Lecture Notes

On

Advance Control Systems

7^{Th} Semester Electrical Engineering

Part 2: Pole Placement and Design of State Observer

By

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Course Objective:

The objective of this course is to equip students with a deep understanding of discrete-time control systems, state variable analysis, and nonlinear system behavior. Students will learn to analyze, design, and implement control strategies using modern techniques, including Z-transform methods, state-space representations, and Lyapunov stability analysis.

Course Outcome:

- **Analyze**: Analyze discrete-time and continuous-time control systems using Z-transform and state-space methods to determine system behavior.
- **Design**: Design control systems utilizing feedback strategies, pole placement, and observer design to achieve specified performance criteria.
- **Evaluate**: Evaluate the stability of linear and nonlinear systems using Routh's criterion and Lyapunov's methods, assessing their robustness.
- **Apply**: Apply techniques for modeling and simulating nonlinear systems, including phase plane and describing function methods, to solve practical engineering problems.

POLE PLACEMENT DESIGN

Poleplacement design is a technique by which closed loop poles of the system can be arbitrarily placed to obtain desired system performance.

The pole-placement design is also k/as pole assignment technique.

For pole-placement design it is required that, all states are measurable and available for feedback.

If the system is completely state controllable, then all closed loop poles may be placed at any desired location by means of state feedback through an appropriate state feedback ~~matrix~~ gain matrix.

Let's consider a control system defined by,

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

SISO s/s

where A = $n \times n$ system matrix
B = $n \times 1$ i/p coupling matrix
C = $1 \times n$ o/p coupling matrix
D = Transmission matrix.
x = state vector (n)
y = o/p signal (scalar)
u = control signal (scalar)

Now choosing control signal to be,

$$u = -Kx$$

It implies the control signal depends on the instantaneous values of state.

$$\dot{x}(t) = Ax + B(-Kx)$$
$$\dot{x}(t) = (A - BK)x(t)$$

The soln of this eqn is given by

$$x(t) = e^{(A-BK)t}\, x(0)$$

where, $x(0)$ is the initial state caused by disturbance.

The complete behaviour of the system depends on the eigen values $(A-BK)$ matrix.

If the proper value of K can be choosen, so that all closed loop poles can be placed at desired location, then $x(t)$ will approach zero as time approaches infinity.

Determination of Matrix K using Transformat^n matrix T

The control system is defined as

$$\dot{X} = AX + BU$$

Let us choose a transformation matrix T, such that A and B will be CCF.

$$T = MW \quad \text{—— (1)}$$

where, M is controllability matrix

$$M = [B \quad AB \quad A^2B \quad \cdots \quad A^{n-1}B]$$

$$W = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ a_{n-3} & a_{n-4} & \cdots 1 & 1 & 0 \\ \vdots & & & & \\ a_1 & 1 & \cdots 0 & 0 & 0 \\ 1 & 0 & \cdots 0 & 0 & 0 \end{bmatrix}$$

The charecteristic eq^n of the given system is

$$|SI - A| = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1}s + a_n \quad \text{— (2)}$$

Define a new state vector,

$$X = T\hat{X}$$

$$\dot{\hat{X}} = T^{-1}AT\hat{X} + T^{-1}BU$$

$$T^{-1}AT = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}, \quad T^{-1}B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Let the desired pole locations be $\mu_1, \mu_2, \mu_3 \cdots \mu_n$.

