Chapter - 9

Morphological Image Processing

The Word Morphology Commonly denotes a branch of biology that deals with the form and structure of animals and plants.

→ Mathematical Morphology as a tool for extracting image components that are useful in the representation and description of region shape, such as boundaries, skeletons, and the convex hull.

Preliminaries

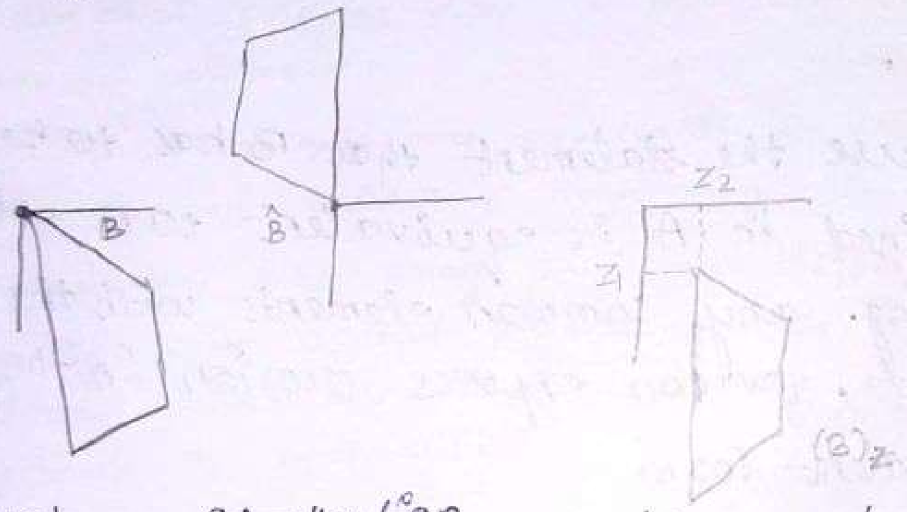
Mathematical morphology is set theory.
Morphology offers a unified and powerful approach to numerous image processing problems. sets in mathematical morphology represent objects in an image

→ The set of all white pixels in a binary image is a complete morphological description of the image.

The concepts of set reflection and translations are used extensively in morphology. The reflection of a set B, denoted $\hat{B}$ is defined as

$$\boxed{\hat{B} = \{ w \mid w = -b, \text{ for } b \in B \}}$$

If B is the set of pixels representing an object in an image, then $\hat{B}$ is simply the set of points in B whose (x, y) have been replaced $(-x, -y)$



(a) A set (b) reflection (c) translation

The translation of a set B by point $z = (z_1, z_2)$ denoted $(B)_z$, is defined by

$$(B)_z = \{ c \mid c = b + z, \text{ for } b \in B \}$$

If B is the set of pixels representing an ~~imag~~ object in an image, then $(B)_z$ is the set of points in B whose (x, y) co-ordinates have been replaced by $(x + z_1, y + z_2)$.

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Erosion and Dilation

Erosion

with A and B as sets in Z^2, the erosion of A by B, denoted $A \ominus B$, is defined as

$$A \ominus B = \{ z \mid (B)_z \subseteq A \}$$

the erosion of A by B is the set of all points Z such that B, translated by Z, is contained in A. set B assumed to be a structuring element.

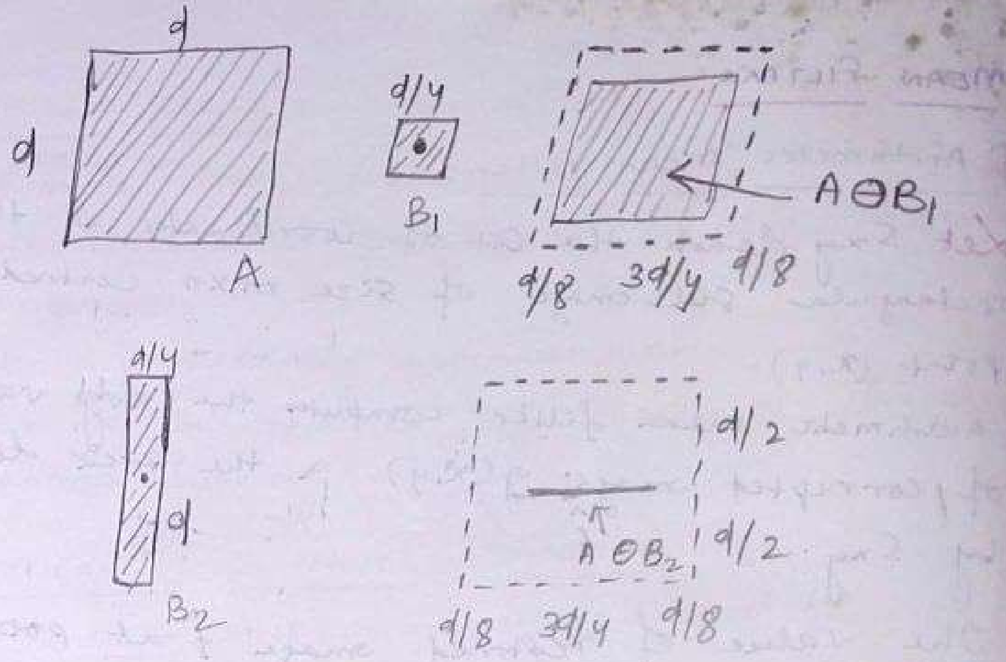
→ Because the statement that B has to be contained in A is equivalent to B not sharing any common elements with the background, we can express erosion in the following equivalent form

$$A \ominus B = \{ z \mid (B)_z \cap A^c = \emptyset \}$$

→ The following figure shows the example.

The elements of A and B are shown shaded and the background is white. The solid boundary(c) is the limit beyond which further displacements of the origin of B would cause the structuring element to cease being completely contained in A. The locus of points within this boundary, constitutes the erosion of A by B.

→



Dilation

With A and B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \}$$

This equation is based on reflecting B about its origin, and shifting this reflection by z such that $\hat{B}$ and A overlap by at least one element. Based on this interpretation,

$$\boxed{A \oplus B = \{ z \mid [(\hat{B})_z \cap A] \subseteq A \}}$$

Keep in mind, however, that dilation is based on set operation and therefore is a non-linear operation; whereas convolution is a linear operation.

