Metal Cutting & Machining

MODULE-2

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Syllabus

Machinability Criteria, Tool life and Taylor's equation, Effect of variables on tool life and surface finish, Measurement of cutting force, Lathe tool dynamometer, and Drill tool dynamometer. Economics of Machining: Minimum Cost, Maximum Production, and Maximum Profit Rate.

Learning Outcomes

After completing Module II, students will be able to:

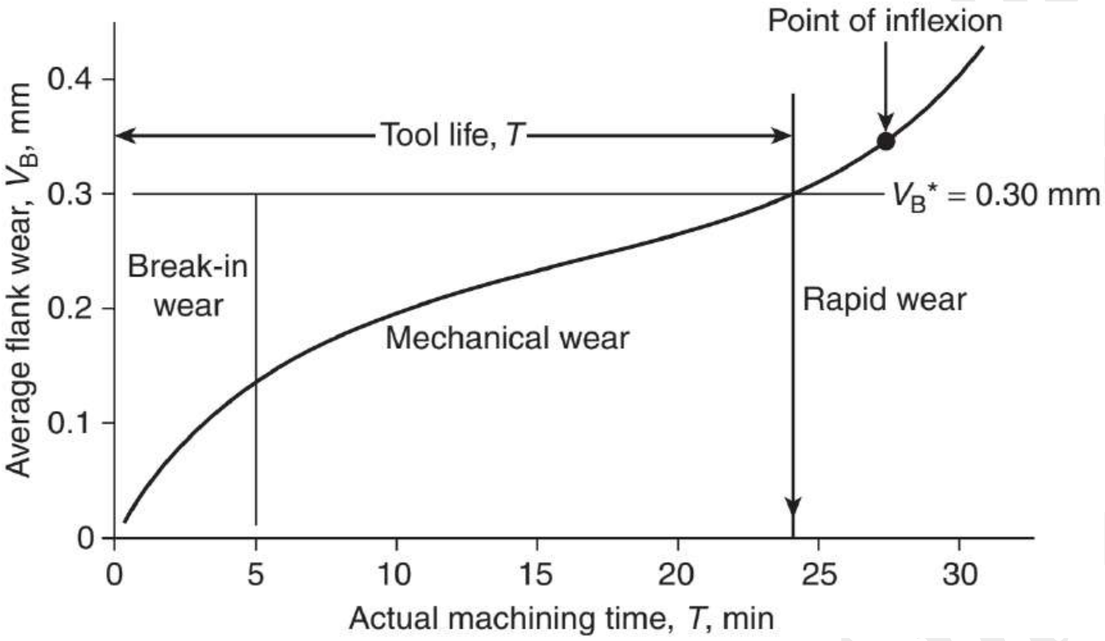
- **Understand machinability criteria** – Students will gain knowledge of how machinability is defined, assessed, and compared across different materials.
- **Predict and evaluate tool life** – They will learn to use Taylor's tool life equation and understand how cutting variables (speed, feed, depth of cut) influence tool wear and surface finish.
- **Measure and analyze cutting forces** – Students will develop the ability to use instruments like lathe tool dynamometer and drill tool dynamometer to measure forces during machining.
- **Link cutting conditions with product quality** – They will understand how machining parameters affect surface finish, productivity, and tool performance.
- **Apply economic considerations in machining** – Students will be able to calculate and compare machining conditions for minimum cost, maximum production rate, and maximum profit rate, bridging technical and economic decision-making.

Machinability

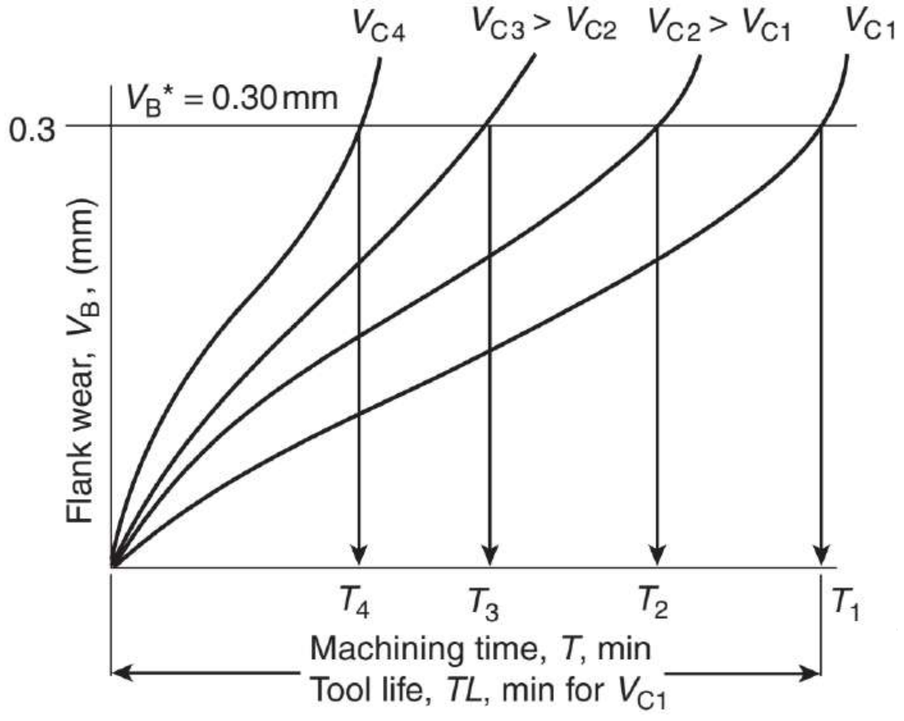
- Machinability simply means 'ease of machining', which is judged by
    1. Magnitude of the cutting forces.
    2. Level of the cutting temperature.
    3. Wear and life of cutting tools.
    4. Surface roughness.
    5. Chip formation.
- Improved machinability means lesser cutting forces, lower cutting temperature, slower tool wear or longer tool life, better surface finish and favourable chip form but without sacrificing productivity.
- Improvement of machinability for a given workpiece requires the following:
    1. Proper design and preforming of the blank.
    2. Proper planning and scheduling of the machining operations.
    3. Appropriate selection of cutting tools - type, material and geometry.
    4. Optimum selection of the values of the process parameters - cutting velocity, feed and depth of cut.
    5. Proper selection and application of environment or cutting fluid for cooling and lubrication at the cutting zone.

Tool Life

- Tool life is defined by the span of actual uninterrupted machining time through which the tool or tool tip renders desired service and satisfactory performance, and after which that tool needs **replacement**, **regrinding**, or **indexing**.
- Whenever a significant parameter or index of tool wear reaches its stipulated value, the tool tip is declared to have failed, and the corresponding span of machining time is considered to be the tool life.
- Globally standardized, that when the value of average flank wear (V_B) reaches **0.30 mm**, the tool tip would be considered to have failed.

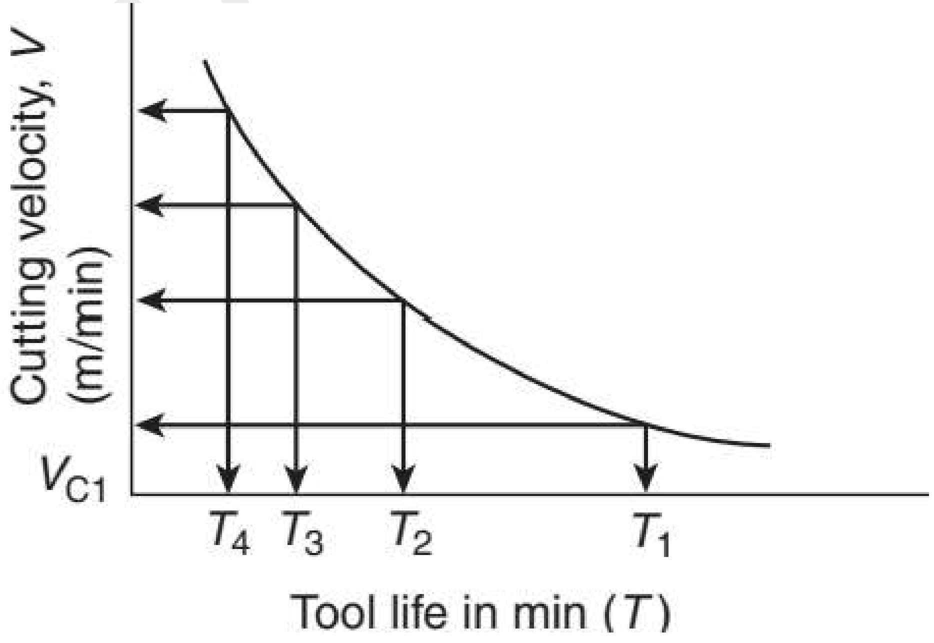


- Systematic wear passes through three stages:
  (a) Rapid break-in wear initiated by attrition.
  (b) A longer span of slowly and uniformly growing mechanical wear.
  (c) Lastly, rapid wear is accelerated by diffusion, grain pull-out, fracturing, flaking, etc.
- The rate of growth of tool wear and hence tool life depends upon the tool and work materials, and the cutting conditions.
- The levels of the process parameters more or less influence the growth of wear.
- Cutting velocity plays the most significant role.
- Application of cutting fluid also plays a substantial role in the control of tool wear and improvement in tool life.



