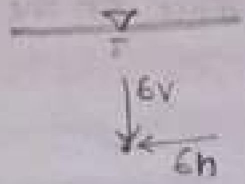
Lateral Earth Pressure and Retaining Structures

Introduction :-

* Incase of fluids, hydrostatic pressure act equally in all directions at any depth below the water surface. thus lateral pressure of the fluid is equal to the products of unit weight of the fluid pressure and depth.



$\sigma_h = \gamma_w \times h = \sigma_v$
equall in all direction at point

* Incase of ~~fluids~~ soils or other material such as grain, coal etc. which possess shearing strength, the lateral pressure is not equal to the vertical pressure but only related to it.

* magnitude of lateral pressure depend upon ~~the~~ a number of factors, such as the
  (i) mode of movement of the wall] wall
  (ii) flexibility of the wall
  (iii) properties of the soil] soil
  (iv) Drainage condition

* magnitude of lateral pressure is requires to design the retaining structure. i.e.
  (i) Retaining walls
  (ii) Sheet pile walls
  (iii) Crib walls
  (iv) Sheeting in excavations
  (v) basement walls

Structure which are used to hold back a soil mass are called retaining structures.

In soil ~~structure~~ structure interaction problem, as the earth pressure are depend upon the flexibility of walls. The earth pressure theories which consider soil-structure interaction are complicated and require a computer ~~for convenience~~.

* For convenience, the retaining wall is assumed to be rigid and the soil-structure interaction effect is neglected.

→ The lateral earth pressure is usually computed using the classical theories proposed by Coulomb and Rankine.
→ the general wedge theory proposed by Terzaghi

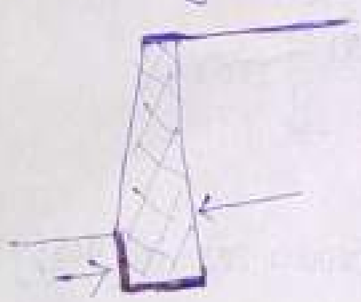
Earth pressure: →
The soil that is retained at a slope steeper that it can sustain by virtue of its shear strength exert a force on the retaining wall. This force is called the earth pressure.

Back fill: →
The material that is retained by the wall is referred to as backfill

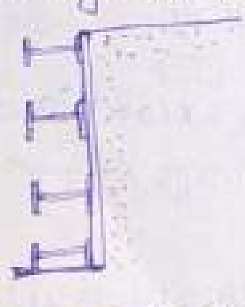
Some retaining structures with their mode of deformation

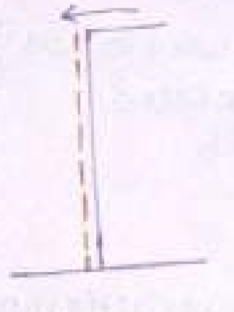
Structure name | failure mode

a. Retaining wall

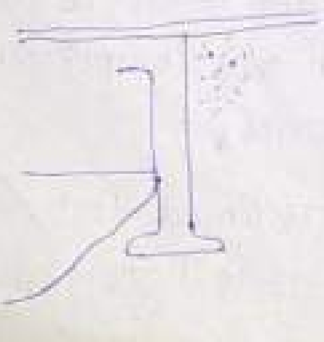


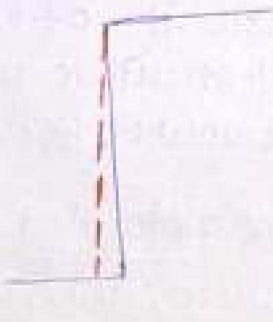
b. Bracing in cuts



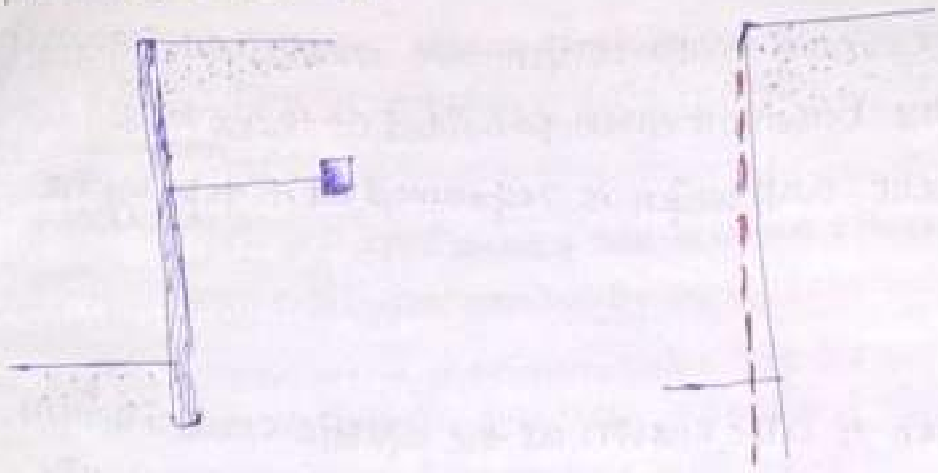


c. Abutment of a bridge

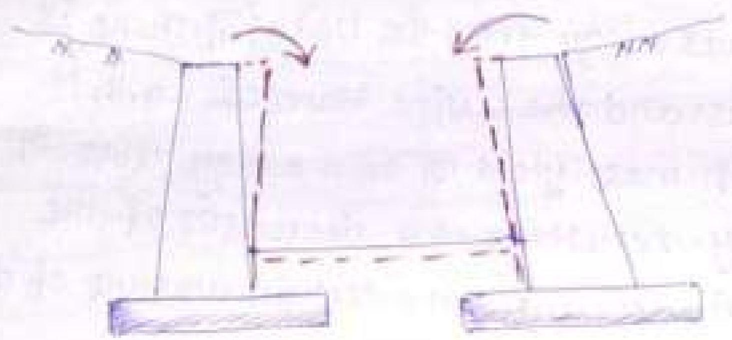




d. Anchored Sheet pile



e. twin retaining walls for a cut



Different types of Lateral earth pressure

Lateral earth pressure grouped into 3 catagories which is depending upon the movement of the retaining wall with respect to the soil retained. Retain soil by retaining structure is known as backfill.

(I) At rest pressure:→

The lateral earth pressure is called at-rest pressure when the ~~soil mass is not~~ wall is rigid and unyielding, the soil mass is ~~a~~ is not subjected to any lateral yielding or movement the soil mass is in a state of rest and there are no deformations and displacement.

~~Examples:→~~

This case occurs when the retaining wall is firmly fixed at its top and is not allowed to rotate or move laterally.

Example:-

(i) Basement retaining walls which are restained against the movement by the basement slab provided at their tops.

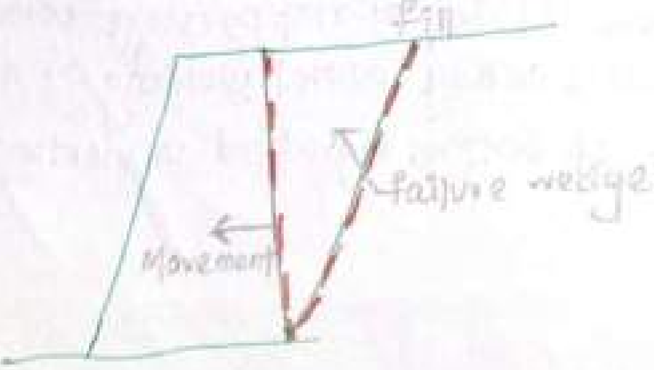
(ii) Bridge abutment wall which is restrained at its top by the bridge slab

At rest condition is also known as the elastic equilibrium, as no part of soil mass has failed and attained the plastic equilibrium.

(2) active pressure:→

* A retaining wall when moves away from the backfill, there is a stretching of the soil mass and the active state of earth is exist. It occurs when the soil mass yield in such a way that it tends to stretch horizontally, resulting in a decreases of the earth pressure, which continues, until at a certain amount of displacement.

* When the wall moves away from the backfill ~~located next a~~ ~~to the retaining wall~~ portion of the backfill located next to the retaining wall tends to break away from the rest of the soil mass and tends to move downwards and outwards relative to the wall. & failure will occur in the backfill and slip surfaces will be developed.

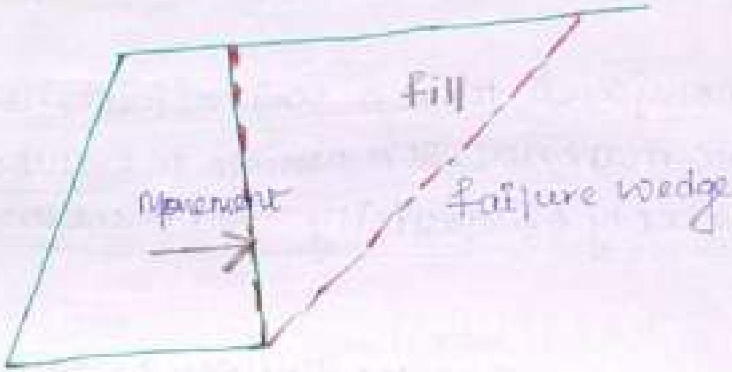


(3) Passive pressure :→

If the wall is pushed towards the back fill the soil is compressed and the soil offers resistance to this movement by virtue of its shearing resistance.