So the characteristic eqn of the system corresponding to the desired pole location will be

$$(s-\mu_1)(s-\mu_2) \cdots (s-\mu_n) = 0$$

$$s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \cdots + \alpha_{n-1} s + \alpha_n = 0 \quad \text{--- (3)}$$

We have choosen, $u = -Kx = -KT\hat{x}$

Let's choose $KT = [\delta_n \; \delta_{n-1} \; \delta_{n-2} \cdots \delta_1]$

so the system can be represented as,

$$\dot{\hat{x}} = T^{-1}AT\hat{x} - T^{-1}BKT\hat{x}$$

so c/s charecteristic eqn can be determined as follows

$$|sI - T^{-1}AT + T^{-1}BKT| = 0$$

$$\left[s\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} [\delta_n \; \delta_{n-1} \cdots \delta_1] \right]$$

$$= \begin{vmatrix} s & -1 & \cdots & 0 \\ 0 & s & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ a_n + \delta_n & a_{n-1} + \delta_{n-1} & & s + a_1 + \delta_1 \end{vmatrix}$$

$$= s^n + (a_1 + \delta_1)s^{n-1} + \cdots + (a_{n-1} + \delta_{n-1})s + (a_n + \delta_n) = 0$$

This characteristic eq^n is obtained for the system with state feedback. So it must be equal to the desired characteristic eq^n.

$$\Rightarrow s^n + (a_1+\delta_1)s^{n-1} + \cdots + (a_{n-1}+\delta_{n-1})s + (a_n+\delta_n)$$
$$= s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1}s + \alpha_n = 0$$

Equating, the co-efficients of equal power of s

$a_1 + \delta_1 = \alpha_1$

$a_2 + \delta_2 = \alpha_2$

$\vdots$

$a_n + \delta_n = \alpha_n$

$$\Rightarrow K = [\alpha_n - a_n \quad \alpha_{n-1} - a_{n-1} \quad \cdots \quad \alpha_1 - a_1]\, T^{-1}$$

Q) Consider the following control system,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

that use the state feedback control, $u = -Kx$. For the given system determine the state feedback gain matrix K such that desired poles will be at $s = -2 + j4$, $s = -2 - j4$ & $s = -10$.

<u>Soln</u>: Determination of K using Transformation matrix.

<u>Step-1</u>: Obtain the controllability matrix M and check the controllability of the system.

Here, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$M = [B \quad AB \quad A^2B]$$
$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix}$$

$|M| = 3$. Hence the system is completely state controllable.

step-2 Obtain the charecteristic eq^n of the given system.

$$|sI - A| = \begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{vmatrix}$$

$$= \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 5 & s+6 \end{vmatrix}$$

$$= s^3 + 6s^2 + 5s + 1$$

comparing the charecteristic eq^n with $s^3 + a_1 s^2 + a_2 s + a_3 = 0$

we get,

$$\boxed{a_1 = 6,\ a_2 = 5 \text{ and } a_3 = 1}$$

step-3 Obtain the desired charecteristic eq^n

$$(s+2-j4)(s+2+j4)(s+10) = s^3 + 14s^2 + 60s + 200$$

comparing it with $s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3 = 0$

$$\boxed{\alpha_1 = 14,\ \alpha_2 = 60 \text{ and } \alpha_3 = 200}$$

step-3 obtain the transformation matrix T.

Since the system is already in CCF so $T = I$

step 4 Determine the state feedback gain matrix.

$$K = [\alpha_3 - a_3 \quad \alpha_2 - a_2 \quad \alpha_1 - a_1]\, T^{-1}$$

$$= [200-1 \quad 60-5 \quad 14-6]$$

$$K = [199 \quad 55 \quad 8]$$

Determination of K using direct substitution method

step-1 $K = [k_1 \quad k_2 \quad k_3]$

step-2 Desired charecteristic eq^n can be obtained from

$$|sI - A + BK|$$

$$= \left| \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [k_1 \ k_2 \ k_3] \right|$$

$$= \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1+k_1 & 5+k_2 & s+6+k_3 \end{vmatrix}$$

$$= s^3 + (6+k_3)s^2 + (5+k_2)s + 1+k_1$$

Step-3 Desired characteristic eq^n as obtained from desired poles locations.

