Engineering Thermodynamics

Module -5

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Pressure Ratio for optimum work,

$$\eta_{compress} = \frac{W_{ideal\ comp}}{W_{normal\ comp}}$$

$$\Rightarrow W_{normal\ c} = \frac{W_{ideal\ comp}}{\eta_{comp}} = \frac{W_c}{\eta_c}$$

Here W_1 & W_2 are work done by normal compressor, W_c = work done by ideal compressor,
η_{c_1} = Efficiency of compressor 1
η_{c_2} = Efficiency of compressor 2.

or $\eta_{c_1} = \frac{W_{c_1}}{W_1}$ & $\eta_{c_2} = \frac{W_{c_2}}{W_2}$

$$\Rightarrow W_1 = \frac{W_{c_1}}{\eta_{c_1}} \quad \& \quad W_2 = \frac{W_{c_2}}{\eta_{c_2}}$$

$$W_{c_1} = \frac{\gamma}{\gamma-1} RT_1 \left\{ \left(\text{Pressure ratio}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}$$

$$W = W_1 + W_2 = \frac{W_{c_1}}{\eta_{c_1}} + \frac{W_{c_2}}{\eta_{c_2}}$$

$$= \frac{1}{\eta_{c_1}} \cdot \frac{\gamma}{\gamma-1} \cdot RT_1 \left\{ \left(\frac{P_x}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} + \frac{1}{\eta_{c_2}} \cdot \frac{\gamma}{\gamma-1} \cdot RT_y \left\{ \left(\frac{P_2}{P_y}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right\}$$

Eqn (A)

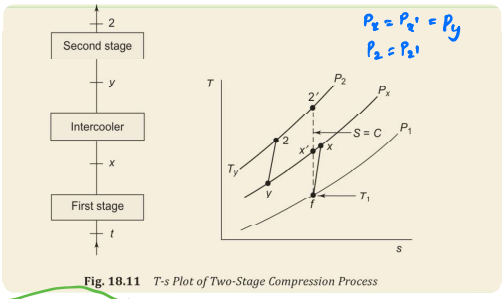


Fig. 18.11 *T-s Plot of Two-Stage Compression Process*

Adiabatic Relations :-

$$\frac{T_{x'}}{T_1} = \left(\frac{P_{x'}}{P_1}\right)^{\gamma-1/\gamma} = \left(\frac{P_x}{P_1}\right)^{\gamma-1/\gamma} \quad \& \quad \frac{T_y}{T_{x'}} = \left(\frac{P_{y'}}{P_{x'}}\right)^{\gamma-1/\gamma} = \left(\frac{P_2}{P_y}\right)^{\gamma-1/\gamma} = \left(\frac{P_2}{P_x}\right)^{\gamma-1/\gamma}$$

Eqn B →

Replacing Eqn B in Eqn A :-

$$W = \frac{1}{\eta_{c_1}} \cdot \frac{\gamma}{\gamma-1} RT_1 \left\{ \frac{T_{x'}}{T_1} - 1 \right\} + \frac{1}{\eta_{c_2}} \cdot \frac{\gamma}{\gamma-1} RT_y \left\{ \frac{T_{2'}}{T_{x'}} - 1 \right\}$$

$$W = \frac{\gamma R}{\gamma-1} \cdot \left[\frac{T_1}{\eta_{c_1}} \cdot \left\{ \frac{T_{x'}}{T_1} - 1 \right\} \right] + \frac{\gamma R}{\gamma-1} \cdot \left[\frac{T_y}{\eta_{c_2}} \cdot \left\{ \frac{T_{2'}}{T_{x'}} - 1 \right\} \right]$$

$$\Rightarrow W = \frac{\gamma R}{\gamma-1} \left[\frac{T_1}{\eta_{c_1}} \left(\frac{T_{x'}}{T_1} - 1 \right) + \frac{T_y}{\eta_{c_2}} \left(\frac{T_{2'}}{T_{x'}} - 1 \right) \right]$$

T_1, $T_{2'}$ & T_y are constant

$\therefore$ $T_{x'}$ is the variable.

$$\frac{dW}{dT_{x'}} = 0$$

$$\Rightarrow \frac{dw}{dT_{x'}} = \frac{d}{dT_{x'}}\left[\frac{\gamma R}{\gamma-1}\left\{\frac{T_1}{\eta_{c_1}}\cdot\left(\frac{T_{x'}}{T_1}-1\right)\right\}\right] + \frac{d}{dT_{x'}}\left[\frac{\gamma R}{\gamma-1}\cdot\left\{\frac{T_y}{\eta_{c_2}}\cdot\left(\frac{T_2'}{T_{x'}}-1\right)\right\}\right]$$

$$0 = \frac{\gamma R}{\gamma-1}\cdot\frac{T_1}{\eta_{c_1}}\cdot\frac{1}{T_1} + \frac{\gamma R}{\gamma-1}\cdot\frac{T_y}{\eta_{c_2}}\cdot T_2'\left\{\frac{-1}{(T_{x'})^2}\right\}$$