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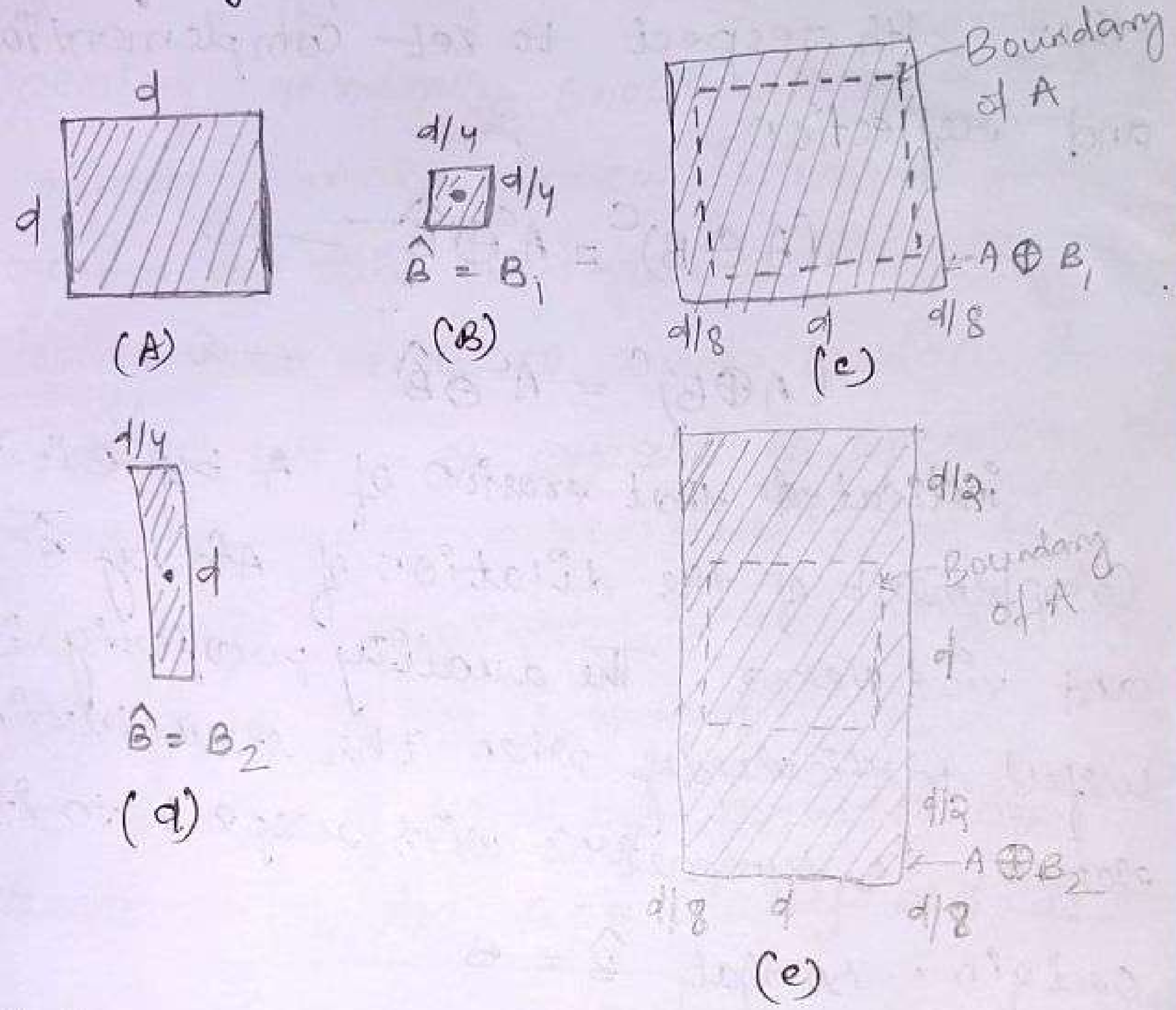
Unlike erosion, which is a shrinking or thinning operation, dilation "grows" or "thickens" objects in a binary image.



(A) → set A

(B) → square structuring element (dot denotes the origin) (B_1)

(c) → Dilation of A by B_1

(d) → Elongated structuring element (B_2)

(e) → Dialation of A using this elements (B_2)

(c) and (e) has the boundary of set A, shown only for reference.

Duality

Erosion and dilation are duals of each other with respect to set complementation and reflection.

$$(A \ominus B)^c = A^c \oplus \hat{B} \quad \text{—} ①$$

$$(A \oplus B)^c = A^c \ominus \hat{B} \quad \text{—} ②$$

indicates that erosion of A by B is the complement of the dilation of A^c by $\hat{B}$, and vice versa. The duality property is useful particularly when the structuring element is symmetric with respect to its origin. so that $\hat{B} = B$

Proof of 1

$$(A \ominus B)^c = \{z \mid (B)_z \subseteq A\}^c$$

If set $(B)_z$ is contained in A, then $(B)_z \cap A^c = \emptyset$

$$(A \ominus B)^c = \{z \mid (B)_z \cap A^c = \emptyset\}^c$$

But the complement of the set of z's that satisfy $(B)_z \cap A^c = \phi$ is the set of z's s.t. $(B)_z \cap A^c \neq \phi$

$(B)_z \cap A^c = \emptyset$

$$(A \ominus B)^c = \{z \mid (B)_z \cap A^c \neq \emptyset\}$$
$$= A^c \oplus \hat{B}$$

Opening and Closing

As we have seen, dilation expands the components of an image and erosion shrinks them.

→ Opening generally smoothes the ~~contour~~ contour of an object, breaks narrow isthmuses, and illuminates thin protrusions.

→ Closing also tends to smooth sections of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.

→ The opening of set A by structuring element B, denoted A o B, is defined as.

$$\boxed{A \circ B = (A \ominus B) \oplus B}$$

→ The opening A by B is the erosion of A by B, followed by a dilation of the result by B

The closing of set A by structuring element B, denoted A • B is

$$\boxed{A \bullet B = (A \oplus B) \ominus B}$$

This geometric fitting property of the opening operation leads to a set theoretic formulation, which states that the opening of A by B is obtained by

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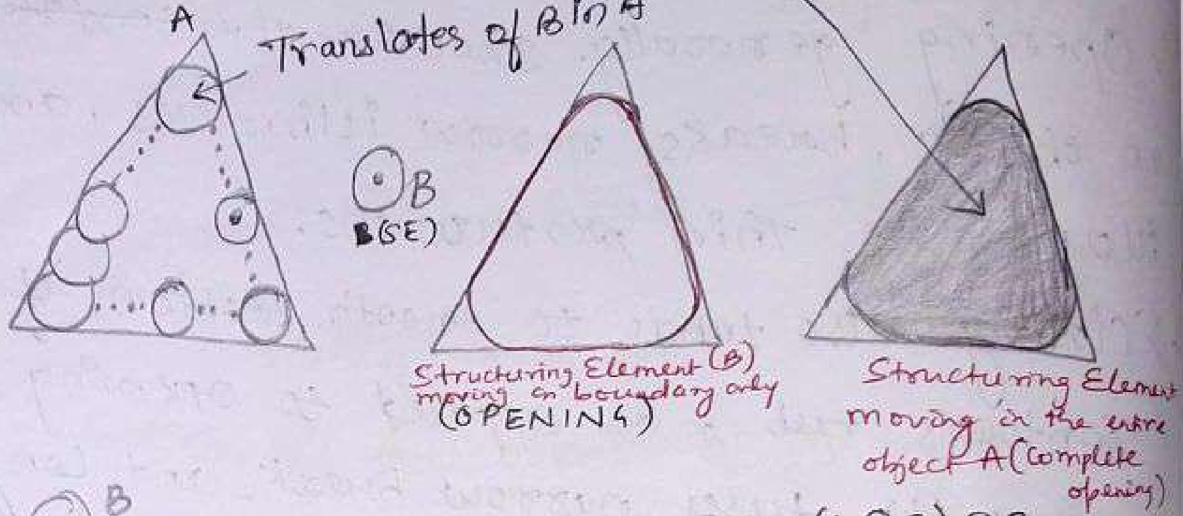
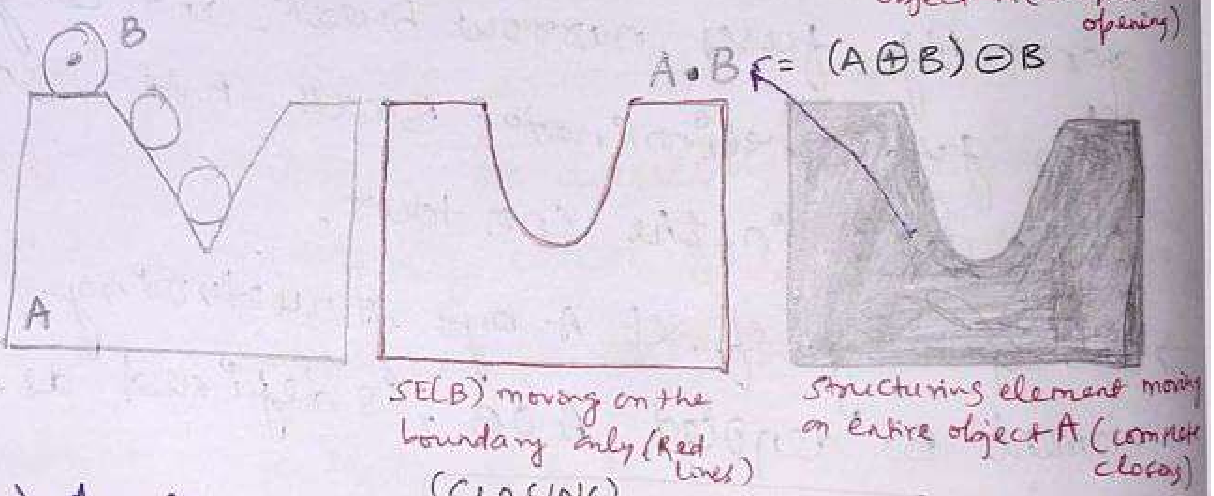
taking the union of all translates of B that fit into A.

