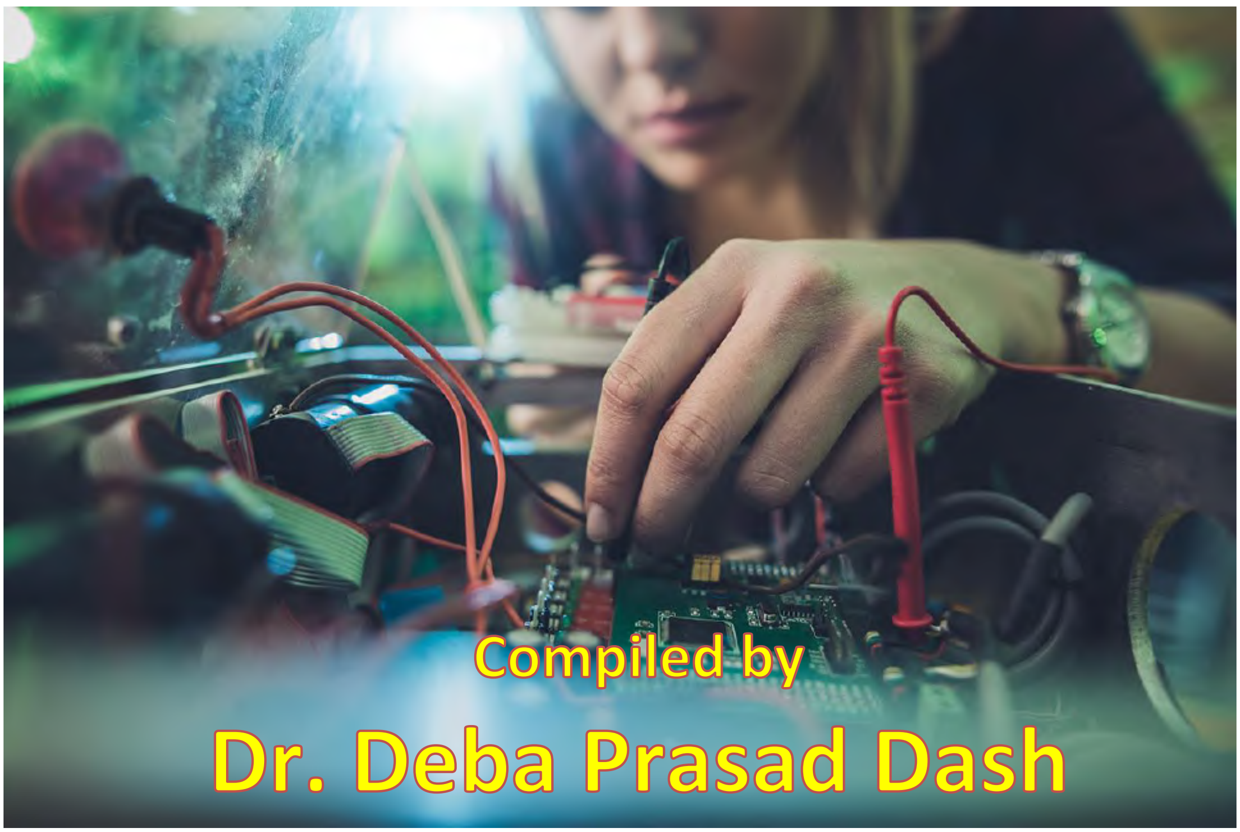
GOVERNMENT COLLEGE OF ENGINEERING, KALAHANDI

Lecture notes

on

BASIC ELECTRICAL ENGINEERING
(Module II)



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MODULE-II

SINGLE PHASE

A.C CIRCUIT

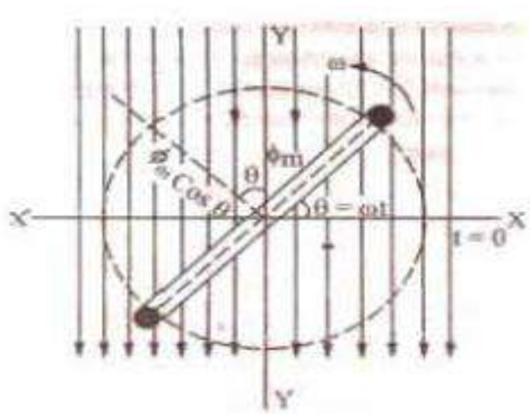
Single phase EMF generation:

Alternating voltage may be generated by rotating a coil in a magnetic field

The value of voltage generated depends upon

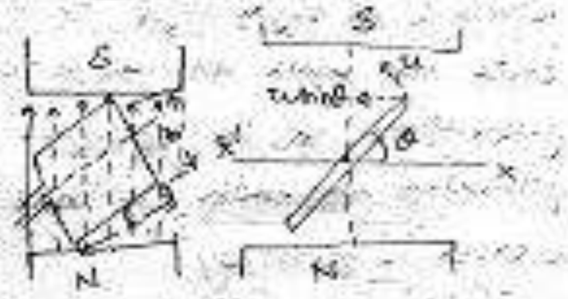
1) No. of turns in the coil 2) field strength 3) speed

Explanation of alternating voltage and current:



An alternating voltage is any voltage that varies in magnitude and polarity with respect to time, similarly, an alternating current is that current that varies in magnitude and direction with respect to time.

Generation of



Generation of Emf in a rotating coil

Consider a rectangular coil rotating in a uniform magnetic field with a constant angular velocity. The axis of rotation of the coil at right angle to the magnetic coil / field.

Let l = length of coil, b = breadth of coil

ω = angular velocity, u = linear velocity

Angular velocity = $\frac{\text{angular distance}}{\text{time}}$

$$\omega = \frac{\theta}{t} \Rightarrow \theta = \omega t \qquad (1)$$

At this instant the velocity u may be resolved into two component $u \sin\theta$ perpendicular to the magnetic field and $u \cos\theta$ parallel to it.

At this instant the velocity of the coil side perpendicular to magnetic field i.e. $u \sin\theta$.

This component of coil velocity, $u \sin\theta$ which is at right angle to magnetic field, cuts the flux and as a result the generation of the voltage. Hence voltage generated in each coil side at any instant is given by

$$V_1 = Blu\sin\theta \qquad \text{——— (2)}$$

The voltage generated in one turn coil at any instant is

$V = 2 \times$ voltage generated in each coil side

$$V = 2Blu\sin\theta$$

As we know Linear velocity = angular velocity × radius

$$u = \omega \cdot \frac{b}{2} \qquad \text{——— (3)}$$

put the value of u in eqn (2) so

$$V = 2Bl\left(\omega \cdot \frac{b}{2}\right)\sin\theta$$

$$V = B(lb)\omega\sin\theta \qquad \text{——— (4)}$$

But $lb = A$ = area of coil

$$V = BA\omega\sin\theta \qquad \text{——— (5)}$$

If the no. of turn is N_c (coil), the generated voltage at any instant is

$$V = BA\omega N_c\sin\theta \qquad \text{——— (6)}$$

But BA = flux linking the coil when the plane of the coil is normal to the magnetic field.

It is maximum value of flux which links the coil so it is denoted ϕ_m

$$\phi_m = BA \quad \text{——— (7)}$$

$$V = \phi_m \omega N_c \sin\theta \quad \text{——— (8)}$$

The above expⁿ. (8) is maximum value, when $\sin\theta = 1$ or $\theta = 90°$ so the maximum value of v is denoted as V_m

put $v = V_m$ and $\sin\theta = 1$ in expⁿ. (8) so

$$V_m = \phi_m \omega N_c \quad \text{——— (9)}$$

The expⁿ. (8) can be written in simple form as

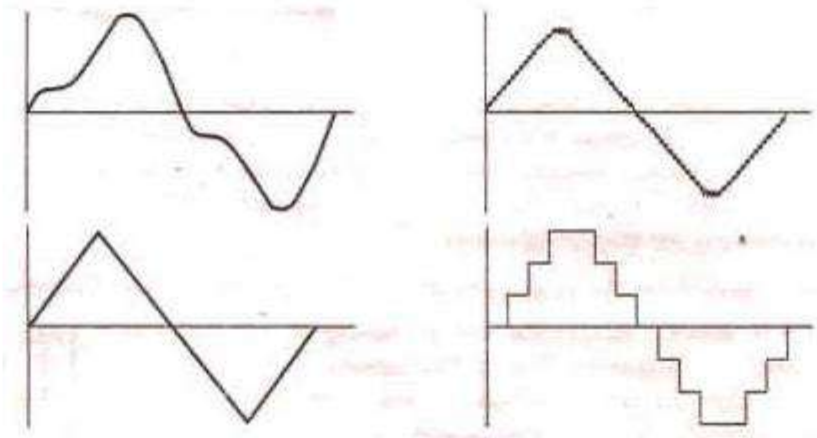
$$\boxed{V = V_m \sin\theta}$$

A simple coil rotating in a magnetic field is the most elementary form of an a.c. generator.

An ac generator is called an alternator.

A.C terms: Cycle: -

A complete set of positive and negative values of an alternating quantity is known as cycle.



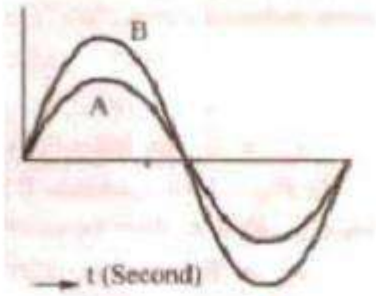
Time period: The time taken by an alternating quantity to complete one cycle is called time T.

