

**LECTURE NOTES ON**  
**POWER PLANT ENGINEERING**

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**MODULE – II**

**FLOW THROUGH NOZZLES**

## **SYLLABUS**

### **FLOW THROUGH NOZZLES**

Types of nozzles and their area of application & related calculation, critical pressure & chocked flow, super saturated flow. Effect of friction and nozzle efficiency

## STEAM NOZZLES

### TECHNICAL TERMS:

- 1. Wet steam:** The steam which contains some water particles in superposition.
- 2. Dry steam / dry saturated steam:**  
When whole mass of steam is converted into steam then it is called as dry steam.
- 3. Super heated steam:** When the dry steam is further heated at constant pressure, the temperature increases the above saturation temperature. The steam so obtained is called super heated steam.
- 4. Degree of super heat:** The difference between the temperature of superheated steam and saturated temperature is called degree of superheat.
- 5. Nozzle:** It is a duct of varying cross sectional area in which the velocity increases with the corresponding drop in pressure.
- 6. Coefficient of nozzle:** It is the ratio of actual enthalpy drop to isentropic enthalpy drop.
- 7. Critical pressure ratio:** There is only one value of ratio ( $P_2/P_1$ ) which produces maximum discharge from the nozzle . then the ratio is called critical pressure ratio.
- 8. Degree of reaction:** It is defined as the ratio of isentropic heat drop in the moving blade to isentropic heat drop in the entire stages of the reaction turbine.
- 9. Compounding:** It is the method of absorbing the jet velocity in stages when the steam flows over moving blades. (i)Velocity compounding (ii)Pressure compounding and (iii) Velocity-pressure compounding
- 10. Convergent nozzle:** The crosssectional area of the duct decreases from inlet to the outlet side then it is called as convergent nozzle.
- 11. Divergent nozzle:** The crosssectional area of the duct increases from inlet to the outlet then it is called as divergent nozzle.

### Flow of steam through nozzles:

The flow of steam through nozzles may be regarded as adiabatic expansion. - The steam has a very high velocity at the end of the expansion, and the enthalpy decreases as expansion takes place. - Friction exists between the steam and the sides of the nozzle; heat is produced as the result of the resistance to the flow. The phenomenon of super saturation occurs in the flow of steam through nozzles. This is due to the time lag in the condensation of the steam during the expansion.

### Continuity and steady flow energy equations

Through a certain section of the nozzle:

$$m.v = A.C$$

m is the mass flow rate,

v is the specific volume,

A is the cross-sectional area and

C is the velocity.

For steady flow of steam through a certain apparatus, principle of conservation of energy states:

$$h_1 + C_1^2 / 2 + gz_1 + q = h_2 + C_2^2 / 2 + gz_2 + w$$

For nozzles, changes in potential energies are negligible,  $w = 0$  and  $q \cong 0$ .

$$h_1 + C_1^2 / 2 = h_2 + C_2^2 / 2$$

### Types of Nozzles:

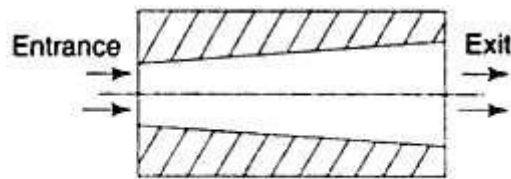
1. Convergent Nozzle
2. Divergent Nozzle
3. Convergent-Divergent Nozzle

### Convergent Nozzle:

A typical convergent nozzle is shown in fig. in a convergent nozzle, the cross sectional area decreases continuously from its entrance to exit. It is used in a case where the back pressure is equal to or greater than the critical pressure ratio.

