Lecture Notes

On

Advance Control Systems

7Th Semester Electrical Engineering

Part 3: Modeling and Design of Compensators

By

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Course Objective:

The objective of this course is to equip students with a deep understanding of discrete-time control systems, state variable analysis, and nonlinear system behavior. Students will learn to analyze, design, and implement control strategies using modern techniques, including Z-transform methods, state-space representations, and Lyapunov stability analysis.

Course Outcome:

- **Analyze**: Analyze discrete-time and continuous-time control systems using Z-transform and state-space methods to determine system behavior.
- **Design**: Design control systems utilizing feedback strategies, pole placement, and observer design to achieve specified performance criteria.
- **Evaluate**: Evaluate the stability of linear and nonlinear systems using Routh's criterion and Lyapunov's methods, assessing their robustness.
- **Apply**: Apply techniques for modeling and simulating nonlinear systems, including phase plane and describing function methods, to solve practical engineering problems.

Compensation

Compensation

Re-design or addition of a suitable device to a control system is called compensation.

A device inserted into the system for the purpose of satisfying the specifications is called compensator. It basically introduce poles or zeros in the open loop TF to modify the performance of the system.

The performance of a system can be specified in terms of time domain specifications or freq. domain specification.

Damping factor or peak-overshoot are the measure of relative stability and rise time, settling time or natural freq. are the measure of speed of response in time domain.

similarly in freq. domain, the measure of relative stability is resonant peak (M_p) or phase margin (ϕ_{pm}) while the measure of speed of response is resonant freq. (ω_r) or bandwidth (ω_b).

Need of Compensation

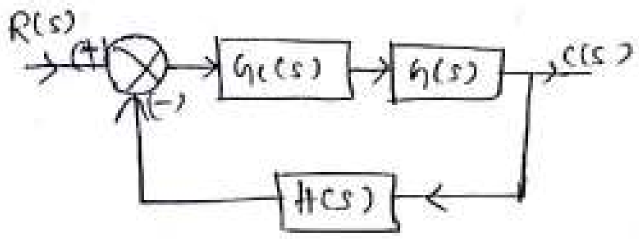
→ If a system is absolutely unstable, then compensation is required to stabilize it as well as to achieve a specified performance.

→ If a stable system is not able to meet the required performance specification, then compensation is needed.

Compensation scheme

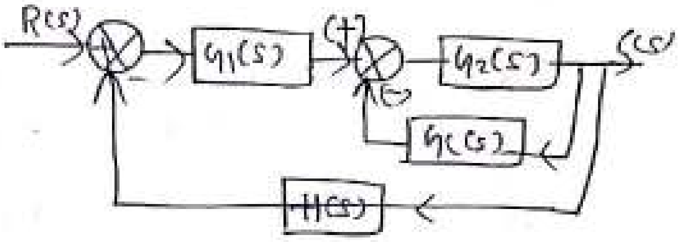
1) Series Compensation

In this compensation scheme, compensator tf will be placed in cascade/seriese with the plant TF.



2) Feedback compensation

In this scheme, the compensator TF will be placed in the feedback path.



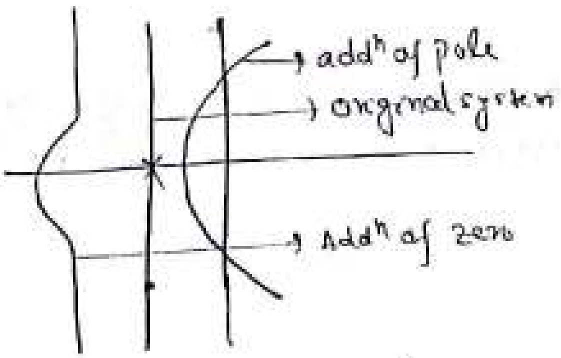
Root-locus approach of Designing compensators

In order to meet the desired specification the root locii are reshaped so that they pass through the point where the dominant closed loop poles are located.

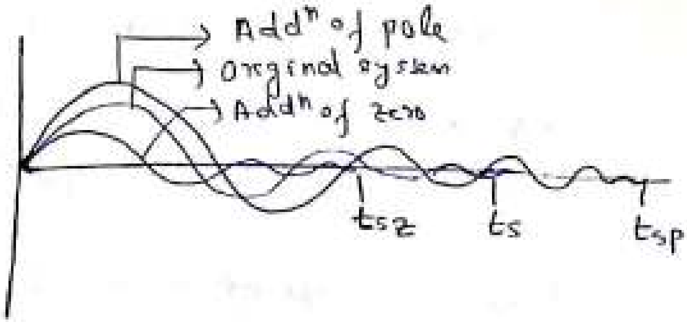
The root locus are reshaped by introducing a compensator. A compensator will add a pole or zero on the open loop TF of the system.

The addⁿ of pole to open loop TF has the effect of pulling the root locus to the right which reduces the relative stability of the system and increases the settling time.

The addⁿ of zero to open loop TF has the effect of pulling the root locus to the left which reduces the settling time and increases relative stability.



Effect of addⁿ of pole & zero on the relative stability



Effect of addⁿ of pole & zero on settling time (ts).

Bode plot approach of designing Compensator

The objective of frequency domain design is to reset the frequency response characteristics so that the desired specifications are met.

Low freq. region of the bode plot provides information regarding the steady state performance and high freq. region provides information regarding the transient state performance. The medium frequency range provides information about relative stability.

Therefore low freq. region of bode plot is reshaped by lag compensation to improve steady state performance. The high freq. region of bode plot is reshaped by lead compensation to improve transient performance.

When the system requires improvement in both steady state and transient performance a lag-lead compensator is used to alter both the low and high freq. region of bode plot.

Correlation between the time domain & freq. domain parameters

$$\text{phase margin} = \tan^{-1} \frac{2\delta}{\sqrt{\sqrt{(4\delta^2+1)} - 2\delta^2}}$$

$$\text{Gain cross-over freq } \omega_{gc} = \omega_n \sqrt{\sqrt{4\delta^4+1} - 2\delta^2}$$

Lag Compensator

A compensator having the characteristics of a lag network is called a lag compensator.

If a sinusoidal signal is applied to a lag network then in steady state, the o/p will have a phase lag wrt i/p.

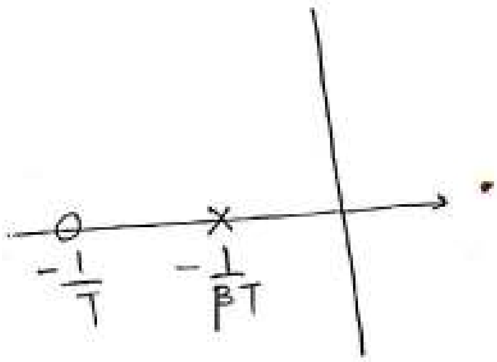
Lag compensator is a low pass filter. so the high freq. noise signals are attenuated. so lag network improves SNR of the system.

Lag compensator results in large improvement in steady state performance. But results in slow slow response due to reduced bandwidth.