- **Frequency**: It is the number of cycles that occur in one second. f = 1/T

f = PN/120 where, P= No. of poles, N= Speed in rpm

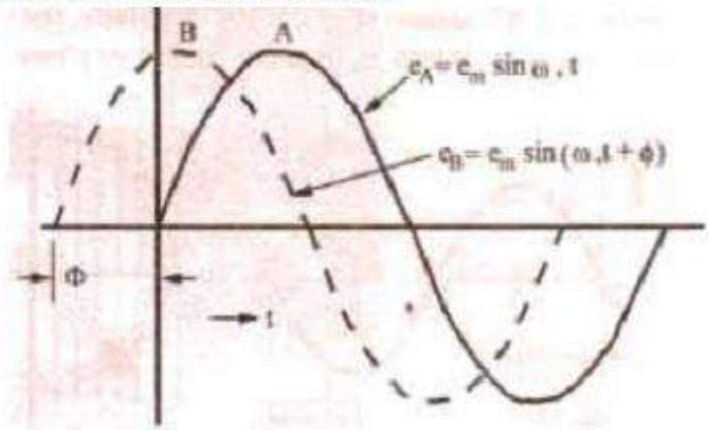
- **Waveform**: A curve which shows the variation of voltage and current w.r.t time or rotation

- Phase & Phase difference:



<u>In phase</u>:
$$e_A = E_{mA} Sin\omega t$$
$$e_B = E_{mB} Sin\omega t$$

<u>Out of phase:</u> i) B leads A

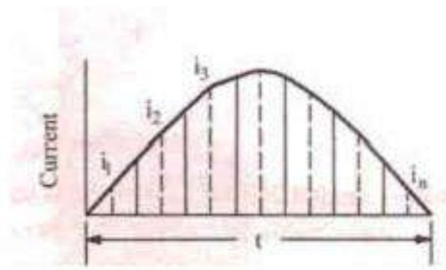


Phase difference Φ.
$$e_A = E_m Sin\omega t$$
$$e_B = E_{mB} Sin(\omega t + \alpha)$$

ii) A leads B or B lags A

$$e_A = E_m Sin\omega t$$
$$e_B = E_m Sin(\omega t - \alpha)$$

Root mean Square (RMS) or effective or virtual value of A.C:-



$I_{rms} = \sqrt{\frac{i_1^{\,2} + i_2^{\,2} + \ldots.. + i_n^{\,2}}{n}}$ = Square root of the mean of square of the instantaneous currents

- It is the square root of the average values of square of the alternating quantity over a time period.

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(\omega t)d(\omega t)}$$

RMS value by integration Method

Root mean square value :→

An alternating current varies from instant to instant, It is required to determine an equivalent direct current that will produce the same heat, in the same time interval, in the same resistor. In other words ac interms of dc which has a constant value is known as root mean square value or effective value.

The effective value of a.c. is equal to that the value of direct current which will produce the same heat in the same time in the same resistor.

Heat produced by an alternating current of instantaneous value i in resistor R in time dt is $= i^2 R\,dt$

The total heat produced in one cycle or in time T is

$$H_{ac} = \int_0^T i^2 dt \qquad \text{—— (1)}$$

Heat produced by the equivalent direct current I in resistor R in time T is

$$H_{dc} = I^2 RT \qquad \text{--- (3)}$$

$$H_{dc} = H_{ac} \qquad \text{--- (3)}$$

$$I^2 RT = \int_0^T i^2 dt \qquad \text{--- (4)}$$

$$I^2 = \frac{1}{T}\int_0^T i^2 dt \qquad \text{--- (5)}$$

$$I = \sqrt{\frac{1}{T}\int_0^T i^2 dt} \qquad \text{--- (6)}$$

the effective value of voltage can be produced from eqn. (A) so

$I = \frac{V}{R}$, $i = v/R$ put this value in eqn. (A) $\Rightarrow$

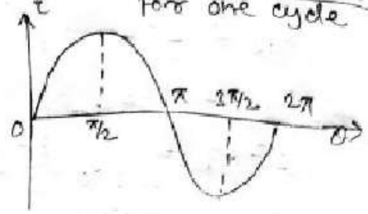
$$\frac{V^2}{R} RT = \int_0^T \frac{v^2}{R} R dt$$

$$V = \sqrt{\frac{1}{T}\int_0^T v^2 dt} \qquad \text{--- (7)}$$

the rms value of an alternating voltage is that value of direct voltage which will produce the same heat in the same resistor in the same time.

Average value of sinusoidal waveform

For one cycle



we know

$i = I_m \sin\theta$

Average value of current for whole cycle is ~~Area~~

$$\frac{\text{Area under the sine curve}}{\text{length of the base of the curve}} = \frac{A}{2\pi}$$

OR

$$I_{av} = \frac{1}{T}\int_0^T i\,dt = \frac{1}{2\pi}\int_0^{2\pi} i\,dt$$

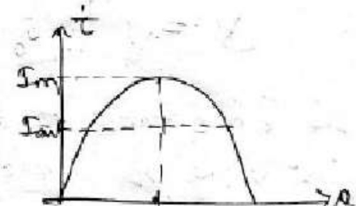
$$\frac{1}{2\pi}\int_0^{2\pi} I_m \sin\theta \, d\theta = 0$$

$I_{av} = 0$

Similarly we can prove that

$V_{av} = 0$

For half cycle



$$I_{av} = \frac{\text{area under sine curve}}{\text{length of base curve}}$$

$$= \frac{1}{\pi}\int_0^{\pi} i\,d\theta = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

$$\text{For } V_{av} = \frac{2V_m}{\pi} = 0.637 V_m$$

RMS value of sine wave

As we know $I^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{2\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{4\pi}\left[\theta\right]_0^{2\pi} - \frac{I_m^2}{8\pi}\left[\sin 2\theta\right]_0^{2\pi}$$

Now $\sin 4\pi = 0$ $\sin 0 = 0$ put this value in below eqn.

$$\Rightarrow I^2 = \frac{I_m^2}{4\pi}(2\pi - 0) - \frac{I_m^2}{8\pi}(\sin 4\pi - \sin 0)$$

$$I^2 = \frac{I_m^2}{2}$$

$$I = \frac{I_m}{\sqrt{2}} \Rightarrow \boxed{I = 0.707 I_m}$$

Form factor $\Rightarrow \dfrac{\text{RMS value}}{\text{Average value}}$

peak factor $= \dfrac{\text{Maximum value}}{0.707 \times \text{maximum value}}$

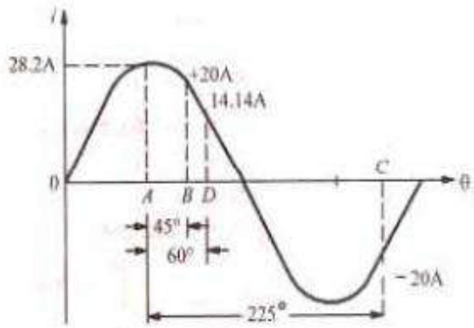
$$= \frac{\text{Maximum value}}{\text{RMS value}}$$

Example: An alternating current varying sinusoidally with a frequency of 50 Hz has an RMS value of 20 A. Write down the equation for the instantaneous value and find this value a) 0.0025 sec b) 0.0125 sec after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A?

$$I_m = 20\sqrt{2} = 28.2\,A$$

Solution:

$$\omega = 2\pi \times 50 = 100\pi \text{ rad/s}$$



The equation of the sinusoidal current wave with reference to point O as zero time point is

i= 28.2 sin 100πt Ampere

Since time values are given from point A where voltage has positive and maximum value, the equation may itself be referred to point A. In this case, equation becomes

i= 28.2 cos 100πt

i) When t= 0.0025 second

i = 28.2 cos 100π X 0.0025 ………….angle in radian

= 28.2 cos 100 X 180 X 0.0025 ………….angle in degrees

= 28.2 cos 45° = 20 A ……………………point B

ii) When t = 0.0125 sec

I =28.2 cos 100 X 180 X 0.0125

= 28.2 cos 225° = 28.2 X (-1/√2)

= -20 A …………………………point C

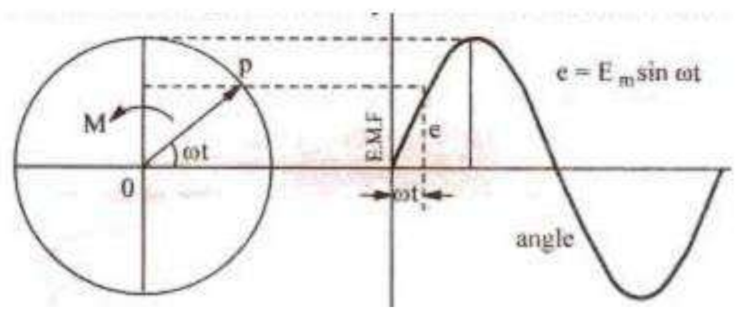
iii) Here $i = 14.14$ A

$14.14 = 28.2 \cos 100 \times 180\, t$

$\cos 100 \times 180\, t = ½$

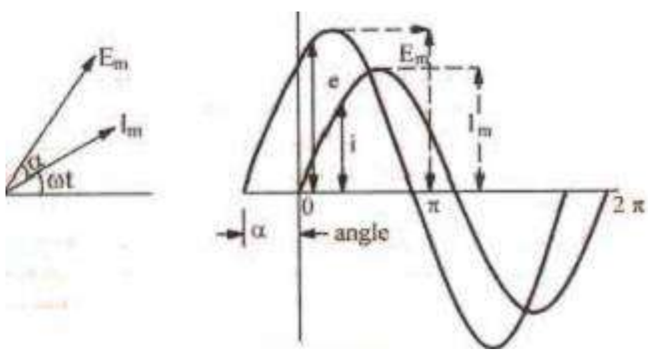
Or, $100 \times 180\, t = \cos^{-1}(1/2) = 60°$, $t = 1/300$ sec …………..point D

Phasor & Phasor diagram:



Phasor: Alternating quantities are vector (i.e having both magnitude and direction). Their instantaneous values are continuously changing so that they are represented by a rotating vector (or phasor). A phasor is a vector rotating at a constant angular velocity.

Phasor diagram: It is one in which different alternating quantities of the same frequency are represented by phasors with their correct phase relationship.