$$s^3 + 14s^2 + 60s + 200$$

$$\Rightarrow s^3 + (6+k_3)s^2 + (5+k_2)s + (1+k_1) = s^3+14s^2+60s+200$$

$$\Rightarrow k_1 = 199, \ k_2 = 55 \ \& \ k_3 = 8$$

$$K = [199 \ \ 55 \ \ 8]$$

Determination of matrix K using Ackermann's formula

Let the given c/s be defined as

$$\dot{x} = Ax + Bu$$

If state feedback control is used,

$$u = -Kx$$

$$\dot{x} = Ax - BKx$$

$$\dot{x} = (A - BK)x$$

Let's define, $\tilde{A} = A - BK$

$$\dot{x} = \tilde{A}x$$

The desired characteristic eq^n, from state feedback,

$$|sI - A + BK|$$

It should be equal to the desired char. eq^n as obtained from desired pole locat^n.

$$(s-\mu_1)(s-\mu_2)\dots(s-\mu_n) = s^n+\alpha_1 s^{n-1}+\dots+\alpha_{n-1}s+\alpha_n$$

$$\Rightarrow |sI-A+BK| = s^n+\alpha_1 s^{n-1}+\alpha_2 s^{n-2}+\dots+\alpha_{n-1}s+\alpha_n$$

According Cayley-Hamilton's Th^m, $\tilde{A}$ satifies its own charecteristic eq^n.

so, $\phi(\tilde{A}) = \tilde{A}^n+\alpha_1\tilde{A}^{n-1}+\dots+\alpha_{n-1}\tilde{A}+\alpha_n I = 0$

for a 3rd order system,

$$\phi(\tilde{A}) = \tilde{A}^3+\alpha_1 \tilde{A}^2+\alpha_2\tilde{A}+\alpha_3 I = 0 \quad \text{---(1)}$$

$I = I$

$\tilde{A} = A-BK$

$\tilde{A}^2 = (A-BK)(A-BK) = A^2-ABK-BK(A-BK)$

$= A^2-ABK-BK\tilde{A}$

$\tilde{A}^3 = (A-BK)^3 = (A-BK)(A^2-ABK-BK\tilde{A})$

$= A^3-A^2BK-ABK\tilde{A}-BK(A^2-ABK-BK\tilde{A})$

$= A^3-A^2BK-ABK\tilde{A}-BK\tilde{A}^2$

Substituting these values in eq^n -(1)

$$\phi(\tilde{A}) = A^3-A^2BK-ABK\tilde{A}-BK\tilde{A}^2+\alpha_1(A^2-ABK-BK\tilde{A}) + \alpha_2(A-BK)+\alpha_3 I = 0$$

$$\Rightarrow \phi(\tilde{A}) = \alpha_3 I+\alpha_2 A+\alpha_1 A^2+A^3-\alpha_2 BK-\alpha_1 ABK - \alpha_1 BK\tilde{A}-A^2BK-ABK\tilde{A}-BK\tilde{A}^2$$

But $\alpha_3 I+\alpha_2 A+\alpha_1 A^2+A^3 = \phi(A) \neq 0$

$$\Rightarrow \phi(\tilde{A}) = \phi(A)-\alpha_2 BK-\alpha_1 BK\tilde{A}-BK\tilde{A}^2 - \alpha_1 ABK-ABK\tilde{A}-A^2BK$$

since $\phi(\tilde{A}) = 0$

$$\phi(A) = B(\alpha_2 K+\alpha_1 K\tilde{A}+K\tilde{A}^2)+AB(\alpha_1 K+K\tilde{A})+A^2BK$$

$$= [B \quad AB \quad A^2B]\begin{bmatrix} \alpha_2 K+\alpha_1 K\tilde{A}+K\tilde{A}^2 \\ \alpha_1 K+K\tilde{A} \\ K \end{bmatrix}$$

$$[B \ AB \ A^2B]^{-1} \phi(A) = \begin{bmatrix} \alpha_2 K + \alpha_1 K\tilde{A} + K\tilde{A}^2 \\ \alpha_1 K + K\tilde{A} \\ K \end{bmatrix}$$

