GOVERNMENT COLLEGE OF ENGINEERING KALAHANDI, BHAWANIPATNA

2nd Semester, All Branches(ME,CE,CSE,EE)

ENGINEERING MECHANICS

By

Dr. Mrutyunjay Rout

Ashutosh K. Tiwari

Anish Kumar Shukla

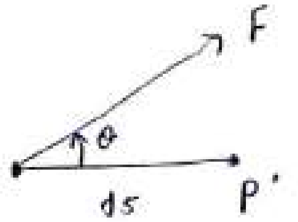
Nilesh Sarkar

Topic - Principle of virtual work

* Basic Terminology.

①. Work of a force. If a particle is subjected to a force 'F' and the particle is displaced by an infinitesimal displacement 'ds' then -

Work of force $(W) = (F\cos\theta)\,ds$.



{ where θ is the angle the line of action of force & displacement. }

→ It is a Scalar quantity which has magnitude & sign but no direction.

→ It's S.I unit is Nm also known as joule (J)

* Note - ⓐ. If $\theta = 0$, $W = F\,ds$

ⓑ. If $W = 0$ then - either $ds = 0$ or $\theta = 90°$

ⓒ. $W = +ve$ when the direction of both displacement & force is same.

②. Virtual Displacement & Virtual work -

When a system of forces acting on a body are in equilibrium then the displacement of the body would be zero and no work is possible. But an imaginary infinitesimal displacement to the body in equilibrium can be assumed to be given. Such displacement is called virtual displacement.

The resulting work done by the forces acting on the body during virtual displacement is called virtual work.

Principle of virtual work - It states that :- "If a rigid body is in equilibrium, the total work of the external forces acting on the rigid body is zero for any virtual displacement of the body consistent with the geometrical condition of the body.

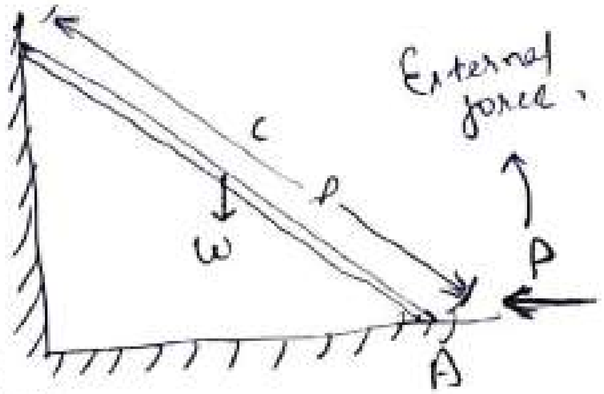
Note - forces which do not work are -
(a). weight of body when its centre of gravity moves in horizontal direction.
(b). The reaction at a frictionless hinge when the body rotates about the hinge.
(c). The reaction at a frictionless surface when the body moves along the surface.
(d). The friction force acting on wheel when it rolls without slipping
(e). The internal forces of the nature of action & reaction (Tension in string & axial forces in bar).

Procedure for application of principle of virtual work -
1. Take any fixed point in the problem as origin & fix co-ordinates and find co-ordinates of all the points where forces are acting.
2. find the virtual displacement.
3. Use principle of virtual work to find unknown forces.

Note - (a). If any force is acting along (+) x axis or (+) y axis then take the force as positive.

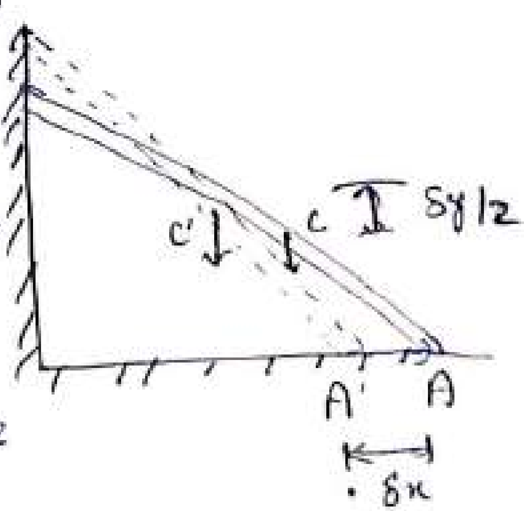
(b). The sign convention for the quadrant of the point will depend on quadrant in which the point is lying.

Example - (1). A ladder AB of length 'l' and weight 'w' stands in a vertical plane supported by smooth surfaces at A & B. Using the principle of virtual work, find the magnitude of horizontal force 'P' to be applied at the end A if the ladder is to be in equilibrium.



Soln → Step 1). Choose fixed point 'O' as origin and the coordinates of A be $(x, 0)$ & B be (o, y).

Let $OA = x$, $OB = y$ & θ is ~~angle~~ angle between ladder and surface at point A.



Step-2). Given a small virtual displacement at A, the corresponding virtual displacement at B is δy & at C is $\frac{\delta y}{2}$.

The angle of inclination θ of the ladder shall changed by $\delta\theta$.

Step 3 - The forces doing the work are the weight of the ladder w and the force P. The reaction at A & B do no work.

For triangle ABO, $\{y = l \cancel{\cos\theta} \sin\theta\}$

$\{x = l\cos\theta\}$.

Differentiating both we get.

$$\delta x = l(-\sin\theta)\, d\theta$$

$$\delta y = l(\cos\theta)\, d\theta$$

The principle of work gives.

$$P\,\delta x = w \times \frac{\delta y}{2} = 0.$$

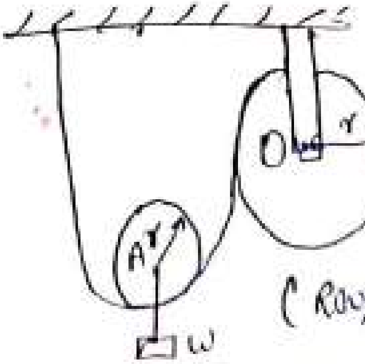
$$P\,\delta x - w\frac{\delta y}{2} = 0.$$

$$P(l\sin\theta\,\delta\theta) - w(l\cos\theta\,\delta\theta) = 0.$$

$$\boxed{P = \frac{w}{2}\cot\theta}$$ - Ans

(2). A weight w of 1000 N is said to be raised by a system as shown in figure. Using the principle of virtual work find the value of force P which can hold the system in equilibrium.