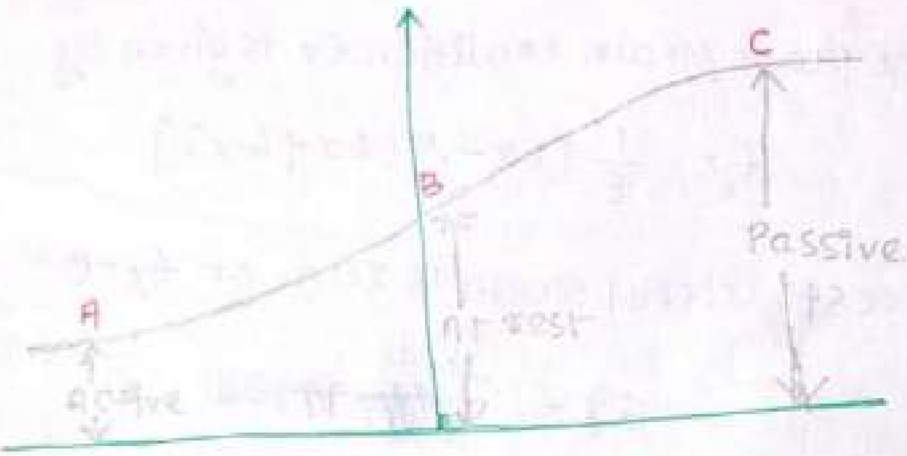
Since the shearing resistance builds up in directions towards the wall, the earth pressure gradually increases.

The pressure reaches a maximum value and the entire shearing resistance has been mobilised and does not increase any more with further wall movement. The pressure is called the passive earth pressure.



Variation of Pressure

~~Shows~~ The variation of earth pressure with the wall movement.



Earth Pressure at Rest

In natural state, an element of soil at a depth z below the ground surface is not subjected to any strain the element is in a condition known as the "at rest" condition. The corresponding lateral pressure called the earth pressure at rest, is expressed in the form

$$\sigma_h / P_0 = K_0 \sigma_v \quad \text{—— (1)}$$

K_0 is called the coefficient of earth pressure at rest.
σ_v / σ_z is the effective vertical stress at depth z.

If the soil mass is considered to be a semi-infinite, homogeneous, elastic and isotropic material, it is possible to evaluate the lateral pressure using the theory of elasticity. Since there are no displacement at all.

Let ϵ_x be the strain in the horizontal direction at depth z on an element of soil and let the poisson ratio and the elastic modulus be μ and E respectively.

For the plane strain condition, ϵ_x is given by

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \mu(\sigma_z + \sigma_x)\right]$$

At rest lateral strain is zero or $\epsilon_x = 0$

$$\frac{1}{E}\left[\sigma_x - \mu(\sigma_z + \sigma_x)\right] = 0$$

$$\sigma_x - \mu(\sigma_z + \sigma_x) = 0$$

$$\sigma_x - \mu\cdot\sigma_z - \mu\cdot\sigma_x = 0$$

$$\Rightarrow \sigma_x(1-\mu) - \mu\cdot\sigma_z = 0$$

$$\Rightarrow \mu\sigma_z = \sigma_x(1-\mu)$$

$$\Rightarrow \sigma_x = \frac{\mu}{1-\mu}\cdot\sigma_z$$

σ_x is commonly designated as the lateral earth pressure at rest P_o. Hence

$$\sigma_h / P_o = \frac{\mu}{1-\mu} \sigma_z \quad \text{—— (4)}$$

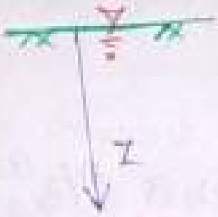
comparing 1 & 4. we have

$$K_o = \frac{\mu}{1-\mu}$$

$$\sigma_h / P_o = K_o \gamma Z.$$

where, γ - unit weight of the soil

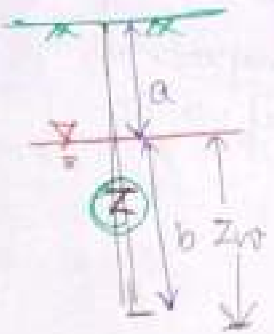
if the soil mass below the ground water table, the lateral pressure at depth Z is given by



$$P_o = K_o(\gamma_{sat} - \gamma_w) Z + \gamma_w Z$$

P_o or P_Z

OR



K.R.AROR

some what confusion

$$P_o = K_o(\gamma Z - \gamma_w Z_w) + \gamma_w Z_w$$

u can't take b.coz some part dry some part saturated

$$P_o = K_o(\gamma \times a + \gamma_{sat} b - \gamma_w b) + \gamma_w . b$$

$$= K_o(\gamma \times a + \gamma' b) + \gamma_w b$$

In above Lateral earth pressure at depth Z

Lateral earth pressure distribution diagram described by above eq^n is Triangular with its value being zero at the top and maximum at the base.

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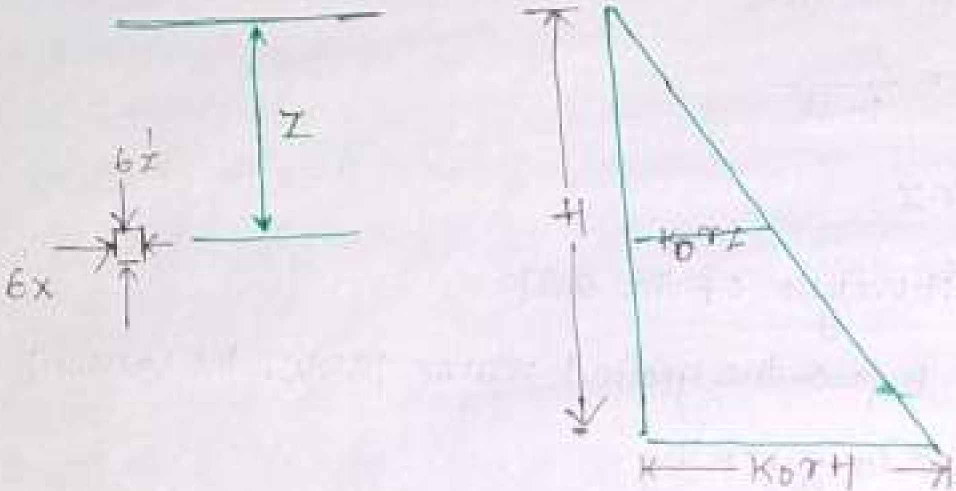
The total pressure P_0 per unit length of a retaining wall of height H can be determined by calculating the area of the lateral pressure distribution diagram



Thus $P_0 = \frac{1}{2} K_0 \gamma H \cdot H = \frac{1}{2} K_0 \gamma H^2$

P_0 the total pressure will act at C.G which is $H/3$ above the base.

K_0 can be evaluate from the poisson's ratio μ is known. However, the behaviour of soils is not realy according to elastic theor and soils do not have well defined poisson's ratio values. theoritical value may vary widely from the real value.

K_0 also find empirically

for Sands and normally consolidated clays, the value of K_0 can be related approximately

$K_0 = (1 - \sin\phi')$ (Jaky 1944)

Rankine's Theory of Earth Pressure

HOPSP

Rankine considered the equilibrium of a soil element within a soilmass bounded by a plane surface.

The assumptions for the derivation of earth pressure

(1) the soilmass is homogeneous and semi-infinite

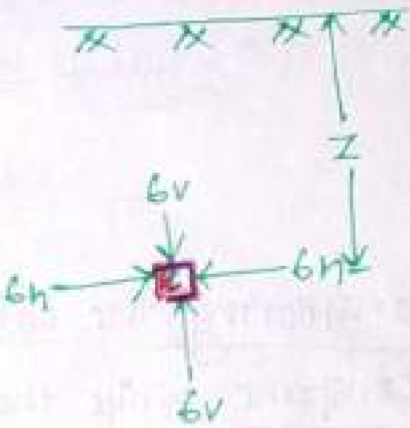
(2) the soil is dry and cohesionless

(3) the ground surface is plane which may be horizontal or inclined

(4) the back of the retaining wall is smooth and vertical.

(5) the soil element is in a state of plastic equilibrium. i.e. at the verge of failures.

* Active earthpressure



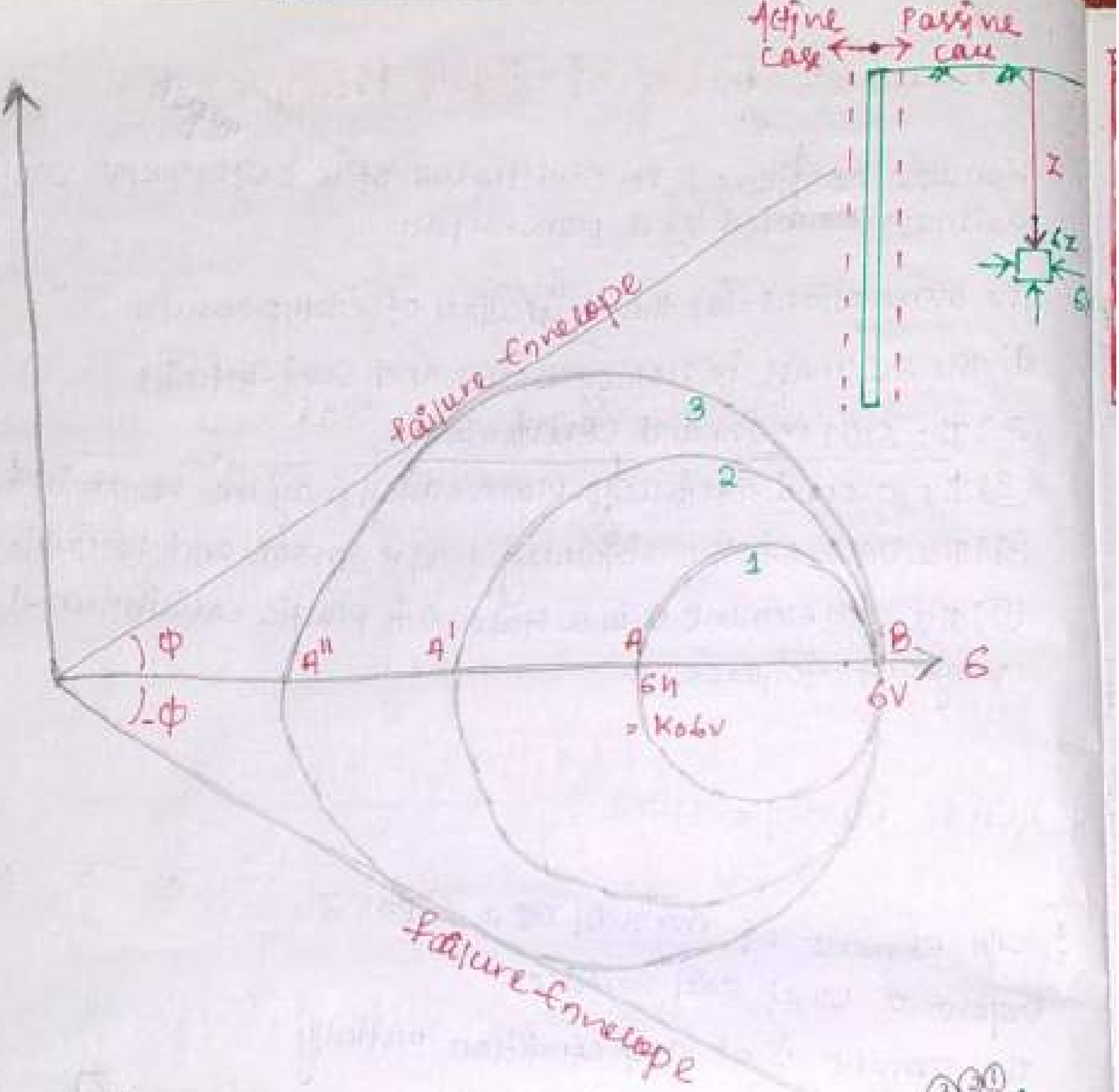
An element of dry soil at a depth Z below a level soil surface. the element is at rest condition initially and the horizontal pressure.

