V_2 = 40 \quad (1)$$

<u>step-2</u> :- (Consider junction-2 and apply KCL)

$$I_3 = I_4 + I_5$$

$$\Rightarrow \frac{V_1 - V_2}{10} = \frac{V_2 - 0}{30} + \frac{V_2 - 0}{30}$$

$$\Rightarrow \frac{V_1}{10} - \frac{V_2}{10} = \frac{V_2}{30} + \frac{V_2}{30}$$

$$\Rightarrow \frac{V_1 - V_2}{10} = \frac{2V_2}{30\,15}$$

$$\Rightarrow \frac{V_1 - V_2}{10} = \frac{V_2}{15}$$

$$\overset{3}{\cancel{15}}(V_1 - V_2) = \overset{2}{\cancel{10}}V_2$$

$$3V_1 - 3V_2 = 2V_2$$

$$3V_1 - 3V_2 - 2V_2 = 0$$

$$3V_1 - 5V_2 = 0 \quad \text{—— (2)}$$

from (1) & (2)

$$11V_1 - 6V_2 = 40 \quad \times 3$$
$$3V_1 - 5V_2 = 0 \quad \times 11$$

$$\cancel{33V_1} - 18V_2 = 120$$
$$\cancel{33V_1} - 55V_2 = 0$$
$$(-) \quad (+) \quad (-)$$

$$37V_2 = 120$$

$$V_2 = \frac{120}{37} = 3.24 \text{ volt}$$

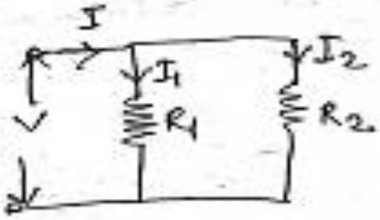
$$V_1 = 5.41 \text{ volt}$$

$$\text{so } I_2 = \frac{V_1 - 0}{20} = \frac{5.41}{20} = 0.27 \text{ Amp.}$$

$$V_{20} = I_2 \times 20 = 0.27 \times 20 = 5.41 \text{ volt}$$
$$P_{20} = V_{20} \times I_2 = 5.41 \times 0.27 = 1.46 \text{ watt}$$

Current - divider equations :→

Let us two resistances R_1 & R_2 are connected in parallel.



Again R_p = total resistance of the parallel ckt.

I = total current of the ckt.

$R_p = \frac{R_1 R_2}{R_1 + R_2}$ ⟹ Total resistance of the ckt.

$I = \frac{V}{R_p}$ Similarly $I_1 = \frac{V}{R_1}$ & $I_2 = \frac{V}{R_2}$

$$\frac{I_1}{I} = \frac{V/R_1}{V/R_p} = \frac{R_p}{R_1} = \frac{1}{R_1} \cdot \left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$\frac{I_1}{I} = \frac{R_2}{R_1 + R_2} \Rightarrow \boxed{I_1 = \frac{R_2 I}{R_1 + R_2}}$$

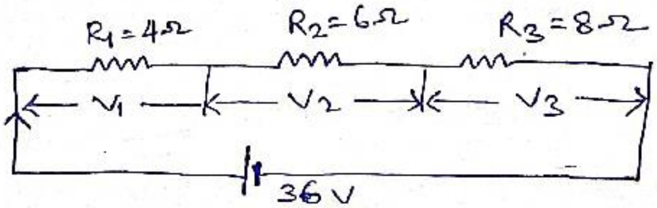
For I_2

$$\frac{I_2}{I} = \frac{V/R_2}{V/R_p} = \frac{R_p}{R_2} = \frac{1}{R_2} \cdot \left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$\boxed{I_2 = \frac{R_1 I}{R_1 + R_2}}$$

* Three resistances of values 4, 6 and 8 Ω are connected in series across a 36V dc supply. Calculate (a) total resistance of the ckt. (b) total current of the ckt. (c) voltage drop across each resistance (d) power dissipated in each resistance (e) total power dissipated in the ckt.

Solⁿ:



(a) total resistance (R_S) $= R_1 + R_2 + R_3 = 4 + 6 + 8 = 18\Omega$

(b) total current of the ckt. $I = \dfrac{V}{R_S} = \dfrac{36}{18}$

$I = 2$ Amp.

(c) Voltage drop across 4Ω resister

$V_1 = R_1 I = 4 \times 2 = 8V$

voltage drop across 6Ω resister

$V_2 = R_2 I = 6 \times 2 = 12V$

voltage drop across 8Ω resister

$V_3 = R_3 I = 8 \times 2 = 16V$

Total voltage $V_S = V_1 + V_2 + V_3 = 8 + 12 + 16 = 36V$

(d) Power dissipated in 4Ω resistance

$P_1 = I^2 R_1 = (2)^2 \times 4 = 16W$

power dissipated in 6Ω resistance

$P_2 = I^2 R_2 = (2)^2 \times 6 = 24W$

power dissipated in 8Ω resistance is

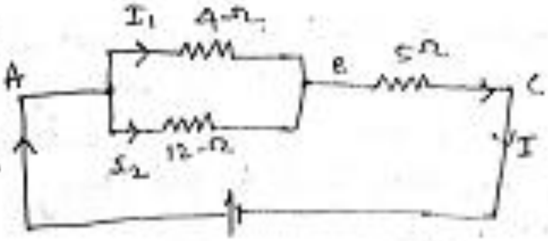
$P_3 = I^2 R_3 = (2)^2 \times 8 = 32W$

(e) Total power dissipated in ckt. is

$P = P_1 + P_2 + P_3 = 16 + 24 + 32 = 72W$

$\boxed{P = 72 \text{ watt}}$ OR $\boxed{P = VI = 36 \times 2 = 72W}$

Q1



For the network determine

(a) voltage drop in each resistor

(b) Current in each resistor.

Solⁿ: 1st calculate the total resistance of parallel ckt. A ll B

$$\frac{1}{R_{AB}} = \frac{1}{4} + \frac{1}{12} = \frac{1}{3} \left[R_{AB} = \frac{4 \times 12}{4+12} = \frac{48}{16} = 3\ \Omega \right]$$

$\boxed{R_{AB} = 3\ \Omega}$, $V_{AC} = I \times R_{AC} = 10 \times 8 = 80\ V$

$R_{AC} = R_{AB} + R_{BC} = 3 + 5 \Rightarrow \boxed{R_{AC} = 8\ \Omega}$

voltage drop across 5 Ω resistor

$V_{BC} = R_{BC}\ I = 5 \times 10 \Rightarrow \boxed{V_{BC} = 50\ V}$

$V_{AC} = V_{AB} + V_{BC}$

$\Rightarrow 80 = V_{AB} + 50 \Rightarrow \boxed{V_{AB} = 30\ V}$

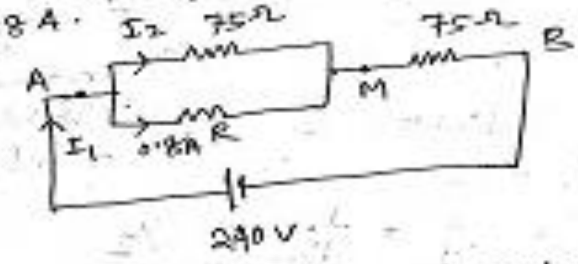
voltage drop across 4 Ω resistor

= voltage drop across 12 Ω resistor = 30 V.

$$I_1 = \frac{V_{AB}}{4} = \frac{30}{4} \Rightarrow \boxed{I_1 = 7.5\ A}$$

$$I_2 = \frac{V_{AB}}{12} = \frac{30}{12} \Rightarrow \boxed{I_2 = 2.5\ A}$$

Q.2 A 150Ω resistance coil AB is connected across 240V dc supply. Calculate the value of the resistance which, when connected between the midpoint of AB and end A, will carry a current of 0.8 A.