### Divergent Nozzle:

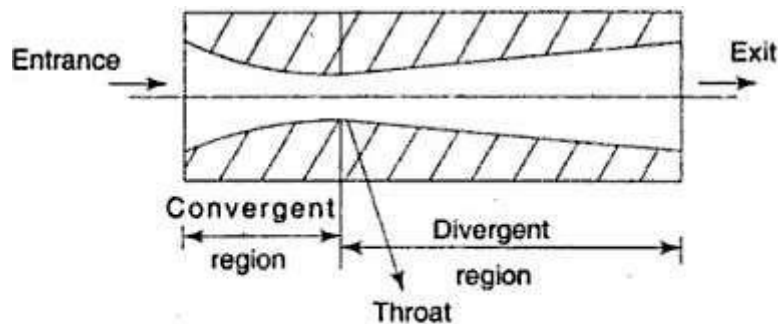
The cross sectional area of divergent nozzle increases continuously from its entrance to exit. It is used in a case, where the back pressure is less than the critical pressure ratio.



Divergent Nozzle:

### Convergent-Divergent Nozzle:

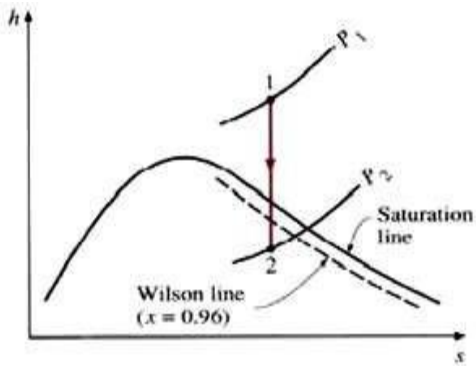
In this case, the cross sectional area first decreases from its entrance to throat, and then increases from throat to exit. It is widely used in many type of steam turbines.



Convergent-Divergent Nozzle

**Supersaturated flow or Meta stable flow in Nozzles:** As steam expands in the nozzle, its pressure and temperature drop, and it is expected that the steam start condensing when it strikes the saturation line. But this is not always the case. Owing to the high velocities, the residence time of the steam in the nozzle is small, and there may not sufficient time for the necessary heat transfer and the formation of liquid droplets. Consequently, the condensation of steam is delayed for a little while. This phenomenon is known as super saturation, and the steam that exists in the wet region without containing any liquid is known as supersaturated steam.

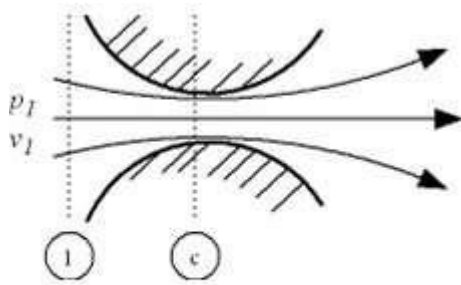
The locus of points where condensation will take place regardless of the initial temperature and pressure at the nozzle entrance is called the Wilson line. The Wilson line lies between 4 and 5 percent moisture curves in the saturation region on the h-s diagram for steam, and is often approximated by the 4 percent moisture line. The super saturation phenomenon is shown on the h-s chart:



The  $h$ - $s$  diagram for the isentropic expansion of steam in a nozzle.

**Critical Pressure Ratio:** The critical pressure ratio is the pressure ratio which will accelerate the flow to a velocity equal to the local velocity of sound in the fluid.

**Critical flow nozzles** are also called **sonic chokes**. By establishing a shock wave the sonic choke establish a fixed flow rate unaffected by the differential pressure, any fluctuations or changes in downstream pressure. A sonic choke may provide a simple way to regulate a gas flow.



Critical flow nozzles

The ratio between the critical pressure and the initial pressure for a nozzle can expressed as

$$P_c / P_1 = [2 / (n + 1)]^{n / (n - 1)}$$

Where,  $P_c$  = critical pressure and  $P_1$  = inlet pressure

$n$  = index of isentropic expansion or compression or polytropic constant

For a perfect gas undergoing an adiabatic process the index ‘ $n$ ’ is the ratio of specific heats  $k = c_p / c_v$ . There is no unique value for  $n$ . Values for some common gases are

- Steam where most of the process occurs in the wet region:  $n = 1.135$ ,  $P_c = 0.577 P_1$
- Steam super-heated:  $n = 1.30$ ,  $P_c = 0.546 P_1$
- Air:  $n = 1.4$
- Methane:  $n = 1.31$

- Helium:  $n = 1.667$

### Effect of Friction on Nozzles:

- 1) Entropy is increased.
- 2) Available energy is decreased.
- 3) Velocity of flow at throat is decreased.
- 4) Volume of flowing steam is decreased.
- 5) Throat area necessary to discharge a given mass of steam is increased.