Role of increase in cutting velocity on the growth of flank wear and tool life

- The rate of growth of tool-wear increases sharply with the increase in cutting velocity.
- Whereas it increases moderately with increase of feed.
- Increase in depth of cut , if not excessive, generally does not influence tool wear and tool life.
- Under a given condition of machining with fixed feed and depth of cut, the values of tool life decreases from T_1 to T_4 due to increase in cutting velocity from V_1 to V_4.



Nature of variation in tool life with the increase in cutting velocity.

Taylor's Tool Life Equation

$$VT^n = C$$

V= Cutting Velocity
T= Tool Life
n= Taylor's exponent
C= Taylor's constant

The values of the Taylor's constants, n and C, depend upon the following:

1. Work material and tool material.
2. Maximum permissible value of V_B (i.e., 0.3, 0.4, 0.6 mm, etc.).
3. Type and method of application of cutting fluid.
4. Levels of feed and also depth of cut t to some extent.

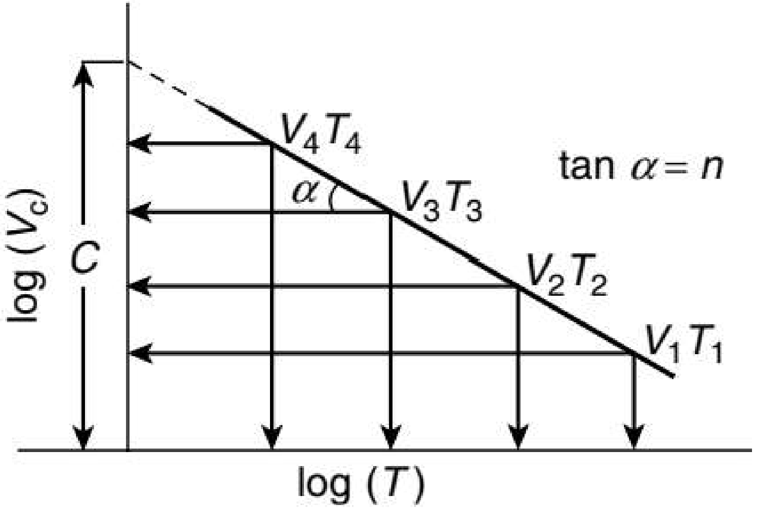
$$VT^n = C \Rightarrow \log(VT^n) = \log C$$

$$\Rightarrow \log V + \log T^n = \log C$$

$$\Rightarrow \log V + n \log T = \log C$$

$$\Rightarrow \log V = -n \log T + \log C$$

$$y = mx + c$$



Tool life versus cutting velocity in log–log scale.

Modified Taylor's Tool Life Equation

Taylor's tool life equation can be modified to the following equation:

$$VT^n {s_o}^x t^y = C$$

Here n, x, y are tool life exponents; s_o is feed rate in mm/rev; t is depth of cut in mm.

Now the equation can be written as

$$T^n = \frac{C}{V {s_o}^x t^y}$$

$$\Rightarrow T = \frac{C^{1/n}}{V^{1/n} . {s_o}^{x/n} . t^{y/n}}$$

In general $1/n > x/n > y/n$; therefore

$$Cutting\ velocity > Feed > Depth\ of\ cut$$

Question: Under a given condition of turning, the tool life was found to decrease from 24 min to 16 min when only the cutting velocity V_c was raised from 200 m/min to 250 m/min. What will be the tool life if the cutting velocity is further increased to 300 m/min under the same machining conditions?

Solution:

Given data;

V_1= 200m/min, T_1= 24min

V_2= 250m/min, T_2= 16 min

V_3= 300 m/min, T_3=?

Taylor's tool life equation; $VT^n = C \Rightarrow V_1 T_1^n = V_2 T_2^n = V_3 T_3^n$

$$\left(\frac{T_2}{T_1}\right)^n = \left(\frac{V_1}{V_2}\right) \Longrightarrow \left(\frac{16}{24}\right)^n = \left(\frac{200}{250}\right) \Longrightarrow n = 0.55$$

Again,

$$\left(\frac{T_3}{T_1}\right)^n = \left(\frac{V_1}{V_3}\right) \Longrightarrow T_3 = T_1 \left(\frac{V_1}{V_3}\right)^{1/n} = 24 \times \left(\frac{200}{300}\right)^{1/0.55} = 11.6\, min$$

Practice Questions

Q1. In a machine shop, single-point turning tools are used at 75m/min to yield a tool life of 30 min. However, war breaks out, and turning tools have to be used at 100 m/min to yield a tool life of 20 ins. Lastly, the machine shop falls into the hands of the enemy army, which starts running the tools at a speed of 125 m/min. What will be the tool life they will get?

Q2. The life of a plain milling cutter of diameter 75 mm was found to decrease from 50 min to 30 min due to an increase in speed (N) of the cutter from 200 to 260 rpm while milling a cast iron plate at a given feed and depth. How much would the life of that cutter be if the speed is 120 rpm, keeping all other parameters unchanged?

Q3. The following data is received for a new carbide tool machining low-carbon steel. Determine the n and C of Taylor's tool life equation given as $TV^n=C$

V (m/min)	40	50	60	70	80
T (min)	40	32	26	20	17

Role of Different Machining Parameters on Tool Life

Work Material

- Metallurgical Factor: composition, microstructure, chemical affinity, chemical aggressiveness towards tool
- Mechanical Properties: High strength, Hardness and Work hardenability
- Tool life decreases with more heat resistivity of the work material

Tool Material

- Chemically inert or stable against the work material, atmospheric gases and cutting fluid, undergo lesser and slower damage
- Finer and refined microstructure of the tool material renders improved tool life
- Tool strength and hot strength, high hardness and hot hardness, high fracture toughness.
- Low thermal conductivity at the surfaces but high thermal conductivity at its core is ideal for prolonged tool life.

Tool Geometry

- Proper selection of primary geometrical features
- Reduction in cutting forces and heat generation
- Better heat dissipation
- Favourable stress distribution
- Prevention of built up edge formation
- More effective cutting fluid action

Process Parameters

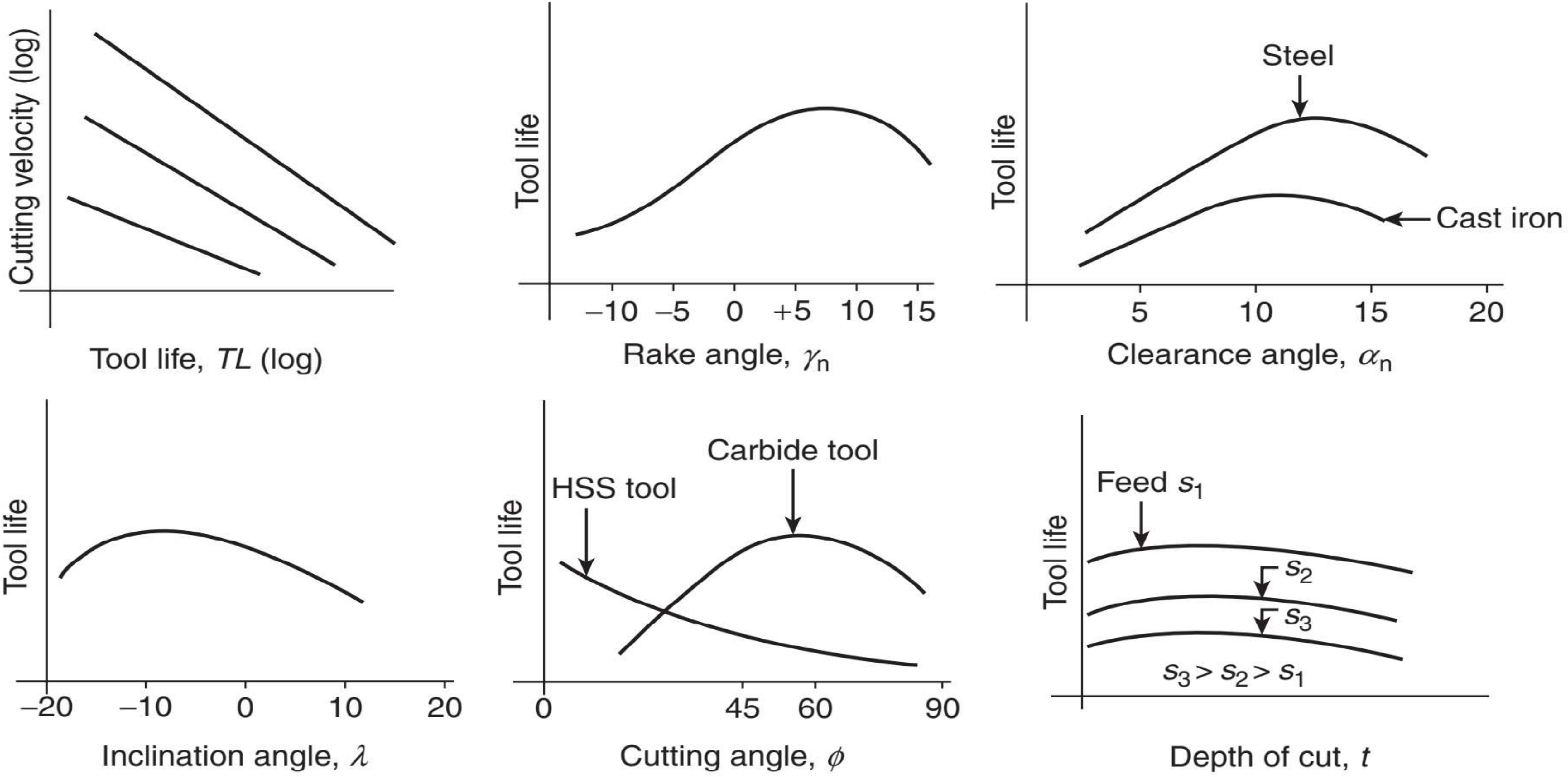
- Among the 3 major process variables cutting velocity affects the tool wear and tool life
- This is due to increased cutting temperature and longer effective sliding length.
- Increase in feed also reduces tool life as it results in increased forces and cutting temperature.