Soln

Let M be the mid point of AB and R be the resistance between A and M and the $I_R = 0.8A$

voltage across $R_{MB} = R_{MB} I_1 = 75 I_1$

$V_{AB} = V_{AM} + V_{MB} \Rightarrow V_{AM} = V_{AB} - V_{MB}$

$V_{AM} = 240 - 75 I_1$

By KCL at point A.

$I_1 = I_2 + 0.8 \Rightarrow I_1 = \frac{V_{AM}}{75} + 0.8$

$I_1 = \frac{240 - 75 I_1}{75} + 0.8$

$\Rightarrow 75 I_1 = 240 - 75 I_1 + 0.8 \Rightarrow I_1 = \frac{300}{150}$

$\boxed{I_1 = 2 A}$

$V_{AM} = 240 - 75 \times 2 \Rightarrow \boxed{V_{AM} = 90 V}$

$R = \frac{V_{AM}}{0.8} \Rightarrow R = \frac{90}{0.8} \Rightarrow \boxed{R = 112.5 \Omega}$

Q.3. A DC ckt. comprises two resistors, A of value 25 Ω, and B of unknown value, connected in ~~parallel~~ parallel, together with a third resistor C of the value 5 Ω connected in series with the parallel group. The potential difference across C is 90 V. If the total power in the ckt. is 4320 W, Calculate -

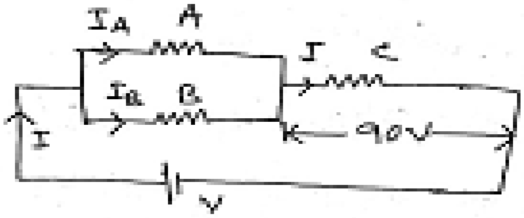
(a) The value of resistor B.

(b) The voltage applied to the ends of the whole ckt.

(c) The Current in each resistor.

Solⁿ

For this ckt.



(a) $I = \frac{90}{5} = 18A$

If R_p is the total resistance of the ckt. then total power will be

$P = I^2 R_p \Rightarrow 4320 = I^2 R_p$

$R_p = \frac{4320}{(18)^2} \Rightarrow R_p = 40/3\ \Omega$

But $R_p = \frac{25B}{B+25} + 5$

so

$\frac{25B}{B+25} + 5 = 40/3 \Rightarrow \boxed{B = 12.5\Omega}$

(b) voltage applied to the end of whole ckt. is $= V = RI = \frac{40}{3} \times 18 = 240V$

$\boxed{V = 240V}$

(c) Current in each resistor.

Current in resistor C $= I = 18$ Amp.

But the total voltage across the whole ckt. $= 240V$.

So the voltage across the parallel branch = Total voltage across whole ckt. — voltage across resistor C.

$\Rightarrow 240 - 90 = 150V$.

voltage across parallel branch $= 150V$.

Current in resistor A is

$$\Rightarrow I_A = 150/25$$

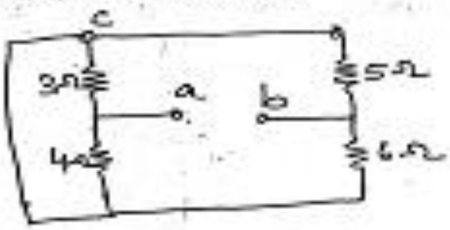
$$\boxed{I_A = 6A}$$

Current in resistor B is

$$\Rightarrow I_B = 150/12.5$$

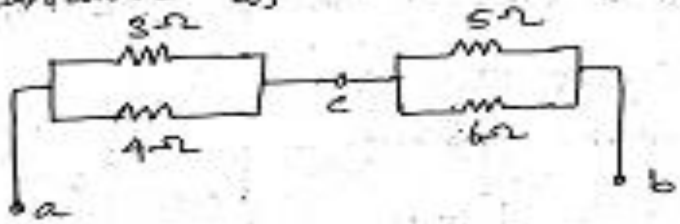
$$\boxed{I_B = 12A}$$

Q.4 Determine the resistance between the terminals a-b in the network.



Sol^n

The network of ~~fig~~ above fig. can be redrawn as



$$R_{ac} = \frac{3 \times 4}{3+4} = 12/7\ \Omega$$

$$R_{bc} = \frac{5 \times 6}{5+6} = 30/11\ \Omega$$

$$R_{ab} = R_{ac} + R_{bc} = \frac{12}{7} + \frac{30}{11}$$

$$\boxed{R_{ab} = 4.44\ \Omega}$$

NETWORK THEOREMS:

SUPERPOSITION THEOREM

THEVENIN'S THEOREM

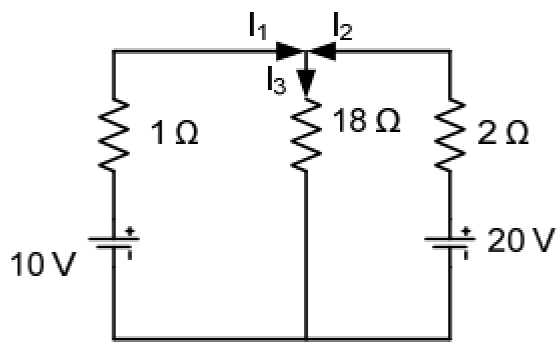
NORTON'S THEOREM

MAXIMUM POWER TRANSFER

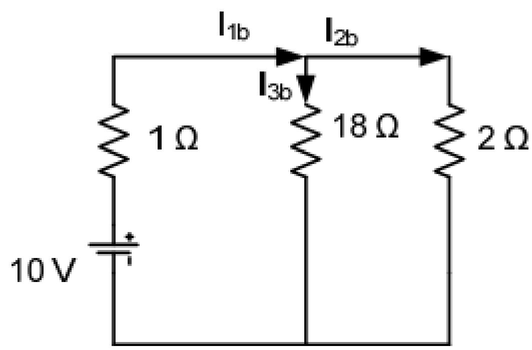
SUPERPOSITION THEOREM:

Statement: - *In a network of linear resistances containing more than one generator (or source of of all the currents which would flow at that point if each generator were considered separately and all the other generators replaced for the time being by resistances equal to their internal resistance.*

Example: - By using superposition theorem, calculate the currents in the network shown.



Step 1. Considering 10 V battery



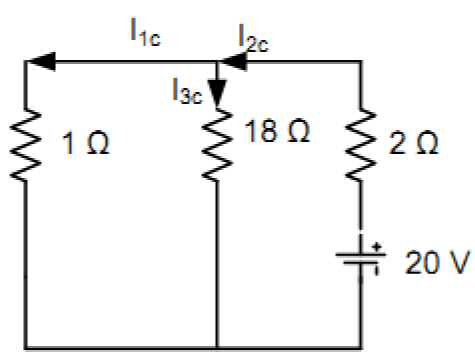
$$R_{eq} = \frac{2 \times 18}{2 + 18} + 1 = 2.8\ \Omega$$