{Here - radius of pulley are same}.



(Rough figure).

Solution -

Step-1) - Choose the fixed point O as origin and the coordinate of 'B' where force P is applying be $(x, 0)$ & the coordinate of A where weight is acting is $(y, 0)$, so

Let y = Distance between the two centre of pulley.

Step-2) - Let a small virtual displacement of point B be δy then the corresponding virtual displacement at A is $\delta y/2$ (for string at B to be moved by δy, the point A has to be moved or pulled $\delta y/2$ because of two strings at the pulley A).

Sign Convention - Here virtual displacement at B = $(-ve)\,\delta y$.
& ,, ,, ,, A = $(+ve)\,\delta y/2$.

Step-3 - The Principle of virtual work gives -

$$+W(\delta y/2) - P\,\delta y = 0$$

$$\boxed{P = 500\,N} \quad - \text{Ans}$$

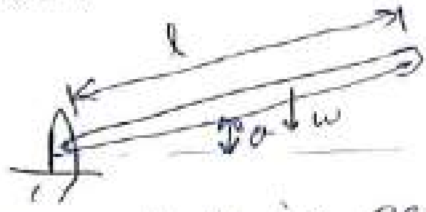
Potential Energy & Equilibrium -

If a System is in equilibrium the derivative of its total P.E is zero. (P.E = Potential Energy)

$$\boxed{\frac{\partial (P.E)}{\partial \theta} = 0}$$

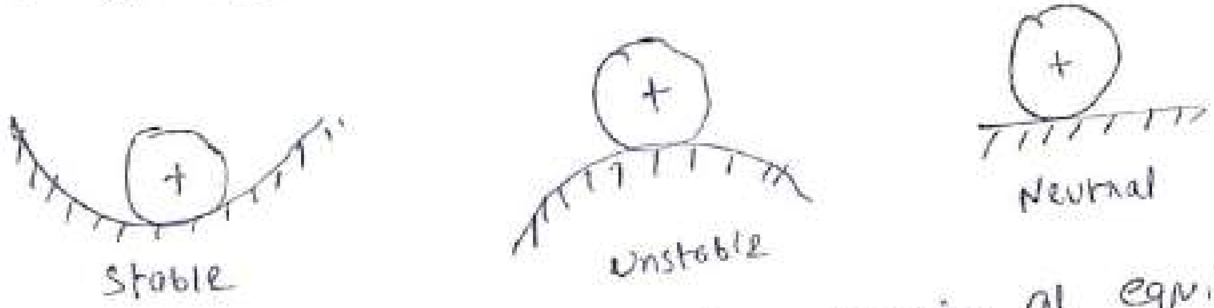
If a bar is hinged at one end, it can rotate about an axis through hinge and angle θ defines the position with reference line. then,

$$P.E = Wh = W\left(\frac{l}{2}\sin\theta\right).$$



$$\left(\frac{P.E}{d\theta}\right) = 0$$ – for system to be in equilibrium.

Stability of equilibrium -



When a system is disturbed from position of equilibrium by the slightest force it produces a small displacement from equilibrium position. If the system returns to original position as soon as disturbing force is removed the equilibrium is known as stable one., If it moves away from equilibrium position then it is unstable equilibrium and if it neither returns or move away from its original position it is called neutral equilibrium.

Note. ① - $\frac{d(P.E)}{d\theta} = 0$ - stable

② $\frac{d^2(P.E)}{d\theta^2}$ = +ve (stable)
= -ve (unstable)
= 0 (Neutral).

AKT

Rectilinear Motion of a Particle (1)

* **Dynamics** - It is a part of mechanics that deals with the analysis of bodies in motion.

Dynamics

- Kinematics (Dealing with displacement, velocity, acceleration without considering the forces causing them).
- Kinetics (Dealing with Displacement, Velocity, acceleration considering the forces causing them).

* **Particle** - The term particle is convenient idealisation of the physical objects which need not to be a small in size. In this idealization, the mass of a body is assumed to be concentrated at a point & the body motion is considered as motion of entire unit neglecting any rotation about its own mass centre.

Note - If there is rotation present in the system then the body is not considered as particle.

Types of motion

- Rectilinear motion (when body is moving in a straight line)
- Curvilinear motion (when body moves along a curved path).

Kinematics (Rectilinear Motion)

We know about displacement, velocity & acceleration & their relations -

$$s = dx$$

$$v = \frac{dx}{dt}$$

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$$

Average velocity $= \frac{\text{Total distance}}{\text{Total time}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$

Remember the expression difference..

Instantaneous velocity $= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

i.e velocity at a particular instant of time.

Similarly Average acceleration $= \frac{\Delta v}{\Delta t}$.

Instantaneous Acceleration $= \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$.

$$\boxed{a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx}}$$

Uniform motion - when $a = 0$, v = constant

Uniform Accelerated motion - when a = constant

Jerk - $\frac{da}{dt}$ = Rate of acceleration is known as Jerk.

If a is constant $\rightarrow$

(1) - $v = v_0 + at$

(2) - $s = v_0 t + \frac{1}{2}at^2$.

(3) - $v^2 = u_0^2 + 2as$.

Q①. Driver of a car Travelling at 72 km/hr observes the light 300m ahead of him turning red. The Traffic light is timed to remain red for 20 seconds before it turns green. Determine -

①. the required Uniform acceleration of car,

②. the speed with which the motorist crossed the traffic light.