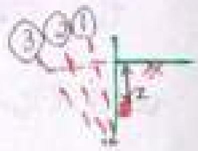
$$6h = K_0 6v$$ 1st condition

where $6v$ = vertical stress at C and
$6h$ = horizontal stress at C

Of course $6v = \gamma . Z$

The stresses (6h) and (6v) are, respectively, the minor and major principal stresses, and are indicated by points A and B in the mohs circle





Considering the case when the vertical stress constant while the horizontal stress is decreased

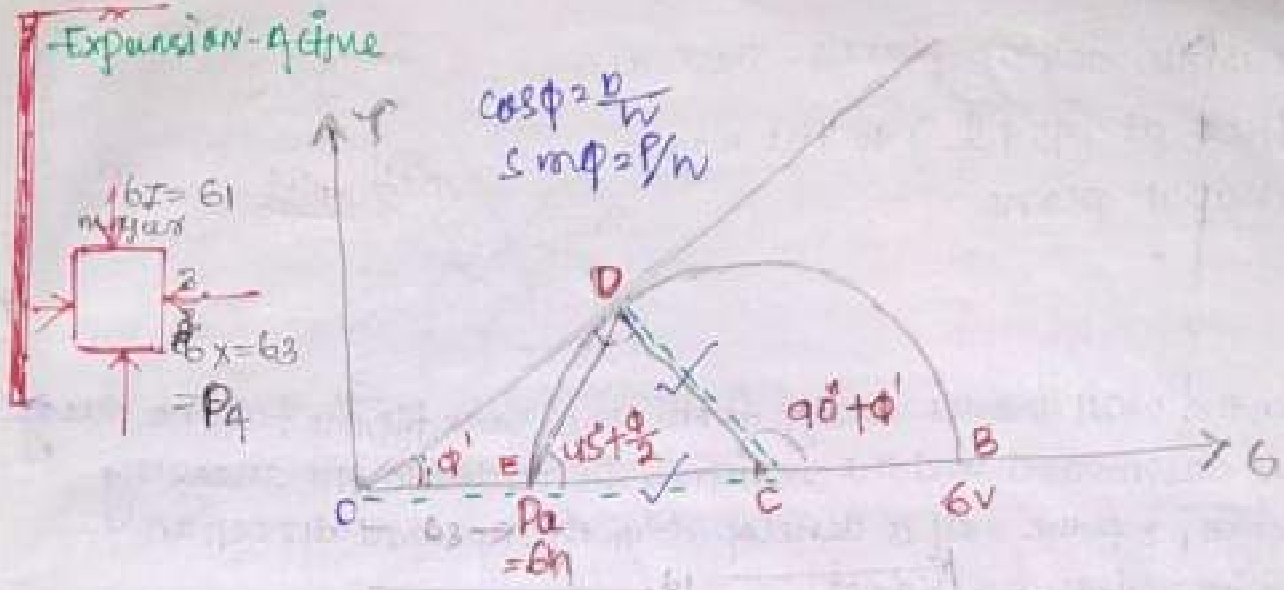
the point A shifts to position A' and the diameter of the mohr circle ~~decreases~~ increases.

In the limiting condition the point A shifts to position A'' when the mohr circle touches the failure envelope.

The soil is at the verge of shear failure. It has attained the rankine active state of plastic equilibrium.

The horizontal stress at that state is the active pressure (Pa

Expansion - Active



$Pa = OE = OC - CE$

'radius of o/e'

$CE = CD = OC\sin\phi'$

$Pa = OC - OC\sin\phi' = OC(1 - \sin\phi')$ —— (1)

radius of o/e $CE = CD = CB$

Also $\sigma_v = OB = OC + CB = OC + OC\sin\phi'$

$\sigma_v = OC(1 + \sin\phi')$ —— (2)

from eqn (1) and 2 $\quad \frac{Pa}{\sigma_v} = \frac{1 - \sin\phi'}{1 + \sin\phi'}$

$$Pa = \left(\frac{1 - \sin\phi'}{1 + \sin\phi'}\right)\sigma_v$$

$$Pa = Ka\,\sigma_v$$

where,

Ka is a coefficient of active earth pressure. It is a function of the angle of shearing resistance (ϕ') and is given by

$$Ka = \frac{1 - \sin\phi'}{1 + \sin\phi'} = \tan^2\left(45 + \frac{\phi'}{2}\right)$$

[Pressure distribution is same in case of at-rest and active condition only Ka is substituted for Ko.]

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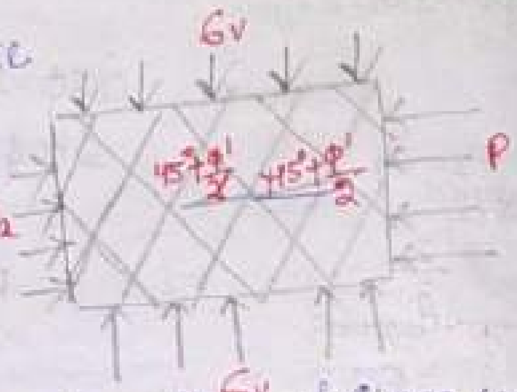
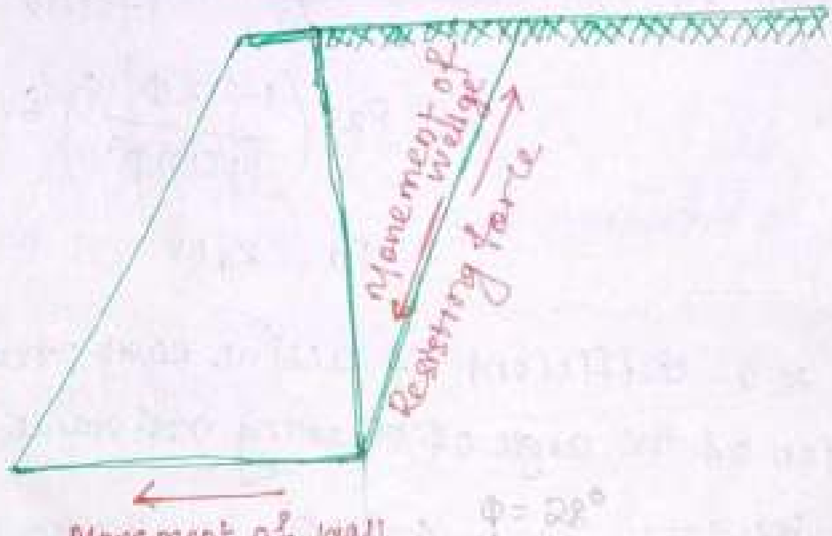
this is the failure planes. These are inclined at $(45° + \frac{\phi}{2})$ to the major principal plane



When the wall moves away from the back fill, the failure wedge moves downward and the resisting force due to the shearing strength of the soil is developed in the upward direction along the failure planes.

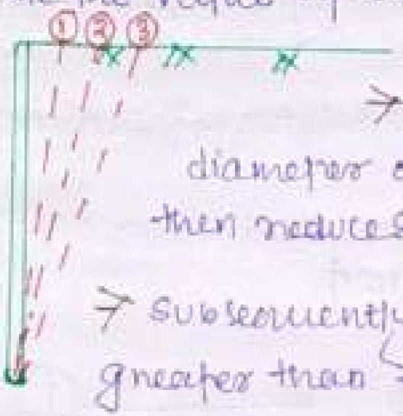
The resisting forces causes a decrease in the earth pressure acting on the wall. The decreases in earth pressure continues till the maximum resistance has been mobilised. The earth pressure does not decrease beyond this point and the active state is reached and the soil has attained plastic equilibrium



* Passive earth pressure: →

→ If the wall moves towards the backfill, there will be a uniform compression in the horizontal direction.

→ This leads to an increase in the value of σ_x from its original value, while the value of σ_z remain constant.