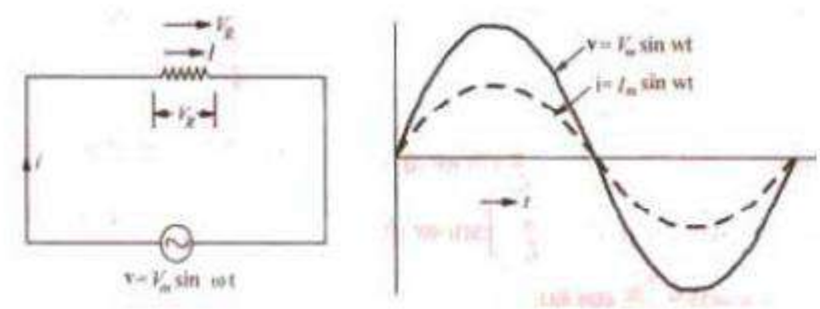
Points to remember: 1. The angle between two phasors is the phase difference

2. Reference phasor is drawn horizontally

3. Phasors are drawn to represent rms values and rotates in anticlockwise direction

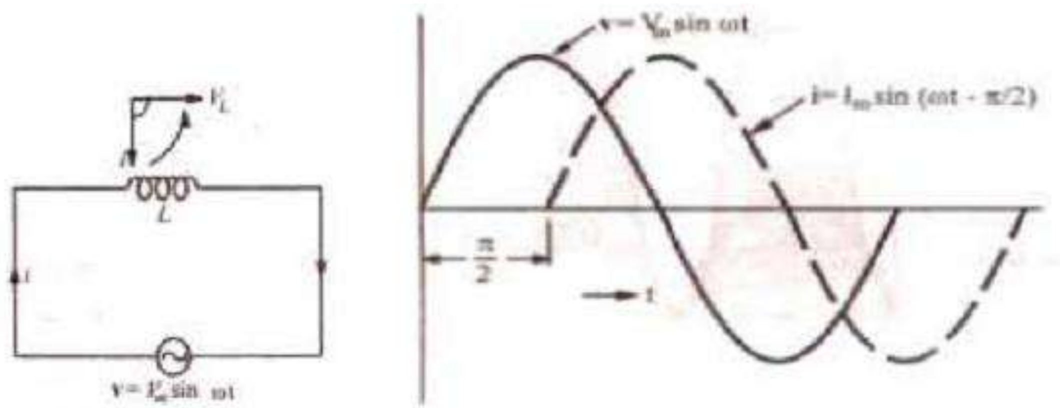
4. Phasor diagram represents a “still position” of the phasors in one particular point.

A.C through pure ohmic resistance only:



$$v = iR \text{ or } i = \frac{v}{R} = \frac{v_m}{R}\sin\omega t\,(\text{in phase})$$

A.C through pure inductance only:



$$v = L\frac{di}{dt} = V_m \sin\omega t$$

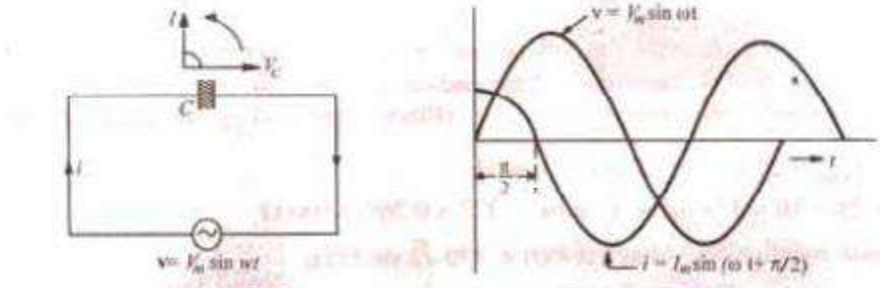
$$i = \frac{V_m}{L}\int \sin\omega t$$

$$i = -\frac{V_m}{\omega L}\cos\omega t$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad (\text{current lags by } 90^\circ)$$

$$\omega L = 2\pi f L = X_L = \text{inductive reactance}(\text{in } \Omega)$$

A.C through pure Capacitance only:



$$i = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin\omega t)$$

$$= \omega C V_m \cos\omega t$$

$$= \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{V_m}{\frac{1}{\omega C}}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= I_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{(current leads by } 90^\circ\text{)}$$

$$\frac{1}{\omega C} = X_C = \frac{1}{2\pi f C} = \text{capacitive reactance (in } \Omega\text{)}$$

Q. Write down the frequency, the rms and peak values of a voltage wave expressed as

$$v = 14.1 \sin 1000 \pi t$$

Calculate the current flowing when this voltage is applied across

(a) 5 Ω resistor (b) 1 mH inductor

(c) 150 μF Capacitor

Sol^n.

$$v = V_m \sin \omega t = V_m \sin 2\pi f t$$

$$v = 14.1 \sin 1000 \pi t$$

$$V_m = 14.1, \quad 2\pi f = 1000 \pi$$

$$f = \frac{1000}{2} = 500\text{ Hz}$$

rms value of voltage is

$$V = \frac{V_m}{\sqrt{2}} = \frac{14.1}{\sqrt{2}} = 10 \text{ volts}$$

(a) Current through 5 Ω resistor is

$$i_R = \frac{V_m}{R} \sin 2\pi f t$$

$$= \frac{14.1}{5} \sin 2\pi \times 500 t$$

$$i_R = 2.82 \sin 1000 \pi t \text{ Amp.}$$

(b) Current Through 1mH inductor is –

$$i_L = \frac{V_m}{X_L} \sin\left(2\pi f t - \pi/2\right)$$

But $L = 1\,mH = 1\times10^{-3}\,H$

$$X_L = 2\pi f L = 2\times\pi\times500\times1\times10^{-3}$$

$$\boxed{X_L = 3.142\,\Omega}$$

$$i_L = \frac{14.1}{3.142} \sin\left(1000\pi t - \pi/2\right)$$

$$i_L = 4.48 \sin\left(1000\pi t - \pi/2\right) \text{ Amp.}$$

(c) Current Through 150 μF Capacitor is

$$i_C = \frac{V_m}{X_C} \sin\left(2\pi f t + \pi/2\right)$$

$$C = 150\,\mu F = 150\times10^{-6}\,F$$

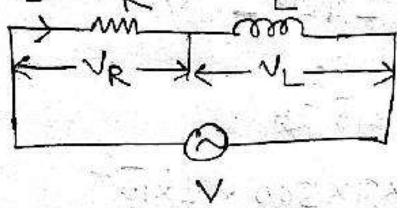
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\times500\times\pi\times150\times10^{-6}}$$

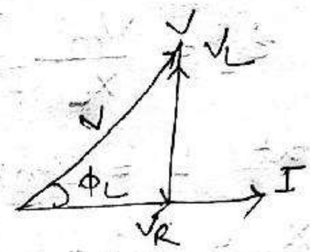
$$\boxed{X_C = 2.12\,\Omega}$$

$$i_C = \frac{14.1}{2.12} \sin\left(1000\pi t + \pi/2\right)$$

$$i_C = 6.65 \sin\left(1000\pi t + \pi/2\right) \text{ Amp.}$$

Series R-L circuit





Consider a ckt. containing a resistance R and inductance L are connected in series.

Let V = Supply voltage, I = ckt. current

V_R = voltage across R $= RI$

V_L = voltage across L $= X_L I$ $= 2\pi f L$

ϕ_L = phase angle between I and V

Now $V = V_R + V_L$

$$V^2 = V_R^2 + V_L^2 = (RI)^2 + (X_L I)^2$$

$$\frac{V^2}{I^2} = R^2 + X_L^2$$

$$V/I = \sqrt{R^2 + X_L^2}$$

$Z = \sqrt{R^2 + X_L^2}$ where Z = impedance of the ckt $= V/I$

$$Z_L = \frac{\text{rms voltage}}{\text{rms current}}$$

From voltage triangle, $\cos\phi_L = \frac{V_R}{V}$

$\sin\phi_L = V_L/V$

$\cos\phi_L = R/Z_L$

$R = Z_L \cos\phi_L$

$\sin\phi_L = X_L/Z_L$

$X_L = Z_L \sin\phi_L$

$$\boxed{\tan\phi_L = \frac{V_L}{V_R}}$$

$$\boxed{\tan\phi_L = X_L/R}$$

Q. A coil has an inductance of 20mH and a resistance of 5Ω. It is connected across a supply voltage $v = 50 \sin 314t$. Write down the expression for supply current.

Soln $L = 20mH = 20 \times 10^{-3}$ H

$R = 5\Omega$, $v = 50 \sin 314t$

But $2\pi f t = 314t \Rightarrow 2\pi f = 314$

$X_L = 2\pi f L = 314 \times 20 \times 10^{-3}$

$X_L = 6.28\Omega$

$Z_L = \sqrt{R^2 + X_L^2} = \sqrt{5^2 + 6.28^2}$

$\boxed{Z_L = 8\Omega}$

$I_m = \frac{V_m}{Z_L} = \frac{50}{8} \Rightarrow \boxed{I_m = 6.25 \text{ Amp}}$

$\cos\phi_L = \frac{R}{Z_L} = 5/8$

$\cos\phi_L = 0.625 \Rightarrow \phi_L = \cos^{-1} 0.625$

$\phi_L = 51.3^\circ$

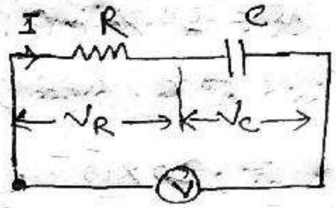
So the ckt. is inductive and the current lags behind the voltage by 51.3° and the supply current can be written as

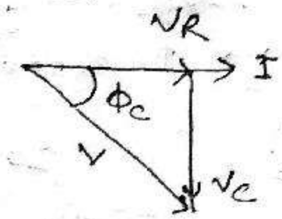
$i = I_m \sin(\omega t - \phi_L) = I_m \sin(314t - \phi_L)$

$i = 6.25 \sin(314t - 51.3^\circ)$

Series R-C circuit

Consider a ckt. containing a resistance R and a capacitance C in series connected.





V_R = voltage drop across R = RI

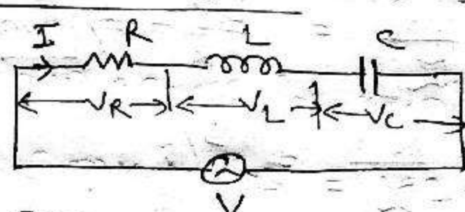
V_C = " across C $= X_C I = \frac{1}{2\pi f C} I$

$V = V_R + V_C$

Impedance $Z = \sqrt{R^2 + X_C^2}$

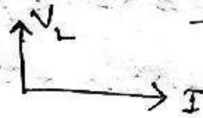
$Z = \frac{V}{I} \Rightarrow V = ZI$, $\tan\phi_c = \frac{V_C}{V_R}$

Series R-L-C ckt.