Soln - Given -

$$u = 72 \frac{km}{hr} = \frac{72 \times 1000}{60 \times 60} = 20 \, m/sec$$

$$u = 20 \, m/sec.$$
$$t = 20 \, sec$$
$$S = 300 \, m$$

$$S = ut + \frac{1}{2}at^2$$

$$a = -0.5 \, m/sec^2 \text{ (Deceleration).}$$

$$v = u + at.$$

$$v = 10 \, m/sec \text{ or } 36 \, km/hr \text{ - Ans}$$

Q2)- Motion of a particle along a straight line is given by eqn.

$$a = t^2 - 2t + 2.$$

where a = acceleration in m/sec^2, t = time in sec.

After 1 second the distance travelled by the particle and the velocity of the article were found to be 14.75 m & 6.33 m/sec. Find - ①- Distance Travelled ②. Velocity ③- Acceleration of particle after 2 seconds.

Solution - Given - at $t = 1$ sec.

$x = 14.75\ m$ & $v = 6.33\ m/sec$.

$\rightarrow$ $a = t^2 - 2t + 2$ — (1)

$v = \int a\,dt = \int (t^2 - 2t + 2)\,dt$

$v = \frac{t^3}{3} - \frac{2t^2}{2} + 2t + C_1$

Here - putting the conditions value at $t = 1$ sec.

$6.33 = \frac{1}{3} - \frac{2}{2} + 2 + C_1$

$C_1 = 5$

so, $v = \frac{t^3}{3} - \frac{2t^2}{2} + 2t + 5$ — (2)

$v = \frac{dx}{dt}$, $x = \int v\,dt$

$x = \int \left(\frac{t^3}{3} - \frac{2t^2}{2} + 2t + 5 \right) dt$

$x = \frac{t^4}{12} - \frac{t^3}{3} + \frac{2t^2}{2} + 5t + C_2.$

Again putting the value of x at $t = 1$ sec we get.

$C_2 = 9.$, $x = \frac{t^4}{12} - \frac{t^3}{3} + t^2 + 5t + 9.$ (3)

To find the x, v & a at $t = 2$ sec, we put the value in (1), (2) & (3). we get.

$x = 21.67\ m$

$v = 7.76\ m/sec$

$a = 2\ m/sec^2$.

3 - A particle starts with velocity v_0. Its acceleration & velocity are related by eqn.

$$a = -kv.$$

where k = constant, v = velocity of particle, a = acceleration of particle

find the displacement time relation.

Soln - we know - $a = -kv$

$$\frac{dv}{dt} = a = -kv.$$

$$\therefore \quad \frac{dv}{v} = -k\,dt$$

Integrating both sides. $\displaystyle\int \frac{dv}{v} = \int -k\,dt$

$$[\ln v]_{u_0}^{v} = [-kt]_0^t$$

$$\ln \frac{v}{v_0} = -kt$$

$$v = v_0 e^{-kt}$$

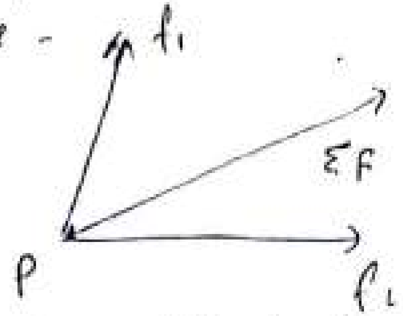
$$\frac{dx}{dt} = u_0 e^{-kt}.$$

Integrating - $\displaystyle\int dx = \int u_0 e^{-kt}\,dt.$

we get - $\boxed{x = \frac{u_0}{k}\left(1 - e^{-kt}\right)}$ - Ans

Kinetics (Rectilinear Motion)

Consider a particle 'P' of mass m having acceleration a when acted upon by several forces i.e F_1 & F_2 then resultant force -



$= m \xrightarrow{} a.$

particle is moving with acceleration a,

Applying Newton's Second law. $\boxed{\Sigma F = ma}$

$$\Sigma F_x = ma_x,$$

$$\Sigma F_y = ma_y.$$

When the body is moving in straight line (x axis). then.

$$\boxed{\Sigma F = ma_x = m\ddot{x}}$$

Equation of Dynamic Equilibrium : D'Alembert's Principle -

$$\Sigma F = ma$$

$$\Sigma F - ma = 0. \rightarrow \text{Eqn of Dynamic equilibrium of } \cancel{\text{body}} \text{ (particle P).}$$

The equation of motion of P can be written as.

$$\Sigma F - ma = 0$$

which means resultant of the external forces (ΣF) & the force (−ma) is zero.

−ma = Inertial force.

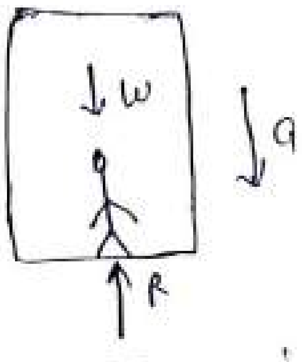
Resistance to the change in the condition of rest or uniform motion of body.

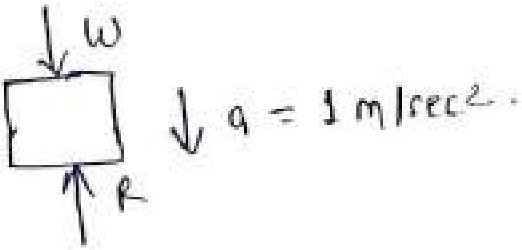
Note - To write the equation of dynamic equilibrium of a particle add a fictious force equal to inertia force to the external forces on the particle and equate the sum (resultant) to zero.

$$\left.\begin{aligned} \Sigma F_x + (-ma_x) &= 0. \\ \Sigma F_x + (-ma_y) &= 0. \end{aligned}\right\} \text{- eqn of dynamic equilibrium of particle.}$$

Que 1 - An Elevator has a downward acceleration of 1 m/sec². What pressure will be transmitted to the floor of the elevator by a man weighing 500N travelling in the lift? Find the pressure if the elevator had an upward acceleration of 1 m/sec².