Environment (Cutting Fluid)

- Proper selection and methods of application of cutting fluid can raise tool life drastically through cooling, lubrication and cleaning of the cutting zonefrom chips and debris.

Special Techniques

- Cryogenic machining
- Dynamic machining
- Hot machining


Cutting velocity (log)
Tool life, TL (log)
Tool life
−10 −5 0 +5 10 15
Rake angle, γn
Steel
Cast iron
Tool life
5 10 15 20
Clearance angle, αn
Tool life
−20 −10 0 10 20
Inclination angle, λ
Carbide tool
HSS tool
Tool life
0 45 60 90
Cutting angle, φ
Feed s1
s2
s3
s3 > s2 > s1
Tool life
Depth of cut, t

Measurement of Cutting Forces

- The cutting forces can be either analytically estimated, approximately, or by experimental measurement, quite accurately and precisely.

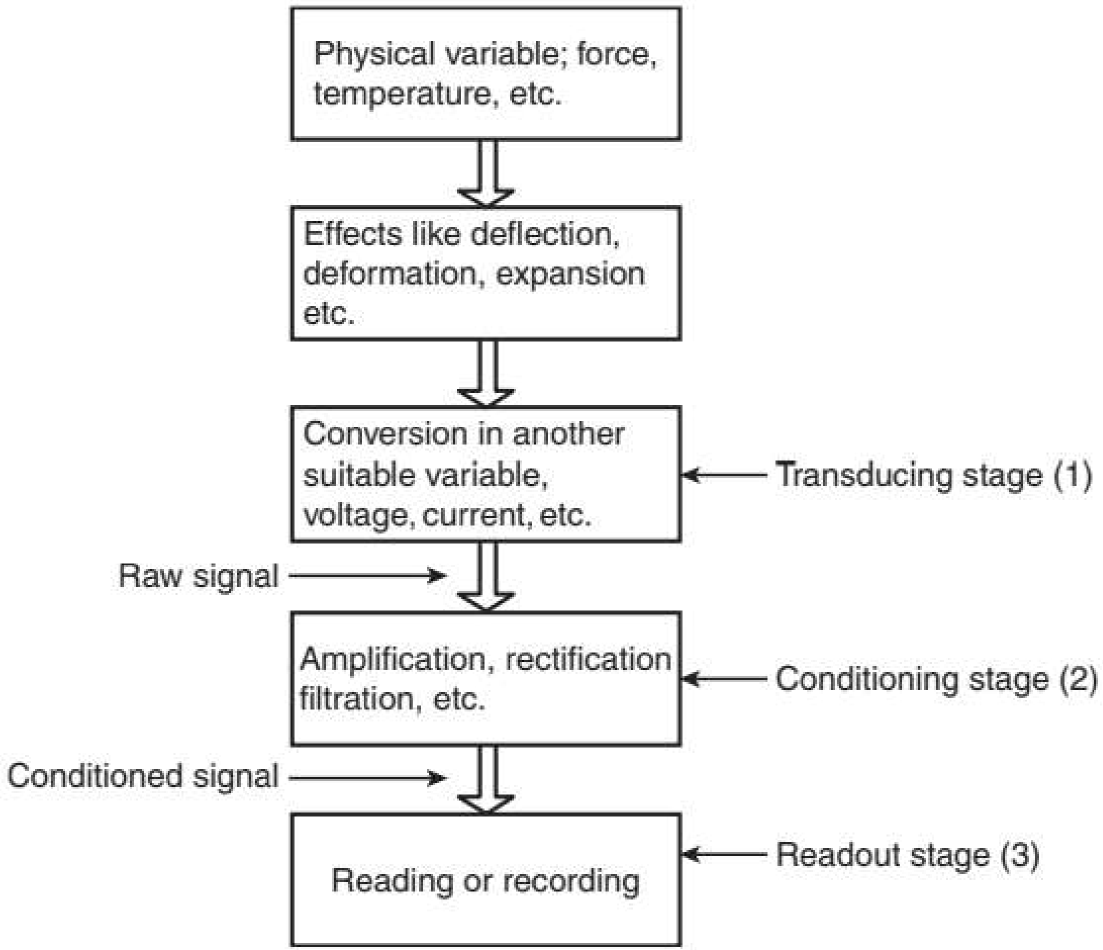
Reasons for using instruments to determine cutting forces

- Determining the magnitude and pattern of the cutting forces accurately, precisely and reliably.
- Directly getting the magnitude of the cutting forces when equations or mathematical models are not available or the available models are not adequate.
- Experimentally verifying any existing or newly proposed models of cutting forces.
- Investigating and evaluating the role of variation in any machining parameters such as process parameters, tool parameters and environment parameters on cutting forces.
- Developing an empirical model related to the magnitude of the cutting forces with the concerned machining variables or parameters.
- Studying and assessing machinability characteristics of any work material under various conditions.
- Assessing directly the relative behaviour and performance of any new work material, tool (type, material and geometry), type and method of application of cutting fluid and any special techniques in respect of cutting forces.
- Indirect but on-line monitoring of the condition of the cutting tool (wear, chipping, fracturing, plastic deformation, etc.) and the machining process (temperature, vibration, dimensional deviation, etc.).

General Process of Measuring Cutting Forces

- **Indirect (Less Accurate)**
  - Cutting Power Consumption
  - Calorimetric Method
- **Direct (More Accurate)**
  - Using suitable Tool-Force Dynamometers

Basic Principle of Force Measurement by Dynamometer



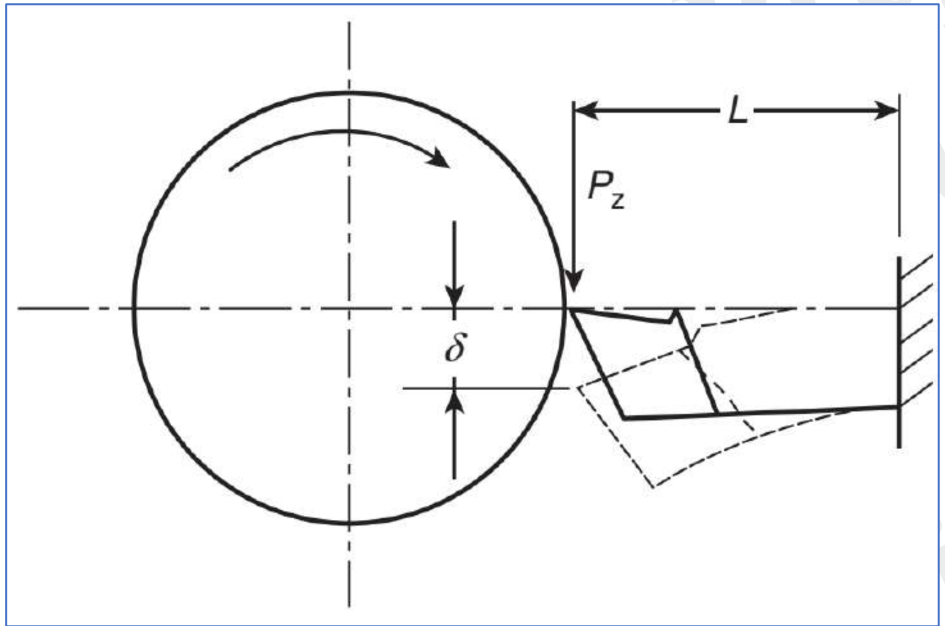
- The existence and level of some physical variables like force, temperature, etc. cannot be detected and quantified directly but are done from their effects only.
- For instance, a force is neither visible nor can be gripped. But that force may be detected and also quantified respectively by its effect(s) and the magnitude of such effects (on some object) like elastic deflection, deformation, pressure, strain, etc. These effects are called **signals**.
- The physical variables that are associated with machining and require measurement include cutting force, cutting temperature, vibration and so on.
- Such variables which cannot be measured as such are converted into another suitable physical variable called a raw signal which is mostly voltage through change in resistance, capacitance, inductance, etc. with the help of an appropriate sensor or transducer.
- The raw signal produced by the transducer needs amplification, rectification, filtering, stabilization, etc. for ease, accuracy and reliability of measurement.
- Finally, the conditioned signal is read, visualized or stored in the readout unit like Galvanometer, Oscilloscope, Strip chart recorder and Computer.