S-plane representation

$Z_c = -\frac{1}{T}$

$P_c = -\frac{1}{\beta T}$



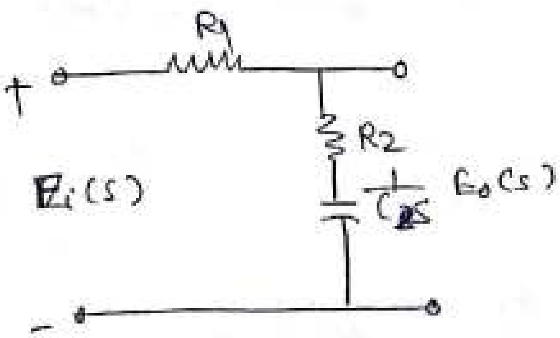
General expression for TF

$$\boxed{g_c(s) = \frac{s+Z_c}{s+P_c} = \left(\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right)} = \beta\left(\frac{1+s\tau}{1+\beta s\tau}\right)$$

where $T > 0$ and $\beta > 1$

$$\boxed{T = \frac{1}{Z_c} \text{ and } \beta = \frac{Z_c}{P_c}}$$

Realization of Lag Network



By applying Voltage division formula,

$$E_o(s) = E_i(s) \times \frac{\left(R_2 + \frac{1}{Cs}\right)}{R_1 + \left(R_2 + \frac{1}{Cs}\right)}$$

$$E_o(s) = E_i(s) \times \frac{(R_2Cs+1)}{(R_1+R_2)Cs+1} = \frac{CR_2\left(s+\frac{1}{R_2C}\right)}{(R_1+R_2)C\left(s+\frac{1}{(R_1+R_2)C}\right)}$$

Dividing by CR_2 throughout

$$E_o(s) = E_i(s) \times \frac{s + \frac{1}{R_2C}}{\frac{R_1+R_2}{R_2}\left(s + \frac{1}{\frac{(R_1+R_2)}{R_2}CR_2}\right)}$$

Assuming $T = R_2C$

$$\beta = \frac{R_1+R_2}{R_2}$$

$$E_o(s) = E_i(s) \times \frac{1}{\beta}\left(\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right)$$

<u>Polar plot</u>

$$g_i(s) = \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} = \frac{j\omega+\frac{1}{T}}{j\omega+\frac{1}{\beta T}}$$

$$|g_c(s)| \angle g_c(s) = \frac{\sqrt{\frac{1}{T^2}+\omega^2}}{\sqrt{\frac{1}{\beta^2T^2}+\omega^2}} \angle \tan^{-1}\omega T - \tan^{-1}\beta\omega T$$

at $\omega = 0$, $M = 1$, $\phi = 0$

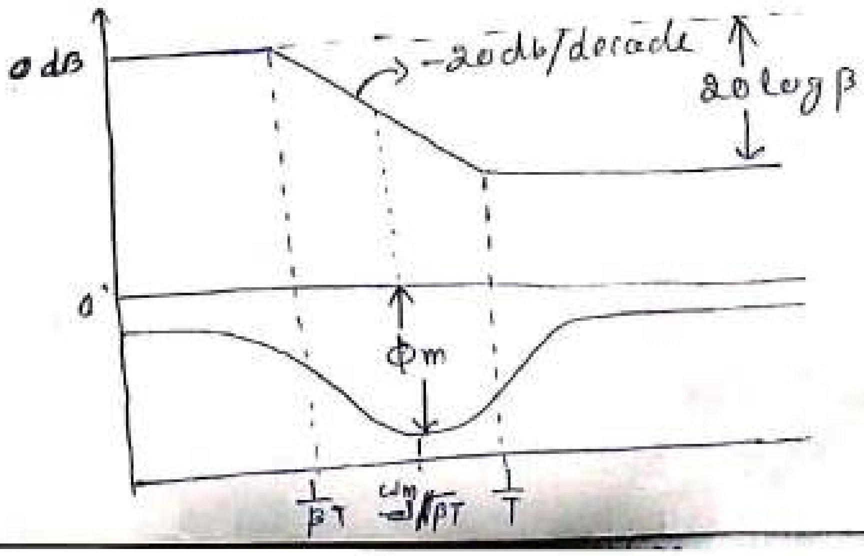
$\omega = \infty$ $M = \frac{1}{\beta}$ & $\phi = 0°$

for $\beta > 1$ and $0 < \omega < \infty$, $\tan^{-1}\omega T < \tan^{-1}\beta\omega T$

<u>Bode plot</u>

$\omega c_1 = \frac{1}{\beta T}$

$\omega c_2 = \frac{1}{T}$



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Lead Compensator

A compensator having the characteristic of a lead network is called a lead compensator.

If a sinusoidal signal is applied to the lead network then in steady state the o/p will have a phase lead wrt i/p.

A lead compensator is basically a high pass filter. So it amplifies high freq. noise signals.

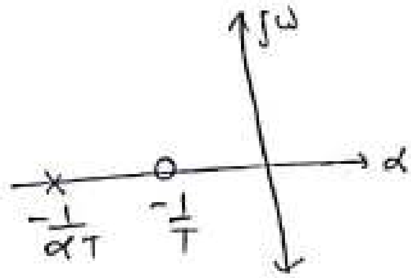
The lead compensation increases the bw of the system. So it improves the speed of the response and also reduces o/p peak overshoot.

Lead compensator improves transient state performance of the system.

S-plane representation

$$z_c = -\frac{1}{T}$$

$$p_c = -\frac{1}{\alpha T}$$



So the generalised TF will be

$$g_c(s) = \left(\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}\right) = \left(\frac{s+z_c}{s+p_c}\right)$$

$$\boxed{\begin{aligned} T &= \frac{1}{z_c} \\ \alpha &= \frac{z_c}{p_c} \end{aligned}}, \quad \alpha < 1 \qquad g_c(s) = \alpha\left(\frac{1+sT}{1+\alpha sT}\right)$$

$$g_c(j\omega) = \alpha\left(\frac{1+j\omega T}{1+j\alpha\omega T}\right)$$

$\omega = 0, \quad M = \alpha$

At zero freq. the network has a gain of $\alpha < 1$ or an attenuation of $1/\alpha$. So to cancel the effect of attenuation an amplifier of gain $1/\alpha$ is introduced.

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So the TF of lead compensator will be,

$$g_c(s) = \left(\frac{1+j\omega T}{1+j\alpha\omega T}\right) = \frac{1}{\alpha}\left(\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}\right) \quad \text{---(1)}$$

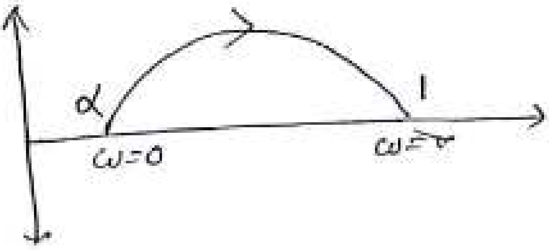
Polar plot

At $\omega = 0$, $M = \alpha$ and $\phi = 0$

$\omega = \infty$ $M = 1$, $\phi = 0$

for $0 < \omega < \infty$ and $0 < \alpha < 1$

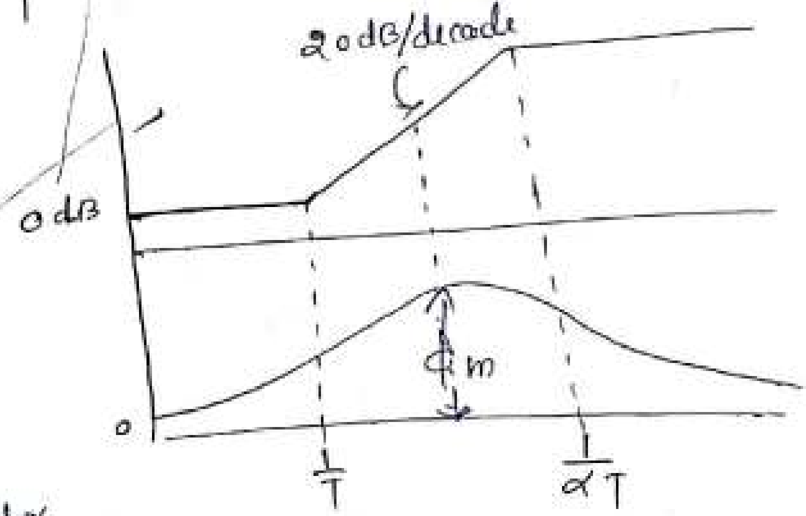
$\tan^{-1}\omega T > \tan^{-1}\alpha\omega T$