$$I_{1b} = \frac{10}{2.8} = 3.57\ A$$

$$I_{2b} = 3.57 \times \frac{18}{20} = 3.21\ A$$

$$I_{3b} = I_{1b} - I_{2b} = 0.36\ A$$

Step 2. Considering 20 V battery



$$R_{eq} = \frac{1 \times 18}{1 + 18} + 2 = 2.95\ \Omega$$

$$I_{2c} = \frac{20}{2.95} = 6.78\ A$$

$$I_{1c} = 6.78 \times \frac{18}{19} = 6.42\,A$$

$$I_{3b} = I_{2c} - I_{1c} = 0.36\,A$$

Step 3. Results

$$I_1 = I_{1b} - I_{1c} = 3.57 - 6.42 = -2.85A$$

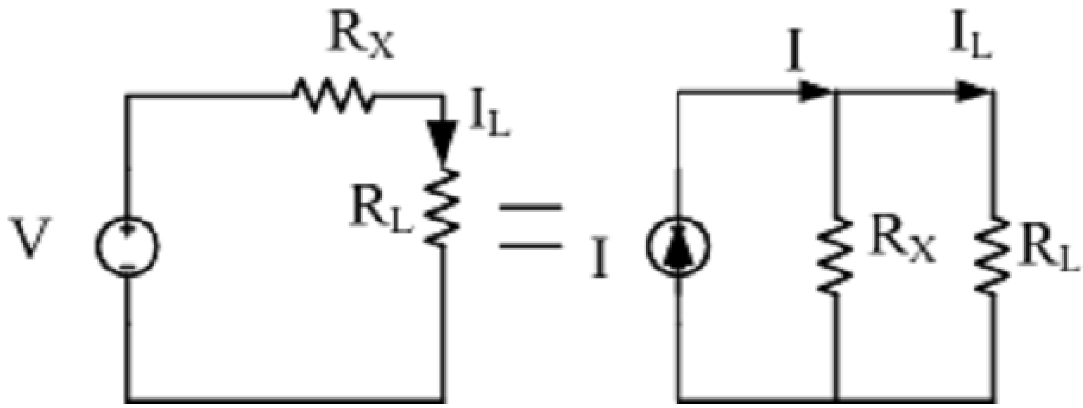
$$I_2 = I_{2c} - I_{2b} = 6.78 - 3.21 = 3.57A$$

$$I_3 = I_{3b} + I_{3c} = 0.36 + 0.36 = 0.72A$$

SOURCE CONVERSION: -

Statement: A voltage source (V) with a series resistance (R) can be converted to a current source (I=V/R) with a parallel resistance (R) and vice-versa.

Explanation:



$$I_L = \frac{V}{R_X + R_L} \qquad (1)$$

$$I_L = I\frac{R_X}{R_X + R_L} \qquad (2)$$

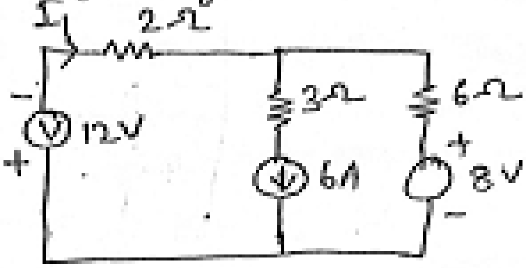
From Eq. (1) & (2)

$$V = IR_X \qquad (3)$$

Summary:

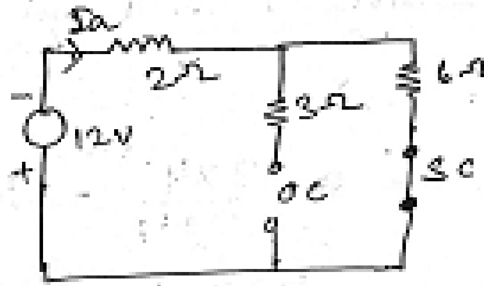
- The two circuits are said to be electrically equivalent if they supply equal load currents with the same resistance connected across their terminals.
- Voltage source having a voltage V and source resistance Rx can be replaced by I(= V/Rx) and a source resistance Rx in parallel with current source.
- Current source I and source resistance Rx can be replaced by a voltage source V (=IRx) and a source resistance Rx in series with V.

Q.5 Determine the branch current I_1 in the given ckt. by using Superposition theorem.



Sol^n There are three sources in the above ckt. we have to calculate current through 2Ω resistor.

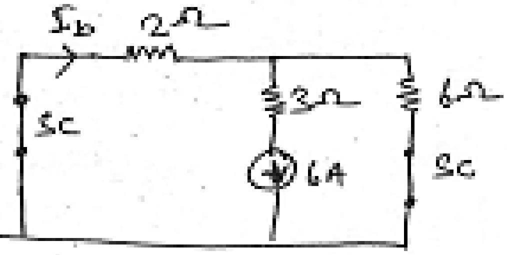
Step-1 Consider only 12V source –



The 6A current source is open-ckted & the 8V voltage is short ckt. The resulting ckt. is given in this fig. There is no current through open ckt. Hence the current through 3Ω resister is zero. By ohm's law

$$I_a = \frac{-12}{2+6} = -1.5 \text{ Amp.}$$

Step-2 : Consider only 6Amp current in the ckt



The 12V voltage source and 8V voltage source is short ckted.

... no current through open ckt. Hence

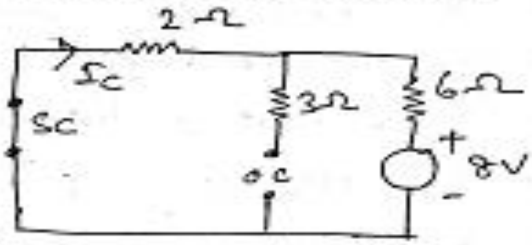
The 6A current source is supplying current to 2Ω and 6Ω resistors in parallel.

By current division rule

$$I_b = \frac{6}{2+6} \times 6 = 4.5\ A.$$

(c) Consider only 8V source in the ckt.

The 6 Amp current source is open ckt. and 12V voltage source is short ckted.



By ohms Law —

$$I_c = \frac{-8}{6+2} = -1\ A.$$

By superposition principle, the total current through the 2Ω resistor is $\Rightarrow I_1 = I_a + I_b + I_c$

$$I_1 = -1.5 + 4.5 - 1$$

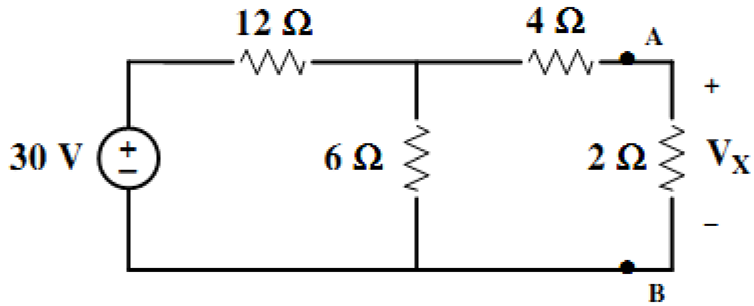
$$\boxed{I_1 = 2\ \text{Amp}}$$

THEVENIN'S THEOREM: -

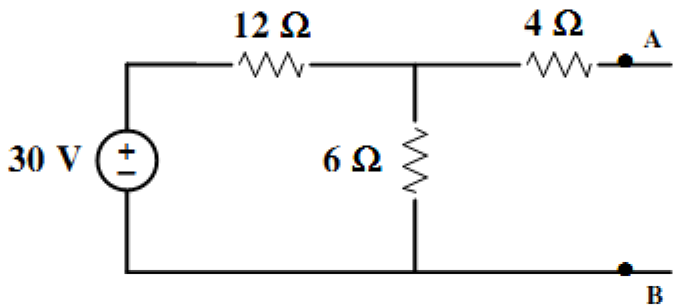
Statement:- *Any pair of terminals AB of a linear active network may be replaced by an equivalent voltage source in series with an equivalent resistance Rth. The value of Vth (called the Thevenin's voltage) is equal to potential difference between the terminals AB when they are open circuited, and Rth is the equivalent resistance looking into the network at AB with the independent active sources set to zero i.e with all the independent voltage*

sources short-circuited and all the independent current sources open- circuited.

Example: - Find V_X by first finding V_{TH} and R_{TH} to the left of A-B.