$$A \circ B = \bigcup \{ (B)_z \mid (B)_z \subseteq A \}$$





→ As in the case with dilation and erosion, opening and closing are duals of each other with respect to set complementation and reflection.

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

$$(A \circ B)^c = (A^c \bullet \hat{B})$$

Opening ~~property~~ operations satisfy following properties:-

(i) $A \circ B$ is a subset of A

(2) If C is a Subset of D, then $C \circ B$ is a subset of $D \circ B$

(3) $(A \circ B) \circ B = A \circ B$

Closing operation property

(1) A is a subset of $A \bullet B$

(2) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$

(3) $(A \bullet B) \bullet B = A \bullet B$

HIT-or-MISS TRANSFORMATION:-

- tool for shape detection
- Let there are two regions A and B. B is composed of D and its background, the matches of B in A denoted as $A \circledast B$ is given by:

$$A \circledast B = (A \ominus D) \cap [A^c \ominus (W-D)] \quad — ①$$

* The regions mentioned in the equation are shown in the figure no. 9.12.

→ Generalising equation ① by letting $B = (B_1, B_2)$ where

B_2 = set of elements from B associated with corresponding background

B_1 = set formed from element of B associated with an object

$\Rightarrow B_1 = D$ and $B_2 = (W-D)$

- Substituting B_1 and B_2 in equation ① we get

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2) \quad — ②.$$

- Using the concept that erosion and dilation are duals of each other and the concept of set difference, we get

$$\boxed{A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)} \quad — ③$$

↑ Equation for Hit/Miss Transformation.

⇒ Example figure 9.12 on next page

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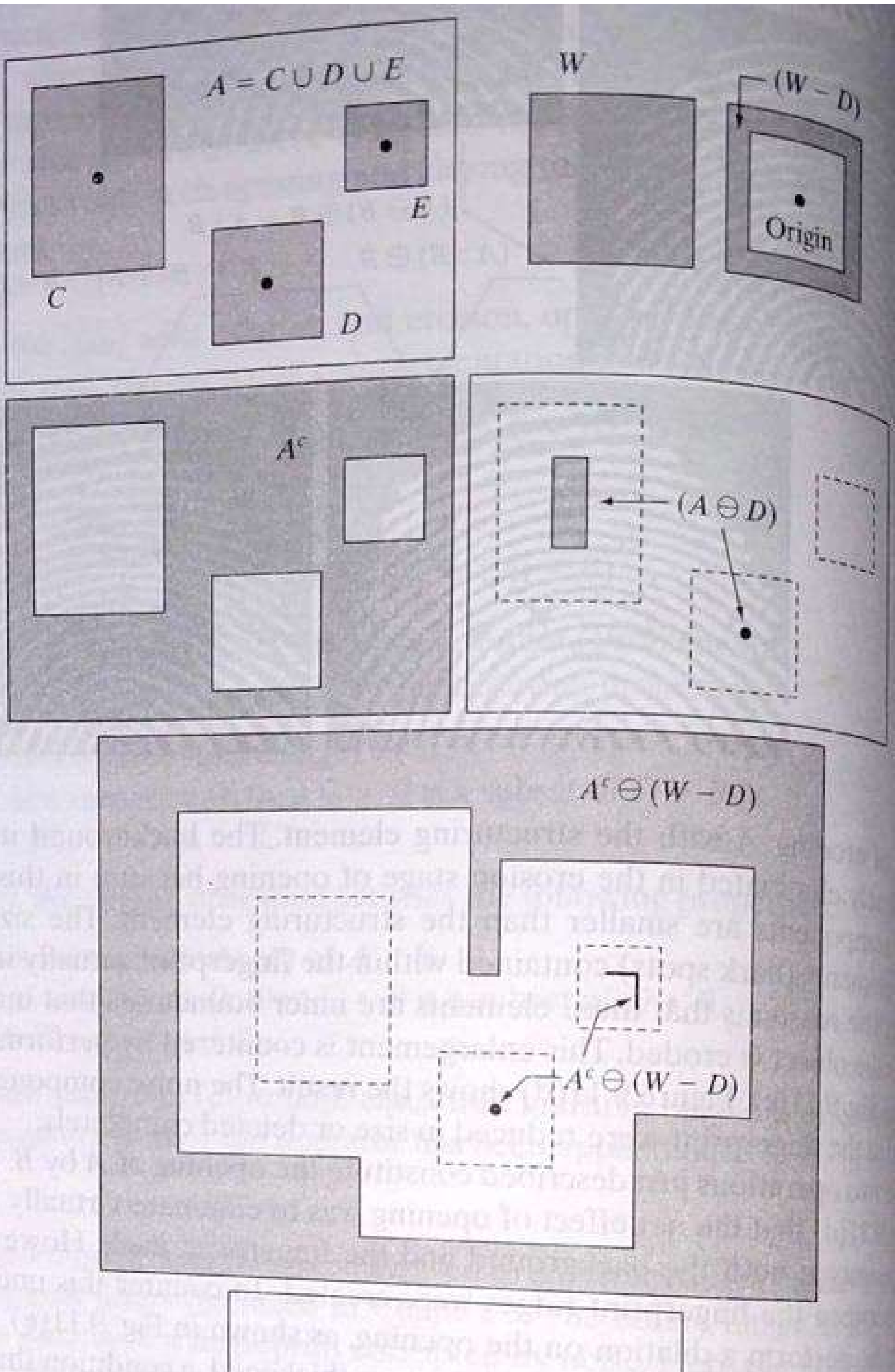
a b
c d
e
f

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.