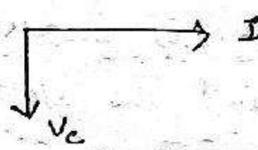


V_R in phase with I =

V_L leading I by 90° =



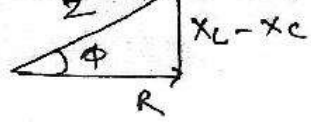
V_C lagging I by 90° =



$V = V_R + V_L + V_C$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$, $\tan\phi = \frac{X_L - X_C}{R}$

$V = ZI$ and $Z = \frac{V}{I}$



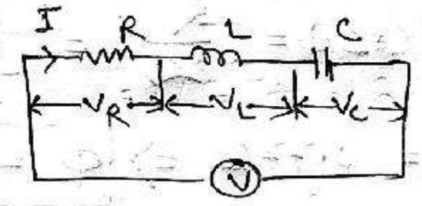
Q. A series ckt. consists of a 300Ω non-inductive resistor, a 7.95μF Capacitor and a 2.06H inductor of negligible resistance. If The supply voltage is 250 V at 50Hz, Calculate (a) The ckt. current (b) ~~phase~~ phase angle (c) voltage drop across each element.

Solⁿ:

$R = 300\Omega$, $L = 2.06H$

$C = 7.95\mu F = 7.95\times10^{-6} F$

Inductive reactance of The ckt. is X_L



$X_L = 2\pi f L = 2\pi \times 50 \times 2.06$

$$\boxed{X_L = 647\Omega}$$

Capacitive reactance of The ckt. $= X_C = \frac{1}{2\pi f C}$

$$X_C = \frac{1}{2\pi \times 50 \times 7.95\times10^{-6}} = 400\Omega$$

Net reactance of The ckt. $\Rightarrow X = X_L - X_C$

$X = 647 - 400 = 247\Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(300)^2 + (247)^2}$$

$$\boxed{Z = 388.6\Omega}$$

Impedance of ckt. $= 388.6\Omega$

(a) Current in the ckt. $I = \frac{V}{Z}$,

$I = \frac{\text{total voltage of ckt.}}{\text{Impedance of the ckt.}}$

$I = \frac{250}{388 \cdot 6} = 0 \cdot 643\,A.$

$\boxed{I = 0 \cdot 643\,A}$

(b) Phase angle of the ckt. (ϕ)

but $\cos\phi = \frac{R}{Z} = \frac{300}{388 \cdot 6}$

$\cos\phi = 0 \cdot 772 \Rightarrow \boxed{\phi = 39 \cdot 46^\circ}$ lagging

Here The ckt. is inductive due to lagging.

(c) Voltage droop across the resistor $= V_R$

$V_R = RI = 300 \times 0 \cdot 643 = 192 \cdot 9V$

Voltage droop across the inductor $= V_L$

$V_L = X_L I = 647 \times 0 \cdot 643 = 416\,V$

Voltage droop across the Capacitor $= V_C$

$V_C = X_C I = 400 \times 0 \cdot 643 = 257 \cdot 2V$

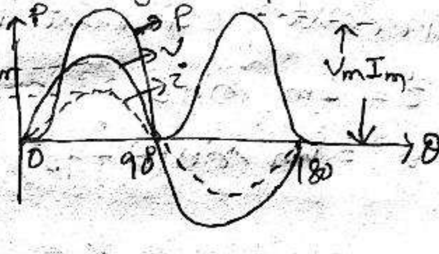
Hence
$$\boxed{\begin{aligned} V_R &= 192 \cdot 9V \\ V_L &= 416\,V \\ V_C &= 257 \cdot 2V \end{aligned}}$$

Power in a pure resistance :→

In a purely resistive ckt, the voltage and current are in phase so it may be expressed as

$v = V_m \sin\theta$

$i = I_m \sin\theta$



Instantaneous power

$$p = vi = V_m I_m \sin^2\theta = \frac{1}{2} V_m I_m (1 - \cos 2\theta)$$

If the voltage & current are negative, their product is still positive. Hence the power flow is only in the direction from source to load resistance R. So the power received by R is called active ~~power~~ energy. The rate of this energy consumption is called active power. It can be expressed as

$$P_R = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} V_m I_m (1 - \cos 2\theta)\, d\theta$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} 1 \cdot d\theta - \frac{V_m I_m}{4\pi}\int_0^{2\pi} \cos 2\theta\, d\theta$$

$$= \frac{V_m I_m}{2} = \frac{(\sqrt{2}\,V)(\sqrt{2}\,I)}{2}$$

$$\boxed{P_R = VI}$$

In purely resistive ckt

$$V = V_R \quad \& \quad V_R = RI$$

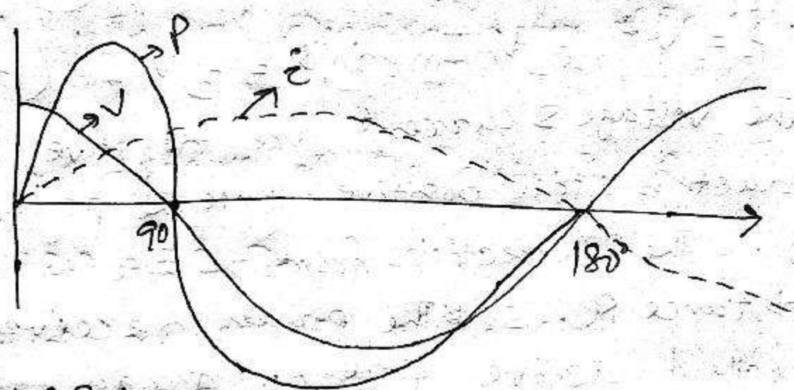
$$\boxed{P_R = V_R I = I^2 R = \frac{V_R^2}{R}}$$

Power in a pure Inductive :→

In purely inductive ckt. voltage leads current / current lags voltage by 90° so

$i = I_m \sin\theta$

$v = V_m \sin(\theta + 90^\circ)$



Instantaneous $p = vi$

$= V_m I_m \sin(\theta + 90^\circ) \cdot \sin\theta = V_m I_m \cos\theta \sin\theta$

$p = \frac{V_m I_m}{2} \sin 2\theta$

Power across inductor $= P_L$

$P_L = \frac{1}{2\pi} \int_0^{2\pi} p \, d\theta$

$\frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\theta \, d\theta$

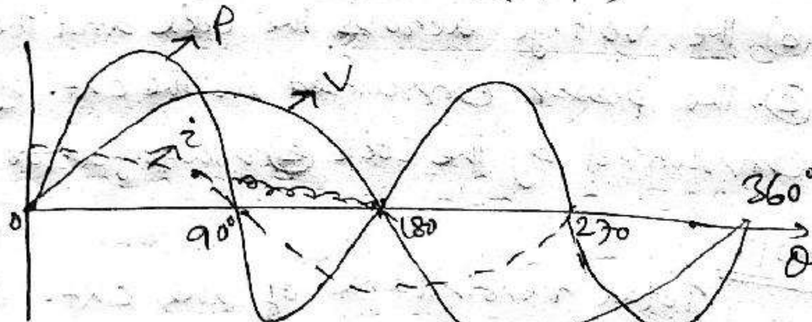
$\boxed{P_L = 0}$

In purely inductive ckt. power over a complete one cycle is zero.

Power in a pure Capacitive :→

In a purely capacitive ckt current leads the voltage by 90° so

$v = V_m \sin\theta$, $i = I_m \sin(\theta + 90°)$



Instantaneous power $P = vi$

$$P = V_m I_m \sin\theta \sin(\theta + 90°)$$
$$= V_m I_m \sin\theta . \cos\theta$$
$$P = \frac{V_m I_m}{2} \sin 2\theta$$

Active power is P_c

$$P_c = \frac{1}{2\pi} \int_0^{2\pi} P \, d\theta$$
$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\theta \, d\theta$$

$$\boxed{P_c = 0}$$

In purely capacitive ckt. power over a complete one cycle is zero.

Q. A coil of resistance 12 Ω and inductance 0.05H, a non-inductive resistor of 20 Ω resistance and a loss-free 40 µF Capacitor are connected in series across a 240V, 50Hz ac supply. Calculate (a) Current (b) The voltage across the coil and the capacitor (c) The power consumed in the ckt. (d) The power consumed by the coil (e) power factor of the ckt.

Sol

Total resistance of the ckt. is R

$R = 12 + 20 = 32\,\Omega$

Inductive reactance of the coil

$$X_L = 2\pi f L$$

$$X_L = 2\pi \times 50 \times 0.05 = 15.7\,\Omega$$

Capacitive reactance of the ~~coil~~ Capacitor

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 40 \times 10^{-6}}$$

$$X_C = 79.5\,\Omega$$

Impedance of the coil is Z_{coil}

$$Z_{coil} = \sqrt{R_{coil}^2 + X_L^2} = \sqrt{12^2 + 15.7^2}$$

$$Z_{coil} = 19.76\,\Omega$$

Impedance of the ckt. is Z

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{32^2 + (15.7 - 79.5)^2}$$

$$\boxed{Z = 71.3\,\Omega}$$

(a) Current of the ckt. $I = \frac{V}{Z} = \frac{240}{71.3}$

$$\boxed{I = 3.37\,A}$$

(b) voltage across the coil ⇒ $V_{coil} = Z_{coil} I$

$$V_{coil} = 19.76 \times 3.37 = 66.6V$$

$$\boxed{V_{coil} = 66.6V}$$

(c) voltage across the Capacitor $= V_c = X_c I$

$$V_c = 79.5 \times 3.37 = 267.9V$$

$$\boxed{V_c = 267.9V}$$

(c) Power consumed by the ckt. $P = I^2 R$

$$P = (3.37)^2 \times 32 = 363.4\,W$$

$$\boxed{P = 363.4W}$$

(d) power consumed by the coil ⇒ $P_{coil} = I^2 R_{coil}$

$$P_{coil} = (3.37)^2 \times 12 = 136.3\,W$$

$$\boxed{P_{coil} = 136.3\,W}$$

(e) power factor of the ckt. is

$$\cos\phi = R/Z = \frac{32}{71.3}$$

$$\boxed{\cos\phi = 0.4488}$$

Hence power factor is leading, $\boxed{\text{so } X_C > X_L}$

Q. A parallel ckt. consists of two branches, one containing a coil of resistance 5 Ω and inductance 38.2 mH, the other non-inductive resistance 16 Ω in series with a capacitor of 300 μF capacitance. The ckt. is connected to a 240V, 50Hz supply. Determine (a) The current in each branch (b) the total current (c) The ckt. phase angle (d) ckt. impedance (e) The component of an equivalent ckt. consisting of a resistance and reactance