Bode plot

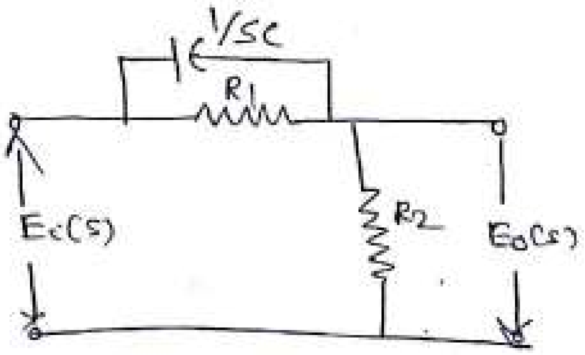
$\omega_{c1} = \frac{1}{T}$

$\omega_{c2} = \frac{1}{\alpha T}$



Realization of lead compensator

Applying voltage divider rule



$$\frac{E_o(s)}{E_i(s)} = \frac{R_2}{R_2 + \frac{\frac{R_1}{sC}}{R_1 + \frac{1}{sC}}}$$

$$= \frac{R_2}{R_2 + \frac{R_1}{(1+R_1Cs)}}$$

$$= \frac{R_2(1+R_1Cs)}{R_1 + R_2(R_1Cs+1)}$$

Dividing throughout by R_1R_2C

$$\frac{\left(s+\frac{1}{R_1C}\right)}{\frac{1}{CR_2} + \frac{1}{R_1C}(R_1Cs+1)} = \frac{s+\frac{1}{R_1C}}{\frac{1}{CR_2}+s+\frac{1}{R_1C}} = \frac{s+\frac{1}{R_1C}}{s+\frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{s+\frac{1}{R_1C}}{s+\frac{1}{C}\left(\frac{R_1+R_2}{R_1R_2}\right)} = \frac{s+\frac{1}{R_1C}}{s+\frac{1}{CR_1}\left(\frac{R_1+R_2}{R_2}\right)}$$

$$= \frac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C \times \left(\frac{R_2}{R_1+R_2}\right)}}$$

Assuming $R_1C = T$

and $\alpha = \frac{R_2}{R_1+R_2}$

$$g_c(s) = \left(\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}\right) = \alpha\left(\frac{1+sT}{1+\alpha sT}\right)$$

From eqn - (1)

Phase angle of the compensator at any freq. is given by,

$$\phi = \tan^{-1}\omega T - \tan^{-1}\alpha\omega T$$

$$\tan\phi = \frac{+\omega T(1-\alpha)}{1+\alpha\omega^2T^2} \quad \text{—(2)}$$

The freq. at which max^m phase-lead will occure can be calculated by $d\phi/d\omega = 0$.

$$\frac{T(1-\alpha)[1+\alpha\omega^2T^2]-[\omega T(1-\alpha)][2\omega T^2\alpha]}{(1+\alpha\omega^2T^2)^2} = 0$$

$$\Rightarrow T+\alpha\omega^2T^3 - T\alpha - \alpha^2\omega^2T^3 - (\omega T-\omega T\alpha)(2\omega T^2\alpha) = 0$$

$$\Rightarrow T+\alpha\omega^2T^3 - T\alpha - \alpha^2\omega^2T^3 - 2\omega^2T^3\alpha + 2\omega^2T^3\alpha^2 = 0$$

$$\Rightarrow T - T\alpha - \omega^2T^3\alpha + \omega^2T^3\alpha^2 = 0$$

$$\Rightarrow T(1-\alpha) = \omega^2(T^3\alpha - T^3\alpha^2)$$

$$\Rightarrow \omega^2 = \frac{T(1-\alpha)}{T^3\alpha(1-\alpha)} \Rightarrow \omega^2 = \frac{1}{T^2\alpha}$$

$$\Rightarrow \boxed{\omega_m = \frac{1}{T\sqrt{\alpha}}}, \quad \omega_m = \sqrt{(1/T)(1/\alpha T)}$$

Hence ω_m is also the geometric mean of the corner freq. of the compensator.

At $\omega = \omega_m$, the max^m phase-lead is given by

$$\tan\phi_m = \frac{(1/T\sqrt{\alpha})T(1-\alpha)}{1+\alpha T^2\left(\frac{1}{T\sqrt{\alpha}}\right)^2}$$

$$= \frac{(1-\alpha)\times \cancel{T}\times\frac{1}{\cancel{T}\sqrt{\alpha}}}{1+\alpha \cancel{T^2}\times\frac{1}{\cancel{T^2}\alpha}} = \frac{1-\alpha}{2\sqrt{\alpha}}$$

or, $\boxed{\sin\phi_m = \frac{1-\alpha}{1+\alpha}}$

or, $\boxed{\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}}$

The magnitude of $G_c(j\omega)$ at $\omega=\omega_m$ is

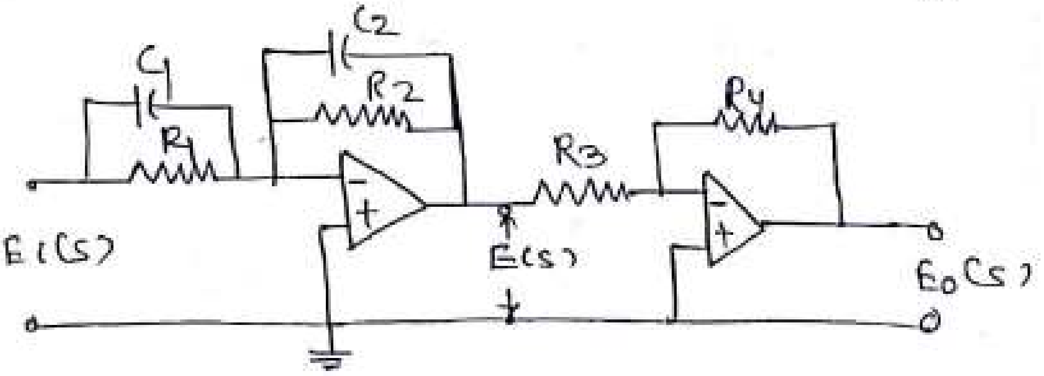
$$|G_c(j\omega)| = \left|\frac{1+j\omega_m T}{1+j\omega_m\alpha T}\right| = \sqrt{\frac{1+(\omega_m T)^2}{1+(\alpha\omega_m T)^2}}$$

$$= \sqrt{\frac{1+\cancel{T^2}\times\frac{1}{\cancel{T^2}\alpha}}{1+\alpha^2\cancel{T^2}\times\frac{1}{\cancel{T^2}\cancel{\alpha}}}} = \sqrt{\frac{1+\frac{1}{\alpha}}{1+\alpha}} = \sqrt{\frac{1+\alpha}{\alpha}\times\frac{1}{1+\alpha}} = \frac{1}{\sqrt{\alpha}}$$

magnitude of compensator expressed in dB

$$|G_c(j\omega)| = 20\log\left(\frac{1}{\sqrt{\alpha}}\right) = 10\log\left(\frac{1}{\alpha}\right).$$

Realization of Lag & Lead Network using Opam ckt.