Dynamometer Working Principle

1. Measurement by Deflection
2. Measurement by Deformation (strain)
3. Measurement by Pressure

1. Determination of Cutting Forces Through Measurement of Elastic Deflection

The amount of deflection δ of the tool-chip will be proportional to the force P_z as



Elastic deflection of turning tool due to P_z.

$$\delta = P_z \left[\frac{L^3}{3EI} \right]$$

L is the length of the cantilever tool

E is the Young's Modulus of elasticity of the material of the tool

I is the plane moment of inertia of the beam section

For a given cutting tool L, E and I are fixed. So,

$$\delta \,\alpha\, P_z \quad \text{or} \quad \delta = kP_z$$

2. Determination of Cutting Forces Through Measurement of Elastic Strain

- The sensitivity and accuracy of cutting force measurement may increase by increasing the elastic deflection of the tool.
- But a large deflection of the tool, if allowed, will affect the machining and dimensional accuracy of the machined product.
- By restricting the tool deflection even to a very small amount, the magnitude of the cutting force can be quite accurately and precisely measured from the elastic strain caused by that force.
- The force (say P_z) produces proportional amounts of stresses as well as strains in an elastic member subjected to that force, as

$$\varepsilon = \frac{\sigma}{E} = \frac{M}{ZE} = \frac{P_z L}{\left(I/Y\right)E} = k.P_z$$

ε = strain

σ = stress

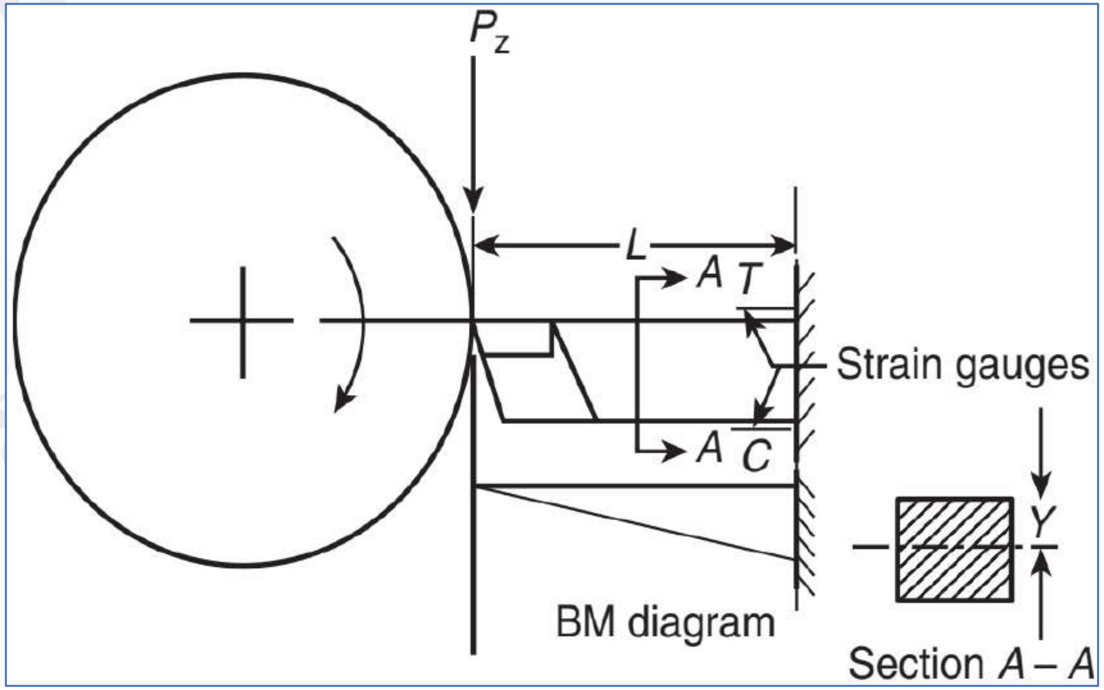
E = Young's Modulus

M = bending moment

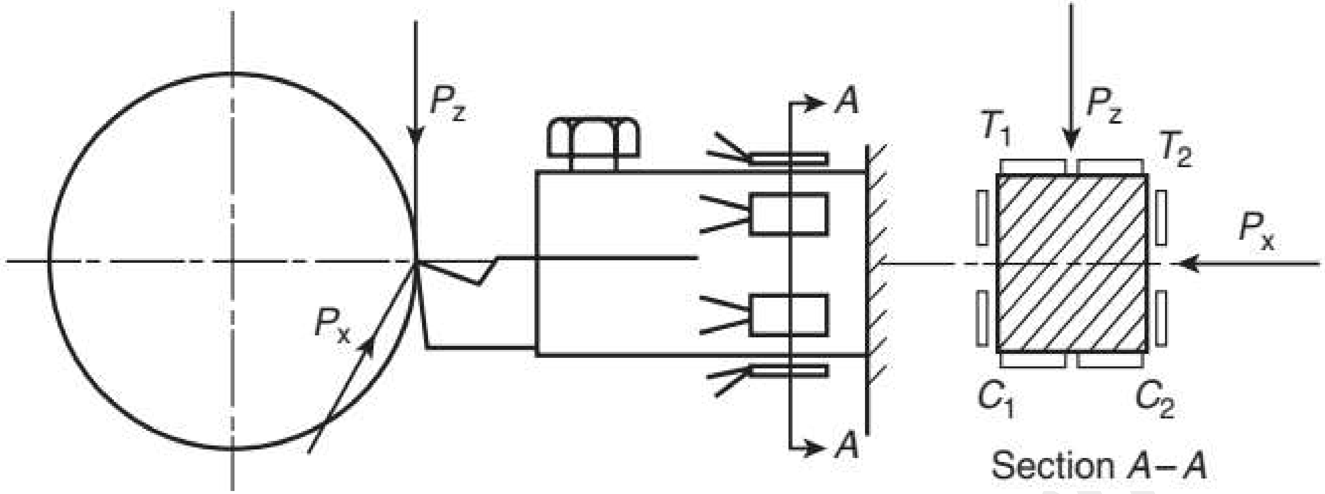
Z = sectional modulus of the beam=I/Y

I = Plane moment of inertia of the tool section

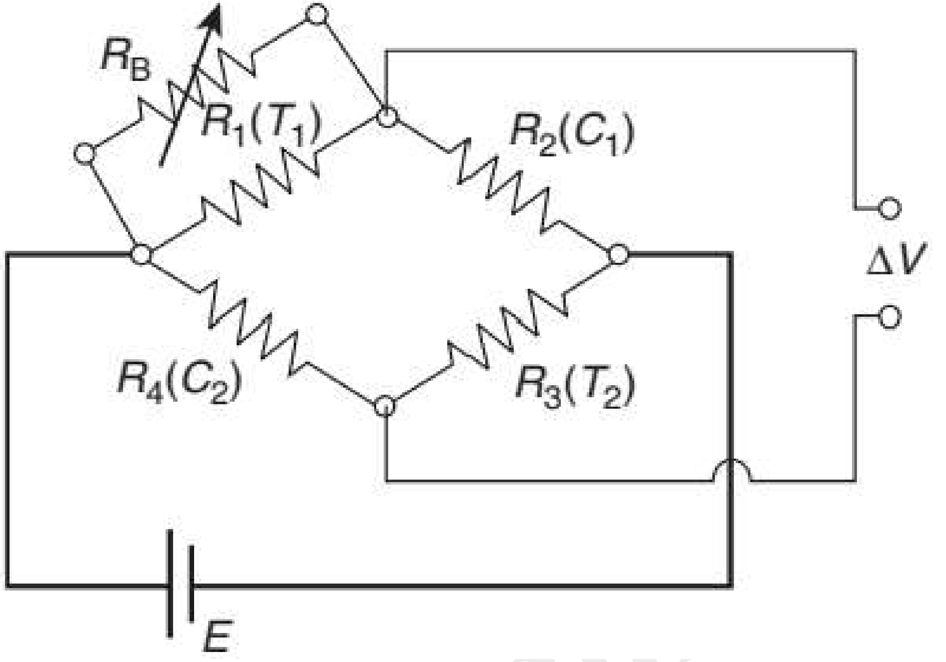
Y = distance of the surface from the neutral plane



Measuring cutting forces by strain gauge type dynamometer.