Soln - Pressure exerted by floor on man (R) = Pressure exerted by the man on the floor.





Downward motion - Dynamic equilibrium eqn of man.

$$\boxed{\Sigma F = W - R}$$

$$\Sigma f = ma$$

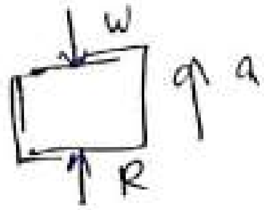
$$\frac{W}{g} a = W - R$$

$$R = W(1 - a/g)$$

$$R = 449 \text{ N} \quad \text{Ans.}$$

Upward motion.

$$\Sigma F = ma$$

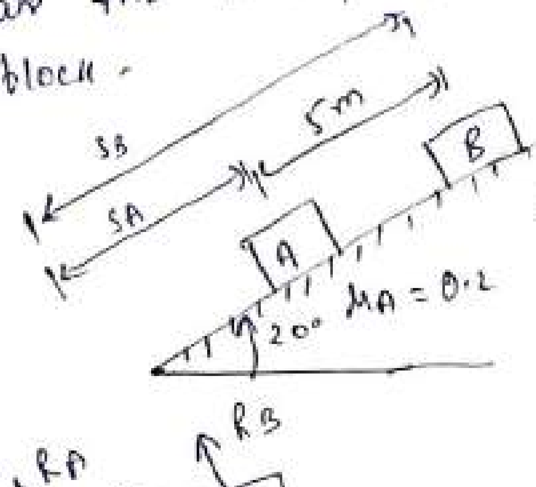


$$\frac{W}{g} a = R - W$$

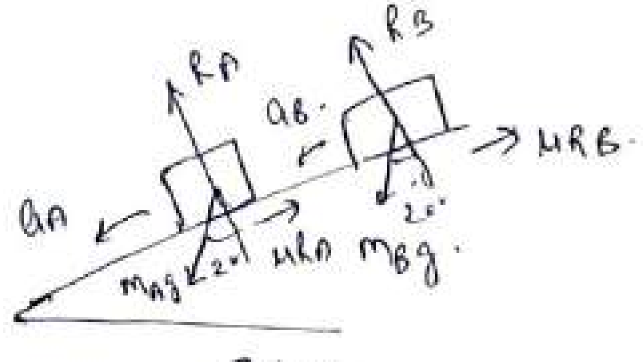
$$R = W\left(1 + \frac{a}{g}\right)$$

$$R = 550.9\,N \quad \underline{Ans}$$

Q.2) - Two Blocks A & B are held on an inclined plane 5 m apart as shown in figure. The coefficient of friction between the block A & B and the inclined plane are 0.2 & 0.1 respectively. If the blocks begin to side down the plane simultaneously, calculate the time & distance travelled by each block.



Soln -



F.B.D

let a_A & a_B be the acceleration of the block A & B.

then - $\Sigma F_x = ma_x$.

(Taking x axis along the plane).

$$m_A a_A = m_A g \sin 20° - \mu_A R_A \quad -① \quad (a_x = a)$$

$$0 = R - m_A g \cos 20° \quad -② \quad (a_y = 0).$$

$\rightarrow \quad \Sigma f_y = m a_y$

Solving from ① & ② we get.

$$a_A = 1.510 \text{ m/sec}^2.$$

Equation of motion for Block B -

$$\Sigma F_x = (a_x) m \quad m_B a_B = m_B g \sin 20° - \mu_B R_B \quad (a_x = a_B).$$

$$m_B a_B = m_B g \sin 20° - 0.1 R_B \quad -③$$

$$\Sigma F_y = m a_y \quad - \quad 0 = R_B - m_B g \cos 20° \quad (a_y = 0) \quad -④$$

Solving from ③ & ④ we get -

$$a_B = 2.43 \text{ m/sec}^2$$

Let the blocks collides after time t.

Distance S_B travelled by block B in time t -

$$S_B = \frac{1}{2} a t^2 + ut \quad (u = 0)$$

$$S_B = \frac{1}{2} a_B t^2.$$

Distance S_A travelled by block A in time t -

$$S_A = \frac{1}{2} a_A t^2 \quad (u = 0).$$

for blocks to collide -

$$S_B - S_A = 5 \text{ m}$$

putting the eqn value & solving we get,

$$t = 3.35 \text{ sec} \quad \underline{Ans}$$

for $t = 3.35$ sec, $\quad S_A = 8.20$ m. $\underline{Ans}$

$S_B = 13.20$ m.

Principle of Dynamics -

Inertial frame of Reference -

A Reference frame in which an object remains either at rest or at a constant velocity unless another force acts upon it. It follows Newton first law.

Non-Inertial frame of Reference -

It is any frame where some external force works upon it. It does not holds the law of inertia.

Momentum & Impulse

Momentum → Consider the motion of particle of mass 'm' acted upon by a force $\vec{F}$:-

$$F_x = ma_x \quad \& \quad F_y = ma_y$$

or
$$F_x = m\frac{dv_x}{dt} \quad \& \quad F_y = m\frac{dv_y}{dt}$$

$$\Rightarrow \quad F_x = \frac{d}{dt}(mv_x) \quad \& \quad F_y = \frac{d}{dt}(mv_y)$$

So it can be written as:-

$$\boxed{\vec{F} = \frac{d}{dt}(m\vec{v})}$$

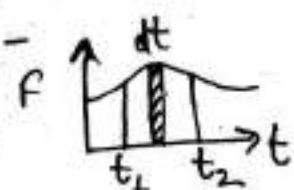
Which states that force $\vec{F}$ acting on a particle is equal to the rate of change of momentum of particle.

→ The vector $m\vec{v}$ is called linear momentum of the particle.

Impulse → When a large force acts over a short period of time, it is called an impulsive force.