$$\frac{E_o(s)}{E_i(s)} = -\left(\frac{R_2}{1+R_2C_2s} \Big/ \frac{R_1}{1+R_1C_1s}\right) = -\frac{R_2}{R_1} \times \frac{1+R_1C_1s}{1+R_2C_2s} \quad -(1)$$

$$\frac{E_o(s)}{E(s)} = -\frac{R_4}{R_3} \quad -(2)$$

Multiplying eq^n -(1) & (2)

$$\frac{E_o(s)}{E_i(s)} = \frac{R_4R_2}{R_3R_1} \times \frac{1+R_1C_1s}{1+R_2C_2s}$$

$$= \frac{R_4\cancel{R_2}}{R_3\cancel{R_1}} \times \frac{\cancel{R_1}C_1}{\cancel{R_2}C_2} \times \frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$$

$$= \frac{R_4C_1}{R_3C_2} \times \frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$$

$$\boxed{\frac{E_o(s)}{E_i(s)} = K_c\alpha \frac{1+sT}{1+\alpha sT} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}}$$

where, $T = R_1C_1$, $\alpha T = R_2C_2$

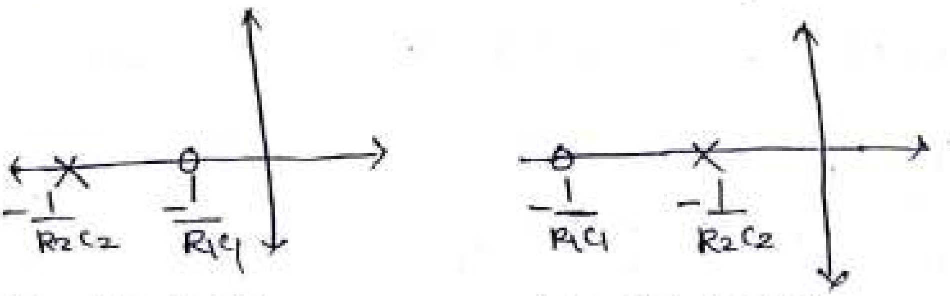
$$\alpha = \frac{R_2C_2}{R_1C_1}, \quad K_c = \frac{R_4C_1}{R_3C_2}$$

If $R_1C_1 > R_2C_2 \Rightarrow \alpha < 1$

Then this network will act as lead network.

If $R_2C_2 > R_1C_1$, $\Rightarrow \beta > 1$

Then this network will act as lag network.



Lead Network   Lag Network.

Lag-Lead compensator

→ Lag-Lead compensator is a combination of both lag compensator and a lead compensator.

→ At lower freq. it acts as a lag compensator and at higher freq. it acts as a lead compensator.

→ Lag-lead compensator is used when both the steady state and transient state performance of a system has to improved.

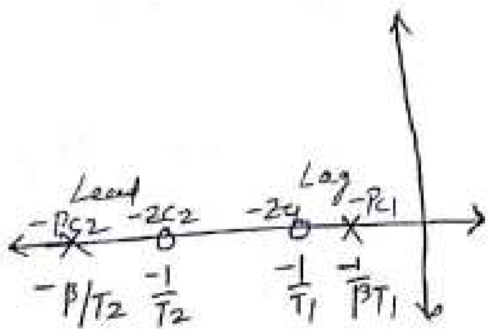
→ The generalised TF of a lag-lead compensator is given by

$$\boxed{G_c(s) = \left(\frac{s+1/T_1}{s+1/\beta T_1}\right)\left(\frac{s+1/T_2}{s+1/\alpha T_2}\right) \quad \beta > 1 \ \& \ \alpha < 1}$$

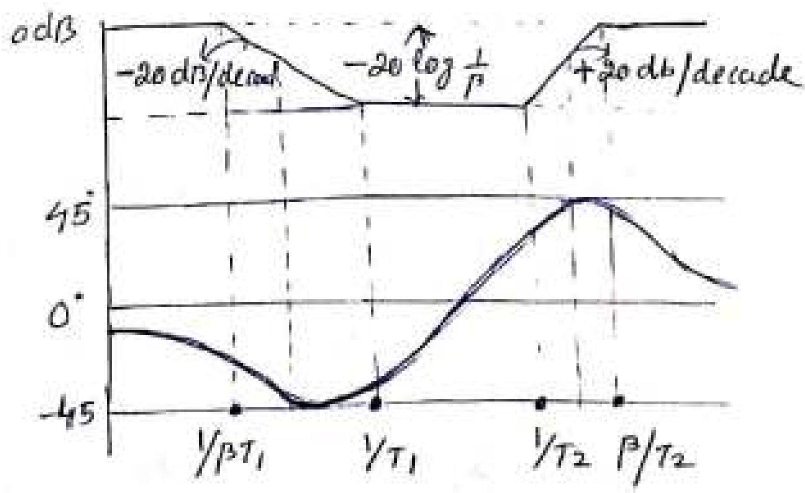
→ If we assume $\alpha\beta = 1$ ⇒ α, & β parameter of the lag-lead network cannot be varied independently then the modified tf of lag-lead compensator will be

$$\boxed{G_c(s) = \left(\frac{s+1/T_1}{s+1/\beta T_1}\right)\left(\frac{s+1/T_2}{s+\beta/T_2}\right)}$$

where, $\boxed{Z_{c1}/P_{c1} = \beta = P_{c2}/Z_{c2}}$, $\beta > 1$

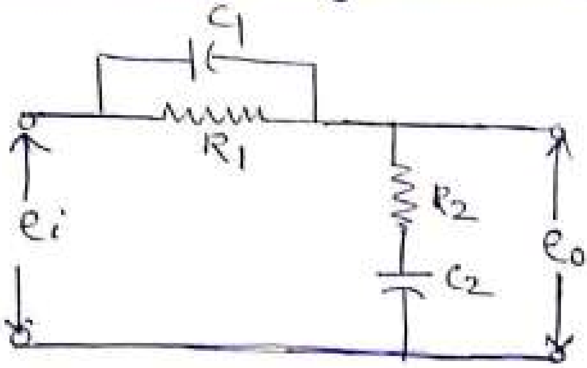


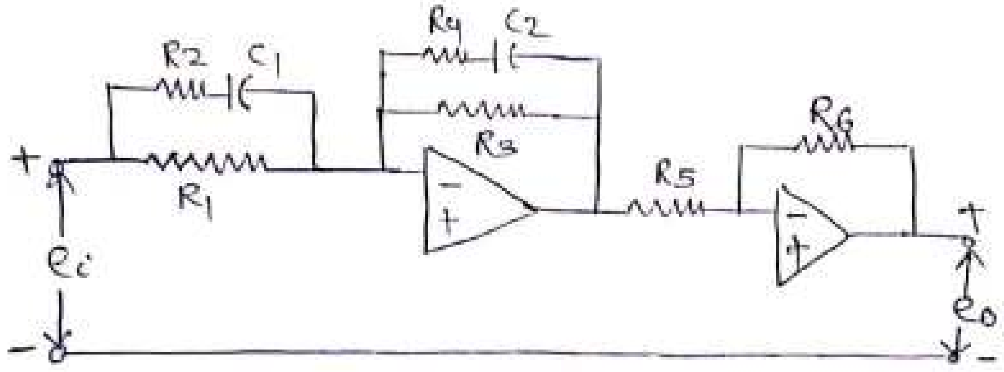
s-plane Representation of lag-lead compensator.



Bode plot representation of Lag-Lead compensator.

Realization of Lag-Lead compensator





Steps for Designing Lead compensator

step-1 : From the given performance indices calculate the required values of δ and ω_n. Then calculate $\omega_d = \omega_n\sqrt{1-\delta^2}$.

Locate the closed loop dominant pole $-\delta\omega_n \pm j\omega_d$

step-2 : Draw the root-locus for the uncompensated system.

step 3 : If the root locus of uncompensated system will pass through the dominant closed loop poles then compensation is not needed. Otherwise go to step 4.

step 4 : Calculate the angle deficiency at the dominant closed loop positions.