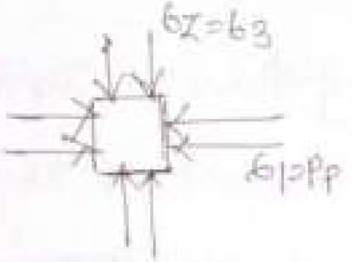


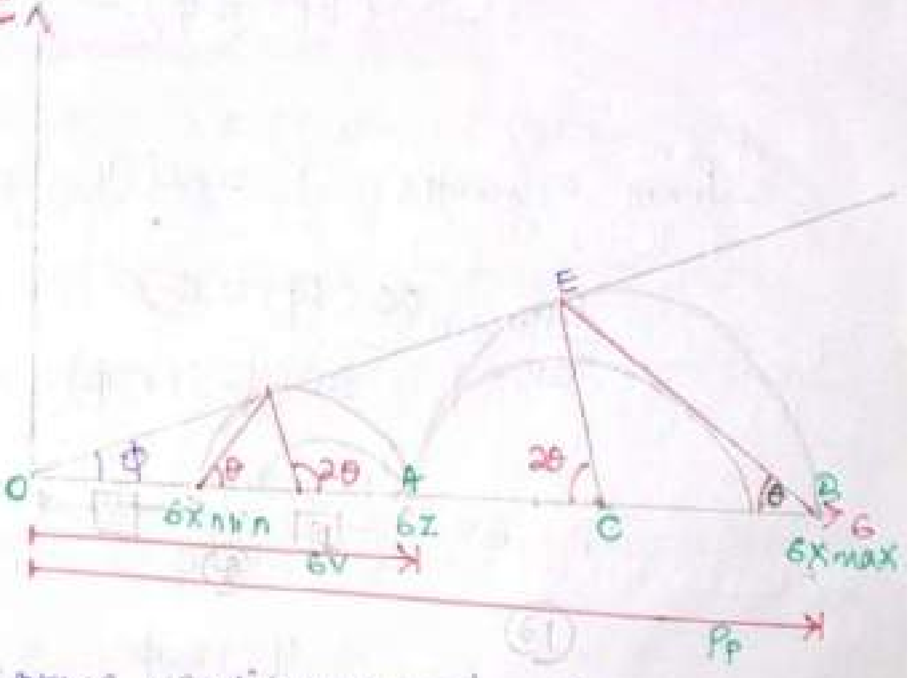
→ As the deformation increases, the diameter of the stress circle becomes smaller then reduces to zero ($\sigma_x = \sigma_z$).

→ Subsequently, the horizontal stress becomes greater than the vertical stress ($\sigma_x > \sigma_z$) and

→ the diameter of the stress circle increases untill a state of plastic equilibrium is reached.

Passive- Compression





For this state σ_x becomes maximum and will thus constitute the major principal stress σ_1, while σ_z will now be the minor principal stress σ_3.

The maximum value of σ_1 is reached when the mohr's circle, drawn with $\sigma_2 = \sigma_3$, touches the strength envelope. The soil is then said to be in the passive Rankine state and the corresponding lateral stress is called the passive earth pressure Pp. Denoting σ_1 by Pp and substituting γz for σ_3 in eqn.

$$OA = OC - AC$$
$$= OC - CE$$
$$= OC - OC\sin\phi$$

$$\sigma_v = \sigma_z = OC(1 - \sin\phi) \text{ ——— (i)}$$

$$\sigma_x = P_p = OC + OB$$
$$= OC + OC\sin\phi$$
$$= OC(1 + \sin\phi) \text{ ——— (ii)}$$

From equation (i) and (ii) we get

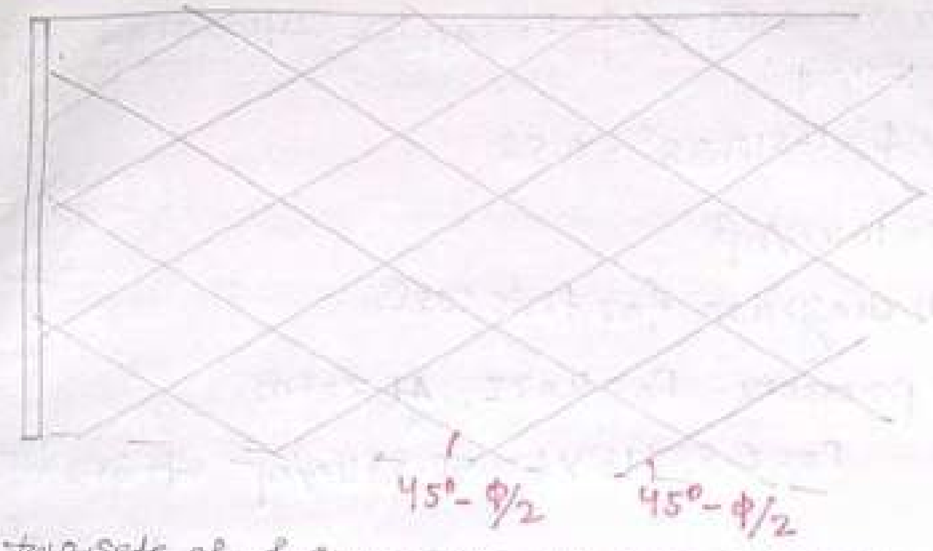
$$\frac{P_p}{\sigma_v} = \frac{OC(1+\sin\phi)}{OC(1-\sin\phi)}$$

$$P_p = \sigma_v \left(\frac{1+\sin\phi}{1-\sin\phi}\right)$$

$$= \gamma z \left(\frac{1+\sin\phi}{1-\sin\phi}\right)$$

$$P_p = \gamma z \times K_p$$

or, $\boxed{P_p = K_p \gamma z}$ where, $K_p = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2(45 + \phi/2)$



two sets of failure planes makes an angle $\theta = 45° - \phi/2$ to the horizontal.

Example:→ A retaining wall with a smooth vertical back retains sand back fill for a depth of 6m. the back fill has a horizontal surface and has the following properties

$$c' = 0,\ \phi' = 28°;\ \gamma = 16\,kN/m^3;\ \gamma_{sat} = 20\,kN/m^3$$

Calculate the magnitude of the total thrust against the wall for the conditions given below:

(a) Back fill fully drained but the top of the wall is restrained against yielding;

(b) Back fill fully drained and the wall is free to yield

(c) wall free to yield. water table at 3m depth and there is no drainage.

Determine the point of application of resultant thrust for case (c).

Solution:→ (A) If the wall is restrained against yielding, the lateral pressure that would develop against the wall would be earth pressure at rest" coefficient of earth pressure at rest K_0 can be calculated from the equation $K_0 = 1-\sin\phi'$

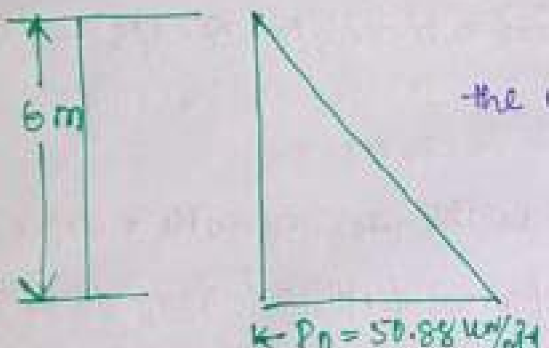
Hence $K_0 = 1-\sin\phi' = 1-\sin 28^\circ = 0.53$

for this case, $\gamma = 16\ kN/m^3$

pressure distribution diagram for this case.

lateral pressure $P_0 = K_0 \gamma z$. At $z=6m$,

$P_0 = 0.53 \times 16 \times 6 = 50.88\ kN/m^2$ at 6m depth



total thrust per meter length of the wall is

$P_0 = \frac{1}{2} \times 50.88 \times 6 = 152.64\ kN$

Ans

(B) for this case again, $\gamma = 16\ kN/m^3$, but the lateral pressure is the active earth pressure.

$\phi' = 28^\circ$; $K_A = \frac{1-\sin\phi}{1+\sin\phi} = 0.36$, $P_A = K_A \gamma . Z$

at 6m $P_A = 0.36 \times 16 \times 6 = 34.56\ kN/m^2$.

and total thrust is $P_A = \frac{1}{2} \times 34.56 \times 6 = 103.68\ kN/m$ length.

(C) the pressure distribution diagram, Above the water table $\gamma = 16\ kN/m^3$ and below table $\gamma = \gamma' = \gamma_{sat} - \gamma_w = 20 - 9.8$
$= 10.2\ kN/m^3$.

the later thrust due to water is shown

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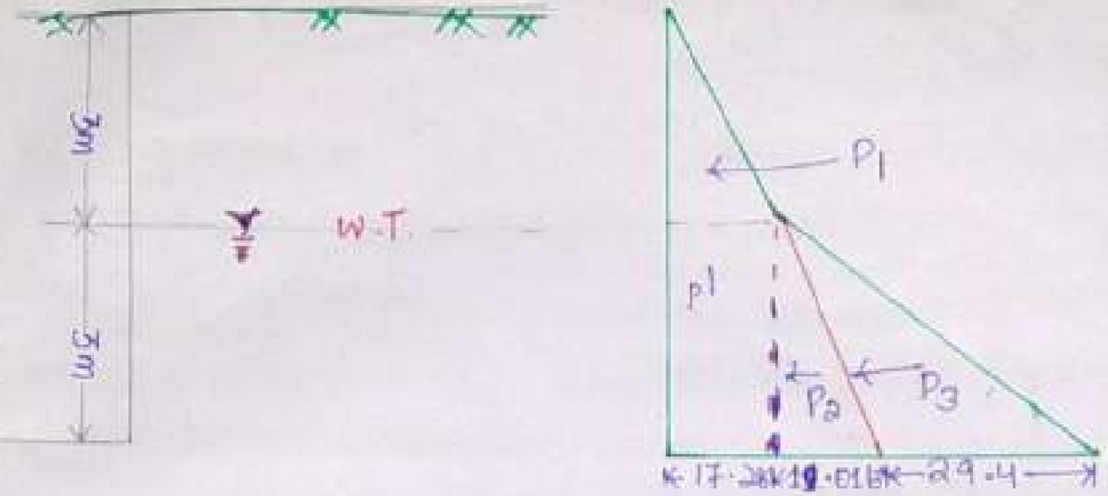


lateral thrust at point

$P_1 = K_a \gamma H_1 = 0.36 \times 16 \times 3 = 17.28\ kN/m^2$

$P_2 = K_a(\gamma H_1 + \gamma' H_2) = 0.36(48 + 30.6) = 28.296$

$P_3 = \gamma_w H_2 = 9.8 \times 3 = 29.4\ kN/m^2$

Total thrust from that point

$P_1 = \frac{1}{2} \times 17.28 \times 3 = 25.92$ kN acting at $(3 + \frac{1}{3} \times 3) = 4m$ from base