Most of the friction occurs in the diverging part of a convergent-divergent nozzle as the length of the converging part is very small. The effect of friction is to reduce the available enthalpy drop by about 10 to 15%. The velocity of steam will be then

$$V_2 = 44.72\sqrt{K(H_1 - H_2)}$$

Where,  $k$  is the co-efficient which allows for friction loss. It is also known as nozzle efficiency.

### Velocity of Steam at Nozzle Exit:

$$V_2^2 = 2000(H_1 - H_2) + V_1^2 \quad \therefore \quad V_2 = \sqrt{2000(H_1 - H_2) + V_1^2}$$

As the velocity of steam entering the nozzle is very small,  $V_1$  can be neglected.

$$\therefore \quad V_2 = \sqrt{2000(H_1 - H_2)} = 44.72\sqrt{(H_1 - H_2)} \text{ m/s}$$

If frictional losses are taken into account then

$$V_2 = 44.72\sqrt{(H_1 - H_2)\eta_n} \text{ m/s}$$

### Mass of steam discharged through nozzle:

$$m = A \sqrt{2000 \frac{n}{n-1} \times \frac{P_1}{v_1} \left[ \left( \frac{P_2}{P_1} \right)^{\frac{2}{n}} - \left( \frac{P_2}{P_1} \right)^{\frac{n+1}{n}} \right]}$$

**Condition for maximum discharge through nozzle:** The nozzle is always designed for maximum discharge

$$\frac{m}{A} = \sqrt{2000 \frac{n}{n-1} \times \frac{P_1}{v_1} \left[ \left( \frac{P_2}{P_1} \right)^{\frac{2}{n}} - \left( \frac{P_2}{P_1} \right)^{\frac{n+1}{n}} \right]}$$

The mass flow per unit area will be maximum at the throat because the throat area is minimum.

It is seen from the above equation that the discharge through a nozzle is a function of  $\frac{P_2}{P_1}$  only, as the expansion index is fixed according to the steam supplied to the nozzle.

Therefore,  $\frac{m}{A}$  is maximum when

$$\left[ \left( \frac{P_2}{P_1} \right)^{\frac{2}{n}} - \left( \frac{P_2}{P_1} \right)^{\frac{n+1}{n}} \right] \text{ is minimum}$$

This means  $m/A = f(P_2/P_1)$ . To get its maximum value we have to differentiate  $m/A$  w.r.t.  $P_2/P_1$  and equate it to zero.

So after manipulation the ratio between the critical pressure and the initial pressure for a nozzle can be expressed as

$$P_2 / P_1 = [2 / (n + 1)]^{n / (n-1)}$$

Putting this value in the mass flow rate equation, we get

$$m = A \sqrt{2000 \frac{n}{n-1} \times \frac{P_1}{v_1} \left[ \left( \frac{P_2}{P_1} \right)^{\frac{2}{n}} - \left( \frac{P_2}{P_1} \right)^{\frac{n+1}{n}} \right]}$$

we know  $\frac{P_2}{P_1} = \left( \frac{2}{n+1} \right)^{\frac{n}{n-1}}$

Putting the value of  $\frac{P_2}{P_1}$  in the above equation

$$m_{\max} = A \sqrt{2000 \frac{n}{n-1} \times \frac{P_1}{v_1} \left[ \left( \frac{2}{n+1} \right)^{\frac{2}{n}} - \left( \frac{2}{n+1} \right)^{\frac{n+1}{n}} \right]}$$