$$[0 \ 0 \ 1][B \ AB \ A^2B]^{-1} \phi(A) = [0 0 1]\begin{bmatrix} \alpha_2 K + \alpha_1 K\tilde{A} + K\tilde{A}^2 \\ \alpha_1 K + K\tilde{A} \\ K \end{bmatrix}$$

$$\boxed{K = [0 \ 0 \ 1][B \ AB \ A^2B]^{-1} \phi(A)}$$

Soln: desired charecteristic eqn:

$$s^3 + 14s^2 + 60s + 200 = 0$$

$$\phi(A) = A^3 + 14A^2 + 60A + 200I$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}^3 + 14\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} + 60\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$$

$$+ 200\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 199 & 55 & 8 \\ -8 & 159 & 7 \\ -7 & -43 & 117 \end{bmatrix}$$

$$[B \ AB \ A^2B] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix}$$

$$[B \ AB \ A^2B]^{-1} = \begin{bmatrix} 5 & 6 & 1 \\ 6 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$K = [0 \ 0 \ 1][B \ AB \ A^2B]^{-1} \phi(A)$$

$$= [0 \ 0 \ 1]\begin{bmatrix} 5 & 6 & 1 \\ 6 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 199 & 55 & 8 \\ -8 & 159 & 7 \\ -7 & -43 & 117 \end{bmatrix}$$

$$K = [199 \ \ 55 \ \ 8]$$

STATE OBSERVERS

STATE OBSERVER

A device (or a computer program) that estimates or observes the state variable is called a state observer or simply observer.

Full Order State Observer

If the state observer observes all state variables of the system, regardless of wheather some state variables are available for direct measurement, it is called a full order state observer.

Reduced Order state Observer

An observer that estimates fewer than 'n' state variables, where n is the dimension of the state vector, is called reduced order state observer or simply a reduced order observer.

Minimum Order state Observer

If the order of reduced order state observer is the minimum possible, then it is called as minimum order state observer.

Mathematical model of state observer

The observer is a subsystem to reconstruct the state vector of the plant. So the mathematical model of the observer is basically the same as that of the plant, except that inclusion of an additional term that estimate error to compensate for inaccuries in matrices A, B and the lack of initial error.

$$\dot{\tilde{x}} = A\tilde{x} + Bu + K_e(y - C\tilde{x})$$

$$\boxed{\dot{\tilde{x}} = (A - K_eC)\tilde{x} + Bu + K_ey}$$

where, $\tilde{x}$ → Observed state of the plant

x → Actual state of the plant

K_e → Observer gain matrix.

Designing of Full order State Observer

If the plant is modelled by,

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

, then

the model of observer will be given by,

$$\dot{\tilde{x}} = A\tilde{x} + Bu + K_e(y - C\tilde{x})$$
$$\dot{\tilde{x}} = A\tilde{x} + Bu + K_e(Cx - C\tilde{x})$$
$$\dot{x} - \dot{\tilde{x}} = Ax + \cancel{Bu} - A\tilde{x} - \cancel{Bu} - K_e(Cx - C\tilde{x})$$
$$\dot{x} - \dot{\tilde{x}} = Ax - K_eCx - A\tilde{x} - K_eC\tilde{x}$$
$$= A(x - \tilde{x}) - K_eC(x - \tilde{x})$$
$$\dot{x} - \dot{\tilde{x}} = (A - K_eC)(x - \tilde{x})$$

$$\dot{x} - \dot{\tilde{x}} = e$$

$$\boxed{\dot{e} = (A - K_eC)e}$$

The dynamic behaviour of the error vector is determined by the eigen values of the matrix $A - K_eC$. If the matrix $A - K_eC$ is a stable matrix, the error vector will converge to zero for any initial error vector $e(0)$. That is $\tilde{x}(t)$ will converge to $x(t)$ regardless of the values of $x(0)$ and $\tilde{x}(0)$. Hence the design of the full order observer becomes that of determining anppropriate K_e such that $A - K_eC$ has desired eigen values. Thus the problem here becomes the same as the pole placement design.