$P' = 17.28 \times 3 = 51.84$ kN acting at $\frac{3}{2} = 1.5m$ from base

$P_2 = \frac{1}{2} \times 11.016 \times 3 = 16.5$ kN acting at $\frac{1}{3} \times 3 = 1m$ from base

$P_3 = \frac{1}{2} \times 29.4 \times 3 = 44.1$ kN acting at $\frac{1}{3} \times 3 = 1m$ from base

It can be seen that the lateral thrust due to water contributes substantially to the total lateral thrust.

total thrust $P = P_1 + P' + P_2 + P_3$

$= 25.92 + 51.84 + 16.5 + 44.1$

$= 138.36$ kN/m length

the distance of the resultant P from the base of the wall can obtained by taking moments about the base

$138.36 \times \bar{H} = 25.92 \times 4 + 51.84 \times 1.5 + 16.5 \times 1 + 44.1 \times 1$

$\bar{H} = \frac{242}{138.36} = 1.75m.$

~~Active pressure on retaining walls:~~ Cohesionless back fill

(a) Dry backfill with no surcharge

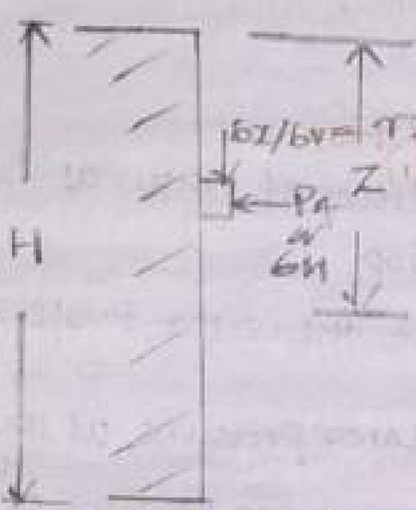


fig.
[retaining wall with vertical back with a dry cohesionless backfill having an unloaded horizontal surface.]

Back of the retaining wall is assumed to be smooth as per Rankine's theory.

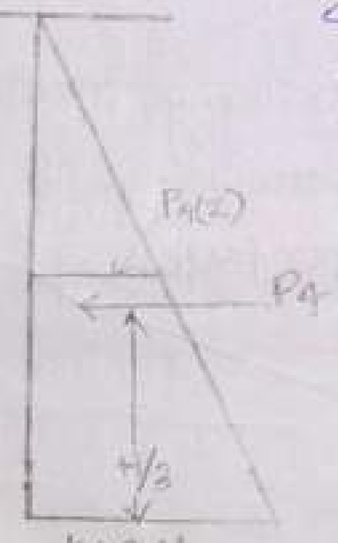
$$P_A = K_A \gamma z$$

where γ is the bulk unit weight of soil above the water table.

At the base of the wall, where $z = H$

$$P_A = K_A \gamma H$$

The force per unit length of the wall due to the active pressure distribution is the total active thrust P_A, and is given by the area of the active pressure distribution diagram and acts through the centre of the area or act at C.G. for



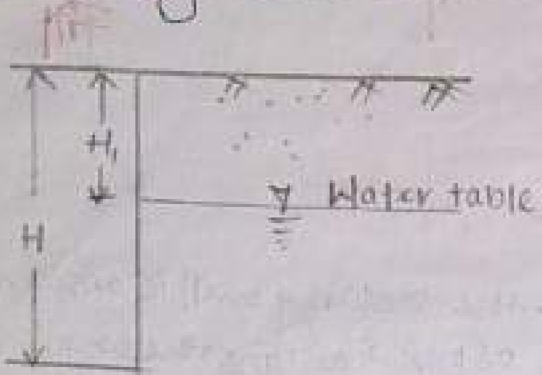
[fig. Active pressure distribution]

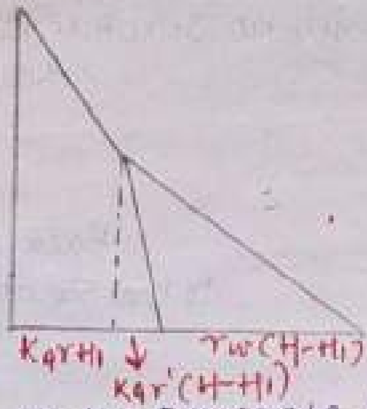
Hence, $P_A = \frac{1}{2} K_A \gamma H . H$

$$= \frac{1}{2} K_A \gamma H^2$$

Acting at a distance $H/3$ above the base of the wall

(b) Submerged backfill





If the backfill is submerged by the presence of natural water table at a depth of H_1 from the top.

γ = bulk unit weight γ of the soil above the water table.

For submerged portion of the backfill, the lateral pressure at any depth is the sum of (i) active earth pressure due to submerged unit weight of the soil mass γ' and (ii) Hydrostatic pressure

$$P_a = K_a \gamma H_1 + K_a \gamma' (H - H_1) + \gamma_w (H - H_1)$$

It has been assumed that there is no change or difference in the value of ϕ (and therefore in the value of K_a) for the soil above and below the water table.

(c) Effect of uniform surcharge

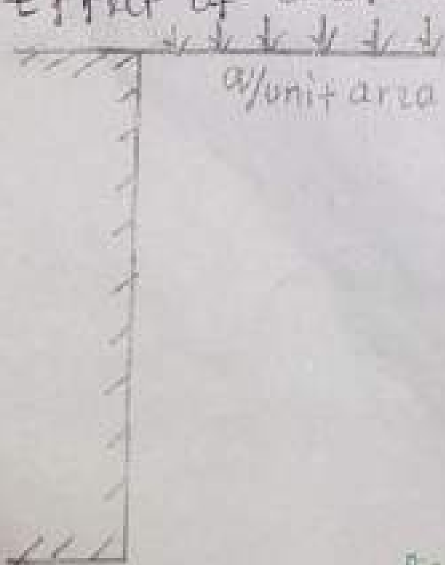


fig. 1

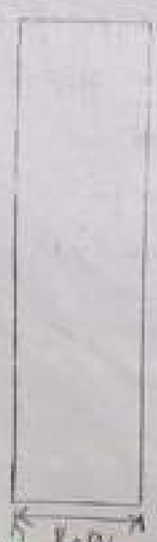


fig. 2

If a uniformly distributed surcharge load of intensity q per unit area is acting over the surface of the backfill, it is assumed that the effective vertical pressure P_v at any depth is increased by q. Hence the increase in active earth pressure is uniform throughout the back of the wall and is equal in magnitude to $K_A q$ fig. 2

vertical pressure $P_v = \gamma z$

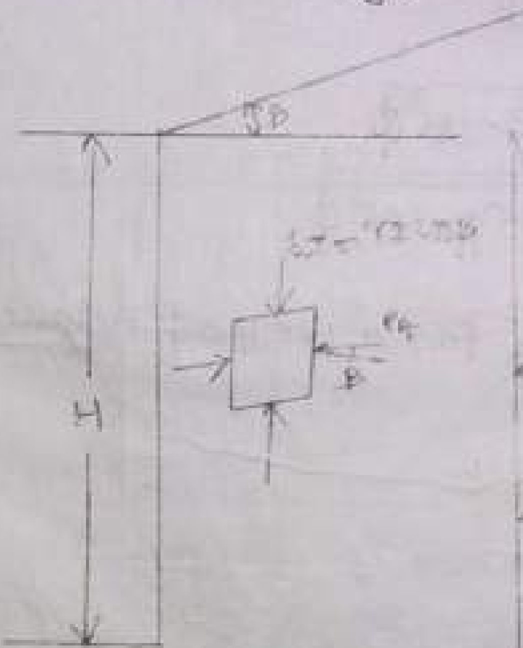
$P_A = K_A \times P_v$ where P_v is the vertical pressure at a given depth z.

Hence if the backfill surface carries a surcharge q,

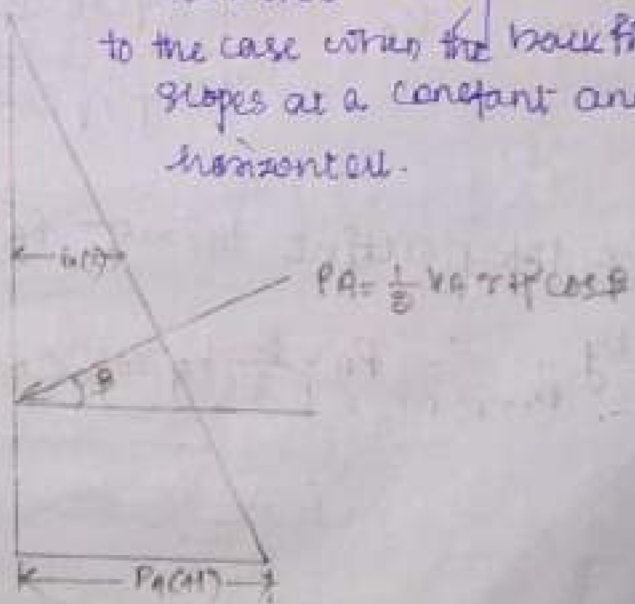
$P_v = \gamma z + q$ and $P_A = K_A \gamma z + K_A q$

which describe the distribution of active pressure combining the effects of soil weights and surcharge.