BASIC MORPHOLOGICAL ALGORITHMS :-

① BOUNDARY EXTRACTION :-

- Boundary of set A denoted by $\beta(A)$ can be obtained by first eroding A by B and then performing the set difference between A and its erosion.

$$\boxed{\beta(A) = A - (A \ominus B)}$$

- Example is shown in the following figure 9.13 and 9.14

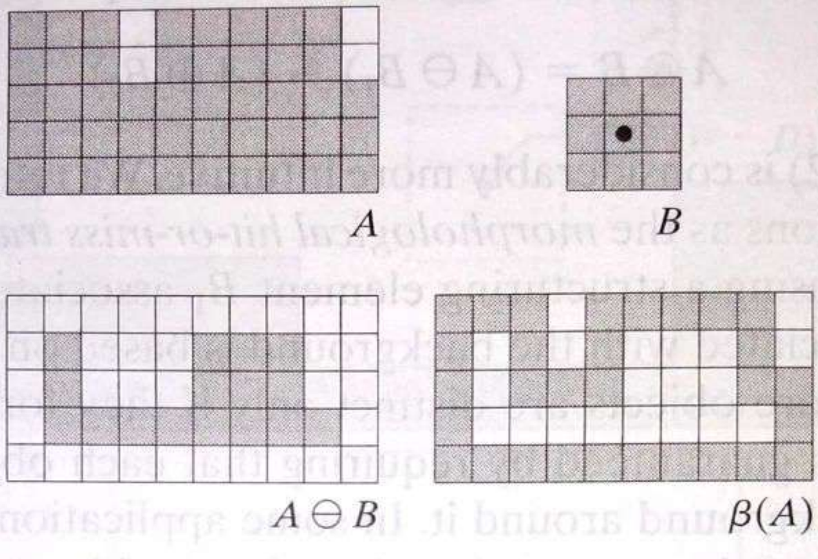
② HOLE - FILLING :-

- Hole is a background region surrounded by a connected border of foreground pixels.
- Holefilling is based on set dilation, complementation and intersection.
- Let A denote a set whose elements are 8-connected boundaries, each boundary enclosing a background region (Hole). Given a point in each hole, the objective is to fill all the holes with 1's.
- Begin by forming an array X_0 of 0's (X_0 has same size as A)
- The following procedure fills all the holes with 1's.

$$\boxed{X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, ..}$$

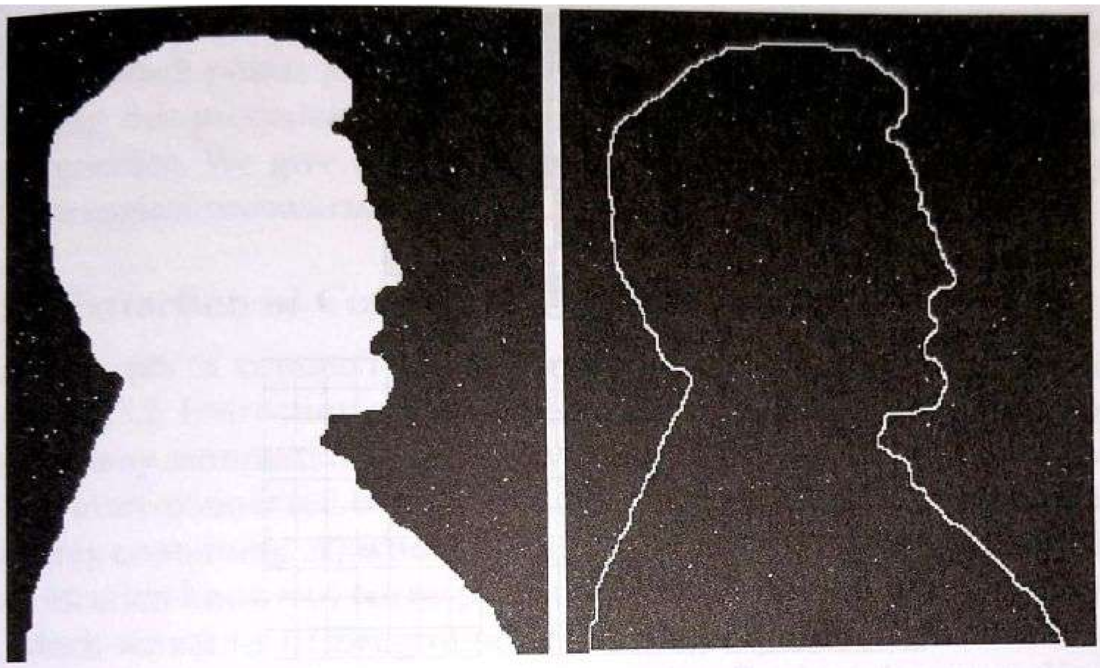
B = structuring element.

- Terminate when $X_k = X_{k-1}$ and X_k contains all the filled holes.
- Example is shown in the figures 9.15 and 9.16.