Soln

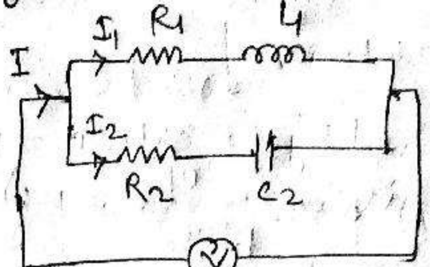


fig ①

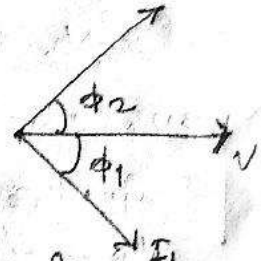


fig. ②

(a) Branch-1 $R_1 = 5\Omega$, $L = 38.2\text{mH} = 38.2\times10^{-3}\text{H}$

Inductive reactance $X_L = 2\pi f L$

$$X_L = 2\pi \times 50 \times 38.2\times10^{-3}\text{H}$$

$$X_L = 12\Omega$$

Impedance $Z_1 = \sqrt{R_1^2 + X_L^2}$

$$Z_1 = \sqrt{5^2+12^2} \Rightarrow Z = 13\Omega$$

$$I_1 = \frac{V}{Z_1} = \frac{240}{13} \Rightarrow I_1 = 18.46\text{A}.$$

$$\cos\phi_1 = \frac{R_1}{Z_1} = \frac{5}{13} = 0.3846$$

$$\Rightarrow \phi_1 = 67.38^\circ \text{ (lag)}$$

Φ_1

Branch-2 $\Rightarrow R_2 = 16\,\Omega,\ C_2 = 300\,\mu F$

Capacitive reactance, $X_{C_2} = \frac{1}{2\pi f C_2}$

$$X_{C_2} = \frac{1}{2\pi \times 50 \times 300 \times 10^{-6}} = 10.61\,\Omega$$

$$\boxed{X_{C_2} = 10.61\,\Omega}$$

Impedance $Z_2 = \sqrt{R_2^2 + X_{C_2}^2} = \sqrt{16^2 + 10.61^2}$

$$Z_2 = 19.2\,\Omega$$

Current $I_2 = \frac{V}{Z_2} = \frac{240}{19.2} \Rightarrow \boxed{I_2 = 12.5\ A}$

$$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{16}{19.2}$$

$$\cos\phi_2 = 0.833 \Rightarrow \boxed{\phi_2 = 33.56\ \text{lead}}$$

Resolving the currents horizontally and vertically in fig ② we get

$$I\cos\phi = I_1\cos\phi_1 + I_2\cos\phi_2$$

$$= 18.46\cos(67.38) + 12.5\cos(33.56)$$

$$I\cos\phi = 17.52$$

$$I\sin\phi = I_1\sin\phi_1 + I_2\sin\phi_2$$

$$= 18.46\sin(67.38) + 12.5\sin(33.56)$$

$$I\sin\phi = -10.13$$

$$I = (I\cos\phi)^2 + (I\sin\phi)^2 = 20.24A.$$

$$\boxed{I = 20.24\ A}$$

(c) ckt. phase angle = tan φ

~~tan φ~~ $\tan\phi = \frac{I \sin\phi}{I \cos\phi} = \frac{-10.13}{17.52}$

$\tan\phi = -0.5782$

$\phi = \tan^{-1}(-0.5782) = -30.04^\circ$

$\phi = 30.04^\circ$ (lagging)

(d) ckt. impedance :—

$Z = \frac{V}{I} = \frac{240}{20.24} \Rightarrow \boxed{Z = 11.86\,\Omega}$

(e) Equivalent resistance of the ckt. is

$R = Z\cos\phi = 11.86 \times \cos 30.04$

$\boxed{R = 10.27\,\Omega}$

Equivalent inductive reactance is X_L

$X_L = Z\sin\phi = 11.86 \sin 30.04^\circ$

$\boxed{X_L = 5.94\,\Omega}$

A complex number **A** is written in the form

$$\mathbf{A} = a + jb$$

where a and b are real numbers and the symbol j is used to denote $\sqrt{-1}$. The symbol j is the unit imaginary number, or imaginary operator. We call a as the real part of **A**. It is noted by the symbol *Re* [**A**] so that

$$\text{Re}\,[\mathbf{A}] = a$$

The number b is called the imaginary part of the complex number **A**. It is denoted by the symbol *Im* [**A**] so that

$$Im\,[\mathbf{A}] = b$$

It is to be noted that the imaginary part of a complex number is a real number, but j times the imaginary part is an imaginary number. Thus, jb is an imaginary number. The imaginary part of **A** is b and *not* jb. It is also to be noted that the real number is a special case of a complex number. A real number is a complex number with its imaginary part equal to zero. For example, real number 5 is equal to $(5 + j0)$. Similarly, an imaginary number is a special case of complex number with its real part equal to zero. For example, the imaginary number $j3$ is equal to $(0 + j3)$.

Since $\quad j = \sqrt{-1}$

$$j^2 = j \,.\, j = -1$$

$$j^3 = j^2 \,.\, j = -j$$

$$j^4 = j^2 \,.\, j^2 = (-1) \times (-1) = 1$$

$$\frac{1}{j} = \frac{j}{j \,.\, j} = \frac{j}{-1} = -j$$

In electrical engineering, the symbol j is used for $\sqrt{-1}$ rather than i as is the norm in mathematics. This is because of the use of the symbol i for current in electrical engineering.

axis. It is the axis of real numbers. The vertical axis Oy is called the *imaginary axis*, *quadrature axis* or *j axis*.

It is the axis of imaginary numbers. There is, of course, nothing "imaginary" about the component of the complex number **A** along the y axis. However, in order that complex number algebra obey all the same laws as real number algebra, that component must be multiplied by the square root of a negative number. Since the square root of all real numbers must be positive, the square root of a negative number is called imaginary.

The plane of the real axis and the quadrature axis is called a *complex plane* as shown in Fig. 13.1.

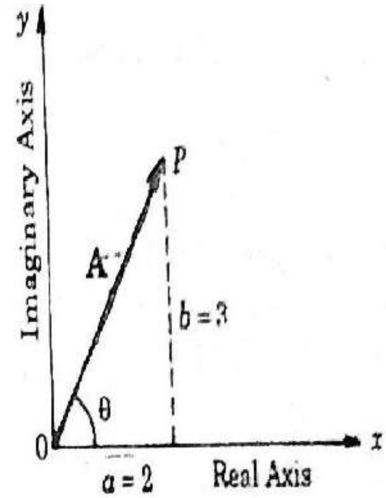


Fig. 13.1. Representation of a complex number **A** in the complex plane.

The complex number $(2 + j3)$ is a sum of two numbers 2 and $j3$. The number 2 is real and $j3$ is imaginary. The point P in Fig. 13.1 to represent complex number $(2 + j3)$ on the complex plane has a horizontal coordinate of 2 and a vertical coordinate of 3.

If a complex number is represented in the form

$$\mathbf{A} = a + jb$$

it is said to be given in the *rectangular* or *Cartesian form*.

13.4 POLAR REPRESENTATION OF COMPLEX NUMBERS

The location of a point in a complex plane may also be represented in terms of the polar coordinates. In this system the radial distance r of the point from the origin, and the angle θ made between the radial line and the positive real axis measured anticlockwise, are specified. In symbolic form it may be written as

$$A = r \angle \theta \qquad \text{(read } r \text{ at an angle } \theta\text{)}$$

This representation is shown in Fig. 13.3.

Here $OP = r$ and $\angle xOP = \theta$; r is called the *magnitude* or *absolute value* or *modulus* of the complex number A and θ is known as the *argument* or *angle* of the complex number A. The magnitude is also written as

$$r = |A|$$

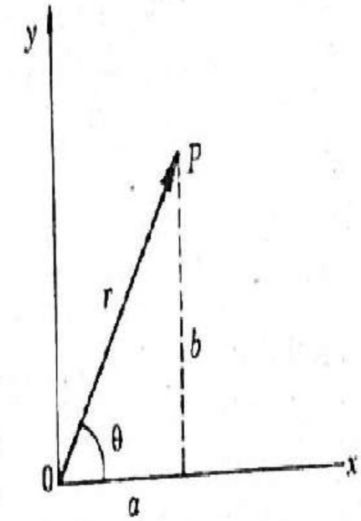


Fig. 13.3. Polar representation of complex number

Polar to Rectangular Conversion:

In Cartesian form

$$A = a + jb$$

In polar form

$$A = r \angle \theta$$

From Fig. 13.3

$$a = r\cos\theta$$

$$b = r\sin\theta$$

$$r = \sqrt{(a^2 + b^2)} = \sqrt{[(\text{real term})^2 + (\text{imaginary term})^2]}$$

$$\tan\theta = \frac{b}{a} = \frac{\text{imaginary term}}{\text{real term}}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

Therefore

$$a + jb = \sqrt{(a^2 + b^2)}\ \underline{/\ \tan^{-1} b/a}$$

Similarly,

$$a - jb = \sqrt{(a^2 + b^2)}\ \underline{/\ \tan^{-1} - b/a}$$

These relations may be used to convert rectangular form into polar form or vice versa.

13.12 USE OF SCIENTIFIC CALCULATOR FOR INTERCONVERSION OF RECTANGULAR AND POLAR FORMS

The use of scientific calculator is very helpful in converting quantities from rectangular to polar form and vice versa. The following examples illustrate the procedure :

Example 13.6

Convert $3 + j4$ to polar form.

$$3 \ \boxed{\text{INV}}\ \overset{R \to P}{\boxed{+}}\ 4 \ \underset{\downarrow \atop 5}{\boxed{=}}\ \underset{\downarrow \atop 53.13}{\boxed{X \leftrightarrow Y}}$$

$\therefore\ 3 + j4 = 5\ \underline{/\ 53.13^\circ}$

Example 13.7

Convert $-6 + j8$ to polar form.

