Engineering Thermodynamics

Module -1.1

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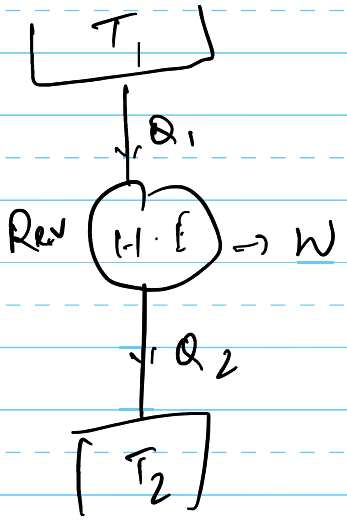
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Clausius Theorem for rev. cycle

Second Law :



By Carnot Theorem :–

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$ (Absolute Thermodynamic Scale of temperature)

or

$$\frac{Q_1}{T_1} = \frac{Q_2}{Q_2} \Rightarrow \frac{Q_1}{T_1} + \left(\frac{-Q_2}{T_2}\right) = 0 \Rightarrow \sum_{cycle} \frac{Q}{T} = 0 \Rightarrow$$

$$\oint \frac{dQ_R}{T} = 0 \Rightarrow$$ Cyclic integral of point function = 0 ∴

∴ $\frac{dQ_R}{T}$ must be a property

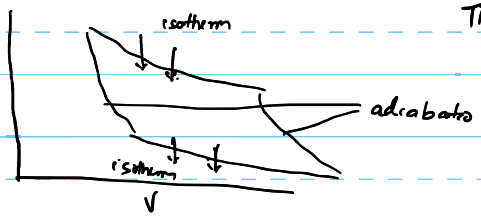
Let it be ds

This is a Carnot cycle :- Reversible heat addition, and R. Heat rejection and can be represented as

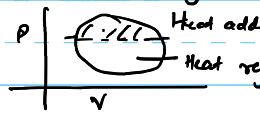
then, $\oint ds = 0 \Rightarrow ds = \frac{dQ_R}{T}$

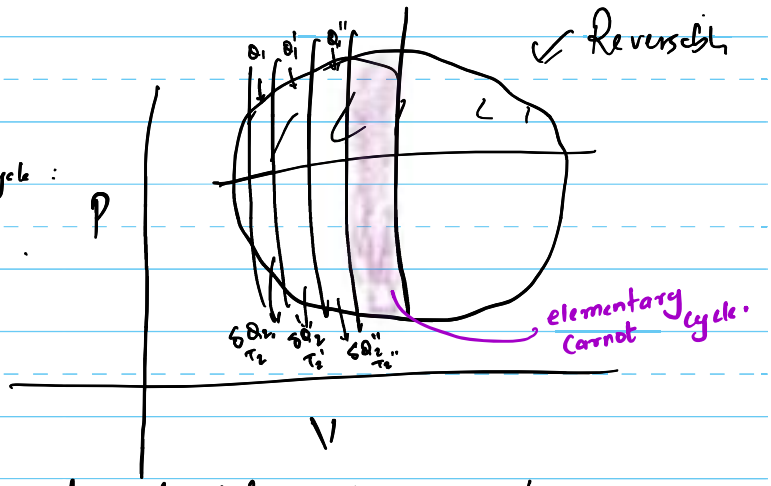
$$\Rightarrow s_2 - s_1 = \int_1^2 \frac{dQ_R}{T}$$

Therefore change in entropy is the quantity $\left.\frac{\delta Q_R}{T}\right|_{Rev.}$



Let us take a general cycle :





or It can be divided into elementary Carnot cycle.

* Construct a number of isotherms
" " " " adiabats.

So these become elementary carnot cycle.