Mounting of Strain Gauges in a typical Turning Dynamometer



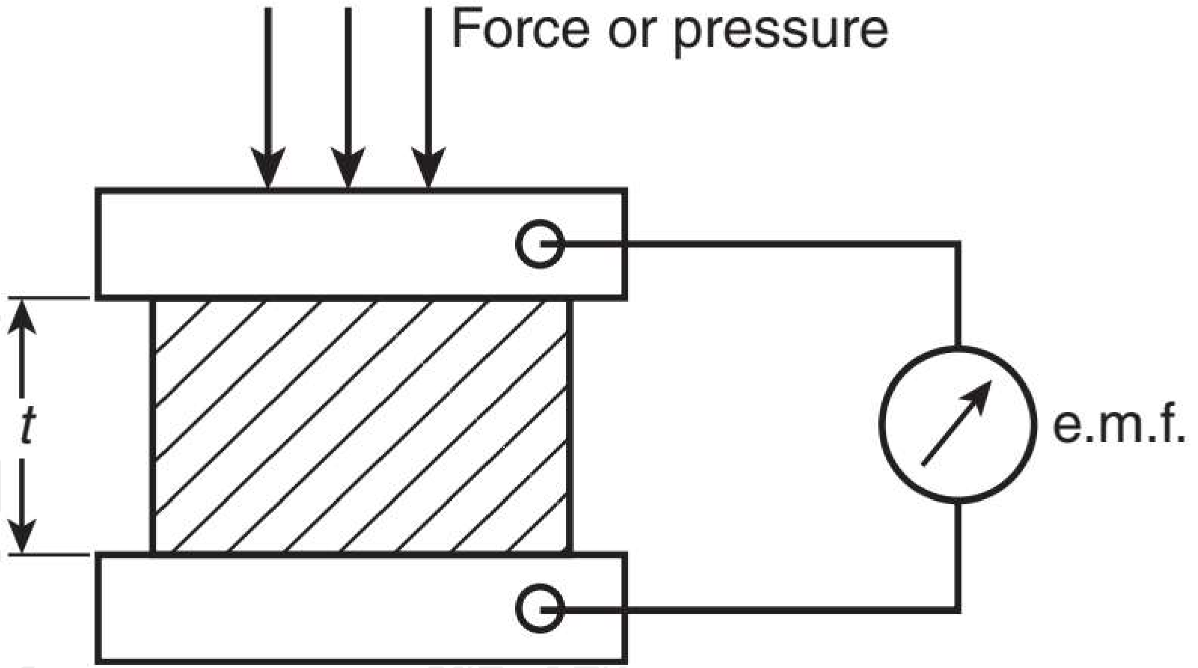
Electrical Circuit with Strain Gauges

- Due to the vertical force P_z, the two upper gauges T_1 and T_2 sense tensile strain and the gauges C_1 and C_2 at the bottom surface sense compressive strain.
- All the four gauges are usually identical and attain the same magnitude of strain (tensile or compressive).
- These four strain gauges (of resistance say R_1, R_2, R_3 and R_4) are connected in the form of a Wheatstone bridge.
- Under null or balanced condition of the Wheatstone bridge, $\frac{R_1}{R_2} = \frac{R_4}{R_3}$. The change in resistance due to straining of the strain gauges produces a voltage output ΔV under the condition $\frac{R_1}{R_2} \neq \frac{R_4}{R_3}$ as $\Delta V = k_s E G(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4)$; where, ε_1 and ε_3 are in tension and ε_2 and ε_4 are in compression and k_s is a constant.

3. Determination of Cutting Forces Using Pressure caused by the Forces

Measurement by Pressure caused by Forces

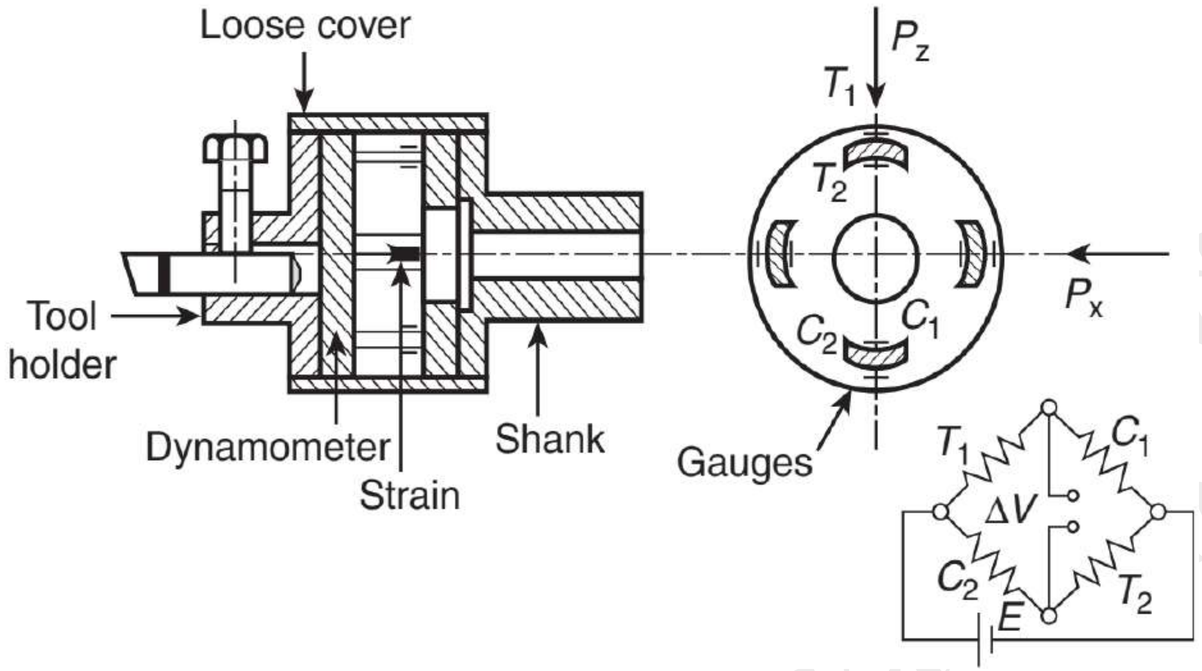
- **Hydraulic Pressure**: The force creates hydraulic pressure (through a diaphragm or piston), which is monitored directly by a pressure gauge.
- **Piezoelectric**: The force presses a piezoelectric crystal and produces a proportional electromotive force (e.m.f.).



$$\text{e.m.f.} = \lambda t p$$

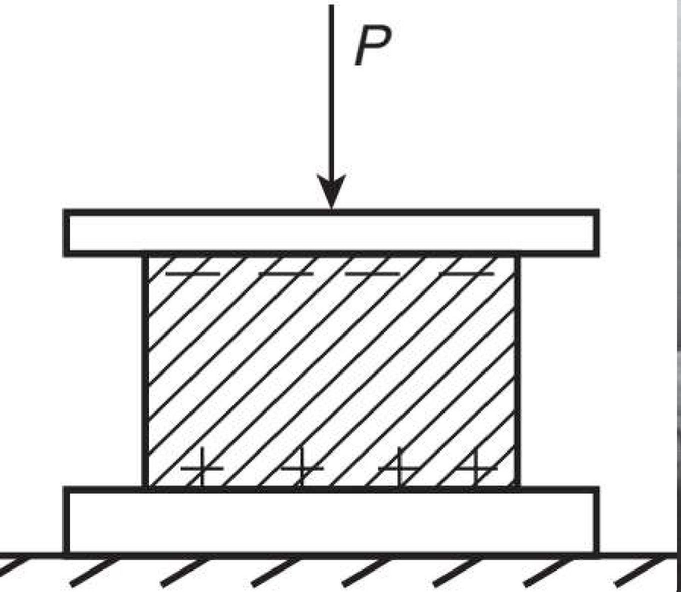
Where, λ is the voltage sensitivity of the crystal, t is the thickness of the crustal and p is the pressure exerted by the force.