a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

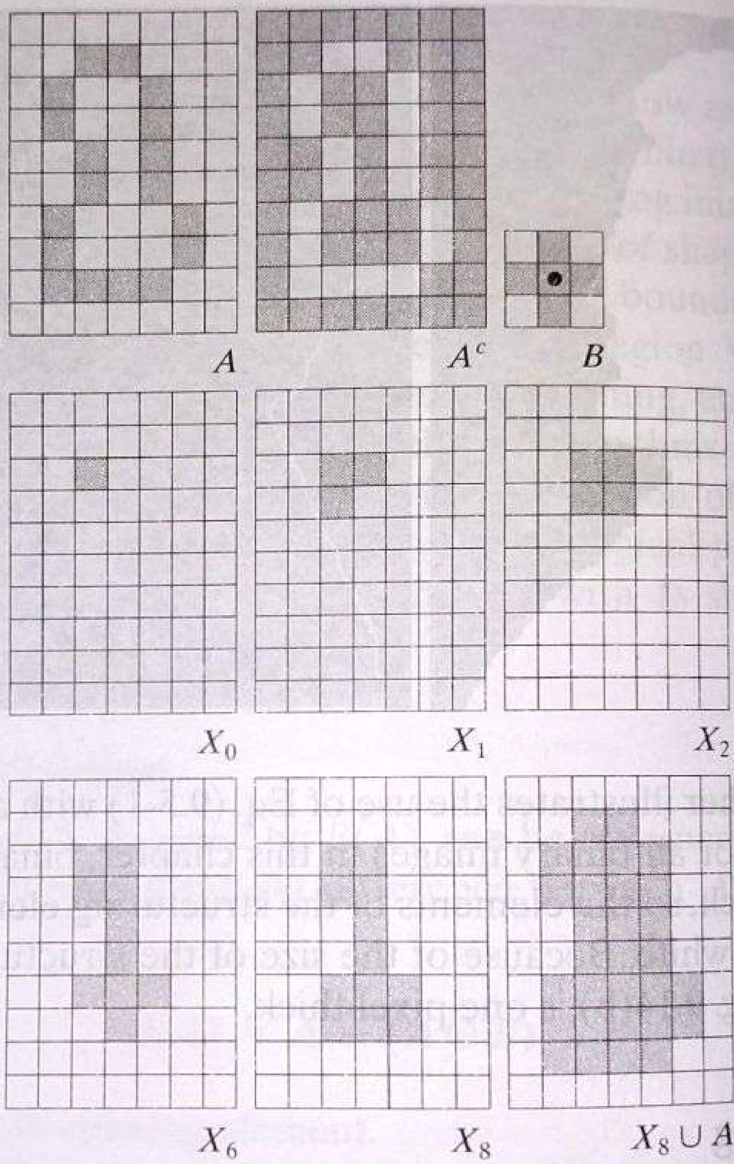


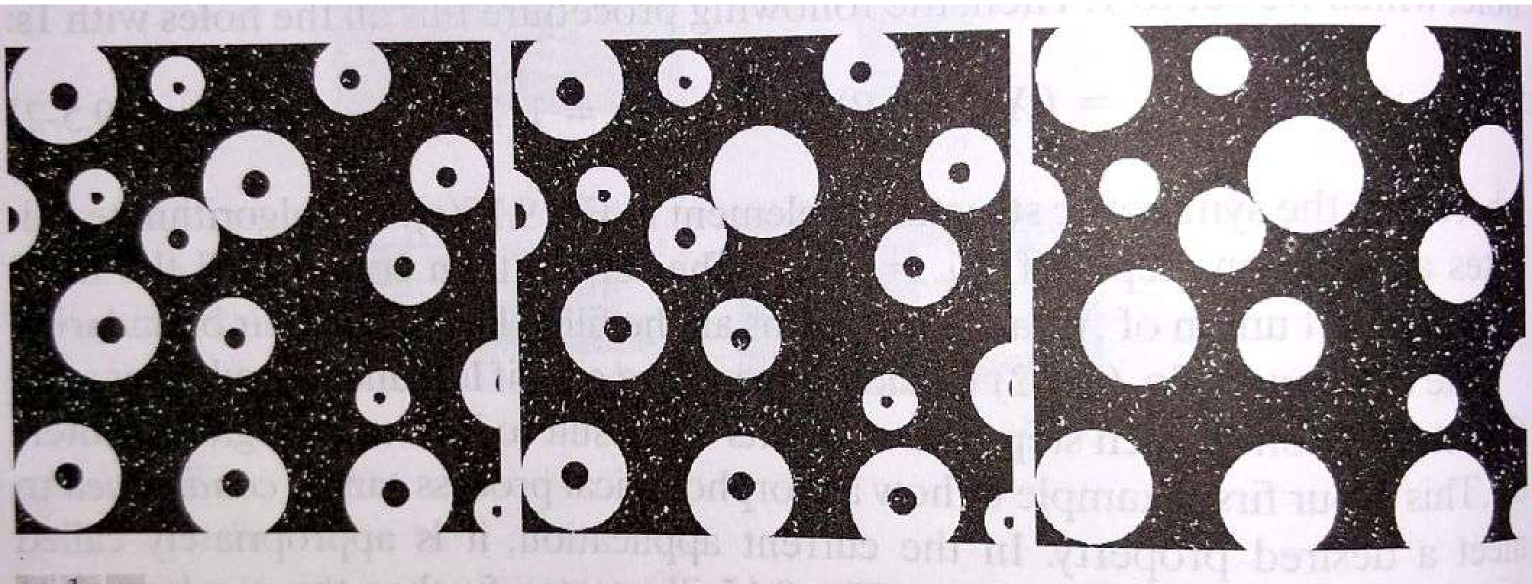
a b

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

a b c
d e f
g h i

FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].





a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

(3) EXTRACTION OF CONNECTED COMPONENTS:-

- Let A be a set containing one or more connected components and form an array X_0 (same as size of A) whose elements are 0's (background values) except at locations of points of connected components which are set to 1.

- objective: Start with X_0 and find all the connected components.

- The iterative equation is given as:

$$\boxed{X_K = (X_{K-1} \oplus B) \cap A \qquad K = 1, 2, 3, \ldots}$$

B = structuring element

- Terminate when $X_K = X_{K-1}$ and X_K contains all the connected components of the input image.

- Procedure is shown in figure (9.17).

- The equation here contains A, ~~exact~~ in comparison to the presence of A^c in Hole-filling because we are looking for foreground points in this case.

- figure (9.18) shows an example application of this process.

(4) THINNING:-

- The thinning of a set A by a structuring element B denoted by $A \otimes B$, can be defined in terms of Hit-or-miss transform as:

$$\boxed{\begin{aligned} A \otimes B &= A - (A \circledast B) \\ &= A \cap (A \circledast B)^c \end{aligned}}$$

- In thinning, no background operation is required as the focus is on pattern matching with the structuring element.

- Another variant of thinning a set A symmetrically is based on the sequence of structuring elements.