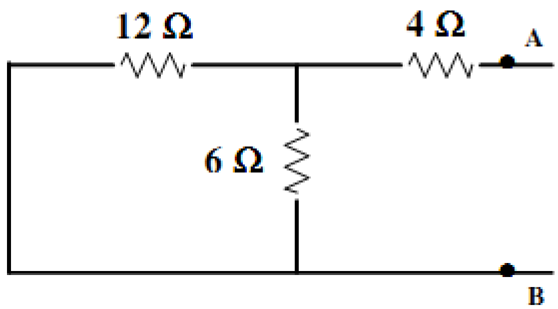
Solution: step1. First remove everything to the right of A-B



$$V_{AB} = \frac{(30)(6)}{6+12} = 10V$$

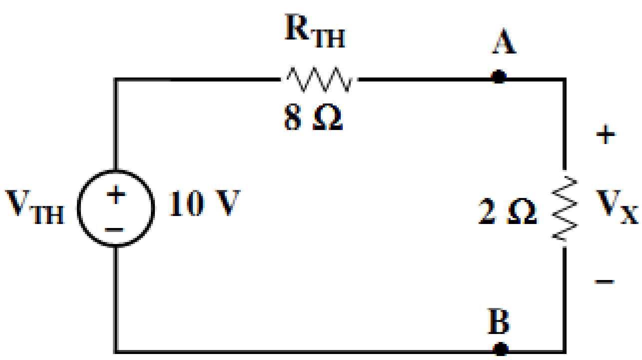
There is no current flowing in the 4 Ω resistor (A-B) is open. Thus, there can be no voltage across the resistor.

Step 2. To find Rth: We now deactivate the sources to the left of A-B and find the resistance seen looking in these terminals.



$R_{TH} = 12 \| 6 + 4 = 8\,\Omega$

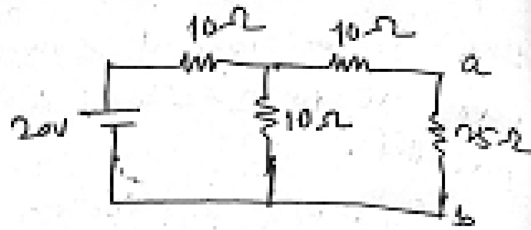
Step 3. To find V_x : After having found the Thevenin circuit, we connect this to the load in order to find V_X.



$$V_X = \frac{(10)(2)}{2+8} = 2V$$

Q.6 Determine the current through and voltage across the 25Ω resistor by using Thevenin's law.

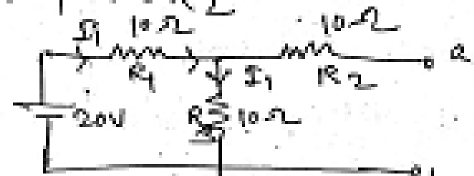
Sol



Step-1 → First disconnect the 25Ω resistor from the original network.

Step-2 → Determination of V_{Th}

Since, the ckt is open between a-b, there is no current will flow in R_2. So that there is no voltage drop in R_2



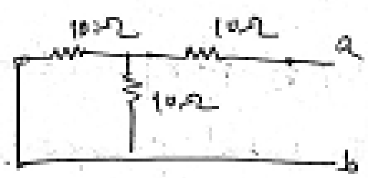
So the V_{ab} = Voltage drop across $R_3 = R_3 I_1$

$$\Rightarrow R_3 \frac{V_s}{R_1+R_3} = 10 \times \frac{20}{10+10} = 10V$$

$\boxed{V_{R_3} = 10V}$

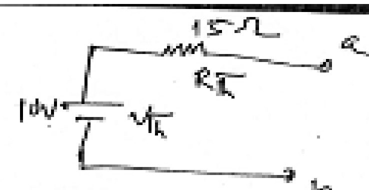
$\boxed{\text{So } V_{Th} = \text{oc voltage} = 10V}$

Step-3 The voltage across the 20V source is shorted.



$$R_{Th} = R_2 + \frac{R_1 \times R_2}{R_1+R_2} = 15\Omega$$

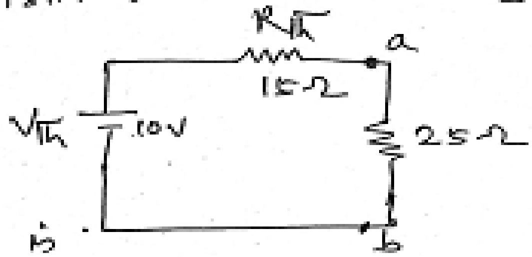
$\boxed{R_{Th} = 15\Omega}$



Step-4

Current Through 25Ω resistor is to be Calculated

. The 25Ω resistor is connected between a-b



Let I be the total current is flowing across 25Ω resistor.

$$I = \frac{V_{Th}}{R_{Th} + V_s} = \frac{10}{15 + 25}$$

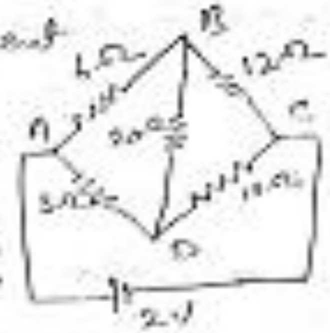
$$\boxed{I = 0.25 \text{ Amp}}$$

Voltage across 25Ω resistor is

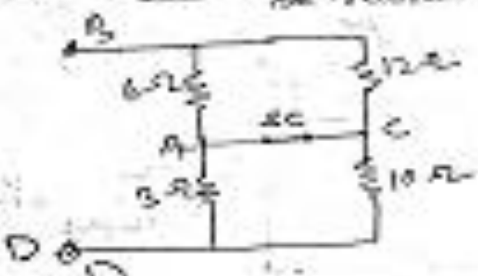
$$= 25 I = 25 \times 0.25 \text{ resistor}$$

$$= \boxed{6.25 \text{ Amp}}$$

Q.7 Determine the current in 20 Ω resistor by using Thevenin's Theorem.



Sol^n Step-I :- The fig. can be redrawn as



Calculate V_{th} :- ~~Removing the 20Ω resistor~~

Let the resistor between B and D is removed

So current through ABC $\Rightarrow I_{AB} = \frac{2}{6+12} = \frac{1}{9}$ A

$I = \frac{V}{R}$

Voltage across R_{AB} is V_{AB}

$$V_{AB} = R_{AB} I_{AB} = 6 \times \frac{1}{9} = 2/3 \, V$$

Current through ADC = $\frac{2}{3+10} = \frac{2}{13}$ A.

$$I_{AD} = \frac{2}{13} \text{ A}.$$

Voltage drop across AD is V_{AD}

$$= R_{AD} I_{AD} = 3 \times \frac{2}{13} = \frac{6}{13} \text{ A}.$$

Apply mesh ABDA by KVL,

$$V_{AB} + V_{BD} + V_{DA} = 0$$

$$V_{BD} = -V_{DA} - V_{AB} \Rightarrow V_{AD} - V_{AB}$$

$$V_{BD} = \frac{6}{13} - \frac{2}{3} = -8/39 \, V$$

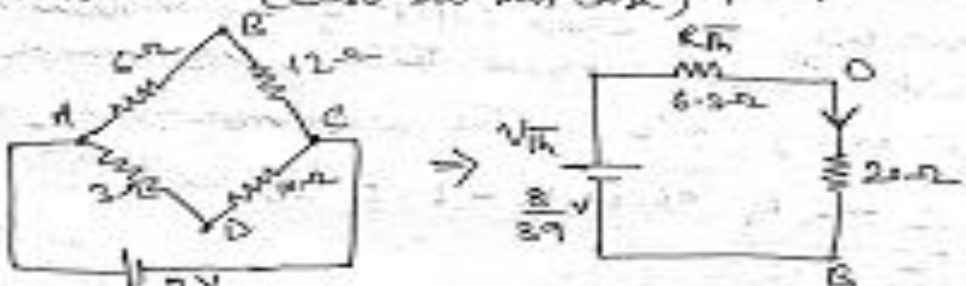
$V_{BD} = -\frac{8}{39}$ V

$$\boxed{V_{DB} = \frac{8}{39}\ V}$$

$$\boxed{V_{Th} = V_{DB} = \frac{8}{39}\ V}$$

Calculate R_{Th}

R_{Th} is the resistance between terminal B and D with 20Ω resistor removed and replacing the battery with its internal resistance (zero in this case).