Turning Dynamometer

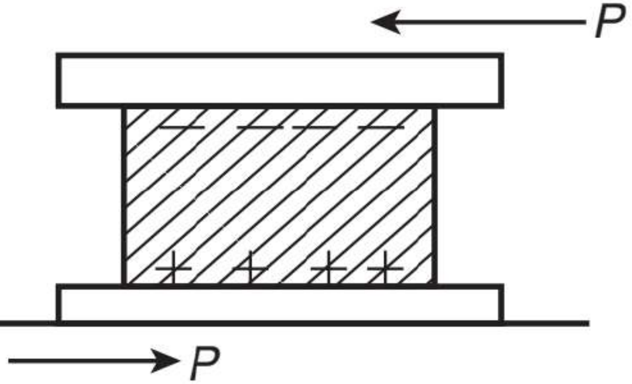


Schematic view of a strain gauge type 2-D turning dynamometer

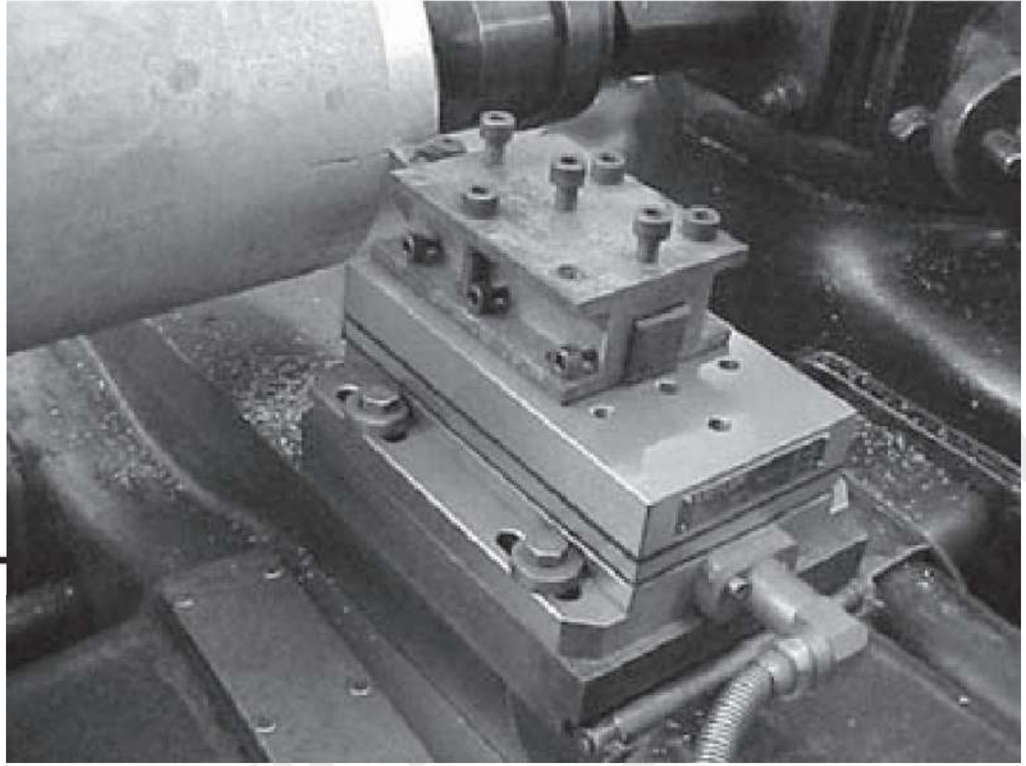
- Both strain gauge and piezoelectric based turning dynamometers of 2-D and 3-D type are available and used to measure the force components P_z, P_x and P_y.
- Strain gauge type 2-D turning dynamometers are popularly used for their simple construction and low cost.
- A typical 2-D turning dynamometer having two full bridges comprising four live strain gauges for P_z and another four for P_x.
- The two channels of P_z and P_x from the dynamometer are connected to the strain measuring bridge (SMB).
- Strain gauge type 3-D turning dynamometers have also been designed for monitoring P_z, P_x and P_y simultaneously.
- Several rings or discs of piezoelectric crystal are properly mounted and connected within the solid and rigid stainless-steel casing.
- The pressures created by the individual forces are directly converted into e.m.f. which is desirably conditioned by charge amplifiers and recorded in an oscilloscope or PC through a data acquisition system.
- Such dynamometers are highly sensitive, robust and reliable but quite expensive.



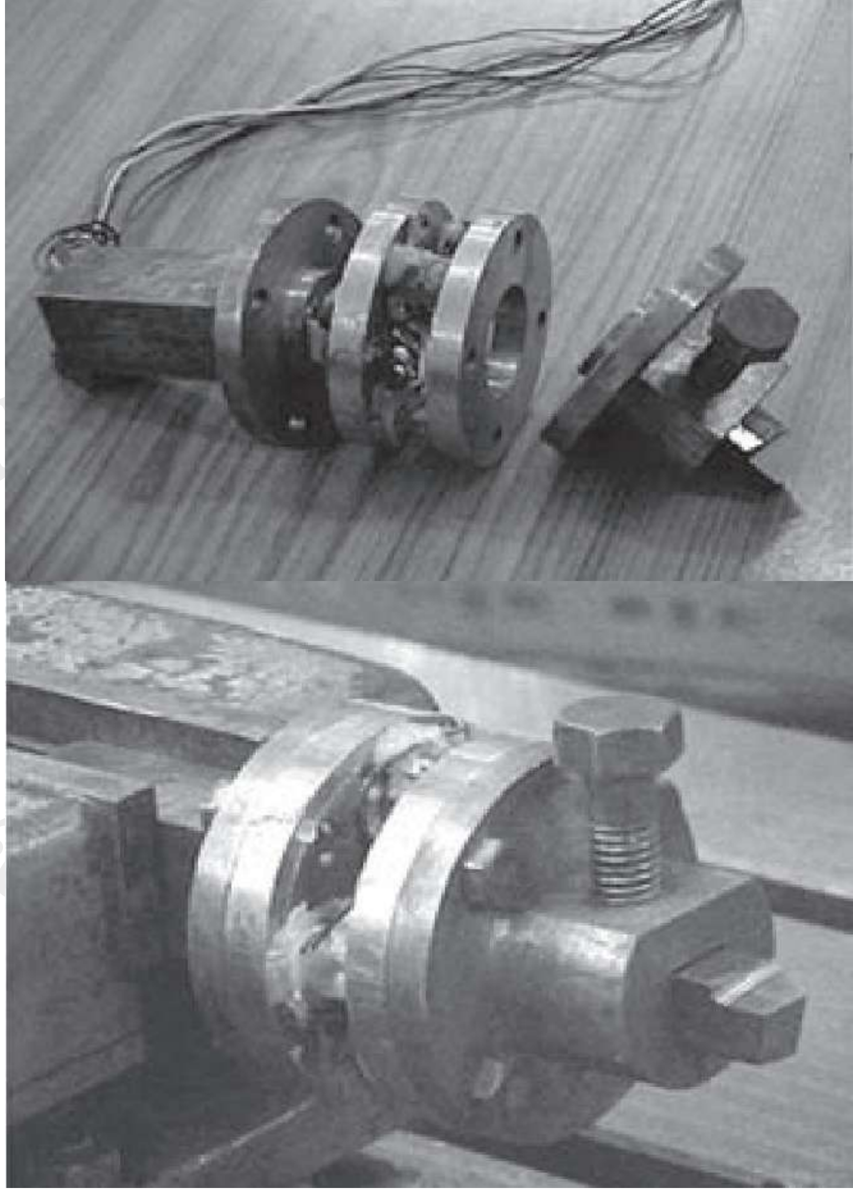
Generation of e.m.f in piezoelectric sensors by longitudinal force effect



Generation of e.m.f in piezoelectric sensors by shearforce effect



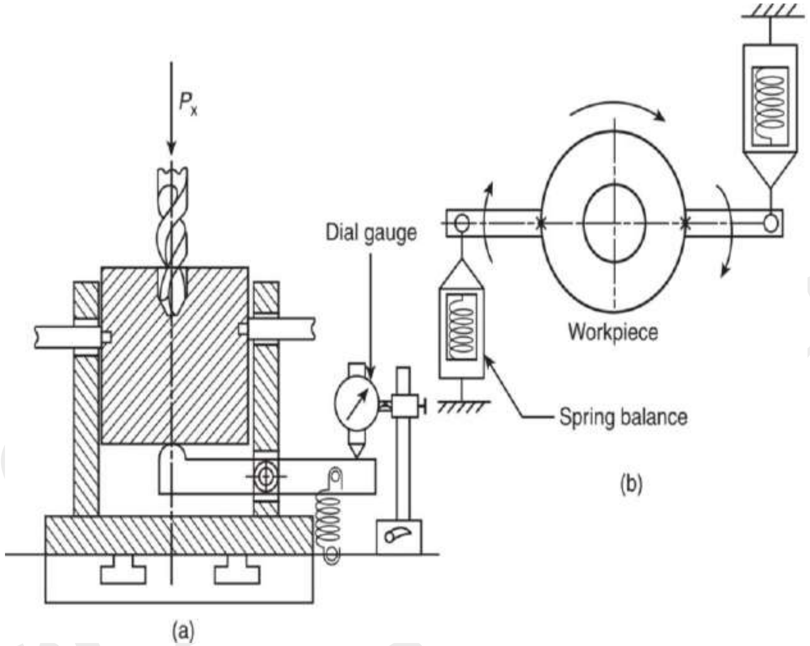
Piezoelectric type 3-D turning dynamometer (KISTLER)



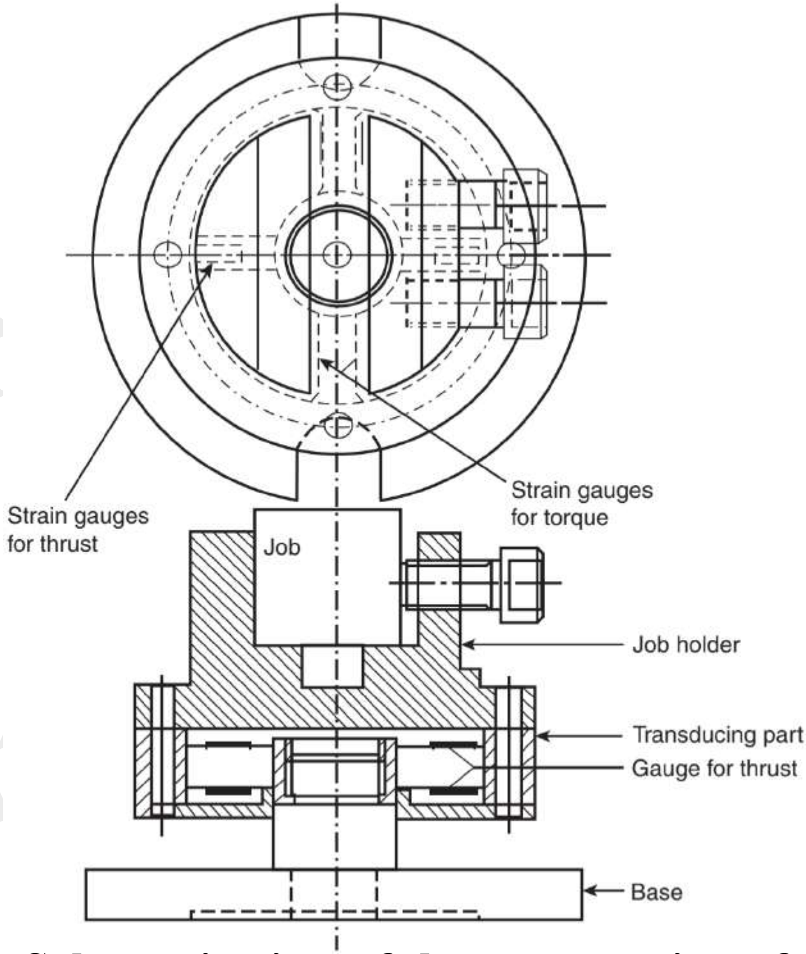
Strain Gauge type 2-D turning dynamometer

Drilling Dynamometer

- Various transducer-based dynamometers ranging from the very simple mechanical type to the modern sophisticated piezoelectric type are being used to measure the drilling torque and thrust.
- The deflection of the beam against a tension spring due to the force P_x may be detected by a dial gauge, and the torque T is detected by two spring balances.
- This crude but simple method could measure P_x and T very roughly and without their dynamic characteristics. Some hydraulic type simple drilling dynamometers were also designed and used earlier.