Thus we can say

$$\frac{\delta Q_1}{T_1} = \frac{\delta Q_2}{T_2} \quad \{\text{By carnot theorem}\}$$

$$\frac{\delta Q_1'}{T_1} = \frac{\delta Q_2'}{T_2} \quad \{(1)\}$$

$$\frac{\delta Q_1''}{T_1''} = \frac{\delta Q_2''}{T_2''}$$

Adding all above:-

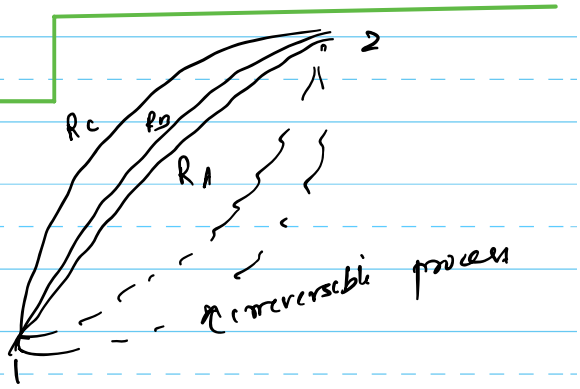
$$\sum \frac{\delta Q_1}{T_1} = \sum \frac{\delta Q_2}{T_2} \quad \text{or} \quad \sum \frac{\delta Q_1}{T_1} + \left(-\sum \frac{\delta Q_2}{T_2}\right) = 0$$

Clausius theorem for reversible cycle:

$$\oint \frac{\delta Q}{T}\bigg|_R = 0 \Rightarrow \oint ds = 0, \text{ where } ds = \frac{\delta Q_R}{T}$$

If cyclic integral of anything is zero, it can be taken as a property.

$\therefore \frac{\delta Q}{T}\big|_R$ is a property and is named as entropy or $\int_1^2 \frac{\delta Q_R}{T} = S_2 - S_1$



For a property, the change in entropy is same irrespective of the path followed

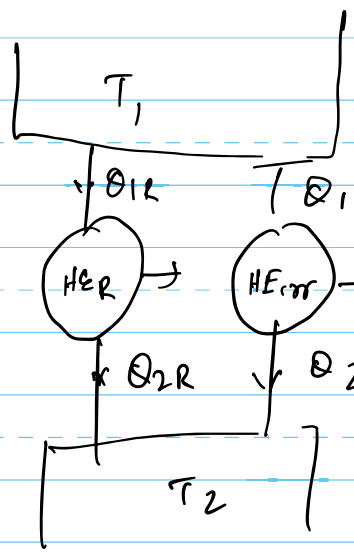
$$R_A\int_1^2 \left(\frac{\delta Q}{T}\right) = R_B\int_1^2 \left(\frac{\delta Q}{T}\right) = R_C\int_1^2 \frac{\delta Q}{T}$$

δQ can be calculated thru these irreversible process

$$\oint_{cyc} \frac{\delta Q}{T} = ?$$

This was answered by Clausius and is known as Clausius inequality.

Derivation for $\oint \frac{\delta Q}{T}\Big|_{cyc} < 0$



* eff of irr. eng $\downarrow$ eff of rev eng $\downarrow$

$\eta = 1 - \frac{Q_2}{Q_1}$, $\eta_r = 1 - \frac{Q_{2R}}{Q_{1R}}$

We know that a reversible heat eng. η_{rev} is greater than an irreversible η.

$\eta_r > \eta$

$1 - \frac{Q_{2R}}{Q_{1R}} > 1 - \frac{Q_2}{Q_1}$

By Carnot theorem :- $\frac{Q_{2R}}{Q_{1R}} = \frac{T_2}{T_1}$

$\Rightarrow \frac{Q_{2R}}{Q_{1R}} < \frac{Q_2}{Q_1}$

$\Rightarrow \frac{T_2}{T_1} < \frac{Q_2}{Q_1}$

$\Rightarrow \frac{Q_2}{Q_1} > \frac{T_2}{T_1}$

$\Rightarrow \frac{Q_1}{Q_2} < \frac{T_1}{T_2}$

$\Rightarrow \frac{Q_1}{T_1} < \frac{Q_2}{T_2}$

$\Rightarrow \frac{Q_1}{T_1} + \left(\frac{-Q_2}{T_2}\right) < 0$

Clausius theorem of inequality

$\Rightarrow \oint_{irr} \frac{\delta Q}{T} < 0$

$$\therefore \quad \Delta S = \int \left(\frac{\delta Q}{T}\right)_R \quad \text{and}$$

for an adiabat process $\delta Q = 0$, but since we dont know if it is a reversible or irreversible $\therefore \Delta S$ can be $>< 0$