Total resistance through BD = R_{BD} = R_{Th}

$$R_{BD} = (R_{AB} \parallel R_{BC}) + (R_{CD} \parallel R_{DA})$$

$$= \frac{6\times12}{6+12} + \frac{3\times10}{3+10} = 6.3\ \Omega$$

$$\boxed{R_{Th} = R_{BD} = 6.3\,\Omega}$$

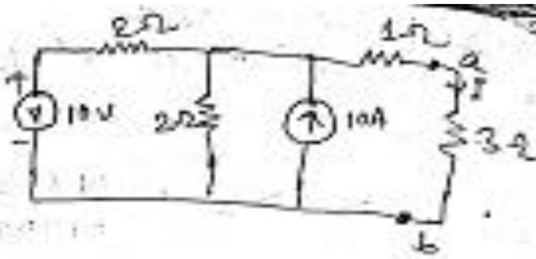
Current through 20Ω resistor $= I_{20} = \frac{V_{Th}}{R_{Th} + R_{BD}}$

$I_{20} = \frac{8/39}{6.3+20} = 7.8$ mA from D to B

$$\boxed{I_{20} = 7.8\ mA}$$

Q.9

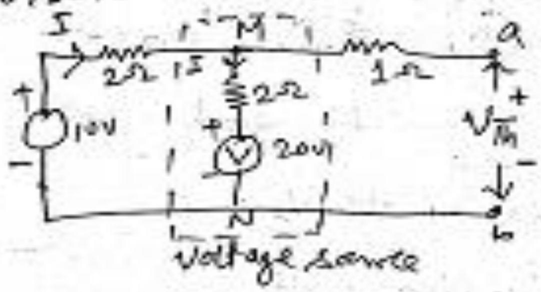
Determine the current I in the network by using Thevenin's Theorem.



Soln Step-1 – Now the 3 Ω resistor through which I is flowing is removed

Step-2 Determination of Thevenin voltage V_T

The 10 A current is parallel with 2 Ω resistor is transformed to a $10 \times 2 = 20V$ source in series with 2 Ω resistor



Here the current through 1 Ω resistor is zero because of the open ckt. between terminal a – b.

Now apply KVL to the left hand side

$$10 - 2I - 2I - 20 = 0$$

$$I = -10/4 = -2.5 \text{ A}.$$

$$\boxed{I = 2.5 \text{ A}}$$

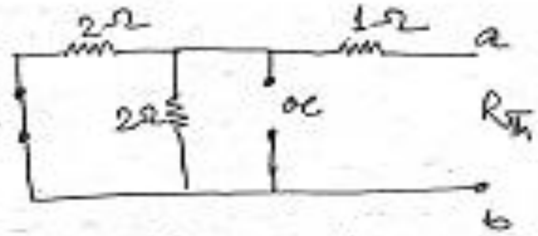
AP Apply KVL to right hand side

$$20 + 2I - V_{Th} = 0$$

$$V_{Th} = 20 + 2I = 20 + 2\times(2{\cdot}5)$$

$$\boxed{V_{Th} = 15\,V}$$

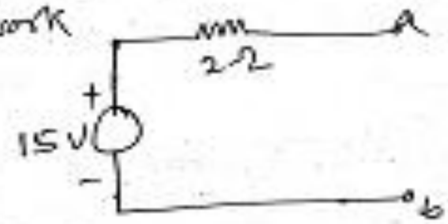
Step-3 Determination of Thevenin resistance R_{Th}.

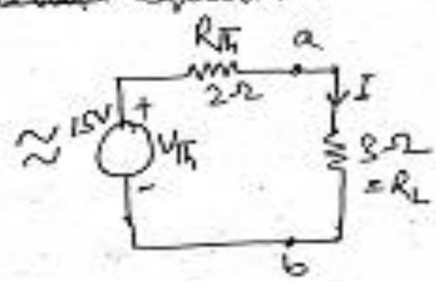


The 10V voltage source is short ckted and the 10 A current source is open ckted. Now Thevenin resistance R_{Th} is = resistance between the terminal a-b

$$= 1 + 2\|2 = 1 + \frac{2\times 2}{2+2} = 2\,\Omega$$

Step-4 Thevenin ~~equivalent~~ equivalent network





Step-5 Determination of current through 3Ω resistor is $\Rightarrow I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{15}{2+3}$

$$\boxed{I = 3\,A}$$

NORTON'S THEOREM:

Statement: *Any two terminal linear active network (containing independent voltage and current sources), may be replaced by a constant current source I_N in parallel with a resistance R_N, where I_N is the current flowing through a short circuit placed across the terminals and R_N is the equivalent resistance of the network as seen from the two terminals with all sources replaced by their internal resistance.*

5.7 APPLICATION OF NORTON'S THEOREM

The procedure for the application of Norton's theorem is as follows :

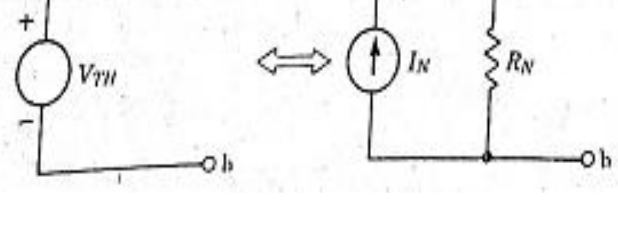
1. Remove the external load R_L and then short-circuit the load terminals a–b with respect to which the Norton equivalent circuit is desired.
2. Determine the current through the terminals when they are short-circuited. This current is I_N or I_{SC}.

   There is *alternative method* of determining I_N.

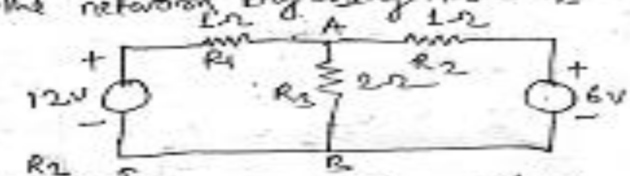
$$I_N = \frac{V_{Th}}{R_{Th}}$$

3. Redraw the network with each ideal voltage source replaced by a short circuit and each ideal current source replaced by an open circuit.
4. Calculate the resistance R_N of the redrawn network at the open-circuited terminals a–b. It is to be noted that $R_N = R_{Th}$.
5. Draw the Norton equivalent circuit between the terminals a–b by connecting the current source I_N in parallel with R_N.
6. The external load R_L that was short-circuited in step 1 is now reconnected across the Norton terminals a–b. It is to be noted that the direction of I_N is such that it circulates the current in the load R_L in the same direction as the original circuit circulated it.
7. The load current I_L through R_L is calculated from the formula

$$I_L = \frac{R_N}{R_N + R_L} I_N$$

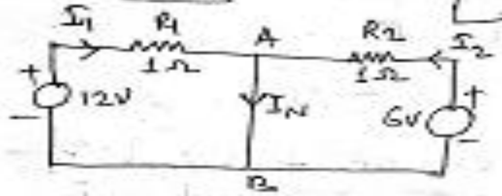


Q. 8 Determine the current through 2Ω resistor in the network by using norton's theorem.



Soln

Step-I



According norton theorem the terminal AB is short ckted

Step-2 Determination of Norton current (I_N)

Norton current I_N is the current through AB when they are short ckted. So I_N is known as short circuit current through AB branch.