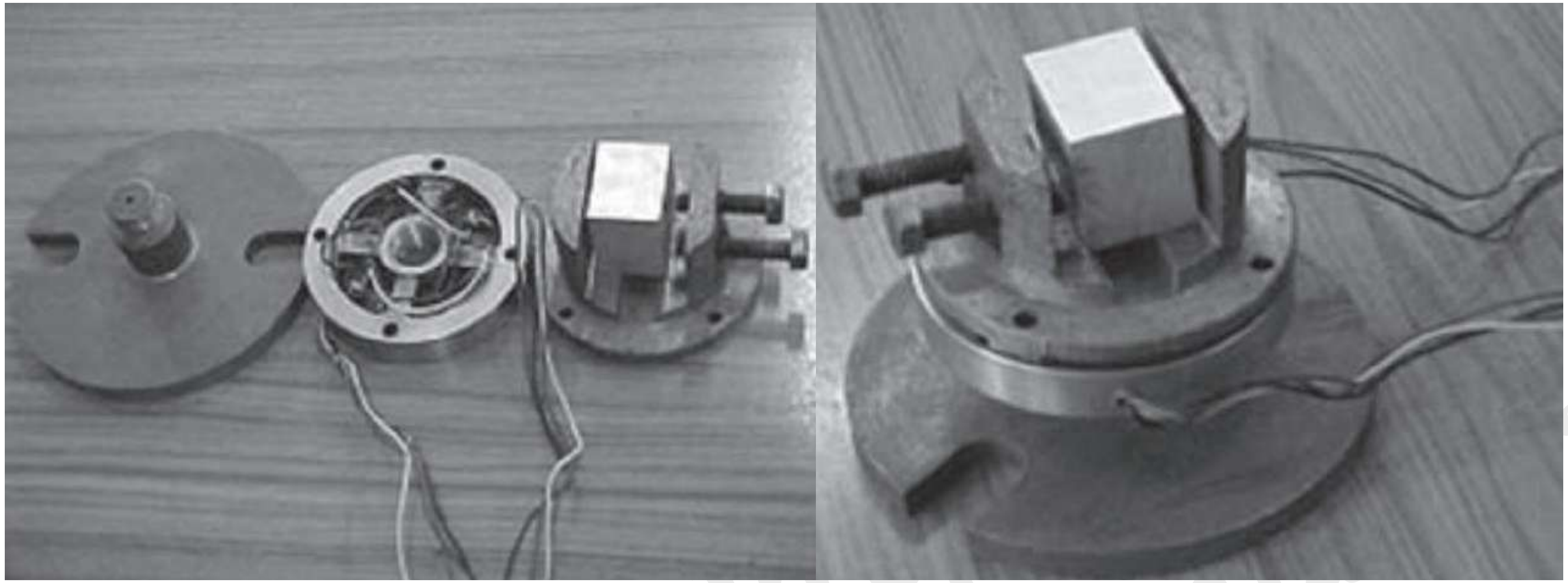


Scheme of mechanical measurement of drilling thrust and torque: (a) Measuring thrust force P_x and (b) measuring torque T.



Schematic view of the construction of a strain gauge-type drilling dynamometer.

- Strain gauge type drilling dynamometers have been popularly used for a long time for the dynamic measurement of the drilling torque and thrust reasonably, accurately and reliably.



Strain gauge type 2-D drilling dynamometer

- The main or transducing ring is fitted in between the job holder and the base plate.
- Four strain gauges are fixed on the upper and lower surfaces of the two opposite ribs for P_x.
- Similarly, another four gauges are mounted on the other two ribs at their side surfaces for the torque.

Piezoelectric type 2-D drilling dynamometer

- The four gauges of each channel are joined in the form of a Wheatstone bridge which is connected to an SMB or a PC or an oscilloscope through a charge amplifier and a data acquisition system.
- However, before actual use, the dynamometer has to be calibrated for both torque and thrust.
- The drilling forces and their dynamic characteristics can be measured highly accurately, precisely and reliably by a piezoelectric type drilling dynamometer.
- The wide use of piezoelectric type drilling dynamometers is restrained by its high cost.

Economy and Optimization of Machining

- The main aim and objective in machining industries, like any other manufacturing industries, are to continuously enhance both volume and rate of production as well as product quality by best utilization of the resources.
- Economy of machining is the method of determination of optimum process parameters for the economic criteria of machining.

Main Economic Objectives

In any machining or manufacturing process, the goals are:

1.Minimize production time → increase production rate
2.Reduce manufacturing cost
3.Maximize profit and profit rate

3 Different Criteria

In economics of machining, the criteria are:

1.Minimum cost criteria
2.Maximum production rate criteria
3.Maximum profit criteria

Factors Needed to Achieve Economic Goals

1.Proper **process planning**

2.Correct **selection of cutting tools** (material, geometry)

3.Appropriate **machine tools and accessories**

4.Selection of **cutting parameters** (cutting velocity V_c, feed s)

5.Proper **machining environment** (cutting fluid selection)

1. Minimum Cost Criteria

Total cost (TC)= $C_1 + C_2 + C_3 + C_4$

Here, **Machining cost**$(C_1) = C_m \times T_m$; C_m is the cost for machining and T_m is the time for machining.

$T_m(in\ minutes) = \frac{L}{s_o N}$; L is length (in mm), s_o is feed (rev/min) and N is rpm.

Also, $V_c = \Pi D N \Rightarrow N = \frac{V}{\Pi D}$; Hence $T_m = \frac{L \times \Pi D}{s_o V} = \frac{\Pi D L}{s_o V}$

$\boldsymbol{Idle\ Cost\ (C_2) = C_m \times T_h}$; T_h is the idle time.

$\boldsymbol{Tool\ Regrinding\ Cost\ (C_3) = C_e \times \frac{T_m}{T}}$; T is the tool life.

Ex: if T_m is 50 mins and T is 10 mins then Regrinding is 50/10=5 times.

$\boldsymbol{Tool\ Changing\ Cost\ (C_4) = C_m \times \frac{T_m}{T} \times T_c}$; T_c is the tool changing time.

From the Taylor's tool life equation,

$$VT^n = C \Rightarrow T^n = {}^{C}/_{V} \Rightarrow T = \left(\frac{C}{V}\right)^{1/n} \Rightarrow {}^{1}/_{T} = \left(\frac{V}{C}\right)^{1/n}$$

Therefore, $\frac{T_m}{T} = \frac{\Pi D L}{s_o V}\left(\frac{V}{C}\right)^{1/n}$

Now, the total cost equation becomes,

$$TC = C_1 + C_2 + C_3 + C_4 \Longrightarrow TC = (C_m \times T_m) + (C_m \times T_h) + \left(C_e \times \frac{T_m}{T}\right) + \left(C_m \times \frac{T_m}{T} \times T_c\right)$$

$$\Longrightarrow TC = \left(C_m \times \frac{\Pi DL}{s_o V}\right) + (C_m \times T_h) + \left(C_e \times \frac{\Pi DL}{s_o V}\left(\frac{V}{C}\right)^{1/n}\right) + \left(C_m \times \frac{\Pi DL}{s_o V}\left(\frac{V}{C}\right)^{1/n} \times T_c\right)$$