$$m_{\max} = A \sqrt{2000 \frac{n}{n-1} \times \frac{P_1}{v_1} \times \left( \frac{2}{n+1} \right)^{\frac{n+1}{n}} \left[ \left( \frac{2}{n+1} \right)^{\frac{2}{n} - \frac{n+1}{n}} - 1 \right]}$$

$$= A \sqrt{2000 \frac{n}{n-1} \times \frac{P_1}{v_1} \times \left( \frac{2}{n+1} \right)^{\frac{n+1}{n}} \left[ \left( \frac{2}{n+1} \right)^{\frac{1-n}{n}} - 1 \right]}$$

$$= A \sqrt{2000 \frac{n}{n-1} \times \frac{P_1}{v_1} \times \left( \frac{2}{n+1} \right)^{\frac{n+1}{n}} \left[ \left( \frac{2}{n+1} \right)^{-1} - 1 \right]}$$

$$= A \sqrt{2000 \frac{n}{n-1} \times \frac{P_1}{v_1} \times \left( \frac{2}{n+1} \right)^{\frac{n+1}{n}} \left[ \frac{n+1}{2} - 1 \right]}$$

$$= A \sqrt{2000 \frac{n}{n-1} \times \frac{P_1}{v_1} \times \left( \frac{2}{n+1} \right)^{\frac{n+1}{n}} \left( \frac{n-1}{2} \right)}$$

$$m_{\max} = A \sqrt{1000n \times \frac{P_1}{v_1} \times \left( \frac{2}{n+1} \right)^{\frac{n+1}{n}}}$$

Where  $P_1$  is the initial pressure of the steam in kpa and  $v_1$  is the specific volume of the steam in  $m^3/kg$  at the initial pressure.

### Problem Solving:

1. Steam at 10.5 bar and 0.95 dryness is expanded through a convergent divergent nozzle. The pressure of steam leaving the nozzle is 0.85 bar. Find i) velocity of steam at throat for maximum discharge, ii) the area at exit iii) steam discharge if the throat area is  $1.2\text{cm}^2$ . assume the flow is isentropic and there are no friction losses. Take  $n = 1.135$ .

#### Given data:

$$P_1 = 10.5 \text{ bar}$$

$$P_2 = 0.85 \text{ bar}$$

$$\text{Area at throat } A_t = 1.2 \text{ cm}^2$$

$$x_1 = 0.95$$

$$n = 1.135$$

solution:

we know that, for  $n = 1.135$

$$\text{Throat pressure } P_t = 0.577 \times P_1 = 0.577 \times 10.5 = 6.06 \text{ bar}$$

Properties of steam from steam tables:

$$\begin{aligned} \text{At } P_1 = 10.5 \text{ bar} \quad h_f &= 772 \text{ KJ/kg} \\ s_f &= 2.159 \text{ KJ/kg} \\ h_{fg} &= 2006 \text{ KJ/kg} \\ s_{fg} &= 4.407 \text{ KJ/kg} \end{aligned}$$

$$\begin{aligned} \text{At } P_t = 6.09 \text{ bar} \quad h_f &= 673.25 \text{ KJ/kg} \\ s_f &= 1.9375 \text{ KJ/kg} \\ h_{fg} &= 2082.95 \text{ KJ/kg} \\ s_{fg} &= 4.815 \text{ KJ/kg} \\ v_f &= 0.01101 \text{ m}^3/\text{kg} \\ v_g &= 0.31556 \text{ m}^3/\text{kg} \end{aligned}$$

$$\begin{aligned} P_2 = 0.85 \text{ bar} \quad h_f &= 398.6 \text{ kJ/kg} \\ h_{fg} &= 2269.8 \text{ kJ/kg} \\ s_f &= 1.252 \text{ kJ/kgk} \\ s_{fg} &= 6.163 \text{ kJ/kgk} \\ v_f &= 0.001040 \text{ m}^3/\text{kg} \\ v_g &= 1.9721 \text{ m}^3/\text{kg} \end{aligned}$$

$$\begin{aligned} \text{So,} \quad s_1 &= s_{f1} + x_1 \times s_{fg} \\ &= 2.159 + 0.95 \times 4.407 = 6.34565 \text{ kJ/kgk} \\ h_1 &= h_{f1} + x_1 \times h_{fg1} \\ &= 772 + 0.95 \times 2006 = 6.34565 \text{ KJ/Kgk} \end{aligned}$$