Reversible adiabatic process:

$$\delta Q_R = 0$$

$$\Delta S = \int \frac{\delta Q_R}{T} = 0$$

** Proof of $\int_1^2 \frac{\delta Q}{T} < \Delta S$

Let $1R_A2$ & $2R_B1$ are reversible process (from fig)

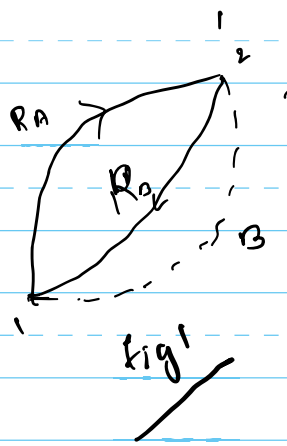


fig 1

On this fig:-

$1R_A2$ & $2R_B1$ are reversible process.

while $2B1$ is irreversible

$$R_A\int_1^2 \frac{\delta Q}{T} + R_B\int_2^1 \frac{\delta Q}{T} = 0 \quad (1)$$

$$R_B\int_1^2 \frac{\delta Q}{T} + B\int_2^1 \frac{\delta Q}{T} < 0 \quad (2)$$

By Clausius theorem

Using $\int_1^2 \frac{\delta Q}{T}$ from eqn 1 in (2), we get

$$\Rightarrow -\int_2^1{}_{R_B} \frac{\delta Q_1}{T} + \int_2^1{}_{B} \frac{\delta Q}{T} < 0$$

$$\Rightarrow \int_2^1{}_{B} \frac{\delta Q}{T} < \int_2^1{}_{R_B} \frac{\delta Q_1}{T}$$

$$\Rightarrow \int_2^1 \frac{\delta Q}{T} < \Delta S \quad \text{or} \quad \Delta S > \int_2^1 \frac{\delta Q}{T}$$

$$\therefore \quad \Delta S \geq \frac{\delta Q}{T} \quad \begin{cases} = \text{for rev proces} \\ > \text{for irrev proces} \end{cases}$$

for adiabatic proces

$$\Delta S \geq 0 \qquad \{\text{as } \delta Q = 0\}$$

A S stem where $\delta Q = 0$, it can be taken as an isolated System.

or $\Delta S_{\substack{\text{isolated} \\ \text{system}}} \geq 0$

from above it proved that : $\Delta S \geq \int \frac{\delta Q}{T}$ ← this is origin of $\Delta S_{\text{isolated}} \geq 0$

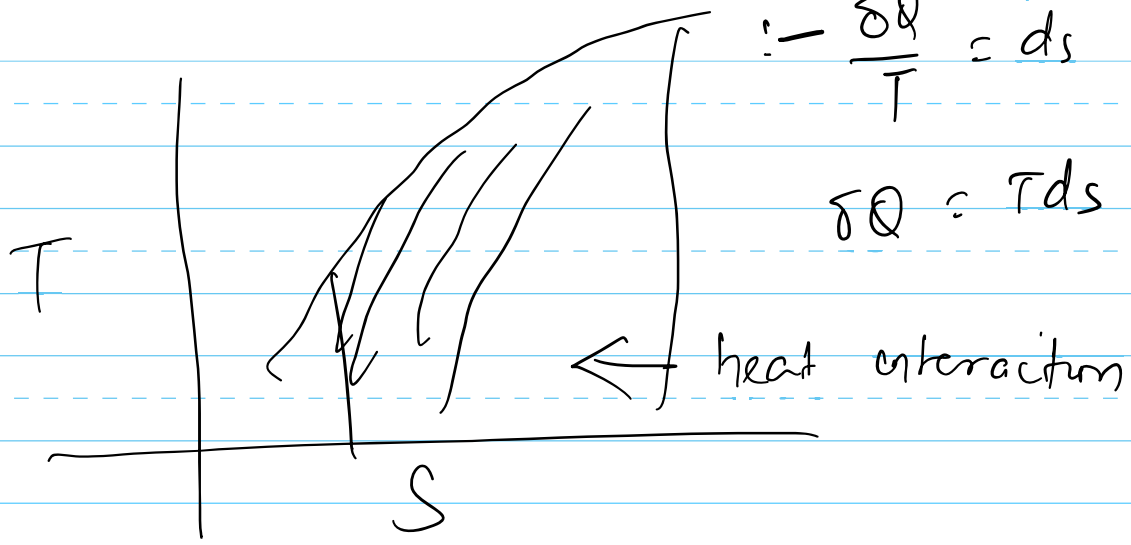
or $\Delta S = \int \frac{\delta Q}{T} + X$ ← Change of entropy due to internal irreversibility