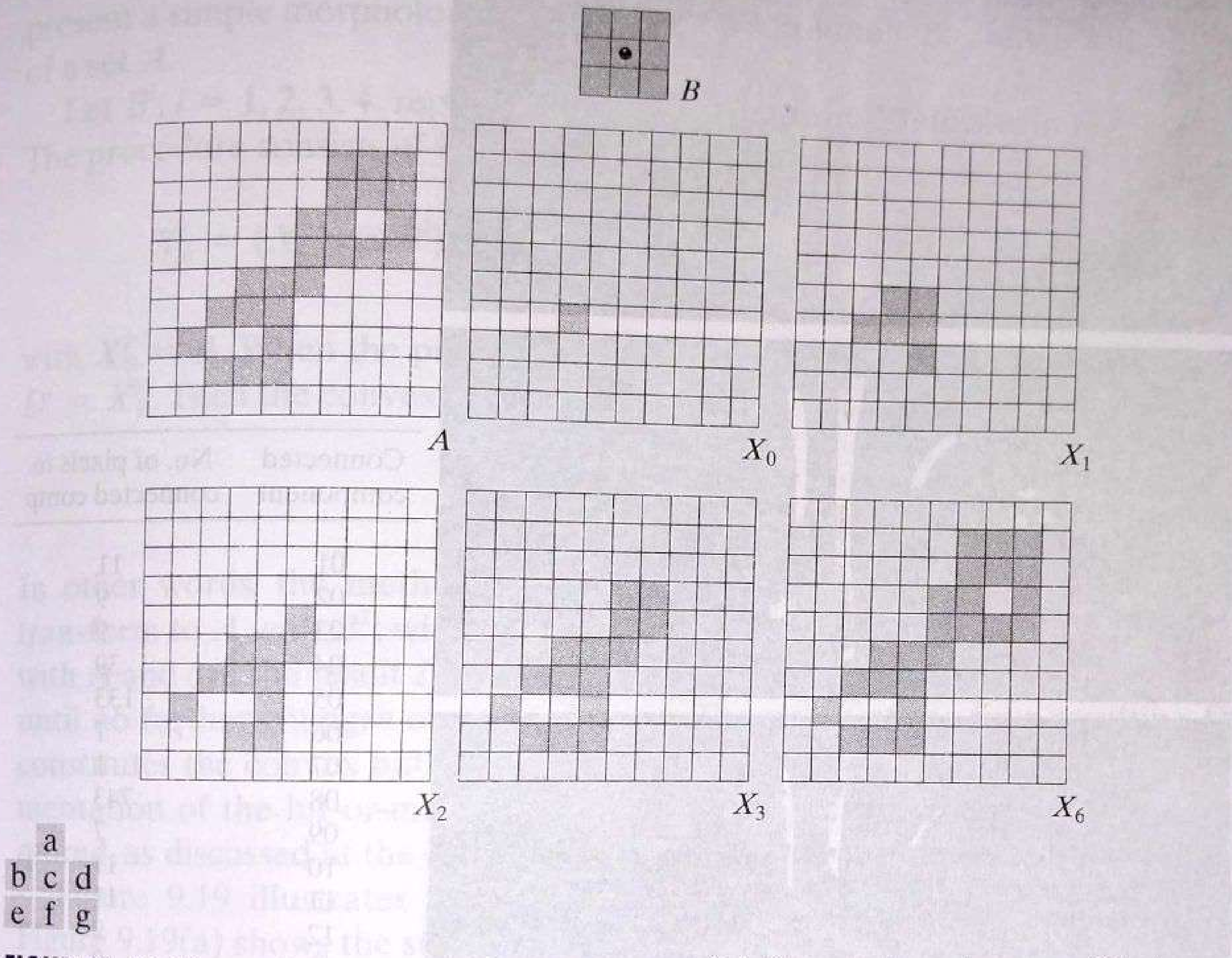


FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).

a
b
c d

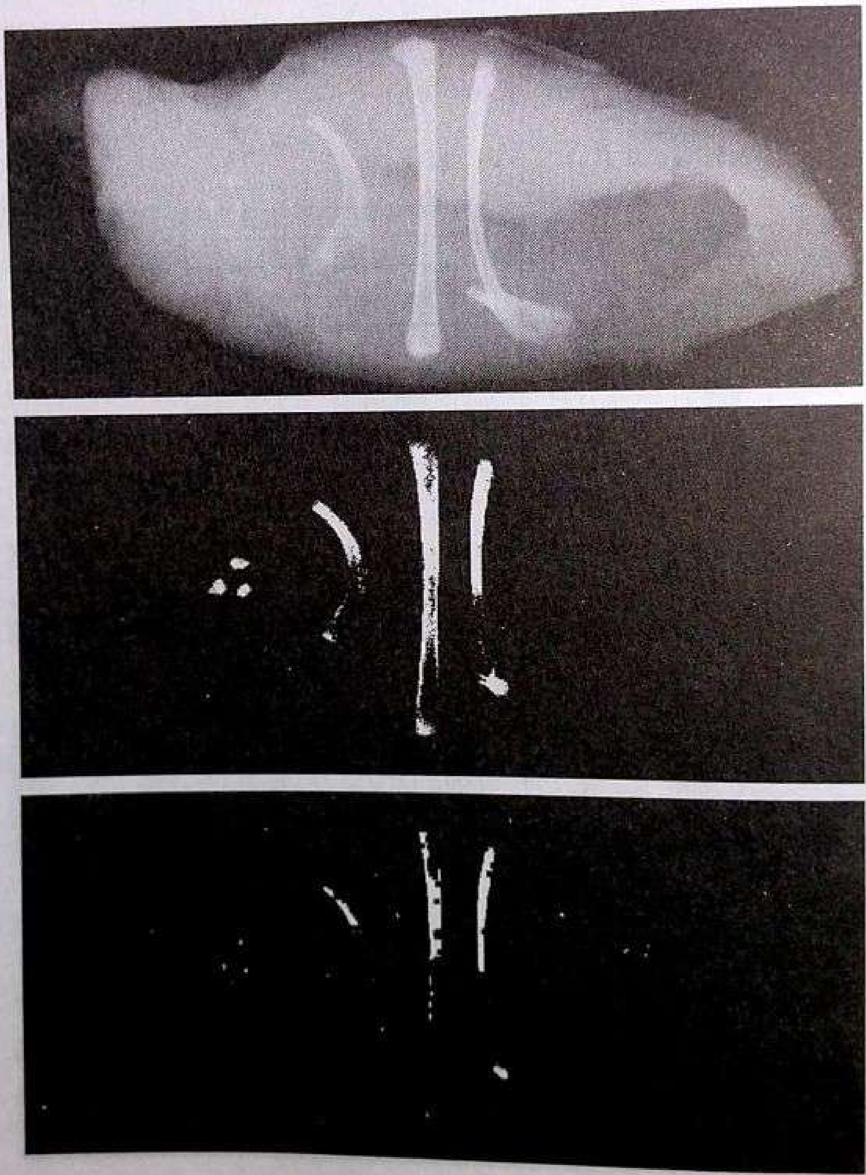


FIGURE 9.18
(a) X-ray image of chicken filet with bone fragments.
(b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s.
(d) Number of pixels in the connected components of (c).
(Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected com
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$ (B^i is a rotated version of B^{i-1})

* Using this concept, thinning is defined as

$$\boxed{A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)}$$

means thin A by B^1, then the result by B^2 and so on until no changes occur in the result.

⇒ fig (9.21) shows the sequential thinning.

(5) THICKENING :-

- It is the morphological dual of thinning and is defined by the expression:

$$\boxed{A \odot B = A \cup (A \circledast B)}$$

$\circledast$ Hit-or-miss Transformation

B = structuring element.

- Thickening can also be defined as sequential operation like thinning:

$$\boxed{A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)}$$

- Structuring elements used for thickening have same form as in case of thinning, but 1's and 0's interchanged.
- Thickening is generally performed using an alternative approach:
    - Thin the background of the set in question
    - Complement the result.

i.e. to thicken set A, form $C = A^c$, thin C and then form C^c. (Shown in example figure (9.22)).

Process is called Background Thinning to achieve Thickening.