(d) Effect of sloping ground surface



Rankine's theory can also be aplied to the case when the backfill surface slopes at a constant angle β to the horizontal.



* An element of soil at depth z bound by two vertical planes || and two plane parallel to the backfill surface.

* it is assumed that the vertical stress and the lateral pressure acting on the soil element are conjugate stresses; that is, the direction of one is parallel to the plane on which the other acts.

the lateral pressure earth pressure on a vertical plane is also assumed to act parallel to the back fill surface.

the relation between the lateral pressure and the vertical stress can be obtained by means of a mohr diagram

The active earth pressure at a depth z, acting parallel to the slope is given by

$$P_a = K_a \gamma z . \cos\beta$$

where $$K_a = \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

the total active thrust P_a for a wall of height H is given by

$$P_a = \frac{1}{2} K_a \gamma H^2 \cos\beta$$

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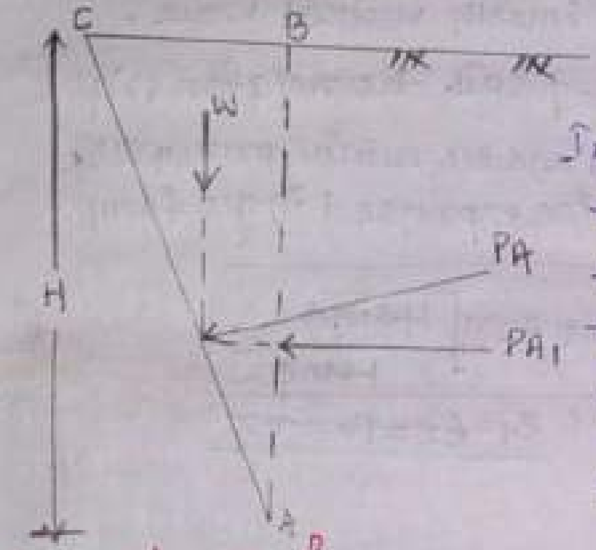
(e) the case of inclined back of wall

when the back of the wall has a batter, the following procedure can be used to determine the active earth pressure

(i) shows the case of an inclined-back wall with a horizontal backfill surface.

(ii) shows an inclined-back wall with a sloping backfill surface.

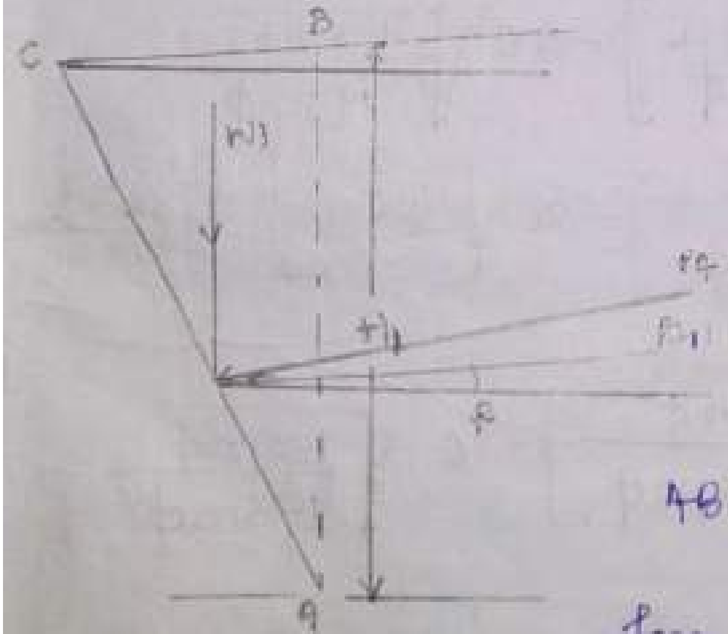


Horizontal Surface

In this case a vertical line is drawn through the base of the wall to intersect the backfill surface at B.

The total active ~~thrust~~ thrust across the vertical plane AB is

AB = H



AB = H1

for Both case W = the weight of ~~the~~ ~~very~~ soil included in the triangle ABC

the active thrust on the wall PA, is then the vector sum of PA1 and W, that is $P_A = \sqrt{P_{A1}^2 + W^2}$

Active cl...

Rankine's earth pressure in cohesive

Active pressure on Retaining Wall - Cohesive Backfill

Rankine's theory considered the case of only cohesionless soils. However the theory has subsequently been extended by Bell (1915) to cover the case of backfills possessing both cohesion and friction.

Retaining wall of height H with a smooth vertical back, retaining a cohesive backfill. For a c-ϕ soil, the relation ship between the major principal stress σ_1 and the minor principal stress σ_3 at plastic equilibrium can be expressed in the form

$$\boxed{\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}}$$

For the case of active earth pressure $\sigma_1 = \sigma_z = P_v = \gamma z$ and $\sigma_3 = P_a$.

$$\gamma z = P_A\left[\frac{1+\sin\phi}{1-\sin\phi}\right] + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$P_A\left[\frac{1+\sin\phi}{1-\sin\phi}\right] = \gamma z - 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$P_A = \frac{\gamma z - 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}}{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$P_A = \gamma z\left[\frac{1-\sin\phi}{1+\sin\phi}\right] - 2c\sqrt{\frac{1-\sin\phi}{1+\sin\phi}}$$

$$= \gamma z\left[\frac{1-\sin\phi}{1+\sin\phi}\right] - 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}} \times \frac{1-\sin\phi}{1+\sin\phi}$$

$$= \gamma z\left[\frac{1-\sin\phi}{1+\sin\phi}\right] - 2c\sqrt{\frac{1-\sin\phi}{1+\sin\phi}}$$

$$P_A = \gamma z K_A - 2c\sqrt{K_A}$$

So At $z = 0$, $P_A = -2c\sqrt{K_A}$ — at which depth $P_A = 0$?

and $P_A = 0$ at $z = z_0 = \dfrac{2c}{\gamma\sqrt{K_A}}$

Active c

$$\gamma z = P_A\left[\frac{1+\sin\phi}{1-\sin\phi}\right] + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$P_A = \left[\frac{1+\sin\phi}{1-\sin\phi}\right] = \gamma z - 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

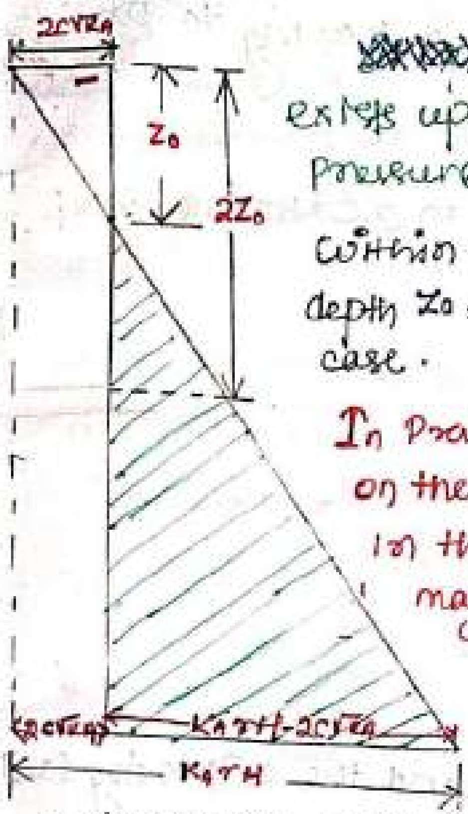
$$P_A = \gamma z \times \frac{1-\sin\phi}{1+\sin\phi} - 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}} \times \frac{1-\sin\phi}{1+\sin\phi}$$

$$= \gamma z \times \frac{1-\sin\phi}{1+\sin\phi} - 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}} \times \sqrt{\frac{1-\sin\phi}{1+\sin\phi}} \times \sqrt{\frac{1-\sin\phi}{1+\sin\phi}}$$

$$= \gamma z \times \frac{1-\sin\phi}{1+\sin\phi} - 2c\sqrt{\frac{\cancel{1+\sin\phi}}{\cancel{1-\sin\phi}} \times \frac{\cancel{1-\sin\phi}}{\cancel{1+\sin\phi}}} \times \sqrt{\frac{1-\sin\phi}{1+\sin\phi}}$$

$$P_A = \gamma z \times K_A - 2c \times \sqrt{K_A}$$

The distribution of active earth pressure is a c-ϕ soil



In this case the negative pressure exists upto Z_0, a depth where the active earth pressure becomes equal to zero.

Within the zone between the ground surface and depth Z_0, the soil is in a state of tension in the "active" case.

In Practice, this tension cannot be taken to act on the wall since tension cracks tend to develop in the soil within the tension zone and the soil may not remain adhered to the wall.