The impulse of a force F acting over a time interval for t_1 to t_2 is given by:-

$$\boxed{I = \int_{t_1}^{t_2} F\,dt} \rightarrow$$



→ When the variation of a force wrt time is unknown, the impulse can also be written as:-

$$\boxed{I = F_{avg} \times \Delta t} \quad (NS)$$

Principle of Impulse & momentum →

This says that the total change in the momentum of a particle during a time interval is equal to the impulse of the force acting during the same interval of time.

i.e. $$\boxed{m\vec{v}_2 - m\vec{v}_1 = \int_0^t \vec{F}\,dt}$$

→ $\boxed{\text{Final momentum} - \text{Initial momentum} = \text{Impulse of } F}$

For a system of particles:-

$$\boxed{\Sigma m v_2 - \Sigma m v_1 = \int_0^t F\,dt}$$

→ The above eqⁿs can be replaced by two component eqⁿs in x & y directions.

Conservation of momentum →

When sum of the impulses due to external forces is zero the momentum of the system remains conserved.

$$\boxed{\Sigma m v_2 = \Sigma m v_1}$$

Q. A ball of mass 100 g is moving towards a bat with a velocity of 25 m/s as shown in fig.
(a) When hit by a bat the ball attains a velocity of 40 m/s. If the bat and ball are in contact for a period of 0.015 s, determine the avg. impulse force exerted by the bat on the ball during the impact.

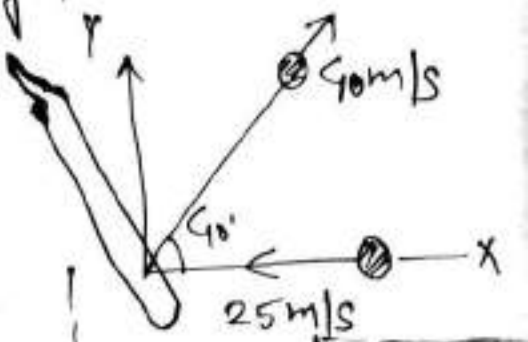
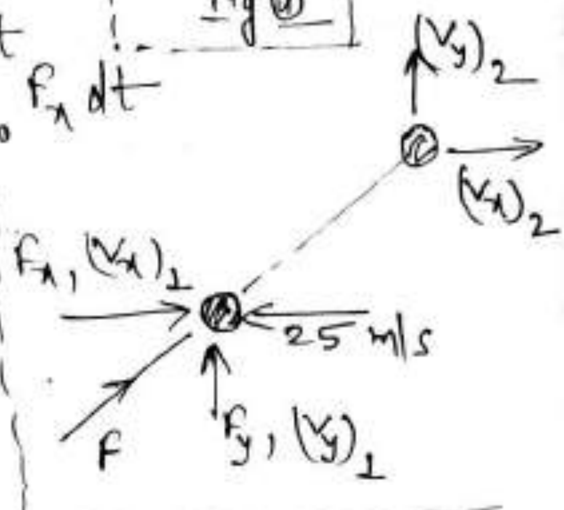


fig (a)

Sol^n: → Applying the principle of impulse and momentum to the ball in X & Y directions:-

$$(mv_x)_2 - (mv_x)_1 = \int_0^t F_x \, dt$$



→ $(V_x)_1 = -25$ m/s

$(V_x)_2 = 40 \cos 40° = 30.64$ m/s

So:- $\int_0^t (F_x)\, dt = (F_x)_{avg} \cdot \Delta t$

$\Rightarrow (F_x)_{avg.} \cdot (\Delta t) = 0.1(30.64) - 0.1(-25)$

$\Rightarrow (F_x)_{avg} = 5.564 / 0.015 = \underline{\underline{370.9}}$ N

The eqn in y dirn: →

$$(mv_y)_2 - (mv_y)_1 = \int_0^t F_y\,dt = (F_y)_{avg} \cdot \Delta t$$

→ $(v_y)_1 = 0$

$(v_y)_2 = 40\sin 40° = 25.72$ m/s

$\Rightarrow (F_y)_{avg} \cdot \Delta t = 0.1(25.72) - 0.1(0)$

$\Rightarrow (F_y)_{avg} = 2.572/0.015 = \underline{\underline{171.5\,N}}$

$\therefore F_{avg} = \sqrt{(F_x)_{avg}^2 + (F_y)_{avg}^2} = \sqrt{370.9^2 + 171.5^2}$

$\Rightarrow F_{avg} = \underline{\underline{408.6\,N}}$ Ans

Impact

(Collision of Elastic Bodies)

The phenomena of collision of two bodies which occurs in a very small interval of time and during which the two bodies exert very large force on each other is called impact.

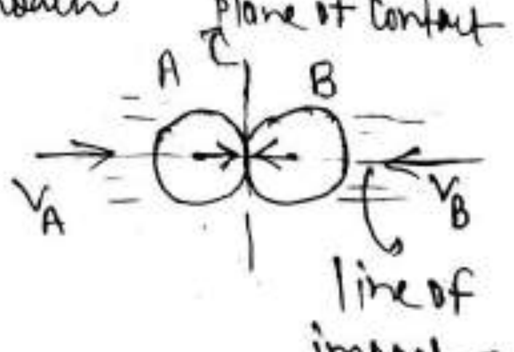
→ Central impact → When direction of motion of the mass centres of the two colliding particles is along the line of impact

→ Oblique impact → When one or both of the particles is at an angle with the line of impact.

* Coefficient of restitution → (e) :-

$$e = (-)\frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$$\boxed{e = \frac{(V_B)_2 - (V_A)_2}{(V_A)_1 - (V_B)_1}}$$



Central impact

Q→ A ball 'A' of mass 1 kg is moving with velocity of 2 m/s, impinges directly on a ball B of mass 2 kg at rest. Find the velocities of two balls after the impact. Assume coeff. of restitution $e = 1/2$.