For minimum cost criteria,

$$\frac{d(TC)}{dV} = 0$$

$$\Longrightarrow \frac{-C_m}{V^2} + \frac{C_e}{C^{1/n}}\left(\frac{1-n}{n}\right)V^{\frac{1-2n}{n}} + C_m T_c \frac{1}{C^{1/n}}\left(\frac{1-n}{n}\right)V^{\frac{1-2n}{n}} = 0 \quad [\because \frac{dV^{1/n-1}}{dV} = \frac{1-n}{n}V^{\frac{1-n}{n}-1} = \frac{1-n}{n}V^{\frac{1-2n}{n}}]$$

$$\Longrightarrow \frac{C_m}{V^2} = \frac{C_e}{C^{1/n}}\left(\frac{1-n}{n}\right)V^{\frac{1-2n}{n}} + C_m T_c \frac{1}{C^{1/n}}\left(\frac{1-n}{n}\right)V^{\frac{1-2n}{n}}$$

$$\Longrightarrow C^{1/n} = \frac{C_e}{C^m}\left(\frac{1-n}{n}\right)V^2 V^{\frac{1-2n}{n}} + T_c\left(\frac{1-n}{n}\right)V^2 V^{\frac{1-2n}{n}}$$

$$\Longrightarrow C^{1/n} = \frac{C_e}{C^m}\left(\frac{1-n}{n}\right)V^{\frac{1}{n}} + T_c\left(\frac{1-n}{n}\right)V^{\frac{1}{n}} \qquad [\because V^2 V^{\frac{1-2n}{n}} = V^{\frac{1-2n}{n}+2} = V^{\frac{1}{n}}]$$

$$\Longrightarrow C^{1/n} = \frac{1-n}{n}.V^{1/n}\left(\frac{C_e}{C_m} + T_c\right)$$

$$\Longrightarrow \boxed{V \times \left\{\left(\frac{1-n}{n}\right)\left(\frac{C_e}{C_m} + T_c\right)\right\}^n = C}$$

Now, the above equation can be written in the form,

$$V \times \left\{\left(\frac{1-n}{n}\right)\left(\frac{C_e}{C_m} + T_c\right)\right\}^n = C \quad \Longrightarrow \quad V_{opt} \times \{T_{opt}\}^n = C$$

Hence,

$$\boxed{T_{opt} = \left(\frac{1-n}{n}\right)\left(\frac{C_e}{C_m} + T_c\right)}$$

$$\boxed{V_{opt} = \frac{C}{\left\{\left(\frac{1-n}{n}\right)\left(\frac{C_e}{C_m} + T_c\right)\right\}^n}}$$

2. Maximum Production Rate Criteria

Rate of Production; $R_P = \frac{1}{T}$

$$Total\ Time(TT) = T_m + T_h + \left(\frac{T_m}{T}\right)T_C = \frac{\Pi DL}{s_o V} + T_h + \frac{\Pi DL}{s_o V}.\left(\frac{V}{C}\right)^{1/n}.T_C$$

For maximum production rate;

$$\frac{d(TT)}{dV} = 0 \Rightarrow -\frac{1}{V^2} + \frac{T_C}{C^{1/n}}\left(\frac{1-n}{n}\right)V^{\frac{1-n}{n}-1} = 0 \Rightarrow -\frac{1}{V^2} + \frac{T_C}{C^{1/n}}\left(\frac{1-n}{n}\right)V^{\frac{1-2n}{n}} = 0$$

$$\Rightarrow \frac{1}{V^2} = \frac{T_C}{C^{1/n}}\left(\frac{1-n}{n}\right)V^{\frac{1-2n}{n}} \Rightarrow C^{1/n} = T_C\left(\frac{1-n}{n}\right)V^2 V^{\frac{1-2n}{n}} = T_C\left(\frac{1-n}{n}\right)V^{\frac{1}{n}}$$

$$\Rightarrow V\left\{T_C\left(\frac{1-n}{n}\right)\right\}^n = C \quad \Rightarrow \quad V_{opt} \times \{T_{opt}\}^n = C$$

$$\boxed{T_{opt} = T_C\left(\frac{1-n}{n}\right)}$$

$$\boxed{V_{opt} = \frac{C}{\left\{T_C\left(\frac{1-n}{n}\right)\right\}^n}}$$

3. Maximum Profit Criteria

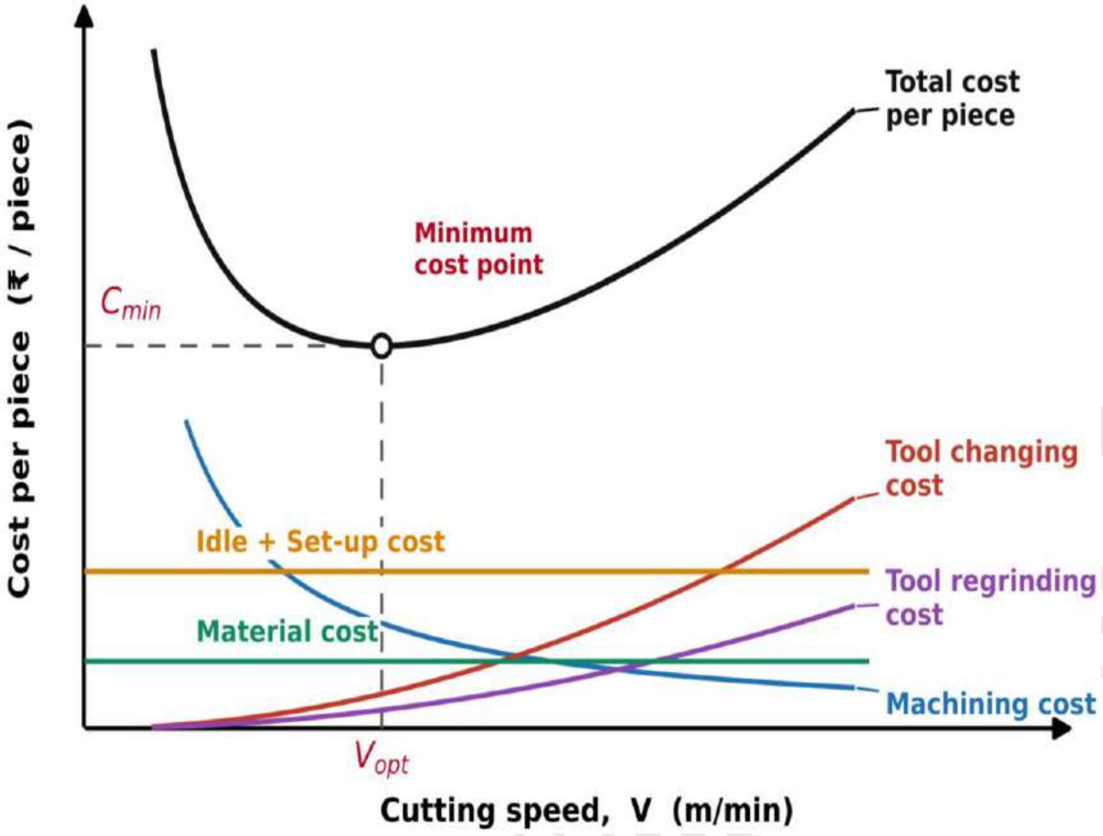
$Profit = S.P. - T.C.$ Where, S.P.=Selling Price; T.C.=Total Cost

$$Maximum\ Production\ Rate = \frac{S.P. - T.C.}{T_m + T_h + \left(\frac{T_m}{T}\right) T_C}$$

$$\boxed{V_{opt}(Production\ Rate) > V_{opt}(Profit\ Rate) > V_{opt}(Minimum\ Cost)}$$

- **Marginal Balance:** Maximum profit occurs at the specific production level where the revenue gained from selling an extra part equals the cost of producing it.
- **Demand Dependency:** Unlike minimum cost or maximum production rate criteria—which assume constant demand and fixed selling prices—the maximum profit criterion incorporates market demand functions and variable selling prices.

Role of Cutting Velocity



- **Components that do not depend on speed**
  - ✓ ***Material cost:*** cost of the blank. Fixed per piece.
  - ✓ ***Idle + set-up cost :*** loading/unloading, tool approach, workpiece indexing, machine set-up charged per piece. Horizontal line, independent of V.
- **Component that falls with speed**
  - ✓ Machining cost$(C_1) = C_m \times T_m$, where $T_m = \frac{L}{s_o N}$. Since N ∝ V, machining time varies as 1/V, so this curve is a rectangular hyperbola. Cutting faster means less machine time is billed per piece.
- **Components that rise with speed**
  - ✓ ***Tool changing cost :*** tool life follows Taylor's equation $VT^n = C$, so life collapses sharply as speed rises. More tool changes per piece means more idle machine time and more labour cost.
  - ✓ ***Tool regrinding (or insert depreciation) cost :*** number of tool failures per piece rises roughly as velocity increases.

- **The total cost curve:**
  - ✓ Adding all five gives the heavy black curve.
  - ✓ At low speed the machining-time term dominates and cost is high; at high speed the tool-related terms dominate and cost climbs again.
  - ✓ Between them lies a shallow minimum at V_{opt}, the optimum cutting speed for minimum cost, giving C_{min}. This is the minimum cost criterion.

Summery

Criterion	Optimum Tool Life (T_{opt})	Optimum Cutting Speed (V_{opt})
Minimum Cost	$\left(\frac{1-n}{n}\right)\left(\frac{C_e}{C_m}+T_c\right)$	$\frac{C}{\left\{\left(\frac{1-n}{n}\right)\left(\frac{C_e}{C_m}+T_c\right)\right\}^n}$
Maximum Production Rate	$T_C\left(\frac{1-n}{n}\right)$	$\frac{C}{\left\{T_C\left(\frac{1-n}{n}\right)\right\}^n}$
Maximum Profit	$T_{opt}(Production\ Rate) < T_{opt}(Profit\ Rate) < T_{opt}(Minimum\ Cost)$	$V_{opt}(Production\ Rate) > V_{opt}(Profit\ Rate) > V_{opt}(Minimum\ Cost)$