Hence, in calculating the total active thrust on the wall, the tension zone is usually ignored and only the area of the pressure distribution diagram between the depth Z_0 and H is considered.

$$\boxed{P_A = \frac{1}{2} K_A \gamma H^2 - 2cH\sqrt{K_A} + \frac{2c^2}{\gamma}}$$

$$\cancel{P_A = \frac{1}{2} \times K_A \gamma H \times (H - Z_0) - \frac{1}{2} \times 2c\sqrt{K_A} \times (H - Z_0)}$$

$$\cancel{=}$$

The net total active thrust is zero for a depth equal to $2Z_0$

$$P_A = \frac{1}{2} \times (K_A \gamma H - 2c\sqrt{K_A}) \times (H - Z_0)$$

$$= \frac{1}{2} \times (K_A \gamma H - 2c\sqrt{K_A}) \times \left(H - \frac{2c}{\gamma\sqrt{K_A}}\right)$$

$$= \frac{1}{2}\{(K_A \gamma H \times H - \frac{2c}{\cancel{\gamma\sqrt{K_A}}} \times \cancel{K_A}^{\sqrt{K_A}} \cancel{\gamma} H$$

$$- 2c\sqrt{K_A} \times H + 2c\sqrt{K_A} \times \frac{2c}{\gamma\sqrt{K_A}}$$

$$= \frac{1}{2}\{(K_A \gamma H^2 - 2cH\sqrt{K_A} H - 2c\sqrt{K_A} H + \frac{4c^2}{\gamma})\}$$

$$= \frac{1}{2} K_A \gamma H^2 - 2cH\sqrt{K_A} + 2\frac{c^2}{\gamma}$$

Active

The net total active thrust is zero for a depth equal to $2z_0$. thus it is implied that in a cohesive soil, a vertical cut can be made upto a depth of $2z_0$ without having to provide any lateral support.

thus the critical depth of vertical cut H_c in a cohesive soil is given by

$$H_c = 2z_0 = \frac{4c}{\gamma\sqrt{K_A}}$$

for $\phi = 0$, $H_c = \frac{4c}{\gamma}$

Passive earth pressure

(a) cohesionless soil

when the back fill surface is horizontal and the soil is dry sand passive earth pressure at a depth z is given by

$$P_{pz} = K_p \gamma z \quad (\text{or } K_p \sigma_v \text{ in the general form})$$

where $K_p = \frac{1+\sin\phi}{1-\sin\phi}$

Hence, the total passive resistance P_p for the full height H of a retaining wall is

$$P_p = \frac{1}{2} K_p \gamma H^2$$

P_p act at a height $H/3$ from the base of the wall.

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If a uniform surcharge load q is applied over the surface, the passive earth pressure is increased by $K_p q$ at every depth.

when the backfill has a sloping surface inclined at β to the horizontal. It can be shown that the passive earth pressure P_p at a depth z is given by

$$P_P = K_P \gamma z \cos\beta$$

where $$K_P = \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$$

the total passive resistance P_P for a wall of height H is given by $$P_P = \frac{1}{2} K_P \gamma H^2 \cos\beta$$

and acts parallel to the slope of the backfill.

(b) Cohesive soil

$$\sigma_1 = \sigma_3 \left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}} \quad \text{—— (1)}$$

for the case of passive earth pressure, $\sigma_3 = \sigma_z = P_v = \gamma z$

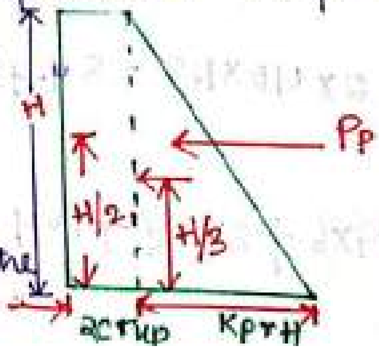
and $\sigma_1 = P_P$

Substituting these in equation (1)

$$P_P = \gamma z \left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

or, $$P_P = \gamma z K_P + 2c\sqrt{K_P}$$

the passive earth pressure distribution is shown in fig



The two component of P_P act as heights of H/3 and H/2 respectively from the base of the wall.

The total passive earth Pressure P_P for the full height H of the retaining wall is given by

$$P_P = \frac{1}{2}\gamma H^2 K_P + 2cH\sqrt{K_P}$$

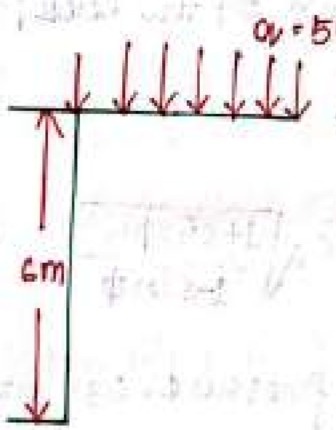
Example:- A retaining wall 6m high, with a smooth vertical back is pushed against a soil mass having $c' = 40\ kN/m^2$ and $\phi = 15°$; $\gamma = 19\ kN/m^3$. What is the total Rankine passive pressure, if the horizontal soil surface carries a uniform load of $50\ kN/m^2$? What is the point of application of the resultant thrust?

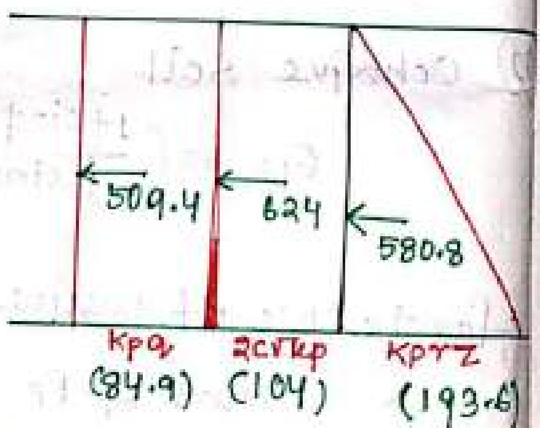
Solution:→

The passive pressure at a depth z in a c-φ soil is given by:

$$P_p = K_p \sigma_v + 2c\sqrt{K_p}$$

where σ_v is the vertical pressure at that depth





for the case of uniform surcharge q acting over the backfill surface. $P_v = \gamma z + q$

$$P_p = K_p \gamma z + K_p q + 2c\sqrt{K_p}$$

$$K_p = \frac{1+\sin\phi}{1-\sin\phi} = \frac{1+\sin 15°}{1-\sin 15°} = 1.698$$

At $z = 0$

$$P_p = 1.698 \times 50 + 2 \times 40 \times 1.3 = 84.9 + 104 = 188.9\ kN/m^2$$

At $z = 6m$

$$P_p = 1.698 \times 19 \times 6 + 84.9 + 104 = 382.5\ kN/m^2$$

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the contribution of surcharge, cohesion and weight of the soil to the passive pressure are indicated separately on the diagram.

they have been calculated by considering the area of the respective pressure distribution.

$$P_P = 509.4 + 624 + 580.8 = 1714.2 \text{ kN/m}$$

the distance of the point of application of the resultant passive thrust, H from the base can be determined by taking moments about the base

$$H = \frac{6 \times 84.9 \times 3 + 6 \times 104 \times 3 + \frac{1}{2} \times 193.6 \times 6 \times \frac{6}{3}}{1714.2}$$

$$= \frac{1528.2 + 1872 + 1161.6}{1714.2}$$

$$= 2.66 \text{m}$$

Coulomb's wedge theory

Rankine back face of
Coulomb retaining space

Coulomb develop a method for the determination of the earth pressure in which he considered the equilibrium of the sliding wedge which is formed when the movement of the retaining wall takes place.

As discussed before, in active case, the sliding wedge moves downward and outward relative to the back fill, whereas in the passive case, the sliding wedge moves upwards.

The lateral pressure on the wall is equal and oposite to the reactive force exerted by the wall in order to keep the sliding wedge in equilibrium.

The analysis is a type of limiting equilibrium method
the following assumptions are made:

1. The back fill is dry, cohesion less, homogeneous, isotropic and ideally plastic ~~[illegible]~~ material.
2. The slip surface is a plane surface which passes through the heel of the wall.
3. The wall surface is rough. The resultant earth pressure on the wall is inclined at an angle δ to the normal to the wall, where δ is the angle of the friction between the wall and the back fill.

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In Coulomb's theory, a plane failure surface is assumed and the lateral force required to maintain the equilibrium of the sliding wedge is found using the principles of statics.

The procedure is repeated for several trial surfaces. The trial surface which gives the largest force for the active case, and the smallest force for the passive case, is the actual failure surface. [for earth force & force only]

This method readily accommodates the friction between the wall and the back fill, irregular back fill, sloping wall, surcharge loads etc.

Although the initial theory was for dry, cohesionless soils, it has now been extended to wet soils and cohesive soils as well.

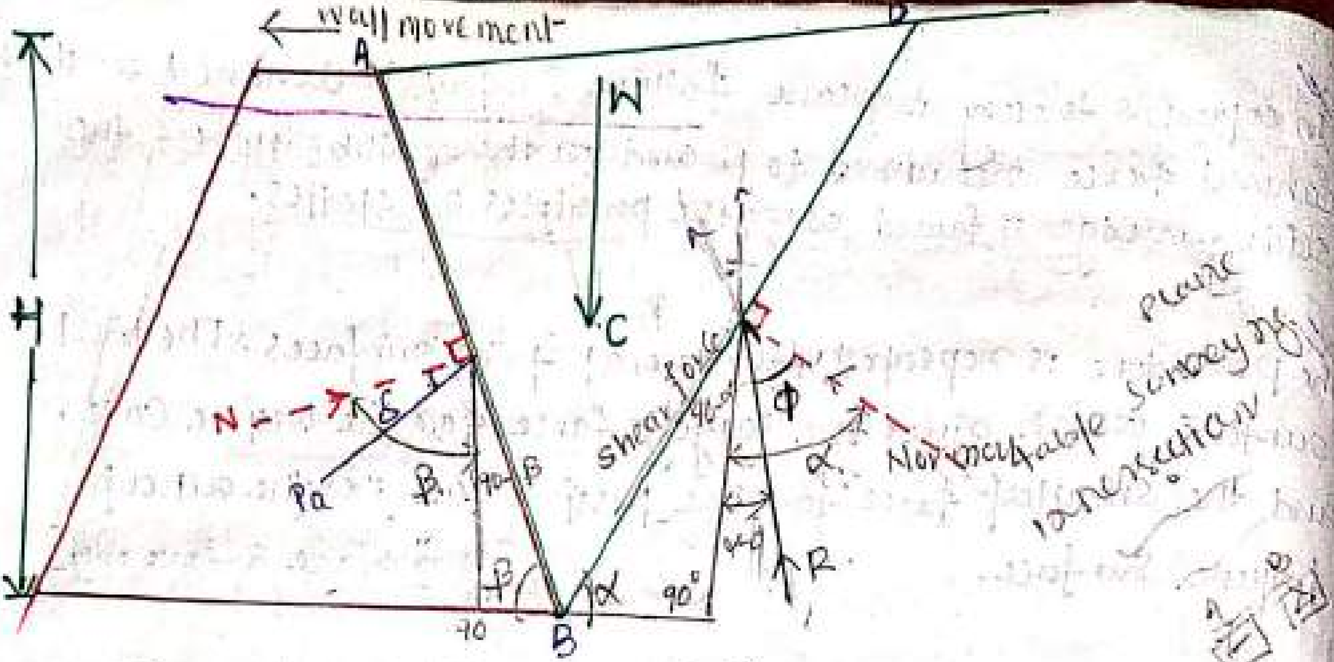
Thus Coulomb's theory is more general than Rankine theory.