Sol:- Since there is no external force involved so from principle of conservation of momentum :-

$$m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$$

$$\Rightarrow \quad 1 \times 2 + 2 \times 0 = 1 \times (v_a') + 2 v_b'$$

$$\Rightarrow \quad \boxed{v_a' + 2v_b' = 2} \quad -(I)$$

Now $e = -\dfrac{(v_b' - v_a')}{v_b - v_a} \Rightarrow 1/2 = \dfrac{-(v_b' - v_a')}{0 - 2}$

$$\Rightarrow \quad \boxed{v_b' - v_a' = 1} \quad -(II)$$

from (I) & (II)

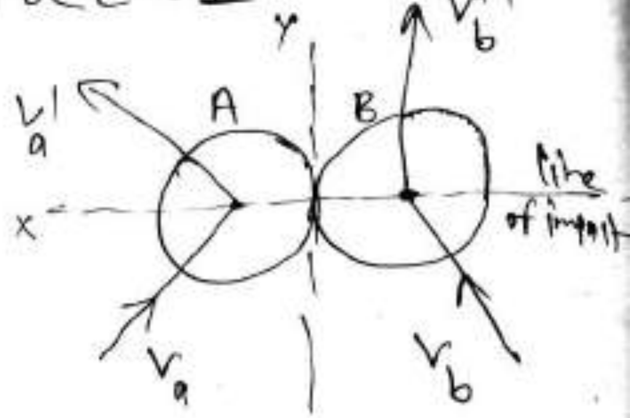
$$v_a' = 0 \ \& \ v_b' = 1 \text{ m/s} \quad \underline{\text{Ans}}$$

Note → In case of perfectly elastic impact $(e=1)$ the energy of the system is also conserved i.e.

$$\boxed{\tfrac{1}{2} m_a v_a^2 + \tfrac{1}{2} m_b v_b^2 = \tfrac{1}{2} m_a v_a'^2 + \tfrac{1}{2} m_b v_b'^2}$$

- Perfectly plastic impact → $e = 0$ → $\boxed{v_b' = v_a' = v'}$
- Inelastic impact → $0 < e < 1$

- Oblique Impact.



In case of oblique impact the total momentum of two bodies along the line of impact is consumed i.e.

$$\boxed{m_a(v_a)_x + m_b(v_b)_x = m_a(v_a')_x + m_b(v_b')_x}$$

↳ Only along the line of impact.

and

Coeff. of restitution:.

$$e = (-)\frac{\text{The component of vel. of separation after the impact along line of impact}}{\text{The component of vel. of approach before impact along line of impact}}$$

$$\boxed{e = -\frac{(v_b')_x - (v_a')_x}{(v_b)_x - (v_a)_x}}$$

- For motion normal to line of impact:-

$$\boxed{(v_a)_y = (v_a')_y \quad \& \quad (v_b)_y = (v_b')_y}$$

Work & Energy

Principle of work & energy →

The work done by a force acting on a particle during its displacement is equal to the change in the kinetic energy of the particle during that displacement.

$U_{1\to 2}$ → work of force 'F' $\boxed{U_{1-2} = T_2 - T_1}$

T_2 → final K.E

T_1 → Initial K.E

- **Work & Energy principle for a system of particles →**

$$\boxed{\Sigma U_{1\to 2} = \Sigma (T_2 - T_1)}$$

$\Sigma U_{1\to 2}$ = Work of all the forces acting on various particles.

- **Principle of Conservation of Energy →**

If a particle moves under the action of conservative force (i.e. gravity), work done is stored as potential energy i.e. $U_{1-2} = -(V_2 - V_1)$

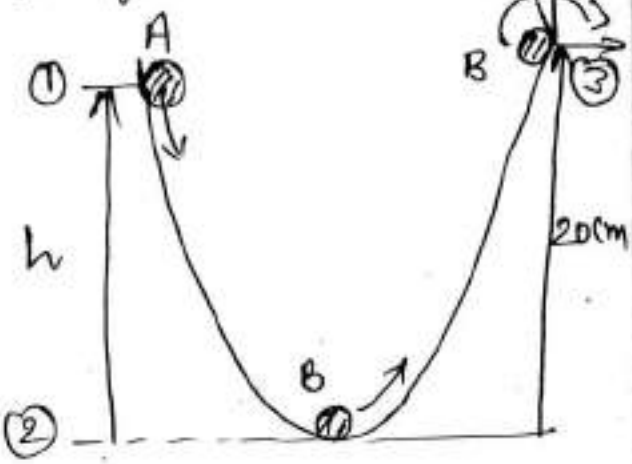
so $U_{1-2} = T_2 - T_1$

$\Rightarrow -(V_2 - V_1) = T_2 - T_1$

$\Rightarrow \boxed{T_1 + V_1 = T_2 + V_2}$

$\boxed{K.E_1 + P.E_1 = K.E_2 + P.E_2}$

A spherical ball A of mass 'm' when released from rest slides down the surface of a smooth bowl and strikes another spherical ball B of mass '$m/4$' resting at bottom the bowl. Determine the height 'h' from which the ball 'A' should be released so that after the impact the ball 'B' just leaves the bowl. The coeff. of restitution may be assumed to be 0.8.



Sol^n:- $m_A = m$; $m_B = m/4$

Since the ball 'A' slides down a height h along a frictionless surface from position 1 to position 2.