Solution

$$6 \ \boxed{+/-}\ \boxed{\text{INV}}\ \overset{R \to P}{\boxed{+}}\ 8 \ \underset{\downarrow \atop 10}{\boxed{=}}\ \underset{\downarrow \atop 126.87}{\boxed{X \leftrightarrow Y}}$$

$\therefore\ -6 + j8 = 10\ \underline{/\ 126.87^\circ}$

Example 13.8

Convert $5 - j12$ to polar form.

Solution

$$5 \ \boxed{\text{INV}}\ \overset{R \to P}{\boxed{+}}\ 12 \ \boxed{+/-}\ \underset{\downarrow \atop 13}{\boxed{=}}\ \underset{\downarrow \atop -67.38}{\boxed{X \leftrightarrow Y}}$$

$\therefore\ 5 - j2 = 13\ \underline{/ -67.38^\circ}$

Example 13.9

Convert $8 \angle 30^\circ$ to rectangular form.

Solution

$$8 \ \boxed{\text{INV}}\ \overset{P \to R}{\boxed{-}}\ 30 \ \underset{\downarrow \atop 6.928}{\boxed{=}}\ \underset{\downarrow \atop 4}{\boxed{X \leftrightarrow Y}}$$

$\therefore\ 8 \angle 30^\circ = 6.928 + j4$

EXAMPLE 13.25

An impedance of $20 \angle 50° \ \Omega$ is connected in series with another impedance of $15 \angle -45° \ \Omega$ and the combination takes a current of 2 A. Calculate the voltage across each impedance.

SOLUTION

$$\mathbf{Z}_1 = 20 \angle 50° = 12.86 + j15.32 \ \Omega$$

$$\mathbf{Z}_2 = 15 \angle -45° = 10.61 - j10.61 \ \Omega$$

Total impedance of the series circuit

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 = (12.86 + j15.32) + (10.61 - j10.61)$$

$$= 23.47 + j4.71 = 23.94 \underline{/11.35°} \ \Omega$$

Taking **I** as reference phasor.

$$\mathbf{I} = 2 \angle 0°$$

By Ohm's law

$$\mathbf{V} = \mathbf{IZ} = (2 \angle 0°)(23.94 \underline{/11.35°})$$

$$= 47.88 \underline{/11.35°} \ \text{V}$$

$$\mathbf{V}_1 = \mathbf{I}_1\mathbf{Z}_1 = (2 \angle 0°)(20 \angle 50°) = 40 \angle 50° \text{V}$$

$$\mathbf{V}_2 = \mathbf{I}_2\mathbf{Z}_2 = (2 \angle 0°)(15 \angle -45°) = 30 \angle -45° \ \text{V}$$

EXAMPLE 13.20

An alternating voltage $(80 + j60)$ V is applied to a circuit and the current flowing is $(-4 + j10)$ A. Find (*a*) the impedance of the circuit, (*b*) the power consumed, and (*c*) the phase angle.

SOLUTION

$$\mathbf{V} = 80 + j60 = 100 \angle 36.87^\circ \text{ V}$$

$$\mathbf{I} = -4 + j10 = 10.77 \angle 111.8^\circ \text{ A}$$

The phase difference between the current **I** and voltage **V** is

$$\phi = 111.8 - 36.87 = 74.93^\circ$$

The current leads the voltage by an angle 74.93°. Therefore, the power factor is leading and is given by

$$\cos\phi = \cos 74.93^\circ = 0.26$$

The impedance of the circuit

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{100 \angle 36.87^\circ}{10.77 \angle 111.8^\circ} = 9.285 \angle -74.9^\circ \ \Omega$$

$$= (2.418 - j8.96)\ \Omega = R - jX_C$$

Power consumed in the circuit

$$P = I^2 R = (10.77)^2 \times 2.418 = 280.47 \text{ W}$$

or

$$P = VI\cos\phi = 100 \times 100.77 \times 0.26 = 280.5 \text{ W}$$

Example: In a particular R-L series ckt. a voltage of 10 V at 50 Hz produces a current of 700 ma while the same voltage at 75 Hz produces 500 mA. What are the values of R and L in the circuit.

Solution:

$$Z = \sqrt{R^2} + (2\pi \times 50L)^2 = \sqrt{R^2 + 98696L^2}$$

$$V = IZ \text{ or } 10 = 700 \times 10^{-3} \sqrt{(R^2 + 98696L^2)}$$

$$\sqrt{(R^2 + 98696L^2)} = 10/700 \times 10^{-3} = 100/7$$

$$\text{or } R^2 + 98696L^2 = 10000/49 \ldots\ldots\ldots\ldots\text{(i)}$$

ii) In the second case

$$Z = \sqrt{R^2 + (2\pi \times 75L)^2} = \sqrt{R^2 + (222066L^2)}$$

$$10 = 500 \times 10^{-3} \sqrt{(R^2 + 222066L^2)}$$

$$\sqrt{(R^2 + 222066L^2)} = 20$$

$$R^2 + 222066L^2 = 400 \ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

subtracting eq(i) from eq(ii), we get

$$222066L^2 - 98696L^2 = 400 - (10000/49)$$

$$123370L^2 = 196$$

$$L = 0.0398 \text{ H} = 40 \text{mH}$$

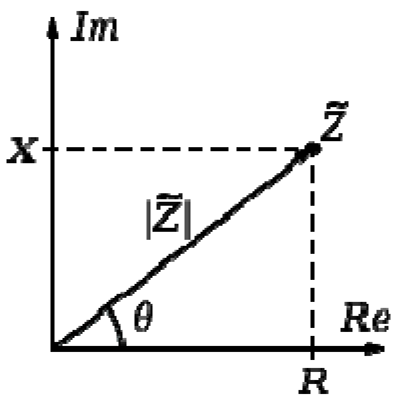
Substituting this value of L in eq(ii),we get

$$R^2 + 222066(0.398)^2 = 400$$

$$R = 6.9 \ \Omega$$

IMPEDANCE IN J NOTATION:

In quantitative terms, it is the complex ratio of the voltage to the current in an alternating current (AC) circuits. Impedance extends the concept of resistance to AC circuits and possesses both magnitude and phase, unlike resistance, which has only magnitude. When a circuit is driven with direct current (DC), there is no distinction between impedance and resistance.



Where X=Total reactance of the network (Both inductive and capacitive)

R=Resistance of the network in ohm.,

θ= Phasor angle in degree/Radian.

.

Z=R+jX, Where Z=impedance of the electrical network in ohm.

: R=Resistance of the network in ohm. , X=Reactance of the electrical network in ohm.

Admittance: In electrical engineering, admittance is a measure of how easily a circuit or device will allow a current to flow. It is defined as the inverse of impedance. The SI unit of admittance is the Siemens (symbol S).

Admittance is defined as: Y = 1/Z

Where

Y is the admittance, measured in Siemens; Z is the impedance, measured in ohms

The unit mho, and the symbol ℧ (an upside-down uppercase omega Ω), are also in common use.

Resistance is a measure of the opposition of a circuit to the flow of a steady current, while impedance takes into account not only the resistance but also dynamic effects (known as reactance). Likewise, admittance is not only a measure of the ease with which a steady current can flow, but the dynamic effects of the material's susceptance to polarization: **Y = G + j B**

Where, G is the conductance, measured in siemen, B is the susceptance, measured in siemen.

Instantaneous and Average Power :

The most general expressions for the voltage and current delivered to an arbitrary load are as follows:

$v(t) = V\cos(\omega t - \theta_V)$

$i(t) = I\cos(\omega t - \theta_I)$

Since the instantaneous power dissipated by a circuit element is given by the product of the instantaneous voltage and current, it is possible to obtain a general expression for the power dissipated by an AC circuit element: $p(t) = v(t)i(t) = V I \cos(\omega t)\cos(\omega t - \theta)$

It can be further simplified with the aid of trigonometric identities to yield

$p(t) = V I/2 \cos(\theta) + V I/2 \cos(2\omega t - \theta)$

where θ is the difference in phase between voltage and current

The average power corresponding to the voltage and current signal can be obtained

by integrating the instantaneous power over one cycle of the sinusoidal signal. Let

$T = 2\pi/\omega$ represent one cycle of the sinusoidal signals.

The average power, P_{av}, is given by the integral of the instantaneous power,

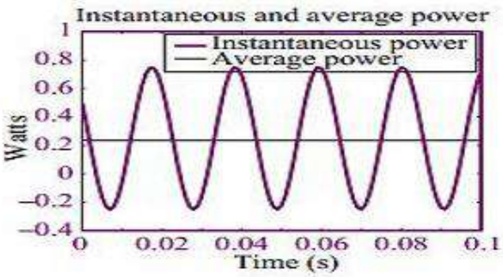
p(t), over one cycle:

$$P_{av} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{VI}{2}\cos(\theta)\,dt + \frac{1}{T}\int_0^T \frac{VI}{2}\cos(2\omega t - \theta)\,dt$$

$$P_{av} = \frac{VI}{2}\cos(\theta) \qquad \text{Average power}$$

since the second integral is equal to zero and cos(θ) is a constant.



In phasor notation, the current and voltage are given by

$$\mathbf{V}(j\omega) = Ve^{j0}$$
$$\mathbf{I}(j\omega) = Ie^{-j\theta}$$

impedance of the circuit elementdefined by the phasor voltage and currentto be

$$Z = \frac{V}{I}e^{-j(\theta)} = |Z|e^{j\theta_Z}$$

The expression for the average power using phasor notation

$$P_{av} = \frac{1}{2}\frac{V^2}{|Z|}\cos\theta = \frac{1}{2}I^2|Z|\cos\theta$$

Power Factor :

The phase angle of the load impedance plays a very important role in the absorption of power by load impedance. The average power dissipated by an AC load is dependent on the cosine of the angle of the impedance. To recognize the importance of this factor in AC power computations, the term cos(θ) is referred to as the power factor (pf). Note that the power factor is equal to 0 for a purely inductive or capacitive load and equal to 1 for a purely resistive load; in every other case,$0<pf<1$.If the load has an inductive reactance, then θ is positive and the current lags (or follows) the voltage. Thus, when θ and Q are positive, the corresponding power factor is termed lagging. Conversely, a capacitive load will have a negative Q, and hence a negative θ. This corresponds to a leading power factor, meaning that the load current leads the load voltage.A power factor close to unity signifies an efficient transfer of energy from the AC source to the load, while a small power factor corresponds to inefficient use of energy .Two equivalent expressions for the power factor are given in the following:

$$\text{pf} = \cos(\theta) = \frac{P_{av}}{\tilde{V}\tilde{I}} \qquad \text{Power factor}$$

where $\tilde{V}$ and $\tilde{I}$ are the rms values of the load voltage and current.