By apply KVL in the left hand mesh,

$12 - I_1 R_1 = 0 \Rightarrow 12 - 1 \times I_1 = 0$

$\boxed{I_1 = 12A}$

Apply KVL to the right hand mesh,

$6 - I_2 R_2 = 0 \Rightarrow 6 - I_2 \times 1 = 0$

$\boxed{I_2 = 6A}$

By KCL at node $A \Rightarrow I_N = I_1 + I_2 = 12+6$

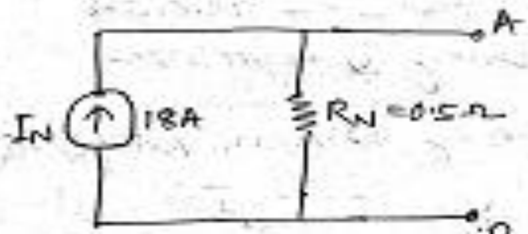
$\Rightarrow \boxed{I_N = 18 \text{ Amp}}$

Step-3 :- Calculate R_N :-

The two terminals A and B are taken out, so Norton resistance $\Rightarrow R_N = R_{AB}$

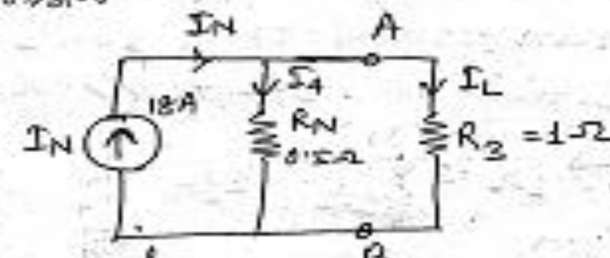
$$R_N = R_1 \parallel R_2 = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{1 \times 1}{1+1} = 0.5\,\Omega$$

Step-4 - Norton equivalent network



The Norton equivalent network between terminal AB is drawn by connecting the current source I_N is parallel with R_N

Step-5 Determination of current through 2Ω resistor.



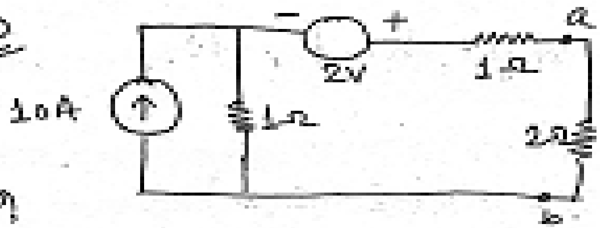
The load resistor R_3 that was removed in step-1 but now it is connected across the Norton's terminals AB.

By the current division rule

$$I_L = \frac{R_N}{R_N + R_3} \cdot I_N = \frac{0.5 \times 18}{0.5 + 2}$$

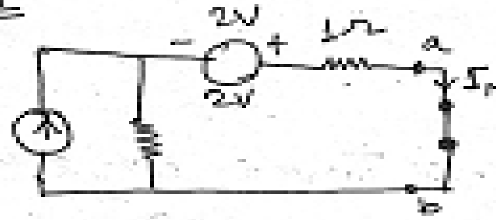
$$\boxed{I_L = 3.6 \text{ Amp}}$$

Q.10



Determine the voltage across 2 Ω resistor by using Norton's theorem.

Soln



Step-1 :- The 2Ω resistor across which voltage is to be calculated and it is short ckted in that branch.

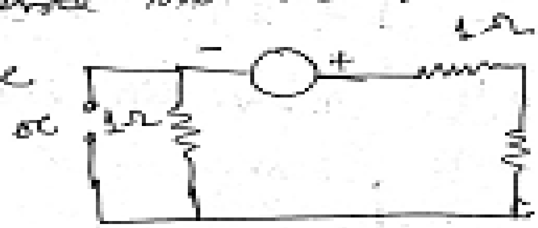
Step-2 Determination of Norton current (I_N)

(a) only 10A current source in the ckt.
The 2V voltage source is short ckted. So by current division rule :-

$$I_{1N} = \frac{1}{1+1} \times 10 = 5A.$$

~~(b) only 2V voltage source in the ckt.~~

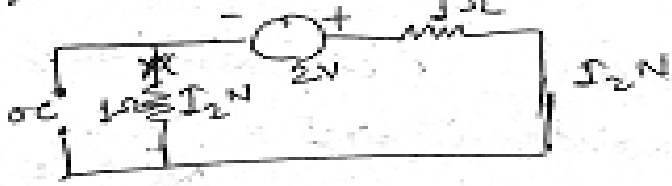
The 10A current source is open - ckt.



~~Applying~~ By the current division rule

$$I_{1N} = \frac{1}{1+1} \times 10 = 5A.$$

(b) Only 2V voltage source in the ckt.



By applying KVL for this fig.

$-I_{2N} \times 1 + 2 - I_{2N} \times 1 = 0$

$\boxed{I_{2N} = 1A}$

But the currents ~~are~~ are flowing in same direction by superposition principle —

$I_N = I_1 N' + I_2 N = 5+1 = 6A.$

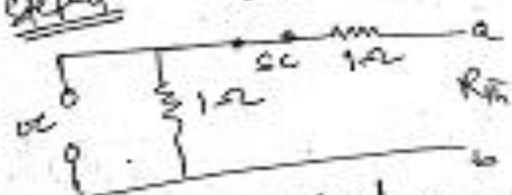
$\boxed{I_N = 6A}$

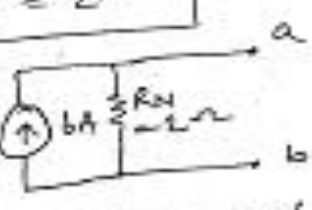
Step-3 Determination of R_N (Norton resistance)

The 10A current source is ~~open~~ oc and 2V voltage source is short ckt. and load is removed

R_N = resistance between terminal a - b.

Step-4 $= 1+1 = 2\Omega$ $\boxed{R_N = 2\Omega}$





In this terminal a-b is drawn by connecting the current source I_N is parallel with R_N so

Step-5

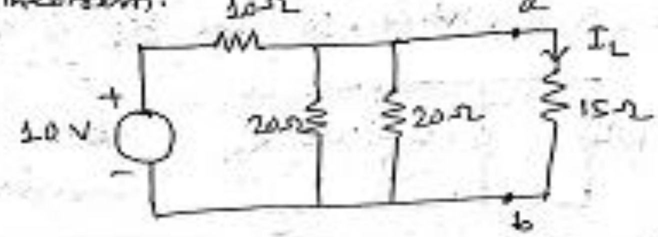
$I_L = \frac{2}{2+2} \times 2$

$I_L = 3A.$

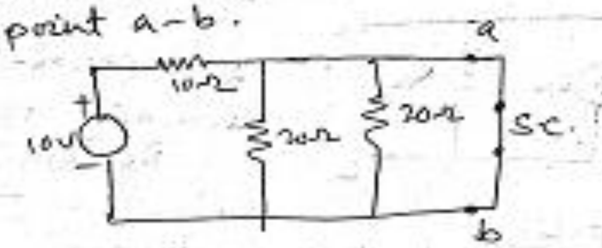
$V_{at\,2\Omega} = I_L R_L \Rightarrow V_L = I_L R_L^2 = 3\times2 = 6$

$\boxed{V_L = 6V}$

Q-13 Determine the current I_L through 15Ω resistor in the given ckt. by Norton's theorem.

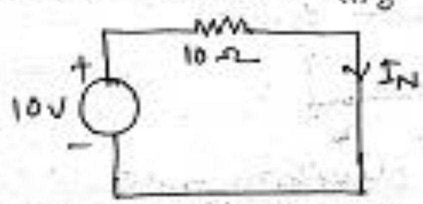


Soln. Step-1 Determine the current through 15Ω resistor. So we have to remove the 15Ω resistor from the original network and short ckt. to the point a-b.