1-t isentropic expansion between inlet and throat

$$\begin{aligned} \text{So} \quad s_1 &= s_t = 6.34564 \text{ kJ/kg} \\ s_t &= s_{ft} + x_t \times s_{fgt} \\ 6.34565 &= 1.9375 + x_t \times 4.815 \\ x_t &= 0.915 \\ h_t &= h_{ft} + x_t \times h_{fgt} \\ &= 673.25 + 0.915 \times 2082.95 \\ &= 2579.15 \text{ kJ/kg} \end{aligned}$$

#### Velocity of steam at throat:

$$\begin{aligned} V_t &= [2000 \times (h_1 - h_t)]^{1/2} \\ &= 443.96 \text{ m/s} \end{aligned}$$

$$\begin{aligned} V_t &= x_t \times v_{gt} \\ &= 0.915 \times 0.31156 = 0.2887 \text{ m}^3/\text{kg} \end{aligned}$$



**Mass of steam discharged:**

$$m = A_t \times V_t / v_t = 1.2 \times 10^{-4} \times 443.96 / 0.28874 \\ = 0.1845 \text{ kg/s}$$

t-2 isentropic expansion between throat and exit

$$s_t = s_2 = 6.34565 \text{ kJ/kgk}$$

$$6.34565 = 1.252 + x_2 \times 6.162$$

$$x_2 = 0.83$$

$$v_2 = 0.83 \times 1.9721 = 1.637 \text{ m}^3/\text{kg}$$

$$h_2 = 398.6 + 0.83 \times 2269.8$$

$$= 2282.534 \text{ kJ/kg}$$

**Velocity of steam at exit**

$$V_2 = \sqrt{2000(h_1 - h_2)}$$

$$= \sqrt{2000(2677.7 - 2282.534)}$$

$$= 889 \text{ m/sec}$$

According to mass balance, steam flow rate of throat is equal to flow rate at exit

$$m_t = m_2 = A_2 \times V_2 / v_2 = A_2 \times 889 / 1.637$$

$$\Rightarrow 0.1845 \times 1.637 / 889 = A_2 = 3.397 \times 10^{-4} \text{ m}^2$$

**Q.2. Dry saturated steam at 2.8 bar is expanded through a convergent nozzle to 1.7 bar. The exit area is 3 cm<sup>2</sup>. Calculate the exit velocity and mass flow rate for, i) isentropic expansion ii) supersaturated flow.**

Given Data :

$$P_1 = 2.8 \text{ bar}, P_2 = 1.7 \text{ bar}, A_2 = 3 \text{ cm}^2$$

Solution :