Coulomb's Active Pressure in Cohesionless Soils

A retaining wall with an inclined back face and a sloping dry granular back fill.

In active case, the sliding wedge ABD moves downward, and the reaction R acts upward and inclined at an angle ϕ' with the normal

the sliding wedge ABD is in equilibrium under three forces



① weight of the wedge (W)

② Reaction R on the slip surface BD. Resultant, R of normal and resisting shear forces along the surface. R will be inclined at an angle φ to the normal drawn to surface.

③ Reaction Pa from the wall
Active force per unit length of wall

It may be noted that, at failure conditions, the shearing resistance on the failure surface is fully mobilised and the reaction R has the maximum obliquity. It is therefore inclined at an angle φ' to the normal to the failure plane.

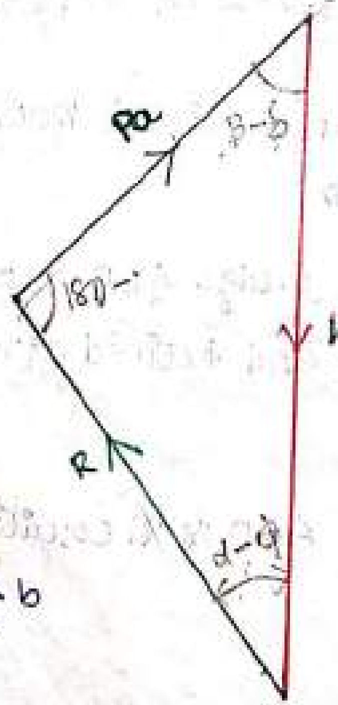


fig. 6

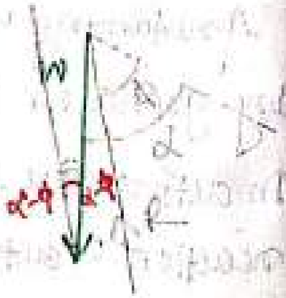


fig 5 shows the force triangle. As the magnitude of one force and the directions of all three forces are known, the force triangle can be completed.

the magnitude of Pa is determined from the force triangle. the pressure acting on the wall is equal and opposite to Pa. the procedure is repeated after assuming an other failure surface. the surface that gives the maximum value of Pa is the critical failure plane, and the corresponding force is the active force.

It is expressed by $K_a = \frac{1}{2} K_a \gamma H^2$

Expression for Active pressure.

from the force triangle, using law of sides

$$\frac{P_a}{\sin(\alpha-\phi)} = \frac{W}{\sin(180-\beta+\delta-\alpha+\phi)}$$

$$\text{or, } \quad P_a = \frac{W \sin(\alpha-\phi)}{\sin(180-\beta+\delta-\alpha+\phi)} \quad \text{—— (6)}$$

where Pa = total active pressure force.

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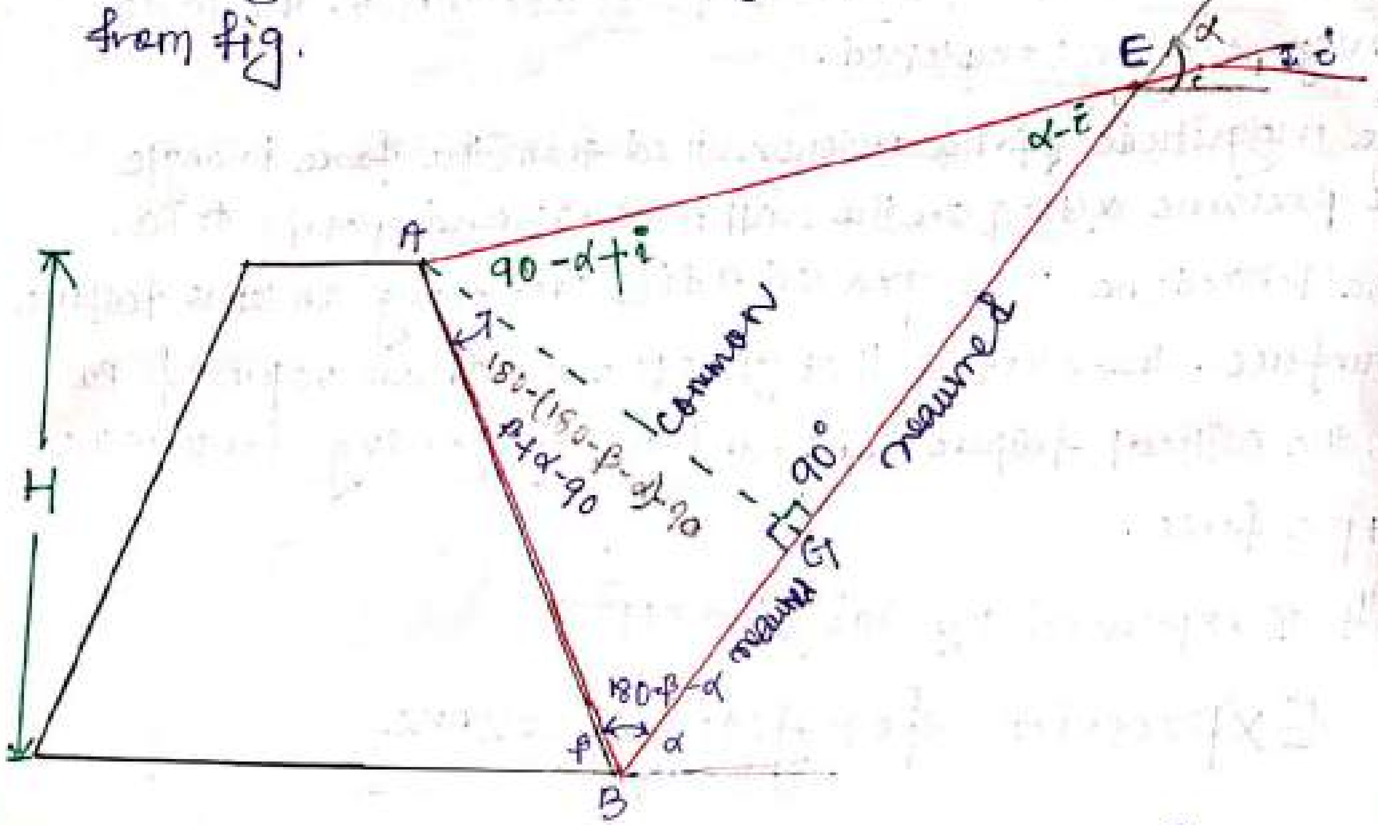
The weight W of the wedge ABO can be determined from fig.



$$W = \left\{\left(\tfrac{1}{2} \times BG \times AG\right) + \left(\tfrac{1}{2} \times GE \times AG\right)\right\} \times \gamma$$

$$= \tfrac{1}{2} \times AG \times BE \times \gamma \quad \text{——— (4)}$$

Taking $AG = m$ and $BE = L$, we have $\frac{1}{2} \times m \times L \times \gamma$

$$W = \tfrac{1}{2}(m \times \gamma) \times (BG + GE) \quad \text{——— (5)}$$

Now, $\sin\beta = \dfrac{H}{AB} \Rightarrow AB = \dfrac{H}{\sin\beta}$

for triangle ABG, $\sin(180-\beta-\alpha) = \sin(180-(\beta+\alpha)) = \dfrac{AG}{AB}$

$$= \frac{m}{AB}$$

Therefore $m = AB\sin(\beta+\alpha)$

$$= \frac{H\sin(\beta+\alpha)}{\sin\beta} \quad \text{——— (1)}$$

From triangle ABG,

$$\frac{BG}{\sin(\beta+\alpha-90)} = \frac{m}{\sin(\beta+\alpha)} \quad \because \text{Law of sines}$$

$$BG = \frac{m \sin(\beta+\alpha-90)}{\sin(\beta+\alpha)} \quad \text{—— (2)}$$

For triangle AGE,

$$\frac{GE}{\sin(90-\alpha+i)} = \frac{m}{\sin(\alpha-i)}$$

$$GE = \frac{m \sin(90-\alpha+i)}{\sin(\alpha-i)} \quad \text{—— (3)}$$

Substituting the value of m, BG and GE from eq^n (1) (2) & (3) we get in equation (5)

$$W = \frac{1}{2} \times \frac{H \sin(\beta+\alpha)}{\sin\beta} \times \gamma \times \left[m \frac{\sin(\beta+\alpha-90^\circ)}{\sin(\beta+\alpha)} + m \frac{\sin(90-\alpha+i)}{\sin(\alpha-i)} \right]$$

$$= \frac{1}{2} \times \frac{H^2 \sin^2(\beta+\alpha)^2 \gamma}{\sin^2\beta} \times \left[\frac{\sin(\beta+\alpha-90^\circ)}{\sin(\beta+\alpha)} + \frac{\sin(90-\alpha+i)}{\sin(\alpha-i)} \right]$$

Substituting the value of W from the above equation in eq^n — (6)