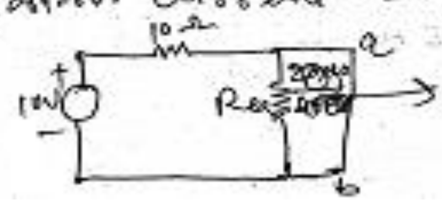
Step-2

Determination of Norton's Current (I_N)
The resistance of S.C. is zero. So the equivalent resistance of the two resistors of 20Ω each and S.C is zero. So the fig will be.



So $I_N = I_{SC} = \frac{10}{10} = 1$ A.

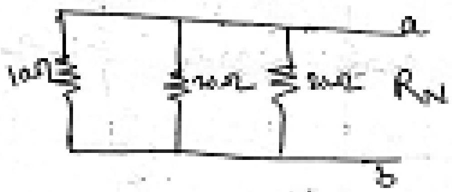
Norton's Current = 1 A.



$\frac{20 \times 20}{20+20}$

Step-3 Determination of Norton's resistance (R_N) :—

Now the 10V voltage source is short circuited. So the fig is reduced to



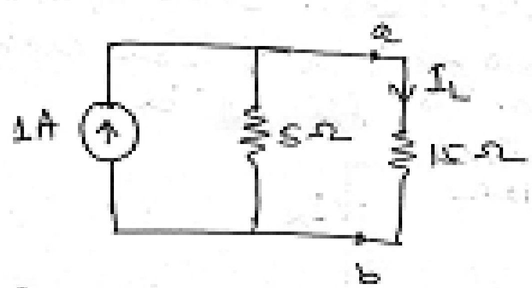
So the equivalent resistance of ckt. is

$$\frac{1}{R_N} = \frac{1}{5} \Rightarrow \boxed{R_N = 5\,\Omega}$$

Step-4 Norton's equivalent ckt.

I_N is parallel to R_N according to Norton's theorem.

Step-5 Determination of current through 15Ω resistor. So the fig. will be



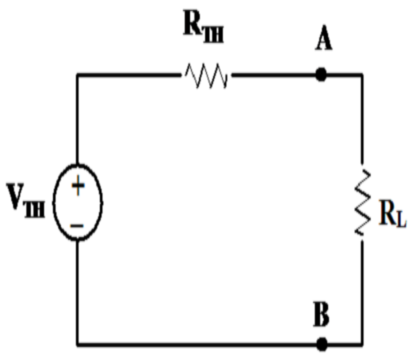
By current division rule:

$$I_L = \frac{5}{5+15} \times 1 = 0.25\,A.$$

$$\boxed{I_L = 0.25\,A}$$

MAXIMUM POWER TRANSFER THEOREM:

• *The maximum power transferred from the power source to the load is when the resistance of the load R_L is equal to the equivalent or input resistance of the power source ($R_{in} = R_{Th}$ or R_N).* • *The process used to make $R_L = R_{in}$ is called impedance matching.*



$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

$$P_L = I^2 R_L = \frac{V_{TH}^2 R_L}{(R_{TH} + R_L)^2}$$

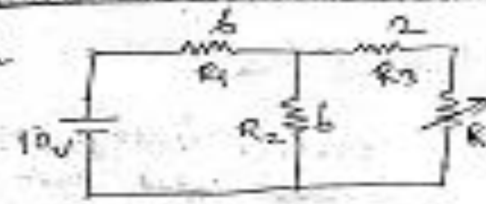
For P_L to be maximum, $\frac{dP_L}{dR_L} = 0$

Or, $R_L = R_{TH}$

So, Maximum power drawn by $R_L = I^2 R_L = \frac{V_{TH}^2 R_L}{(2R_L)^2} = \frac{V_{TH}^2}{4R_L}$

Power supplied by the source $= \frac{V_{TH}^2}{(R_{TH} + R_L)}$

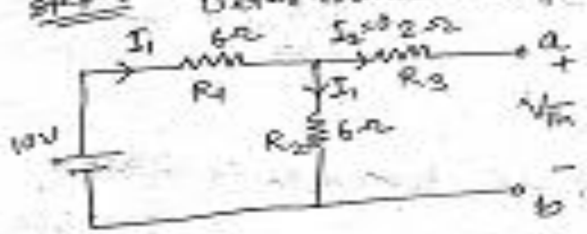
Q-11



In this network (a) determine (a) the value of the load resistance to give maximum power transfer (b) power delivered to the load.

Soln In order to determine maximum power transfer, we have to determine the Thevenin equivalent network

step-1 Determination of V_{Th}



Disconnect the load resistance R_L from original network. Now the ckt. is open between terminal a-b, there is no current through R_3 and the current through R_1 & R_2 is same.

By KVL to the left hand mesh,

$10 - I_1 R_1 - I_1 R_2 = 0 \Rightarrow I_1 = 5/6 \text{ A}$.

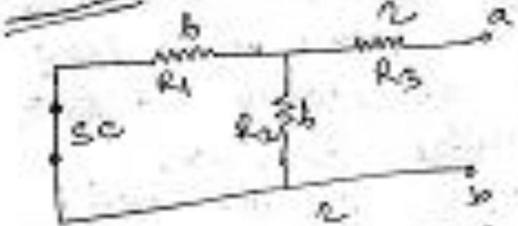
Now KVL to the right hand mesh

$I_1 R_2 - I_2 R_3 - V_{Th} = 0 \Rightarrow I_1 \times 6 - 0 \times 2 - V_{Th} = 0$

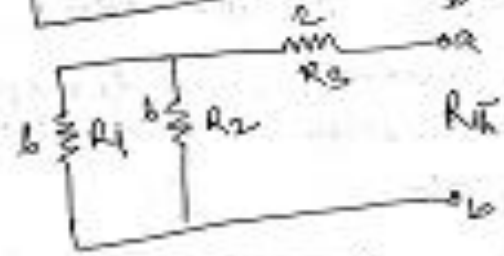
$V_{Th} = 6 I_1 \Rightarrow V_{Th} = 6 \times 5/6 = 5A$.

$$\boxed{V_{Th} = 5A}$$

Step-2. Determination of R_{Th}



The 18V voltage source is short circuited. Now the fig can be redrawn as



$$R_{Th} = R_1 \| R_2 + R_3$$

$$= \frac{R_1 \times R_2}{R_1 + R_2} + R_3$$

$$= \frac{6 \times 6}{6+6} + 2$$

$$\boxed{R_{Th} = 5\,\Omega}$$

For maximum power transfer of CKt-s

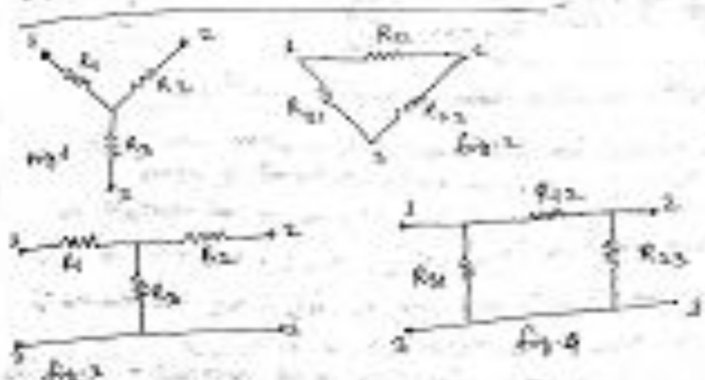
$$R_L = R_{Th} = 5\,\Omega$$

$$P_{L\,max.} = I_L^2 R_{Th} = \left(\frac{V_{Th}}{R_{Th} + R_{Th}}\right)^2 \times R_{Th}$$

$$= \frac{V_{Th}^2}{4 R_{Th}} = \frac{5^2}{4 \times 5} = 1.25W$$

$$\boxed{P_{L\,max} = 1.25W}$$

Star - Delta Transformation



From above fig(3) and (4)