Complex Power:

The expression for the instantaneous power may be further expanded to provide further insight into AC power. Using trigonometric identities, we obtain the

$$\begin{aligned} p(t) &= \frac{\tilde{V}^2}{|Z|}[\cos\theta + \cos\theta\cos(2\omega t) + \sin\theta\sin(2\omega t)] \\ &= \tilde{I}^2|Z|[\cos\theta + \cos\theta\cos(2\omega t) + \sin\theta\sin(2\omega t)] \\ &= \tilde{I}^2|Z|\cos\theta(1 + \cos(2\omega t)) + \tilde{I}^2|Z|\sin\theta\sin(2\omega t) \end{aligned}$$

Recalling the geometric interpretation of the impedance Z

$|Z| \cos\theta = R$ and $|Z|\sin\theta = X$ are the resistive and reactive components of the load impedance,

The instantaneous power is:

$$\begin{aligned} p(t) &= \tilde{I}^2 R(1 + \cos(2\omega t)) + \tilde{I}^2 X\sin(2\omega t) \\ &= \tilde{I}^2 R + \tilde{I}^2 R\cos(2\omega t) + \tilde{I}^2 X\sin(2\omega t) \end{aligned}$$

Since P_{av} corresponds to the power absorbed by the load resistance, it is also called the real power, measured in units of watts (W). On the other hand, Q takes the name of reactive power, since it is associated with the load reactance. The units of Q are volt-amperes reactive, or VAR. Note that Q represents an exchange of energy between the source and the reactive part of the load; thus, no net power is gained or lost in the process, since the average reactive power is zero. In general, it is desirable to minimize the reactive power in a load.

The computation of AC power is greatly simplified by defining a fictitious but very

useful quantity called the complex power, S: $S = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^*$

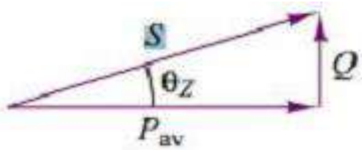
where the asterisk denotes the complex conjugate You may easily verify that this definition leads to the convenient expression

$$S = \tilde{V}\tilde{I}\cos\theta + j\tilde{V}\tilde{I}\sin\theta = \tilde{I}^2R + j\tilde{I}^2X = \tilde{I}^2Z$$

or

$$S = P_{av} + jQ$$

The complex power S may be interpreted graphically as a vector in the complex S plane.



$$|S| = \sqrt{P_{av}^2 + Q^2} = \tilde{V} \cdot \tilde{I}$$

$$P_{av} = \tilde{V}\tilde{I}\cos\theta$$

$$Q = \tilde{V}\tilde{I}\sin\theta$$

The magnitude of S, |S|, is measured in units of volt-amperes (VA) and is called

apparent power, because this is the quantity one would compute by measuring the rms load voltage and currents without regard for the phase angle of the load. The complex power may also be expressed by the product of the square of the rms current through the load and the complex load impedance:

$$S = \tilde{I}^2Z$$

or

$$\tilde{I}^2R + j\tilde{I}^2X = \tilde{I}^2Z$$

or, equivalently, by the ratio of the square of the rms voltage across the load to the

complex conjugate of the load impedance:

$$S = \frac{\tilde{V}^2}{Z^*}$$

Active, Reactive and Apparent Power:

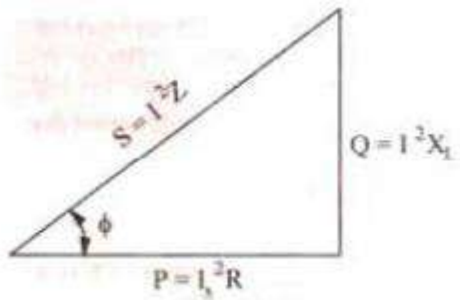


Fig. Power Triangle

$$S^2 = P^2 + Q^2$$

$$S = P + jQ$$

Apparent power, S: It is the product of rms values of the applied voltage and

circuit current. It is also known as wattless (idle) component

$S=VI=IZ x I=I^2Z$ Volt-amp

Active power or true power, P: It is the power which actually dissipated in the circuit resistance. It is also known as wattful component of power.

$P=I^2R=I^2 Z\cos\Phi=VI \cos\Phi$ watt

Reactive power, Q:- It is the power developed in the reactance of the circuit.

$Q=I^2X=I^2 Z\sin\Phi= VI\sin\Phi$ VAR

Introduction to resonance in series & parallel circuit:

Resonance: An AC circuit is said to be in resonance when the circuit current is in phase with the applied voltage. So, the power factor of the circuit becomes unity at resonance and the impedance of the circuit consists of only resistance.

Series Resonance: In R-L-C series circuit, both XL and XC are frequency dependent. If we vary the supply frequency then the values of XL and XC varies. At a certain frequency called resonant frequency (fr), XL becomes equal to XC and series resonance occurs.

At series resonance, $X_L = X_C$

$2\pi f_r L = 1/2\pi f_r C$

$f_r = 1/2\pi\sqrt{LC}$

Impedance of RLC series circuit is given by:

$$Z=\sqrt{R^2+(X_L-X_C)^2} \quad (\text{Since}, X_L = X_C)$$

$$Z=\sqrt{R^2}$$

$$Z=R$$

$$\cos\varphi=\frac{R}{Z}=\frac{R}{R}=1$$

Properties of series resonance:-

In series resonance,

- The circuit impedance Z is minimum and equal to the circuit resistance R.
- The circuit current I= V/Z = V/R and the current is maximum
- The power dissipated is maximum, $P=V^2/R$
- Resonant frequency is fr = $1/2\pi\sqrt{LC}$
- Voltage across inductor is equal and opposite to the voltage across capacitor
- Since power factor is 1, so zero phase difference. Circuit behaves as a purely resistive circuit.

Example: A series RLC circuit having a resistance of 50Ω, an inductance of 500 mH and a capacitance of 400 μF, is energized from a 50 Hz, 230 V, AC supply. Find a) resonant frequency of the circuit b) peak current drawn by the circuit at 50 Hz and c) peak current drawn by the circuit at resonant frequency.

Solution:

$$a) f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{500\times10^{-3}\times400\times10^{-6}}} = 11.25\ Hz$$

b) $R = 50\Omega$

$$X_L = \omega L = 2\pi\times50\times500\times10^{-3} = 157\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\times50\times400\times10^{-6}} = 7.9\Omega$$

$$X = X_L - X_C = 157 - 7.9 = 149.1\Omega$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{50^2 + 149.1^2} = 157.26\Omega$$

Peak supply voltage, $V_m = \sqrt{2}\ V_{rms} = \sqrt{2}\ (230) = 325.26\ V$

Hence peak current at 50Hz $I_m = \frac{V_m}{Z} = \frac{325.26}{157.26} = 2.068$

c) At resonance, $Z_0 = R = 50\Omega$

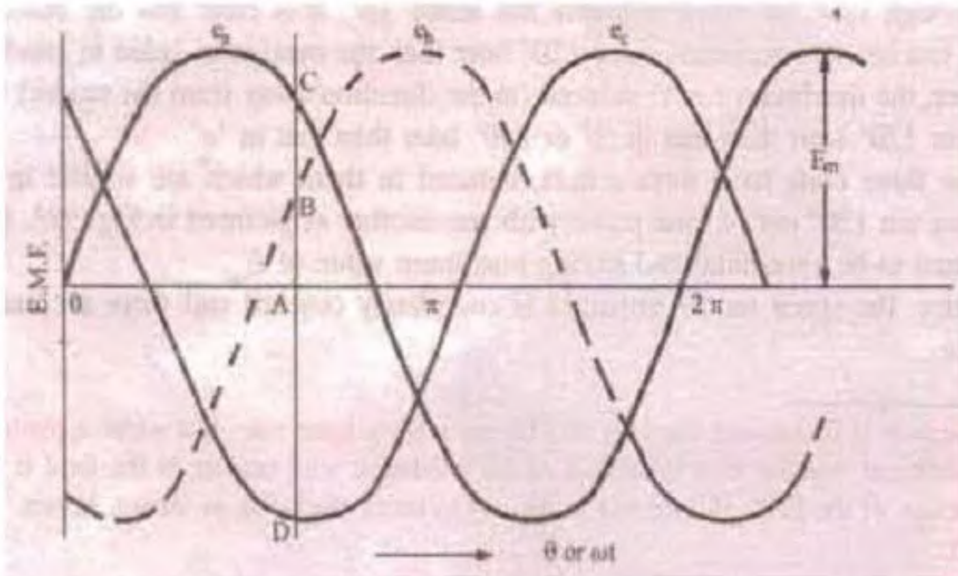
So, peak current during resonance, $I_{mo} = \frac{V_m}{R} = \frac{325.26}{50} = 6.5025A$

Parallel resonance: Points to remember:

- Net susceptance is zero, i.e 1/XC =XL/Z^2, XL x XC =Z^2 or L/C =Z
- Admittance equal to conductance
- Reactive or wattles component of line current is zero
- Dynamic impedance = L/CR Ω
- Line current at resonance is minimum V/L/CR but is in phase with applied voltage
- Power factor of the circuit is unity