$\phi = -180 +$ (Angle contributed by open loop zeros at dominant closed loop pole − Angle contributed by open loop poles at dominant closed loop pole)

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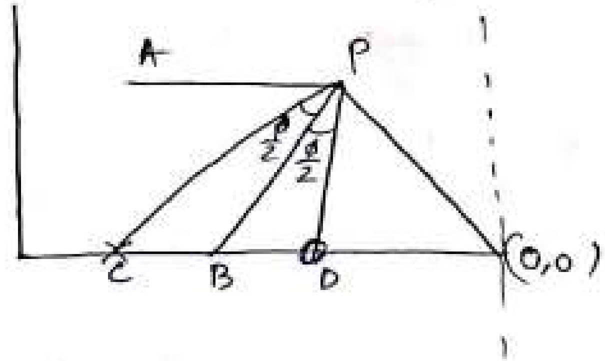
Step-5: Locate the pole and zero of the lead compensator as follows.

→ If P represents dominant closed loop pole location then draw a line joining P to origin and draw another line AP parallel to real axis as shown in fig

→ Let PB be the angle bisector of $\angle APO$ and meet the real axis at B.

→ Draw two lines PC and PD making an angle $\phi/2$ with PB, which intersects real axis at C & D point respectively.

→ The point C will give the location of pole (P_c) and point D will give the location of zero (z_c) of the compensator.



Step 6: Calculate the overall gain of the compensated system. Then determine the gain of the compensator $K_c = \frac{K_{cc}}{K_{uc}}$

The tf of lead compensator will be $K_c\left(\frac{s+z_c}{s+P_c}\right)$

Step 7: Calculate the parameters of equivalent electronic ckt.

$K_{cc} = K_c\alpha$, $R_1C_1 = 1/z_c$, $R_2C_2 =$

Q) Design a suitable compensator for the system whose open loop TF is $G(s) = \frac{10}{s(s+1)}$. So that the compensated system has an undamped natural frequency $\omega_n = 3$ rad/sec. and damping ratio $\delta = 0.5$.

Sol^n:

<u>Step-1</u> As we have to improve the transient performance, so we must design a lead compensator.

<u>Step-2</u> (Locate desired closed loop pole)

$\omega_n = 3$ rad/sec.

$\delta = 0.5$

$\omega_d = \omega_n\sqrt{1-\delta^2} = 3 \times \sqrt{1-0.5^2} = 2.59$

$S_d = -\delta\omega_n \pm j\omega_d = -0.5 \times 3 \pm j\,2.59 = -1.5 \pm j\,2.59$

<u>Step-3</u> Draw the root locus for the uncompensated system.

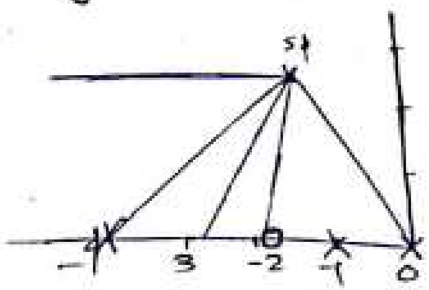
As the root locus is not passing through the dominant closed loop poles, a lead compensator has to be designed.

<u>Step-4</u> Calculate angle deficiency at the dominant poles

$\phi = -180 \pm$ (Angle contributed by syste zeros − Angle contributed by system poles at sd)

$= -180 \pm (0 - (101 + 120)) = -180 + 221 = 41^\circ$

So the angle to be contributed by the compensator is 41°



<u>Step-5</u> Calculate the location of compensator pole & zero.

$P_c = -3.95$ and $Z_c = -1.85$

Step 6: Assuming the TF of lead compensator to be

$$G_c(s) = K_c\alpha\left(\frac{1+sT}{1+\alpha sT}\right) = K_c\frac{(s+\frac{1}{T})}{(s+\frac{1}{\alpha T})}$$

Now the required lead compensator TF becomes

$$G_c(s) = K_c\left(\frac{s+1.85}{s+3.95}\right)$$

$$T = \frac{1}{1.85} = 0.54$$

$$\alpha = \frac{1.85}{3.95} = 0.468$$

Step 7: Now the open loop TF of the compensated system will be

$$G^c(s) = G_c(s) \times G(s)$$
$$= K_c\frac{(s+1.85)}{(s+3.95)} \times \frac{10}{s(s+1)}$$

$$G^c(s) = K\left(\frac{s+1.85}{s(s+1)(s+3.95)}\right)$$

where $K = 10K_c$.

Step 8: Determine the gain of the compensated system at the desired dominant pole location.

$$\left|\frac{K(s+1.85)}{s(s+1)(s+3.95)}\right|_{s=-1.5+j2.59} = 1$$

$$\Rightarrow K = \frac{\sqrt{(1.5)^2+(2.5)^2}\sqrt{(0.5)^2+(2.59)^2}\sqrt{(2.45)^2+(2.59)^2}}{\sqrt{(0.35)^2+(2.59)^2}}$$

$$\Rightarrow K = \frac{3 \times 1.3 \times 3.57}{2.62} = 1.36$$

$$\Rightarrow K_c = \frac{K}{10} = \frac{1.36}{10} = 0.136$$

So the compensator TF is $0.136 \times \frac{(s+1.85)}{(s+3.95)} = G_c(s)$

$$G_c(s) = 0.0636\left(\frac{1+0.54s}{1+0.25s}\right) \text{ in time domain form.}$$

Step : 9 (Evaluating the parameters of electronic lead compensator)

From time-constant form

$R_1C_1 = 0.54$

$R_2C_2 = 0.25$

$K_c\alpha = 0.0636 \simeq 0.064$

Assuming $C_1 = C_2 = 10\mu F$

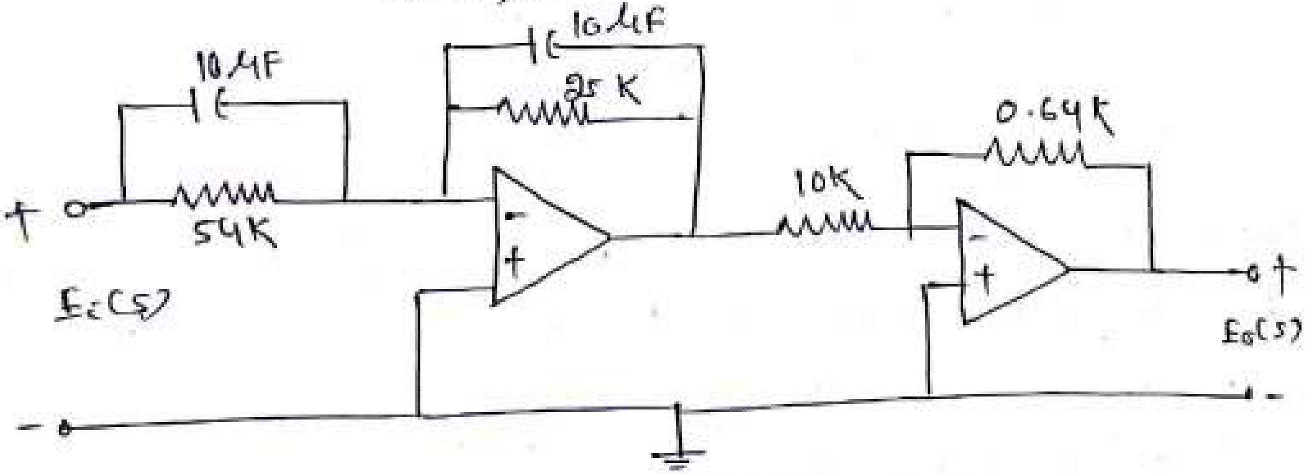
$R_1 = \dfrac{0.54}{10\times10^{-6}} = 54k$

$R_2 = \dfrac{0.25}{10\times10^{-6}} = 25K$

Again $K_c\alpha = \dfrac{R_2R_4}{R_1R_3} = \dfrac{25\times10^3\times R_4}{54\times10^3\times R_3} = 0.064$

Assuming $R_3 = 10k\Omega$

$R_4 = \dfrac{0.064\times10\times\cancel{10^3}\times54}{25\times\cancel{10^3}} = 640\,\Omega$



$R_1 = 54K$

$R_2 = 25K$

$R_3 = 10K$

$R_4 = 0.64K$

$C_1 = 10\mu F$

$C_2 = 10\mu F$

Since $R_1C_1 > R_2C_2$, it will act as lead compensator.