The resistance between terminals 1 and 2 for star network = resistance between terminal 1 and 2 for delta network

$$R_1 + R_2 = R_{12} \parallel (R_{23} + R_{31})$$

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \quad \text{—(1)}$$

Similarly, the resistance between terminal 2 & 3 for star network = resistance between terminal 2 & 3 for delta network

$$R_2 + R_3 = R_{23} \parallel (R_{31} + R_{12})$$

$$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \quad \text{—(2)}$$

Similarly the resistance between the terminal 3 and 1 for star network = resistance between terminal 3 & 1 for delta network

$$R_3 + R_1 = \frac{R_{31} \parallel (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}}$$

$$R_3 + R_1 = \frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} \quad \text{---} (3)$$

Delta to Star Transformation

To convert the delta network to star network, it is necessary to express each element of star network (R_1, R_2, R_3) in terms of the element of delta network (R_{12}, R_{23}, R_{31}). Equation (1) to (3) can be used to find out R_1, R_2 & R_3 in terms of R_{12}, R_{23}, R_{31}. Addition of these equation (1), (2) & (3) and division of the result by 2 gives

$$R_1 + R_2 + R_3 = \frac{R_{12}R_{23} + R_{23}R_{31} + R_{31}R_{12}}{R_{12}+R_{23}+R_{31}} \quad \text{---} (4)$$

Subtraction of eqn (2) from eqn (4) =>

$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{---} (5)$$

Subtraction of eqn (3) from eqn (4) =>

$$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}} \quad \text{---} (6)$$

Subtraction of eqn (1) from eqn (4) =>

$$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{---} (7)$$

$$\text{Star resistance} = \frac{\text{Product of adjacent delta resistance}}{\text{Sum of delta resistances}} \quad \text{---} (8)$$

Star to delta Transformation :-

Delta - connected resistances can be converted in terms of star connected resistance - For we have to find R_{12}, R_{23} and R_{31} in terms of R_1, R_2 and R_3. From the eqn. ⑥, ⑦ and ⑧

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}^2 R_{23} R_{31} + R_{23}^2 R_{31} R_{12} + R_{31}^2 R_{12} R_{23}}{(R_{12} + R_{23} + R_{31})^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \quad \text{—— ⑨}$$

Division of eqn. ⑨ by eqn. ⑤ we get

$$R_{12} = \frac{1}{R_3}(R_1R_2 + R_2R_3 + R_3R_1)$$

$$R_{12} = \frac{\Sigma R_1 R_2}{R_3} \quad \text{—— ⑩}$$

where $\Sigma R_1 R_2 = R_1R_2 + R_2R_3 + R_3R_1$

Divide the eqn. ⑨ by eqn. ⑥ we get

$$R_{23} = \frac{1}{R_1}(R_1R_2 + R_2R_3 + R_3R_1)$$

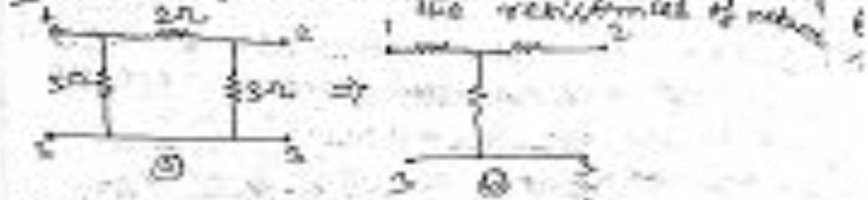
$$R_{23} = \frac{\Sigma R_1 R_2}{R_1} \quad \text{—— ⑪}$$

Divide the eqn. ⑨ by eqn. ⑧ we get

$$R_{31} = \frac{1}{R_2}(R_1R_2 + R_2R_3 + R_3R_1)$$

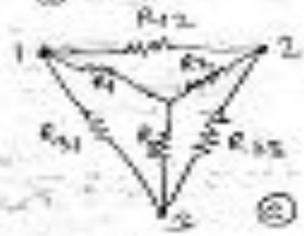
$$R_{31} = \frac{\Sigma R_1 R_2}{R_2} \quad \text{—— ⑫}$$

Q.12 Using Delta - Star conversion find the resistance of network



(a) Delta to star Transformation

The superimposed delta and star network are shown in fig (c)



$$\Sigma R_\Delta = R_{12} + R_{23} + R_{31} = 2+3+5 = 10\,\Omega$$

$$R_1 = \frac{R_{12} R_{31}}{\Sigma R_\Delta} = \frac{2\times 5}{10} = 1\,\Omega$$

$$R_2 = \frac{R_{12} R_{23}}{\Sigma R_\Delta} = \frac{2\times 3}{10} = 0.6\,\Omega$$

$$R_3 = \frac{R_{23} R_{31}}{\Sigma R_\Delta} = \frac{3\times 5}{10} = 1.5\,\Omega$$

(b) star to Delta Transformation

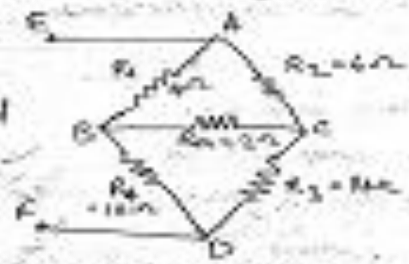
$$\Sigma R_1 R_2 = R_1 R_2 + R_2 R_3 + R_3 R_1 = 3\,\Omega$$

$$R_{12} = \frac{\Sigma R_1 R_2}{R_3} = \frac{3}{1.5} = 2\,\Omega$$

$$R_{23} = \frac{\Sigma R_1 R_2}{R_1} = \frac{3}{1} = 3\,\Omega$$

$$R_{31} = \frac{\Sigma R_1 R_2}{R_2} = \frac{3}{0.6} = 5\,\Omega$$

Q.19. Find the resistance between the terminals E – F of the bridge ckt given in below fig. by using delta-star transformation.



Sol^n Here the two delta triangles are connected to each other by common resistor 2Ω.

We shall transform the delta network of terminals A, B, C to star network.

In star connection, $R_A = \dfrac{R_1 R_2}{\Sigma R_\Delta}$

$\Sigma R_\Delta = R_1 + R_2 + R_m = 4+6+2 = 12\Omega$

$\boxed{\Sigma R_\Delta = 12\Omega}$ $R_A = \dfrac{R_1 R}{\Sigma R_\Delta} = \dfrac{4\times 6}{12}$

$\boxed{R_A = 2\Omega}$

$R_B = \dfrac{R_1 \times R_m}{\Sigma R_\Delta} = \dfrac{4\times 2}{12} = 2/3\ \Omega$

$\boxed{R_B = \frac{2}{3}\Omega}$

$R_C = \dfrac{R_2 \times R_m}{\Sigma R_\Delta} = \dfrac{6\times 2}{12} = 1\Omega$

$\boxed{R_C = 1\Omega}$

After the completion of delta to star transformation, the bridge ckt will be reconstructed as.

From this fig. we shall get the R_c and R_2 are in series connected.

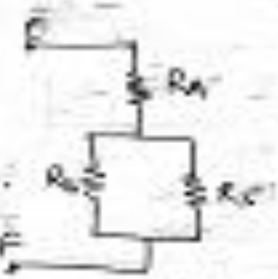
So the total resistance $R_5 = R_c + R_2 = 1 + 14 = 15\Omega$

$$\boxed{R_5 = 15\Omega}$$

Similarly R_B and R_4 are in series connected. So the total resistance

$$R_6 = R_B + R_4 = \frac{2}{3} + 10$$

$$\boxed{R_6 = \frac{32}{3}\Omega}$$



From final fig. we shall conclude that R_5 is parallel to R_6 and the output R_A is series connected. So the total resistance between EF is R_{EF}

$$R_{EF} = R_A + \frac{R_5 \times R_6}{R_5 + R_6} = 2 + \frac{15 \times 32/3}{15 + 32/3} = 8.234\Omega$$

$$\boxed{R_{EF} = 8.234\Omega}$$

End