Hence designing a observer is same as the pole-placement problem for its dual system.

If a system is represented by,

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Then its dual system will be,

$$\dot{z} = A^* z + C^* v$$
$$n = B^* z$$

So, the necessary and sufficient condition for state observation is, the original system should be completely observable.

Transformation Approach to Obtain Ke

$$Q = (WN^*)^{-1}$$

$$N = [\, C^* \quad A^* C^* \; \dots \; (A^*)^{n-1} C^* \,]$$

$$W = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ \vdots & & & & \\ a_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$K_e = Q \begin{bmatrix} \alpha_n - a_n \\ \alpha_{n-1} - a_{n-1} \\ \vdots \\ \alpha_1 - a_1 \end{bmatrix}$$

Ackerman's Formula to Obtain Ke

$$K_e = \phi(A) \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

Q) Design a full order state observer for the system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where, $A = \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [0 \quad 1]$

Assume that the desired eigenvalues of the observer matrix are $\mu_1 = -10$ and $\mu_2 = -10$.

Solⁿ:

$C^* = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $A^* C^* \cancel{= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix}} =$

$$A^* C^* = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$N = [C^* \quad A^* C^*] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Since the ~~orde~~ rank of N is 2, so the system is observable.

<u>Using transformation matrix</u>

Since the system is OCF, so, $Q = I$

Desired poles, $(s+10)(s+10) = s^2 + 20s + 100$

$$\boxed{\alpha_1 = 20 \text{ and } \alpha_2 = 100}$$

The characteristic eq^n of the given system is

$$|sI - A| = \begin{vmatrix} s & -20.6 \\ -1 & s \end{vmatrix} = s^2 - 20.6$$

$$\boxed{a_1 = 0, \quad a_2 = -20.6}$$

$$\text{so, } K_e = Q\begin{bmatrix} \alpha_2 - a_2 \\ \alpha_1 - a_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 120.6 \\ 20 \end{bmatrix}$$

$$K_e = \begin{bmatrix} 120.6 \\ 20 \end{bmatrix}$$

<u>Using direct substitution</u>

The characteristic eq^n of the observer is

$$|sI - A + K_e C| = 0$$

Define $K_e = \begin{bmatrix} K_{e1} \\ K_{e2} \end{bmatrix}$

The desired characteristic eq^n is

$$(s-10)(s-10) = s^2 + 20s + 100 = 0$$

$$\left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} K_{e1} \\ K_{e2} \end{bmatrix}[0 \quad 1] \right| = \begin{vmatrix} s & -20.6 + K_{e1} \\ -1 & s + K_{e2} \end{vmatrix}$$

$$= s^2 + K_{e2}s - 20.6 + K_{e1} = 0$$

Comparing it with the desired charecteristic eqⁿ.

$K_{e1} = 120.6$ and $K_{e2} = 20$

$$K_e = \begin{bmatrix} 120.6 \\ 20 \end{bmatrix}$$

Using Ackermann's Formula

$$K_e = \phi(A) \begin{bmatrix} C \\ CA \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\phi(s) = (s-\mu_1)(s-\mu_2) = s^2 + 20s + 100$$

$$\phi(A) = A^2 + 20A + 100I$$

$$= \begin{bmatrix} 120.6 & 412 \\ 20 & 120.6 \end{bmatrix}$$

$$\begin{bmatrix} C \\ CA \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$K_e = \begin{bmatrix} 120.6 & 412 \\ 20 & 120.6 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$K_e = \begin{bmatrix} 120.6 \\ 20 \end{bmatrix}$$

So expression for state-observer will be,

$$\dot{\tilde{x}} = (A - K_eC)\tilde{x} + BU + K_e y$$

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \left[\begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 120.6 \\ 20 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \right] \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 120.6 \\ 20 \end{bmatrix} y$$