Procedure for designing Lag compensator

Step 1: Select a dominant pole, so that it satisfies the required specification.

Step 2: Assume that the following t.f is the lag compensator.

$$G_c(s) = K_c\beta\left(\frac{1+sT}{1+\beta sT}\right) = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}, \quad \beta > 1$$

Step 4: Determine the value of β as

$$\beta = \frac{\text{Required static error constant}}{\text{Existing static error constant}}$$

Step 3: Calculate the gain of the system at the dominant root s_d, and evaluate the corresponding error constant.

Step 5: Select zero of the compensator sufficiently close to origin. As a guide rule, we may construct a line making an angle 10° (or less) with the desired δ-line from s_d. The intersection of this ~~axis~~ line with the real axis gives location of the compensator zero.

Step 6: The location compensator pole will be

$$-P_c = -Z_c/\beta.$$

Step 7: Check the angle contributed by the compensator at the desired dominant pole. It should be within 5°. If angle contributed by the lag compensator at s_d is $> 5°$, then lag compensator should be re-designed.

Step 8: Determine the gain of the compensated system at the dominant pole.

Step 9: Calculate the parameters of electronic lag compensator.

Q(2) Design a suitable compensator for a system whose open loop TF is $G(s) = \frac{16}{s(s+4)}$. So that the static velocity error constant K_v is $20\ sec^{-1}$ without appreciably changing the original location of pole.

Step 1: Since we have to improve the steady state performance while preserving the transient performance of the system, a lag compensator need to be designed.

Step 2: From the given TF $G(s) = \frac{16}{s(s+4)}$, the charecteristic eqn will be $1+G(s)H(s) = 0$

$$s^2+4s+16 = 0 \Rightarrow \omega_n = 4 \ \& \ \delta = 0.2, \ \omega_d =$$

so the location of dominant closed loop poles will be

$$S_d = -\delta\omega_n \pm j\omega_d = -2 \pm j3.46$$

Step 3: Let the TF of the lag compensator be,

$$G_c(s) = K_c\beta\left(\frac{1+sT}{1+\beta sT}\right) = K_c\left(\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right)$$

Step 4: The velocity error constant of the uncompensated system is

$$K_v^{UC} = \lim_{s\to 0} sG(s)H(s)$$

$$= \lim_{s\to 0} \cancel{s} \times \frac{16}{(s+4)\cancel{s}} = 4\ sec^{-1}$$

Step 5: $K_v^{C} = 20\ sec^{-1}$

$$\text{so, } \beta = \frac{K_v^{C}}{K_v^{UC}} = \frac{20}{4} = 5$$

Step 6: Let us choose the zero of the compensator at,

$-Z_c = -0.01$.

Then location of pole of the compensator will be

$$-P_c = -\frac{0.01}{5} = -0.002.$$

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Step 7: The angle contributed by z_c at $s_d = 119.88$

The angle contributed by P_c at $s_d = 119.98$

Hence net angle contributed by the lag compensator at s_d

$$= 119.88 - 119.98 = |-0.1'| = 0.1' < 5'$$

Hence the location of pole-zero pair of lag compensator is acceptable.

Thus the TF of lag compensator will be,

$$G_c(s) = K_c\left(\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right) = K_c\frac{s+0.01}{s+0.002}$$

TF of the compensated system will be,

$$G_c(s)G(s) = K_c\left(\frac{s+0.01}{s+0.002}\right)\left(\frac{16}{s(s+4)}\right)$$

$$G_c(s)G(s) = \frac{K(s+0.01)}{(s+0.002)(s+4)s} \quad \text{where } K = 16K_c$$

Step 8: The gain of compensated system at s_d is

$$\left|\frac{K(s+0.01)}{s(s+4)(s+0.002)}\right|_{s=-2+j3.46} = 1$$

$$K = \left|\frac{s(s+4)(s+0.002)}{s+0.01}\right|_{s=-2+j3.46}$$

$$K = \frac{\cancel{\sqrt{\omega^2}\sqrt{4^2+\omega^2}\sqrt{\omega^2+(0.002)^2}}}{\cancel{\sqrt{\omega^2+(0.01)^2}}}\Bigg|_{\cancel{\omega=-2-}}$$

$$K = \left|\frac{(-2+j3.46)(2+j3.46)(-1.998+j3.46)}{(-1.99+j3.46)}\right|$$

$$= 16$$

Hence $K_c = \frac{16}{16} = 1$

Hence $G_c(s) = \frac{s+0.01}{s+0.002} = 5\left(\frac{1+100s}{1+500s}\right)$

Step 9:

$K_c\beta = \frac{R_2 R_4}{R_1 R_3} = 5$

$R_1C_1 = T = 100$

$R_2C_2 = \beta T = 500$

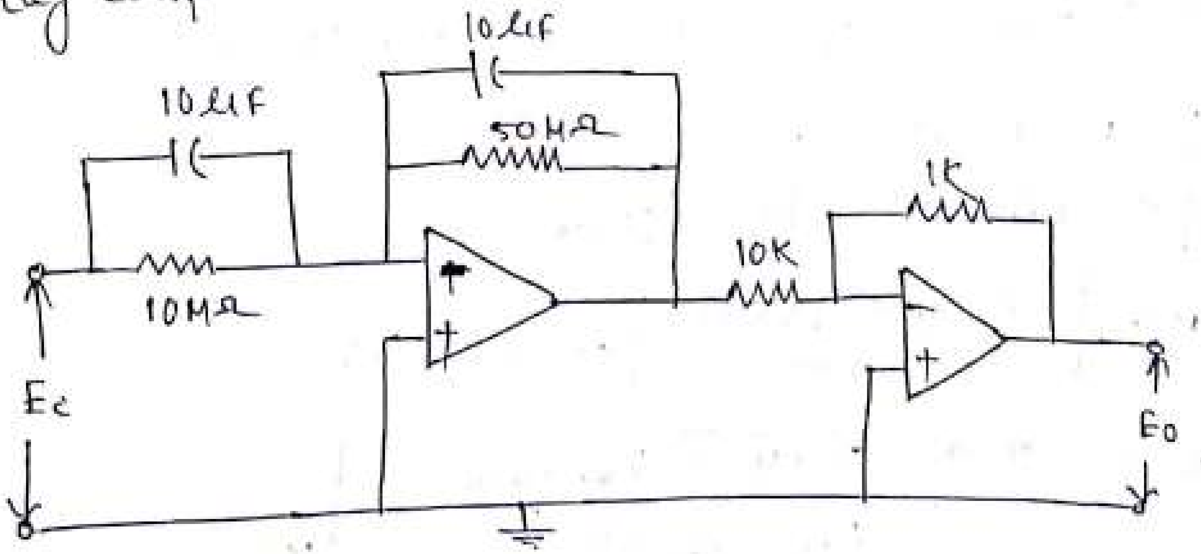
Assuming $C_1 = C_2 = 10\mu F$, $R_3 = 10k\Omega$

$R_1 = \frac{100}{10 \times 10^{-6}} = 10\,M\Omega$

$R_2 = \frac{500}{10 \times 10^{-6}} = 50\,M\Omega$

$R_4 = \frac{5 \times R_1 \times R_3}{R_2} = \frac{5 \times 10^7 \times 10^3}{50 \times 10^6} = 10^3 = 1K$

Here $R_1C_1 < R_2C_2$. Hence the ckt will act as a lag compensator.