Entropy transfer due to heat transfer

Only for reversible proces

$$\therefore \frac{\delta Q}{T} = ds$$

$$\delta Q = T ds$$



Engineering Thermodynamics

Module -1.2

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Concept of an isolated System.

{By clausius inequality}

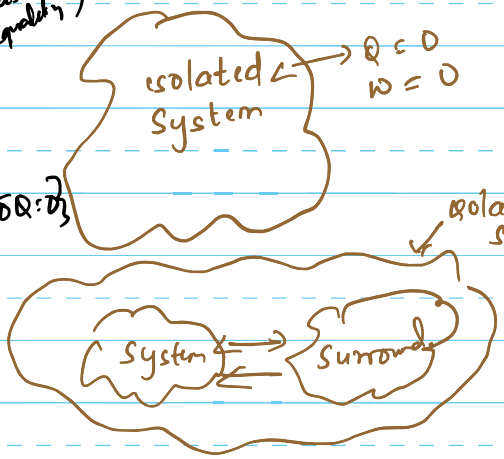
$\Delta S \geq \int \frac{\delta Q}{T}$

for adiabatic: $\therefore \delta Q = 0$

$\Delta S \geq 0$

or $\Delta S_{isolat} \geq 0$

$\because$ for isolated sys, $\delta Q = 0$



or all interacting systems taken together

$\Delta S_{iso} \geq 0$

{ If processes inside the isolated system reversible, then $= 0$
If processes inside the isolated system is irreversible, then $\Delta S > 0$

or $\Delta S_{univ} \geq 0$

$\therefore \Delta S_{iso} = \Delta S_{sys} + \Delta S_{surroundings} = \Sigma (\Delta S)_{\text{all interacting systems.}}$

These isolated system's is sometimes called Universe.

ΔS

$\Delta S_{iso\ sys} \geq 0$

$\Delta S_{univ} \geq 0$

Principle of increase in entropy.

{ universe can be looked at as an isolated System

Cases for each $\Delta S_{sys} > 0$:—

$\Delta S\big|_{system} > \int \frac{\delta Q}{T}$ { By Clausius inequality }

or $(\Delta S)_{System} = \int \frac{\delta Q}{T} + \Delta S_i$ ($\Delta S_i > 0$ for an irreversible process, $= 0$ for an reversible process) — (eqn 1)

Now analysing eqn 1 for cases when $\Delta S_{sys} > 0$

when $\Delta S_{system} > 0$ { ← when do you get $\Delta S_{sys} > 0$)?

$\Rightarrow \int \frac{\delta Q}{T} + \Delta S_i > 0$

Case 1 for an reversible process :-

$\Delta S_i = 0$, Therefor for the $\Delta S_{sys} > 0$, we need $\frac{\delta Q}{T} > 0$

i.e. $\int \frac{\delta Q}{T} > 0 \Rightarrow \delta Q > 0$

$\therefore \delta Q > 0$, $\therefore$ heat has to be added to the System. to increase the entropy of the System.

Case 2 for an irreversible process :

when is $\Delta S > 0$?