$$\Rightarrow \frac{\gamma R}{\gamma-1}\left[\frac{1}{\eta_{c_1}} - \frac{T_y T_2'}{\eta_{c_2}}\left(\frac{1}{T_{x'}^2}\right)\right] = 0$$

$$\Rightarrow \frac{1}{\eta_{c_1}} = \frac{T_y T_2'}{\eta_{c_2}}\left(\frac{1}{T_{x'}^2}\right)$$

$$\Rightarrow T_{x'}^2 = \frac{\eta_{c_1}}{\eta_{c_2}}\cdot T_y T_2'$$

$$\Rightarrow \frac{T_{x'}^2}{T_1^2} = \frac{\eta_{c_1}}{\eta_{c_2}}\cdot\frac{T_y}{T_1}\cdot\frac{T_2'}{T_1}$$

$$= \frac{T_{x'}}{T_1} = \left(\frac{\eta_{c_1}}{\eta_{c_2}}\cdot\frac{T_y}{T_1}\cdot\frac{T_2'}{T_1}\right)^{1/2} \quad \text{— (1)}$$

Now, $\frac{T_{x'}}{T_1} = \left(\frac{P_{x'}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_x}{P_1}\right)^{\gamma-1/\gamma}$ — (2)

& $\frac{T_2'}{T_1} = \left(\frac{P_2'}{P_1}\right)^{\gamma-1/\gamma} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma}$ — (3)

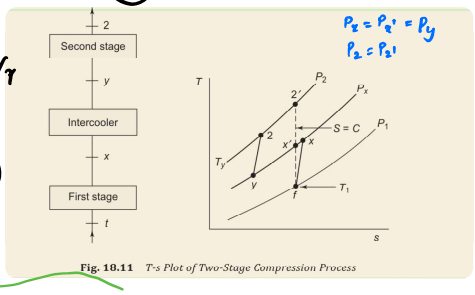


Fig. 10.11 *T-s Plot of Two-Stage Compression Process*

From eqn (2)

$$\left(\frac{P_x}{P_1}\right)^{\gamma-1/\gamma} = \frac{T_{x'}}{T_1} \Rightarrow \frac{P_x}{P_1} = \left(\frac{T_{x'}}{T_1}\right)^{\gamma/\gamma-1} \quad \text{— (4)}$$

Replacing Eqn ① in eqn ④ we get

$$\frac{P_x}{P_1} = \left(\frac{T_x'}{T_1}\right)^{\gamma/\gamma-1} = \left(\frac{\eta_{c_1}}{\eta_{c_2}} \cdot \frac{T_y}{T_1} \cdot \frac{T_2'}{T_1}\right)^{\frac{1}{2}\cdot\frac{\gamma}{\gamma-1}}$$

$$= \sqrt{\left(\frac{\eta_{c_1}}{\eta_{c_2}} \cdot \frac{T_y}{T_1}\right)^{\gamma/\gamma-1} \cdot \left(\frac{T_2'}{T_1}\right)^{1/2 \cdot \gamma/\gamma-1}}$$

From eqn ③, $\frac{T_2'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma}$ $\therefore \frac{P_x}{P_1} = \sqrt{\left(\frac{\eta_{c_1}}{\eta_{c_2}} \cdot \frac{T_y}{T_1}\right)^{\gamma/\gamma-1} \cdot \left\{\left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma}\right\}^{\frac{1}{2}\cdot\frac{\gamma}{\gamma-1}}}$

$$\Rightarrow \frac{P_x}{P_1} = \sqrt{\left(\frac{\eta_{c_1}}{\eta_{c_2}} \cdot \frac{T_y}{T_1}\right)^{\gamma/\gamma-1} \cdot \left(\frac{P_2}{P_1}\right)^{1/2}}$$

$$\frac{P_x}{P_1} = \sqrt{\left(\frac{\eta_{c_1}}{\eta_{c_2}} \cdot \frac{T_y}{T_1}\right)^{\gamma/\gamma-1} \cdot \frac{P_2}{P_1}}$$

Now for the Special Case:- $T_y = T_1$ and $\eta_{c_1} = \eta_{c_2}$, we get

$$\frac{P_x}{P_1} = \sqrt{\frac{P_2}{P_1}}$$ { Requirement for minimum Work }

For multistage compression :-

$$\frac{P_{x_1}}{P_1} = \frac{P_{x_2}}{P_{x_1}} = \frac{P_2}{P_{x_2}} = \left(\frac{P_2}{P_1}\right)^{1/3}$$ (3 stage Compression)

or for 3 stage compression :-

Optimum compression ratio :-

$$\frac{P_x}{P_1} = \left(\frac{P_d}{P_s}\right)^{1/3}$$

P_d = Delivery pressure

P_s = Suction pressure.

For N stage :-

$$\frac{P_x}{P_1} = \left(\frac{P_d}{P_s}\right)^{1/N}$$

$$W = \frac{N\,n\,R\,T_1}{n-1}\left\{\left(\frac{P_d}{P_s}\right)^{\frac{n-1}{Nn}} - 1\right]$$