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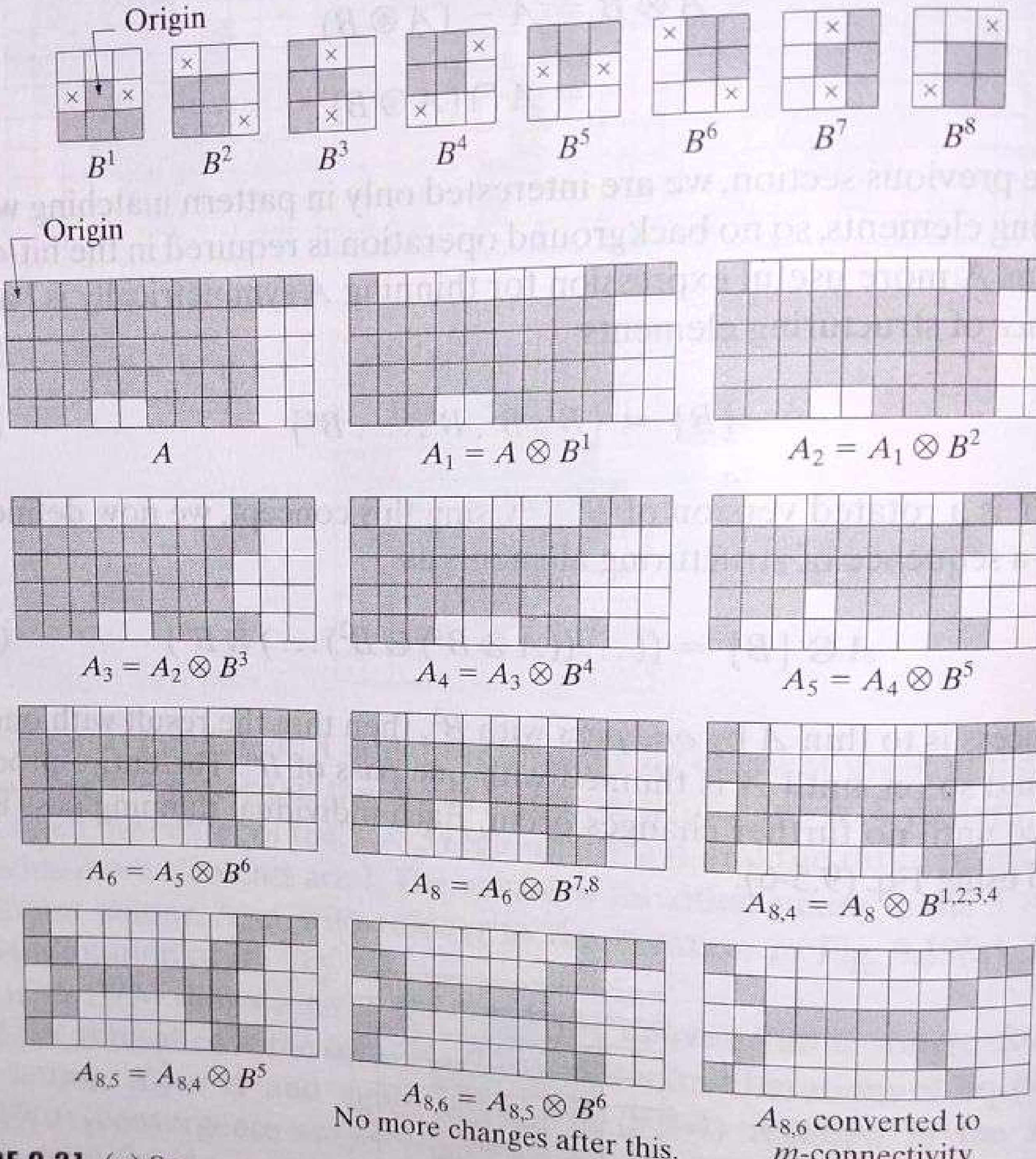

	a	
b	c	d
e	f	g
h	i	j
k	l	m

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

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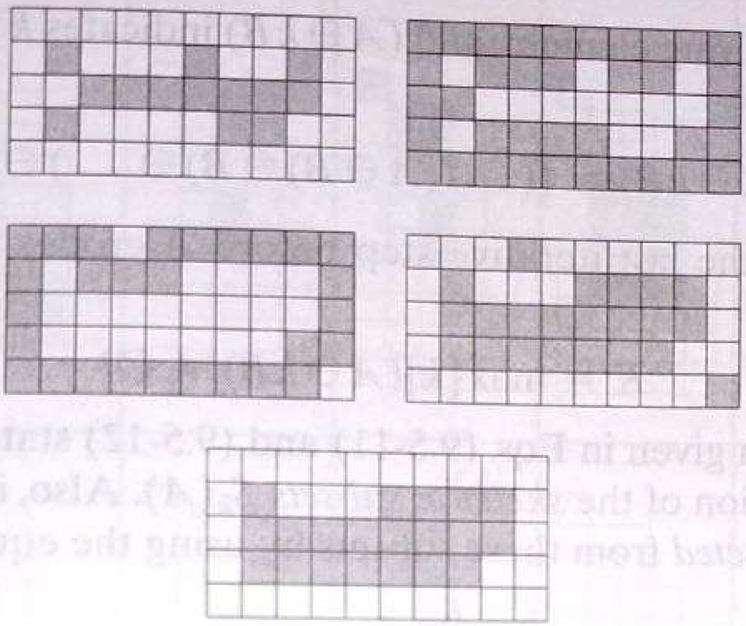


a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

6 PRUNING :-

- It is a type of complementary algorithm to processes like thinning and skeletonization.
- Example application can be the removal of non-uniformities caused to different strokes used in handwriting character recognition. Unwanted branches out of the character are to be removed.
- Steps in pruning:-
  - i) Thinning of input set A with a sequence of structuring elements to detect end points.

$$\boxed{X_1 = A \otimes \{B\}}$$

{B} is used for successive thinning

  - ii) Restore the object to original form but with the unwanted components removed.

$$\boxed{X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)}$$

$\circledast$ - ~~thickening~~ Hit or miss Transformation.

B^k - Shown in fig 9.25

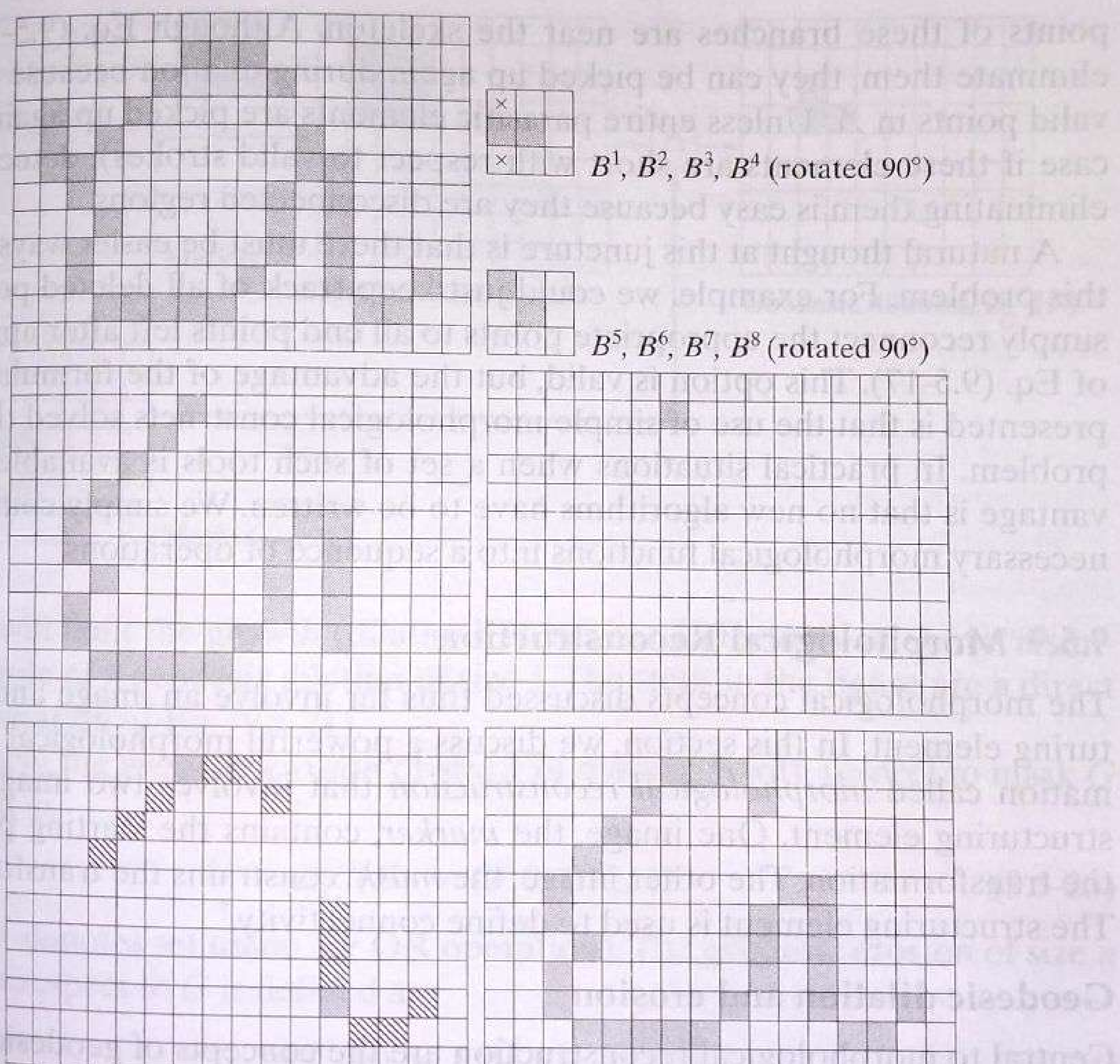
  - iii) Dilation of end-points.