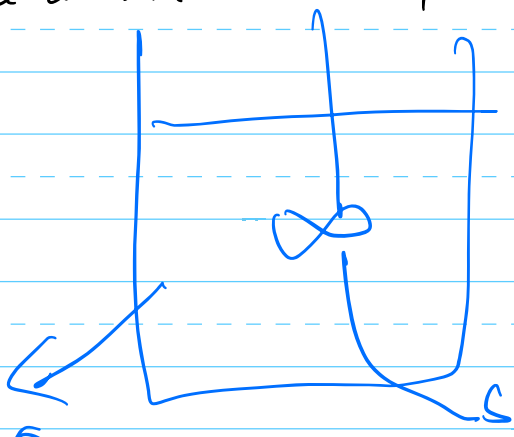
Case 2.1 :- Case 1 :- when $\delta Q > 0$

Case 2.2 :- $\int \frac{\delta Q}{T} < 0$ but $\left|\frac{\delta Q}{T}\right| < \Delta S_i$, then $\Delta S > 0$

That is, for an irreversible process, even if you extract the heat, if the entropy generation is more than the loss in entropy due to heat loss, then $\Delta S > 0$

In both case, its a heating process.

Example of a adiabatic irreversible process :-



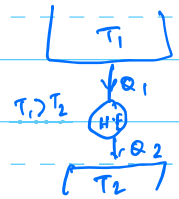
→ In this case $\Delta S_i > 0$, $\frac{\delta Q}{T} < 0$ (as heat is lost to the surr)

$\therefore \Delta S_i + \left(\frac{\delta Q}{T}\right) > 0$ & $\Delta S_i > \left|\frac{\delta Q}{T}\right|$

$5 + (-4) > 0$ $\quad 5 > |-4|$

$\Delta S_{universe} \geq 0$ and $\oint \frac{\delta Q}{T} \leq 0$ } Both these equation are synonymous

↑ principle of increase of entropy ↓ Clausius inequality



By clausius inequality

$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0 \quad \oint \frac{\delta Q}{T} = 0$

$\frac{Q_1}{T_1} < \frac{Q_2}{T_2}$ or $\oint \frac{\delta Q}{T} < 0$ — (1)

By principle of increase of entropy :— $\Delta S_{univ} \geq 0$ {for the figure} — (2)

$\Delta S_A = -\frac{Q_1}{T_1}$, $\Delta S_B = \frac{Q_2}{T_2}$. $\Delta S_{H.E} = 0$

$\Delta S_{univ} : \quad -\frac{Q_1}{T_1} + 0 + \frac{Q_2}{T_2}$

$= -\left(\frac{Q_1}{T_1} - \frac{Q_2}{T_2}\right) = -\oint \frac{\delta Q}{T} \quad \therefore \Delta S_{uni} = -\oint \frac{\delta Q}{T} \geq 0$ {from eqn 2}

or $\oint \frac{\delta Q}{T} \leq 0$ which is Clausius inequality

∴ Clausius inequality and 2nd entropy principle are same.

Starts from Carnot theorem :-

$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$ ⇒ This gives rise to $\oint \frac{\delta Q}{T} = 0$, the cyclic integral for rev. cycle $\frac{\delta Q}{T}$

$\delta Q = du + \delta W$ ⇒ 1) if write this law for a reversible process for a closed system with only pdv work

↑ No restriction to this Expression. This can be irr, rev, electri. any work. Heat transfer is 1 type

then $Tds = du + pdv$:- This eqn. has only properties. and is valid for rev & irr process.

Concept is when process is reversible and work done is pdv

∴ 1st law and this eqn. are same.

i.e $\delta Q = Tds$ & $\delta W = pdv$

But for irrev. process

$dQ \neq Tds$ and $W \neq pdv$

∴ it is not the first law

$H = U + pV \Rightarrow dH = dU + pdv + vdp \Rightarrow Tds + vdp \Rightarrow Tds = dH - vdp$

∴ These 2 relations are property relations Valid for rev & irr process.

But for rev. process.

1st eqn is equivalent to 1st law

(

Imp.