The vel. of ball A (v_a) when it strikes the ball B is to be determined from principle of conservation of energy i.e.

$$mgh = \frac{1}{2} m v_a^2 \rightarrow v_a = \sqrt{2gh}$$

Now let vel. of ball A after the impact is $= v_a'$

The initial vel. of ball B $= v_b = 0$

The vel. of ball B after the impact $= v_b'$

Now since the ball B must rise to hight 20 cm after the impact so min. vel. required for this job is found by conservation of energy for ball 'B' b/w (2) & (3) locations:-

$$\text{i.e.}\quad \tfrac{1}{2}\left(\tfrac{m}{4}\right) v_b'^2 = \frac{m}{4} g\,(0.2)$$

$$K.E \quad = \quad P.E$$

$$\Rightarrow \quad v_b' = \sqrt{2g(0.2)} = \sqrt{0.4g}$$

Now applying the principle of conservation of momentum to the impact of balls A & B:-

$$m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$$

$$\Rightarrow \quad m\sqrt{2gh} + 0 = m v_a' + \frac{m}{4}\sqrt{0.4g}$$

$$\Rightarrow \quad -v_a' + \sqrt{2gh} = 0.495 \quad \text{— (I)}$$

Now from coeff. of restitution:-

$$e = -\frac{v_b' - v_a'}{v_b - v_a} = 0.8$$

$$\Rightarrow \quad 0.8\left(0 - \sqrt{2gh}\right) = \sqrt{0.4g} + v_a' \quad \text{— (II)}$$

from (I) & (II)

$$\Rightarrow \quad v_a' = 0.8\sqrt{2gh} = 1.981 \quad \text{— (III)}$$

from (I) & (III)

$$\Rightarrow \quad h = 0.104\,m = \underline{10.4\,cm} \quad \underline{\underline{\text{Ans}}}$$

Curvilinear Translation

When a moving particle describes a curved path it is said to have a curvilinear motion. And if the curved path lies in a plane it is termed a plane curvilinear motion.

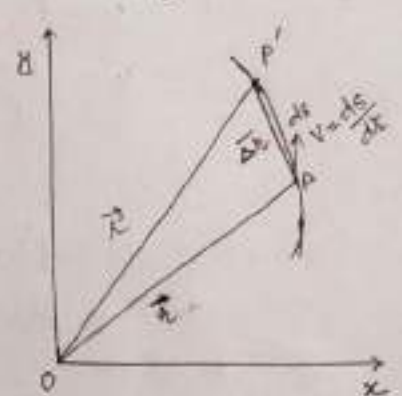
The particle moves in a curved path when –

(a) the direction of the force acting on a particle varies

(b) when the particle has some initial motion in a dirn that does not coincide with the dirn of the force acting on the particle.

Position Vector, Velocity and Acceleration

Position Vector:- To define the position of a particle moving along a curved path at any instant we need to know its coordinates along the x-axis as well as y-axis.



position vector at time $t = \vec{r}$

position vector at time $t + \Delta t = \vec{r}'$

The vector $\vec{\Delta r}$ joining P and P' represents the change in the position vector $\vec{r}$ during the time interval Δt.

$$\vec{r} + \vec{\Delta r} = \vec{r}'$$

$$\vec{\Delta r} = \vec{r}' - \vec{r}$$

$\vec{\Delta r}$ represents a change in magnitude as well as change in the dirn of the position vector $\vec{r}$.

Velocity:- Instantaneous Velocity of a particle can be defined as

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$$

As the Δt and Δr become smaller, the points P and P' gets closer and the vector $\vec{v}$ obtained at the limit becomes tangent to the past at P.

As $\Delta t \to 0$ $\qquad \Delta r = \Delta s$

The magnitude of the velocity v (speed) is thus obtained as

$$v = \lim_{\Delta t \to 0} \frac{PP'}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Acceleration:-

Components of acceleration: Normal & Tangential.

$$a = a_t + a_n$$

Tangential Component:- (a_t):

$$a_t = \frac{dv}{dt}$$

Tangential acc^n component of acc^n is equal to the rate of change of the speed of the particle.

Tangential acc^n (a_t) is considered to be positive in the dir^n of the tangent coinciding with the sense of the motion.

Normal Acceleration (a_n):-

$$a_n = \frac{v^2}{\rho}$$

Normal acc^n of a particle is equal to the square of the speed divided by the radius of curvature of the path at that pt.

The direction of the normal acc^n is such that it is always directed towards the centre of curvature of the path.

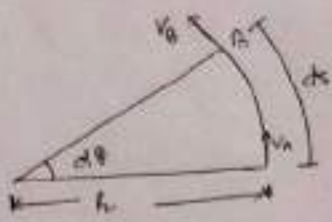
The normal acc^n is also called the centripetal acc^n.

$a = a_t + a_n$ (vector sum)

$a_t = \frac{dv}{dt}$ $a_n = \frac{v^2}{\rho}$

$a = \sqrt{a_t^2 + a_n^2}$ $\theta = \tan^{-1}\left(\frac{a_n}{a_t}\right)$

D'Alembert's Principle in Curvilinear Motion



v_A = tangential velocity at A v_B = tangential velo. at B.

$v_A = v_B = v$

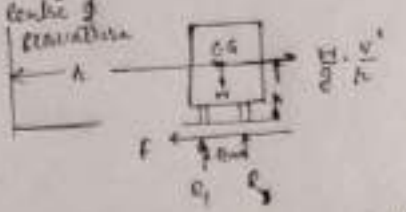
Now $dv = v\,d\theta = v\frac{ds}{r} = \frac{v}{r}ds$

acceleration $= \frac{dv}{dt} = \boxed{\frac{v^2}{r}}$

So when a body moves with uniform velocity along a curved path of radius r, it has a radial inward acceleration of magnitude $\frac{v^2}{r}$

Applying D'Alembert's principle to set equilibrium condition an inertia force of magnitude $\frac{W}{g}\cdot a = \frac{W}{g}\cdot\frac{v^2}{r}$ must be applied in outward dir^n. it is known as Centrifugal force.

Motion on a level road:-



Consider a body is moving with uniform velocity on curvilinear path of radius r. let the road is flat.