$$\boxed{X_3 = (X_2 \oplus H) \cap A}$$

H = 3X3 SE of 1's

  - iv) Final result is obtained as:

$$\boxed{X_4 = X_1 \cup X_3}$$



a b
c
d e
f g

FIGURE 9.25
(a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

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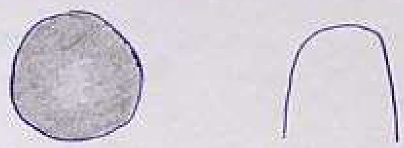
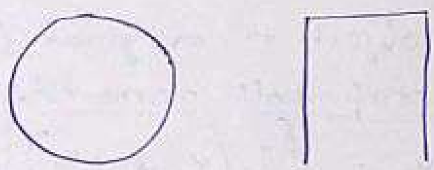
DILATION AND EROSION IN GRAY SCALE IMAGES :-

- $f(x,y)$ = Input image
  $b(x,y)$ = structuring element
- structuring element are of 2-types in gray scale morphology : (a) Non-flat SE



Structuring Element (SE) Intensity Profile

(b) Flat SE



(SE) Intensity Profile

Erosion and Dilation using flat SE :-

- The Erosion of f by a flat structuring element b at any location (x,y) is defined as the minimum value of the image in the region coincident with b when the origin of b is at (x,y).
- Mathematically, it is denoted as :

$$[f \ominus b](x,y) = \min_{(s,t)\in b} \{f(x+s, y+t)\}$$

(x,y) are varied such that origin of b visits each and every pixel in f.

- If the SE is of size 3X3, then the minimum valued pixel is selected from 9 pixels (8 neighbours and the current pixel which coincides with the centre of SE).
- The process seems equivalent to spatial correlation.

...P.T.O.

- Dilation of f by a flat SE b at any location (x,y) is defined as the maximum value of the image in the window outlined by $\hat{b}$ when the origin of $\hat{b}$ is at (x,y).

- Mathematically, it is given as:

$$[f \oplus b](x,y) = \max_{(s,t) \in b} \{ f(x-s, y-t) \}$$

$\hat{b} = b(-x,-y)$

- Explanation is similar to erosion, but maximum is selected and the SE is reflected about its origin.
- The process is equivalent to spatial convolution.

Erosion and Dilation using Non-Flat SE:-

- Erosion of f by non-flat SE b_N is defined as:

$$[f \ominus b_N](x,y) = \min_{(s,t) \in b_N} \{ f(x+s, y+t) - b_N(s,t) \}$$

- Values are subtracted from f to determine erosion at any point.
- This has added computational complexity.
- Dilation using a non-flat SE b_N is defined as:

$$[f \oplus b_N](x,y) = \max_{(s,t) \in b_N} \{ f(x-s, y-t) + b_N(s,t) \}$$

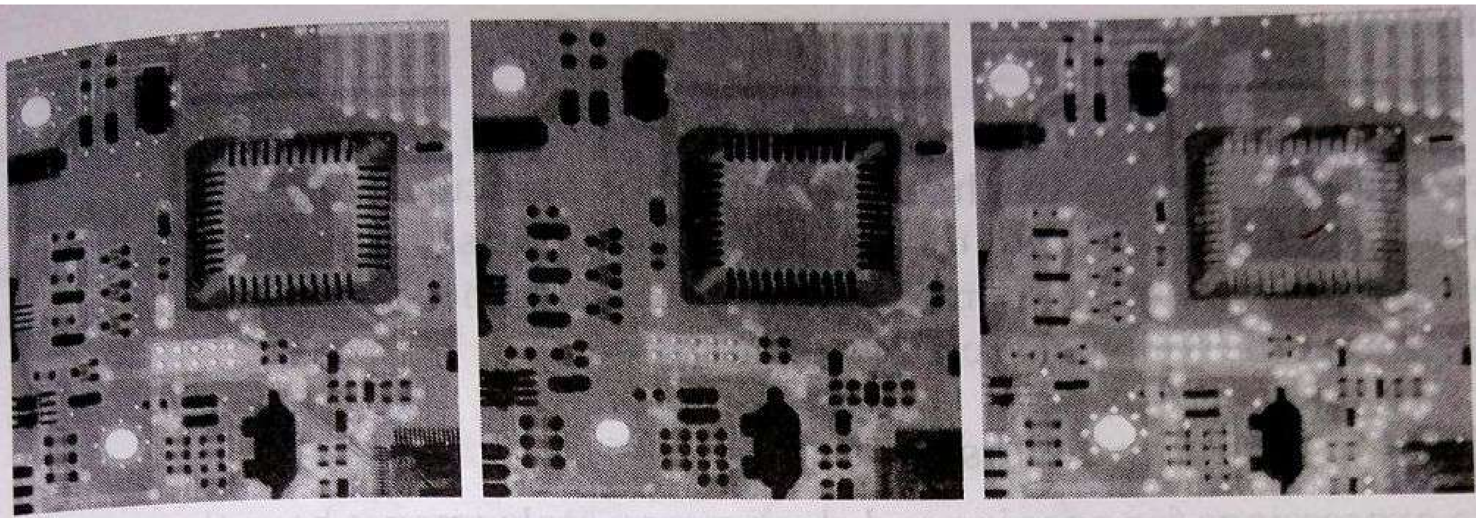
* Also, like binary case of Erosion and Dilation, the gray scale Erosion and Dilation are also duals of each other.

$$(f \ominus b)^c(x,y) = (f^c \oplus \hat{b})(x,y)$$

$f^c = -f(x,y)$, $\hat{b} = b(-x,-y)$

* Generalizing for both flat and non-flat SE,

$$(f \ominus b)^c = (f^c \oplus \hat{b})$$
$$(f \oplus b)^c = (f^c \ominus \hat{b})$$



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 $\times$ 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)