$R_1 = 10\,M\Omega$

$R_2 = 50\,M\Omega$

$R_3 = 10\,K$

$R_4 = 1K$

$C_1 = C_2 = 10\mu F$.

Steps for designing a Lag-lead compensator

Step-1: Determine dominant pole from the given specification

Step-2: Determine K_c from the given velocity error constant.

Step-3: Assume the TF of the compensator as

$$G_c(s) = K_c \frac{\left(s + \frac{1}{T_1}\right)\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{1}{\alpha T_1}\right)\left(s + \frac{1}{\beta T_2}\right)}$$

Assuming $\alpha\beta = 1 \Rightarrow \alpha = \frac{1}{\beta}$

$$G_c(s) = K_c \frac{\left(s + \frac{1}{T_1}\right)\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{\beta}{T_1}\right)\left(s + \frac{1}{\beta T_2}\right)}$$

Step-4: Find the angle deficiency ϕ for the dominant closed loop poles to be part of root locus of the given uncompensated system.

Step-5: Determine T_1 and β from the magnitude ~~rectifier~~ criteria.

Step-6: Choose large enough T_2 so that lag compensating part has approximately unity magnitude.

Step-7: Check the gain and phase angle of the lag compensating part so that its magnitude is approximately equal to one and phase angle lies between 0-5.

Q) Design a lag-lead compensator for the system whose open loop TF $G(s)$ is $\frac{5}{s(1+0.5s)}$ so that damping ratio of the dominant closed loop pole is equal to 0.5 and to increase the undamped natural freq to 5 rad/sec and static velocity error constant to 50 sec^{-1}. Design an appropriate compensator to meet all the performance specifications.

Solⁿ: Since both the transient and steady state performance of the given system has to be improved, a Lag-lead compensator has to be designed.

Step-1: Let us assume the TF compensator to be,

$$G_c(s) = K_c \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{\left(s+\frac{\beta}{T_1}\right)\left(s+\frac{1}{\beta T_2}\right)}$$

Step-2: TF of the given system is $\frac{10}{s(s+2)}$

$$K_v = \lim_{s\to 0} s\,G(s) = \lim_{s\to 0} \cancel{s} \times \frac{10}{\cancel{s}(s+2)} = 5\ \text{sec}^{-1}$$

Required velocity error constant = 50 sec⁻¹

Velocity error constant of compensated system

$$K_v = \lim_{s\to 0} s\,G_c(s)\,G(s)$$

$$= \lim_{s\to 0} s\,G_c(s) \times \lim_{s\to 0} s\,G(s)$$

$$= K_c \times 5$$

$$\Rightarrow 5 \times K_c = 50 \quad \Rightarrow \boxed{K_c = 10}$$

Step-3: Locate the dominant closed loop pole.

$\omega_n = 5$, $\delta = 0.5$

$\omega_d = 5\sqrt{1-0.5^2} = 4.33$

$S_d = -\delta\omega_n \pm j\omega_d = -2.5 \pm j\,4.33$

Step-4: Calculate the angle deficiency.

$\phi = -180 -$ Angle contributed by the system at S_d

$= -180 - [0 - (120 + 97)] = 37^\circ$

Step-5: Locate the pole & zero of the ~~lag~~ lead portion of the network.

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From the graph, $-z_c = -3.3$ and $-p_c = -7.2$

so the TF of Lead portion of the network is

$$K_c\left(\frac{s+z_{c_1}}{s+p_{c_1}}\right) = 10\left(\frac{s+3.3}{s+7.2}\right)$$

$$\Rightarrow \frac{1}{T_1} = 3.3 \Rightarrow T_1 = 0.303$$

$$\beta = \frac{p_{c_1}}{z_{c_1}} = \frac{7.2}{3.3} = 2.25 .$$

<u>Step-6</u>: Assuming a higher value of $T_2 = 10$.

$$\frac{1}{\beta T_2} = \frac{1}{2.25\times 10} = 0.044$$

$$\frac{1}{T_2} = \frac{1}{10} = 0.1$$

So Lag portion of the compensator will be,

$$\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) = \left(\frac{s+0.1}{s+0.044}\right)$$

So the complete TF of lag-lead compensator will be

$$\boxed{G_c(s) = 10\left(\frac{s+3.3}{s+7.2}\right)\left(\frac{s+0.1}{s+0.044}\right)}$$ Pole zero form

$$\boxed{G_c(s) = 10.4\left(\frac{1+0.3s}{1+0.14s}\right)\left(\frac{1+10s}{1+22.7s}\right)}$$ Time constant form.

<u>Procedure for Designing Lead compensator using bode plot</u>

<u>Step-1</u>: Determine the loop gain K to satisify specified error constant.

<u>Step-2</u>: Using this value of K, determine the phase margin of the uncompensated system.

<u>Step-3</u>: Determine the phase-lead required using the relation.

$$\phi_l = \phi_s - \phi_1 + \epsilon$$

$\phi_1 \rightarrow$ phase margin of the fixed part of the system.

$\phi_s \rightarrow$ specified phase margin

$\phi_l \rightarrow$ phase margin to be introduced by lead compensator.

$\epsilon \rightarrow$ a margin of safety required by the fact that the cross-over freq. will increase due to compensation.

Step-4: Let $\phi_m = \phi_l$ and determine the α-parameter of the network from.

$$\boxed{\alpha = \frac{1 - \sin\phi_m}{1 + \sin\phi_m}}$$

Step-5: Locate the frequency at which the uncompensated system has a gain of $-10\log(1/\alpha)$. This is the cross-over frequency $\omega_{c2} = \omega_m$ of the compensated system.

Step-6: Compute the two corner freq of the network as
$\omega_1 = 1/T = \omega_m\sqrt{\alpha}$ and $\omega_2 = 1/\alpha T = \omega_m/\sqrt{\alpha}$

Step-7: Draw the magnitude and phase plots of the compensated system and check the resulting phase margin. If the phase margin is still low, raise the value of ϵ repeat from step 3.

NB: If ϕ_m calculated in step-4 is $> 60^\circ$, then design two identical lead compensator each contributing angle of $\phi_m/2$.

Q(4) Consider a unity feedback system with an open-loop Transfer function $G(s) = \frac{K}{s(s+1)}$. Design a suitable lead compensator so that the compensated system will have $K_v = 12\ sec^{-1}$ and $\phi_{pm} = 40^\circ$.

Step-1: $G(s) = \frac{K}{s(s+1)}$

$K_v = \lim_{s \to 0} s \times \frac{K}{s(s+1)} = K$

$\boxed{K_v = K = 12}$

Step-2: Draw the bode plot for the system $\frac{12}{s(s+1)}$.

From the bode plot phase margin of the system is calculated to be $\phi_1 = 15^\circ$.

Step-3: Required phase lead $\phi_L = \phi_s - \phi_1 + \epsilon$

$\phi_s = 40,\ \phi_1 = 15,\ \epsilon = 5^\circ$ (Assume)

$\phi_L = 40 - 15 + 5 = 30$

$\boxed{\phi_e = \phi_m = 30^\circ}$

Step-4:

$\alpha = \frac{1 - \sin\phi_m}{1 + \sin\phi_m} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = 0.333$

Step-5:

$10 \log(1/\alpha) = 10 \log(1/0.333) = 4.8\ dB$

Calculate the freq. at which the uncompensated system has magnitude -4.8 dB.

from the bode plot, $\omega_m = \omega_{c2} = 4.6$ rad/sec.