Let W = wt. of the body

and inertia force is given by $= \frac{W}{g}\cdot a = \frac{W}{g}\cdot\frac{v^2}{r}$

Condition for skidding:-

Let W = wt. of vehical

R_1, R_2 = reation at wheels

F = frictional force

$\frac{W}{g}\cdot\frac{v^2}{R}$ = inertial force

skidding takes place when the frictional forces reaches limiting value i.e $F = \mu W$

Then permissible speed to avoid skidding

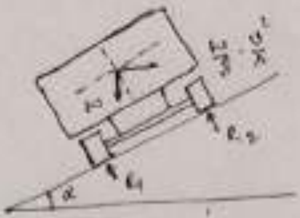
$$v = \sqrt{g\frac{r}{2}\cdot\frac{B}{h}}$$

The distance between inner and outer wheel is equal to the gauge of railway track and expressed as G.

so. $$v = \sqrt{g\frac{R}{2}\cdot\frac{G}{h}}$$

Designed speed and angle of banking.

sum of all forces in the inclined plane

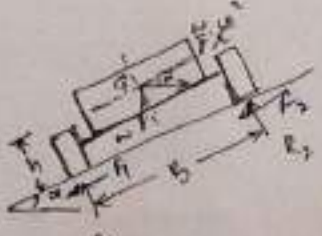


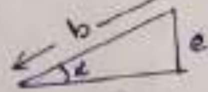
$$\frac{W}{g}\cdot\frac{v^2}{r}\cos\alpha - W\sin\alpha = 0$$

$$\Rightarrow \tan\alpha = \frac{v^2}{gr}$$

Relation between the angle of banking and designed speed is $\boxed{\tan\alpha = \frac{v^2}{gr}}$.

Condition for skidding and overturning:-





(a) Condition for skidding

$$v = \sqrt{\tan(\alpha+\phi)\times gr}$$

where α = angle of inclination

$\tan\phi = \mu$

g = gravitational acceleration.

r = radius of curve.

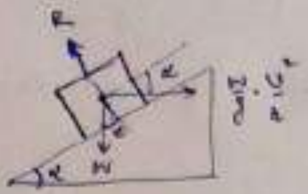
Vehical will skid if the velocity is more than this value.

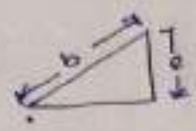
(b) Condition for overturning

$$\boxed{v = \sqrt{gr\cdot\frac{G+(2he/G)}{2h-e}}}$$

Example 1:- Find the proper super elevation 'e' for 7.2 m highway curve of radius r = 600 m in order that a car travelling with a speed of 80 km/h will have no tendency to skid sidewise.

Sol^n:-





$b = 7.2\ m$ $\quad r = 600\ m$ $\quad v = 80\ kmph = 22.23\ m/s$

Resolving along inclined plane

$$w \sin\alpha = \frac{W}{g}\cdot\frac{v^2}{r}\cdot\cos\alpha$$

$$\boxed{\tan\alpha = \frac{v^2}{rg}}$$

From the geometry $\sin\alpha = \frac{e}{b}$. Since α is very small

let $\sin\alpha \simeq \tan\alpha$

$$\therefore \frac{v^2}{rg} = \frac{e}{b} \Rightarrow e = \frac{bv^2}{rg} = \frac{7.2\times 22.23^2}{600\times 9.81}$$

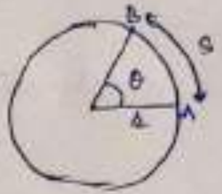
$$e = 0.604\ m \text{ (Ans)}$$

Rotation of Rigid Bodies

Angular motion:-

→ The rate of change of angular displacement with time is called angular velocity and denoted by ω.

$$\boxed{\omega = \frac{d\theta}{dt}} \quad -(1)$$



→ The rate of change of angular velocity with time is called angular acceleration and denoted by 'α'

$$\boxed{\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}} \quad -(2)$$

Angular acceleration may also be expressed as

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\cdot\frac{d\theta}{dt}$$

$$\boxed{\alpha = \omega\,\frac{d\omega}{d\theta}}$$

Relation between Angular and Linear motion

from fig. 1. $s = r\theta$

$$\boxed{v = \frac{ds}{dt} = \frac{d}{dt}(r\theta) = r \cdot \frac{d\theta}{dt}} \quad \text{--- (4)}$$

linear acceleration

$$\boxed{a_t = \frac{dv}{dt} = r\frac{d^2\theta}{dt^2}} \quad \text{--- (5)}$$

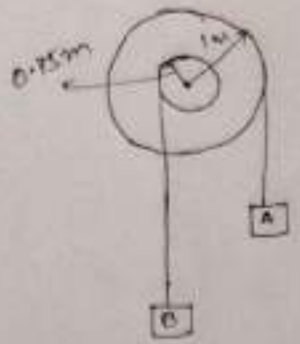
if $\frac{v^2}{r}$ = radial acceleration = a_n

Then
$$\boxed{a_n = \frac{v^2}{r} = r\omega^2} \quad \text{--- (6)}$$

uniform angular velocity (ω)

$$\boxed{\omega = \frac{2\pi N}{60} \text{ rad/sec}}$$

Example 2:- A step pulley starts from rest and accelerate at 2 rad/s^2. How much time is required for block A to move 20m. And also the velocity of A and B at that time.



Sol^n:- when A move 20 m, the angular displacement of pulley θ is given by

$$r\theta = s$$

$$1 \times \theta = 20 \quad \Rightarrow \theta = 20 \text{ rad.}$$

$$\alpha = 2 \text{ rad/s}^2 \quad \text{and } \omega_0 = 0$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$20 = 0 \times t + \frac{1}{2} \cdot 2 \times t^2 \quad \Rightarrow \boxed{t = 4.472 \text{ sec}}$$

Velocity of pulley at this time

$$\omega = \omega_0 + \alpha t$$
$$= 0 + 2 \times 4.472 = \boxed{8.944 \text{ rad/s.}}$$

Velocity of block A $v_A = 1 \times 8.944$ m/s
$$= \boxed{8.944 \text{ m/s}}$$

Velocity of block B $v_B = 0.75 \times 8.944$ m/s
$$= \boxed{6.708 \text{ m/s}}$$