Properties of steam table at 2.8 bar

$$h_1 = 2721.5 \text{ KJ/kg}$$

$$s_1 = 7.014 \text{ KJ/kgK}$$

$$v_1 = 0.64600 \text{ m}^3/\text{kg}$$

$$P_2 = 1.7 \text{ bar}$$

$$h_f = 483.2 \text{ KJ/kg}$$

$$h_{fg} = 2215.6 \text{ KJ/kg}, s_f = 1.475 \text{ KJ/kgK}$$

$$s_{fg} = 5.706 \text{ KJ/kgK}$$

$$v_f = 0.001056 \text{ m}^3/\text{kg}$$

$$v_g = 1.0309 \text{ m}^3/\text{kg}$$

For isentropic flow

$$s_1 = s_2 = 7.014 \text{ J/kgK}$$

$$s_2 = s_{f2} + x_2 \times s_{fg2}$$

$$7.014 = 1.475 + x_2 \times 5.706$$

$$x_2 = 0.97$$

$$h_2 = h_{f2} + x_2 \times h_{fg2}$$

$$= 483.2 + 0.97 \times 2215.6$$

$$h_2 = 2634.152 \text{ KJ/kg}$$

$$v_2 = x_2 \times v_{g2}$$

$$= 0.97 \times 1.0309 = 1.00 \text{ m}^3/\text{kg}$$

**Velocity of steam at exit**

$$V_2 = \sqrt{2000(h_1 - h_2)}$$

$$= \sqrt{200(2721.5 - 2631.15)}$$

$$V_2 = 418 \text{ m/sec}$$

**Mass flow rate at exit**

$$m_2 = \frac{A_2 \times v_2}{v_2}$$

$$= \frac{3 \times 10^{-4} \times 418}{1.00}$$

$$= 0.1257 \text{ m}^3/\text{kg}$$

**For super saturated flow**

$$V_2 = \sqrt{\frac{2n}{n-1}} \times p_1 \times v_1 \left[ 1 - \left( 1 - \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]$$

$$V_2 = \sqrt{\frac{2 \times 1.3}{1.3-1}} \times 2.8 \times 10^5 \times 0.6460 \left[ 1 - \left( 1 - \frac{1.7}{2.8} \right)^{\frac{1.3-1}{1.3}} \right]$$

$$V_2 = 413 \text{ m/sec}$$

**Mass flow rate at exit**

$$m_2 = \frac{A_2 \times v_2}{v_2} = \frac{3 \times 10^{-4} \times 413}{0.94827}$$

$$= 0.1306 \text{ kg/sec.}$$

**Q.3. Dry saturated steam at a pressure of 8 bar enters a C-D nozzle and leaves it a pressure of 1.5 bar. If the steam flow process is isentropic and if the corresponding expanding index is 1.135, Find the ratio of cross sectional area at exit and throat for maximum discharge.**

**Given Data:**  $P_1 = 2.8 \text{ bar}$   $P_2 = 1.7 \text{ bar}$   $n = 1.135$

**Solution:**

We know that  $n = 1.135$

**Throat pressure =  $P_t = 0.577 \times P_1 = 0.577 \times 8 = 4.62 \text{ bar}$**

Properties of steam at steam table

At 8 bar:

$$\begin{aligned} h_1 &= 2769.1 \text{ KJ/kg} \\ s_1 &= 6.6628 \text{ KJ/kgK} \\ v_1 &= 0.2404 \text{ m}^3/\text{kg} \end{aligned}$$

**At 4.62 bar**

$$\begin{aligned} h_f &= 626.7 \text{ KJ/kg} \\ h_{fg} &= 2117.2 \text{ KJ/kg} \\ s_f &= 1.829 \text{ KJ/kgK} \\ s_{fg} &= 5.018 \text{ KJ/kgK} \\ v_f &= 0.001090 \text{ m}^3/\text{kg} \\ v_g &= 0.40526 \text{ m}^3/\text{kg} \end{aligned}$$

**At 1.5 bar**

$$\begin{aligned} h_f &= 467.11 \text{ KJ/kg} \\ h_{fg} &= 2226.5 \text{ KJ/kg} \\ s_f &= 1.4336 \text{ KJ/kgK} \\ s_{fg} &= 5.7897 \text{ KJ/kgK} \\ v_f &= 0.001053 \text{ m}^3/\text{kg} \\ &= 626.7 + 0.963 \times 2117.2 \\ h_t &= 2666.18 \text{ KJ/kg} \\ v_t &= x_t \times v_{gt} \\ &= 0.963 \times 0.40526 = 0.39 \text{ m}^3/\text{kg} \end{aligned}$$