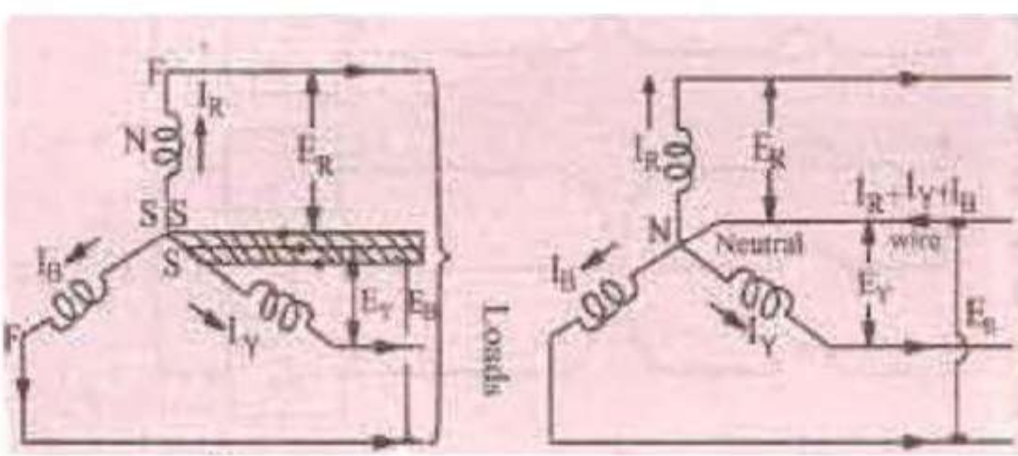
THREE PHASE AC CIRCUIT

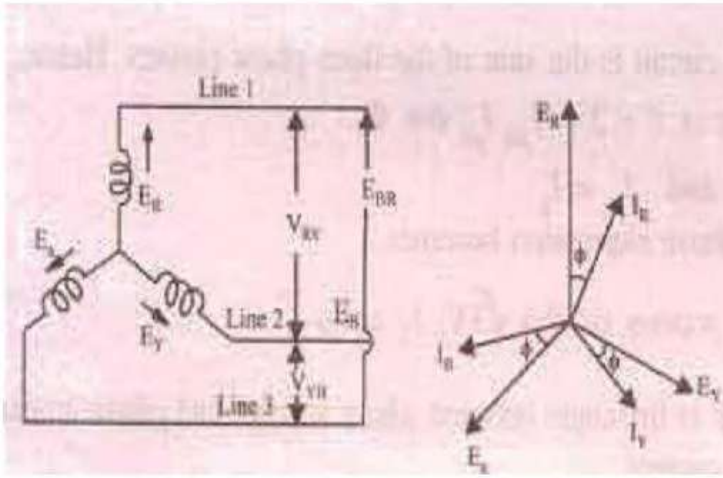
Three phase EMF Generation: -



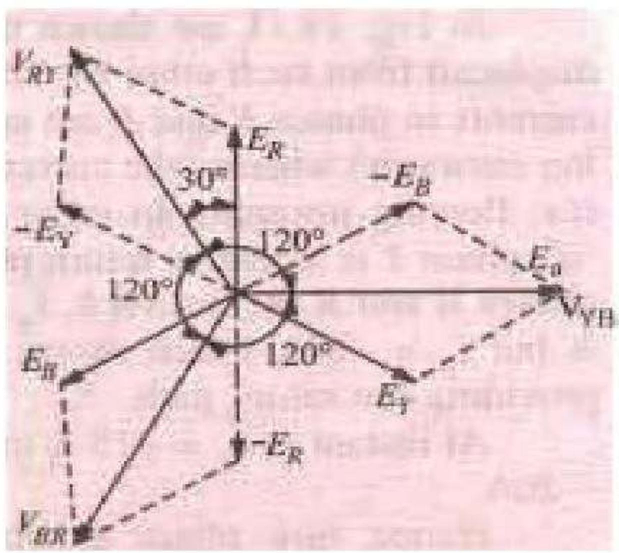
If the 3-coil windings W_1, W_2 and W_3 arranged at 120° apart from each other on the same axis are rotated, then the emf induced in each of them will have a phase difference of 120°. In other words if the emf (or current) in one winding (w_1) has a phase of 0°, then the second winding (w_2) has a phase of 120° and the third (w_3) has a phase of 240°.

Star (Y) connection:-





Phasor diagram:-



Here, E_R, E_Y, E_B are phase voltages and V_{RY}, V_{YB}, V_{BR} are line voltages

$$V_{RY} = \sqrt{E_R^{\,2} + E_Y^{\,2} + 2E_R E_Y \cos 60^\circ}$$
$$= \sqrt{E_R^{\,2} + E_R^{\,2} + 2E_R E_R \cos 60^\circ}$$
$$= \sqrt{3}\, E_R$$

Hence,

- Line voltage =√3 x phase voltage
- Line current = phase current
- Line voltages are also 120° apart
- Line voltage are 30° ahead of respective phase voltages
- The angle between line voltage and line current is (30°+Φ)

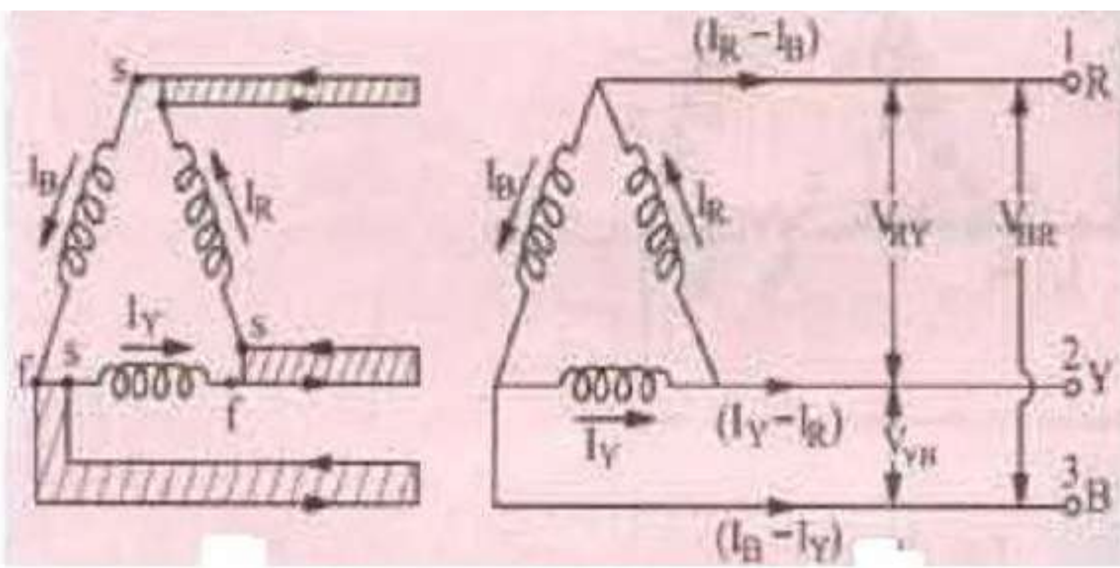
Power: Total power = 3 x phase power

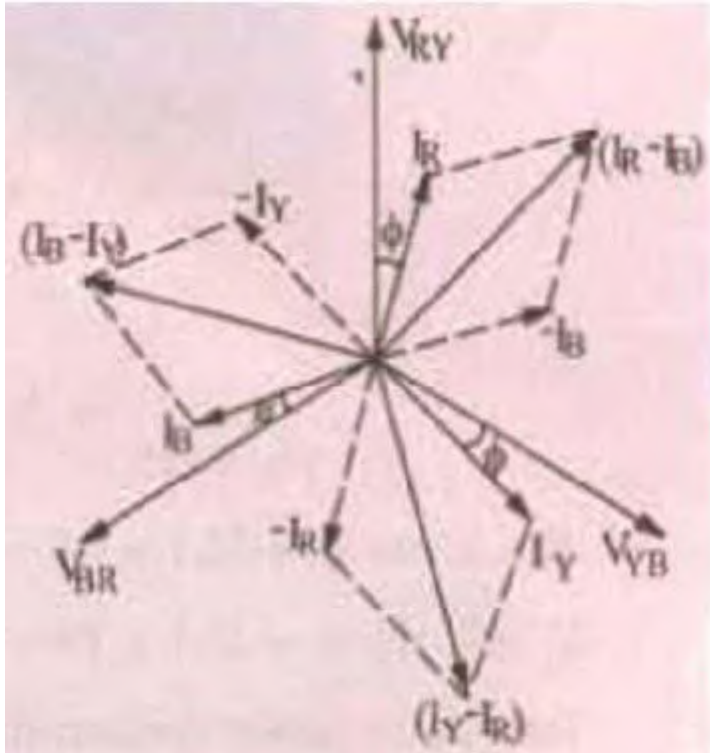
$=3 \times V_{ph} \times I_{ph} \times \cos\Phi$

$=\sqrt{3}\, V_L\, I_L \cos\Phi$

Φ is the angle between phase voltage and current

Delta-connection:





Phasor diagram

$$I_L = I_R - I_B$$

$$I_L = \sqrt{I_R^{\ 2} + I_B^{\ 2} + 2 I_R I_B \cos 60^\circ} = \sqrt{I_R^{\ 2} + I_R^{\ 2} + 2 I_R I_R \cos 60^\circ} = \sqrt{3} I_R$$

Hence,

- Line current =√3 phase current, Line currents are also 120° apart
- Line voltage = phase voltage, Line currents are 30° behind the phase currents
- Angle between line current and line voltage is 30°+Φ

Power: Total power = 3 x phase power

= 3 x V_{ph} I_{ph} cosΦ

= 3 x V_L x I_L/√3 x cosΦ

= √3 V_L I_L cosΦ

Active & True power = √3 V_L I_L cosΦ

Reactive power =√3 V_L I_L sinΦ

Apparent power =√3 V_L I_L

Example: A balanced star connected load of (8+j6) Ω per phase is connected to a balanced 3-phase 400 V supply. Find the line current, power factor, power and total volt-amperes.

Solution:

$$Z_{ph} = \sqrt{8^2 + 6^2} = 10\Omega$$

$$V_{ph} = \frac{400}{\sqrt{3}} = 231\,V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{10} = 23.1\text{ A}$$

$$\text{(i) } I_L = I_{ph} = 23.1\text{ A}$$

$$\text{(ii) p.f} = \cos\Phi = \frac{R_{ph}}{Z_{ph}} = \frac{8}{10} = 0.8\text{(lag)}$$

$$\text{(iii) Power } P = \sqrt{3}\ V_L I_L \cos\Phi$$

$$= \sqrt{3} \times 400 \times 23.1 \times 0.8$$

$$= 12{,}800\text{ W [Also, } P = 3{I_{ph}}^2 R_{ph} = 3(23.1)^2 \times 8 = 12{,}800\text{ W]}$$

(iv) Total volt-amperes,

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 400 \times 23.1 = 16{,}000\text{ VA}$$

End