Step-6:

Lower corner freq. of the network $\omega_1 = \frac{1}{\tau} = \omega_m\sqrt{\alpha} = 2.65$ rad/sec.

Upper corner freq. of the network $\omega_2 = \frac{1}{\alpha\tau} = \omega_m/\sqrt{\alpha} = 7.8$ rad/sec

The transfer function of the lead network will become

$$G_c(s) = \left(\frac{s+2.65}{s+7.8}\right) = 0.339 \times \left(\frac{1+0.377s}{1+0.128s}\right)$$

So the open loop TF of the compensated system will be

→ amplifier

$$G(s) = \frac{12 \times 0.339}{s(s+1)} \times \left(\frac{0.377s+1}{0.128s+1}\right) \times \left(\frac{1}{0.339}\right)$$

$$\boxed{G(s) = \frac{12}{s(s+1)} \times \frac{1+0.377s}{1+0.128s}}$$

Procedure for Designing Lag compensator using Bode plot

Step 1: Determine the open loop gain necessary to satisfy the specified error constant

Step 2: Find the frequency ω_{c2} where the uncompensated system makes a phase margin contribution of

$$\phi_2 = \phi_s + \epsilon$$

Step 3: calculate the freq. ω_{c2} at ϕ_2.

Step 4: Measure the gain of the system at ω_{c2} and equate it to the required high freq network attenuation $(20 \log \beta)$. calculate β.

Step 5: ~~Calculate~~ choose upper corner freq. of compensator to be one decade below ω_{c2}.

$$\omega_2 = \frac{1}{T} = \frac{\omega_{c2}}{10} \Rightarrow T = \frac{10}{\omega_{c2}}$$

Step 6: calculate the lower corner freq, $\omega_1 = \frac{1}{\beta T}$

Q) Design a suitable compensator for system having open-loop TF $G(s) = \frac{K}{s(s+1)(s+4)}$ to meet the following specification.

Damping ratio $\delta = 0.4$

Settling Time $t_s = 10$ sec.

Velocity error constant $K_v \geq 5$ sec^{-1}.

Step:- Since transient performance of the system have to improve,

Given $\delta = 0.4$

$$t_s = \frac{3}{\delta \omega_n} = 10 \Rightarrow \omega_n = \frac{3}{10 \times 0.4}$$

$$\phi_s = \tan^{-1} \frac{2\delta}{\sqrt{\sqrt{4\delta^2+1} - 2\delta^2}} \Bigg|_{\delta=0.4} = 43^\circ$$

$$\omega_b = \quad 1.02 \text{ rad/sec.}$$

Expressing system TF in time-domain form,

$$G(s) = \frac{K/4}{s(1+s)(1+0.25s)}$$

$$K_v = \lim_{s \to 0} s\,G(s) = \lim_{s \to 0} \frac{K/4}{s(1+s)(1+0.25s)} = \frac{K}{4} = 5$$

$\Rightarrow$ The specification on K_v is met by choosing $K = 20$

$$G(s) = \frac{5}{s(1+s)(1+0.25s)}$$

Step-2: Draw the bode plot for the system $G(s)$.

From the bode plot, $\omega_{c_1} = 2.25$ rad/sec. (Gain cross-over freq.)

$\phi_1 = -4^\circ$ (phase margin)

Step-3: $\phi_2 = \phi_s + \epsilon = 43 + 12 = 55^\circ$

The required phase-margin of 55° is found at freq $= 0.52$ rad/sec.

Step-4: Choose the upper corner freq. $\omega_2 (1/\tau)$ one decade below the new gain cross-over freq.

$$\omega_{c_2} = \frac{0.52}{10} = 0.052$$

Step-5 :

compare the magnitude of the system at $\omega c_2 = 0.52$ rad/se. and equate it with $20 \log \beta$.

$$20 \log \beta = 20 \Rightarrow \beta = 10$$

Step-6:

The lower corner freq. $= \frac{1}{\beta T} = \frac{1}{10 \times 0.52} =$

Lag-Lead Compensator

Q) Open loop TF of the uncompensated system is $G(s) = \frac{K}{s(s+1)(s+2)}$. Compensate the system by cascading suitable compensator so that the compensated system has the static velocity error constant of 10 sec, the phase margin of 45°.

Step-1 : Let us assume the TF of lag-lead compensator to be $G_c(s) = K_c\left(\frac{s+\frac{1}{T_1}}{s+\frac{\beta}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)$ where $\alpha = \frac{1}{\beta}$

Step-2: Calculate the static error constant of the uncompensated system and equated it to specified K_v

$$K_v = \lim_{s \to 0} \cancel{s} \times \frac{K}{\cancel{s}(s+1)(s+2)} = \frac{K}{2}$$

$$\Rightarrow \frac{K}{2} = 10 \Rightarrow K = 20$$

Step-3 : Expressing the TF of the system in time-domain form

$$G(s) = \frac{K}{s(s+1)(s+2)} = \frac{20/2}{s(1+s)(1+0.5s)}$$

$$G(s) = \frac{10}{s(1+s)(1+0.5s)}$$

Step-4 : Draw the bode plot for the system

$$G(s) = \frac{10}{s(1+s)(1+0.5s)}$$

From the plot phase margin is calculated to be $\phi_{pm} = -32°$ (unstable).

Step-5: Choose the new gain crossover freq at phase angle −180°

$\omega_{gc}' = 1.5$ rad/sec. (calculated from bode plot)

Gain at $\omega_{gc}' = 1.5$ rad/sec is −13 dB.

Upper corner freq of Lag portion of the network will be

$$\frac{1}{T_1} = \omega_2 = \frac{\omega_{gc}'}{10} = \frac{1.5}{10} = 0.15$$

Lower corner freq. of Lag portion of the network will be

$$\omega_1 = \frac{1}{\beta T} = \frac{0.15}{\beta} \quad -(1)$$

Step-6:

The required phase margin, $\phi_m = 45^\circ$

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

But $\alpha = \frac{1}{\beta} \Rightarrow \sin\phi_m = \frac{1-\frac{1}{\beta}}{1+\frac{1}{\beta}} = \frac{\beta-1}{\beta+1}$

$$\Rightarrow \sin 45^\circ = \frac{\beta-1}{\beta+1} \Rightarrow \beta = 5.83$$

choosing a slight higher value of $\beta = 10$.

$$\omega_1 = \frac{0.15}{10} = 0.015.$$

So the TF of log portion of network will be

$$\left(\frac{s+0.015}{s+0.15}\right).$$

Step-7:

Draw a line at a slope of 20 dB/decade through the point (1.5 rad/sec, −13 dB). The point of intersection of this line with 0 dB line will give the location of pole of the lead compensator.

~~$\omega_4 = 7$ rad sec $\Rightarrow$~~ $T_4 = \frac{1}{\omega_4} = \frac{1}{7} = 0.142$

$$\omega_4 = \frac{1}{\alpha T} = \frac{\beta}{T} = 7$$

$\cancel{\frac{\beta}{T} = 10 \times 7} = 70 \qquad \omega_3 = \frac{1}{T} = \frac{7}{10} = 0.7.$

$\left(\frac{s+0.7}{s+7}\right)$ So the overall TF of lag-lead compensator is

$$\frac{(s+0.7)(s+0.15)}{(s+7)(s+0.015)} = \left(\frac{1+1.43s}{1+0.143s}\right)\left(\frac{1+6.67s}{1+66.67s}\right)$$

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