**Velocity of steam at throat**

$$\begin{aligned} V_t &= \sqrt{2000(h_1 - h_t)} \\ &= \sqrt{2000(2769.1 - 2666.18)} \\ &= 477.749 \text{ m/sec} \end{aligned}$$

t-2 isentropic expansion

$$\begin{aligned} s_t &= s_2 = 6.6628 \text{ KJ/kgK} \\ s_2 &= s_{f2} + x_2 \times s_{fg2} \\ 6.6628 &= 1.4336 + x_2 \times 5.7897 \\ x_2 &= 0.903 \\ v_2 &= x_2 \times v_{g2} \\ &= 0.903 \times 1.1593 = 1.04695 \text{ m}^3/\text{kg} \\ h_2 &= h_{f2} + x_2 \times h_{fg2} \\ &= 467.11 + 0.903 \times 2226.5 \\ h_2 &= 2477.6395 \text{ KJ/kg} \end{aligned}$$

**Velocity of steam at exit**

$$V_2 = \sqrt{2000(h_1 - h_2)}$$

$$=\sqrt{200(2769.1-2477.639)}$$
$$=763.5 \text{ m/sec}$$

**According to mass balance**

Mass flow rate of steam at throat = Mass flow rate of steam at exit

$$m_t = m_2$$

$$\frac{A_t \times v_t}{v_t} = \frac{A_2 \times v_2}{v_2}$$

$$\frac{A_2}{A_t} = \frac{1.04695 \times 477.749}{763.5 \times 0.39} = 1.68$$

**Q.4. Steam enters a group of CD nozzles at 21 bars and 270°C. The discharge pressure of the nozzle is 0.07 bars. The expansion is equilibrium throughout and the loss of friction in convergent portion of the nozzle is negligible, but the loss by friction in the divergent section of the nozzle is equivalent to 10% of the enthalpy drop available in that section. Calculate the throat and exit area to discharge 14 kg/sec of steam.**

$$P_1 = 21 \text{ bar}$$

$$T_1 = 270^\circ\text{C}$$

$$P_2 = 0.07 \text{ bar}$$

$$m = 14 \text{ kg / s.}$$

since loss by friction is 10%.

The efficiency  $\eta = 90\%$ .

**Solution :**

**Properties of steam (from Mollier Diagram)**

$$h_1 = 2980 \frac{\text{KJ}}{\text{kg}} \text{ (at 21 bar and } 270^\circ\text{C)}$$

Since the expansion is isentropic from

$$h_1 = \frac{2980 \text{ kJ}}{\text{kg}} \text{ draw a vertical line in the mollier diagram up to 0.07 bar pressure line}$$

now note the following values at that point.

$$h_2 = 2052.213 \text{ kJ/kg (at 0.07 bar)}$$

$$v_2 = 16.1 \text{ m}^3/\text{kg}$$

The critical pressure ratio when steam is initially super heated,

$$p_t / p_1 = 0.546$$

**Throat pressure**

$$p_t = 0.546 * p_1 = 0.546 * 21 = 11.466 \text{ bar}$$

**Properties of steam at throat**

$$v_t = \sqrt{2000(h_1 - h_t)} = \sqrt{2000(2980 - 2805)} = 591.6 \text{ m/s}$$

Velocity of steam at exit:

$$v_2 = \sqrt{2000(h_t - h_2)} * \eta = \sqrt{2000(2980 - 2052.213)} * 0.9 = \frac{1292.29 \text{ m}}{\text{s}}$$

**Throat area of nozzle**

$$A_2 = m * v_2 / V_2 = (14 * 16.1) / 1292.29 = 0.174 \text{ m}^2 = 1744.4 \text{ cm}^2$$

**TEXT BOOKS:**

1. Power plant Engineering ; – By P.K. Nag (2nd edition) TMH
2. Power Plant Engineering by Arora and Domkundwar, Dhanpat Rai publications

**REFERENCE:**

1. Power Plant Engineering by Rajput
2. Power plant technology : By E.I. Wakil TMH