$$P_a = \frac{1}{2} \gamma H^2 \frac{\sin(\beta+\alpha)^2}{\sin^2\beta} \times \frac{\sin(\alpha-\phi)}{\sin(180-\beta+\delta-\alpha+\phi)}$$

$$\times \left[\frac{\sin(\beta+\alpha-90)}{\sin(\beta+\alpha)} + \frac{\sin(90-\alpha+i)}{\sin(\alpha-i)}\right]$$

The active pressure Pa will be a maximum when the failure plane makes an angle α with the horizontal such that $\frac{\partial P_a}{\partial \alpha} = 0$

The maximum value of of Pa thus obtained is Coulomb's active force, given by

$$P_a = \frac{1}{2} K_a \gamma H^2$$

where Ka = Coulomb's active earth pressure coefficient given by

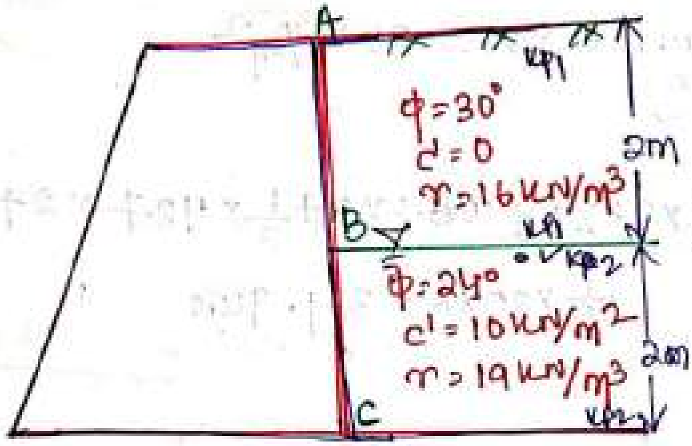
$$K_a = \frac{\sin^2(\beta+\phi')}{\sin^2\beta \cdot \sin(\beta-\delta)\left[1+\sqrt{\frac{\sin(\phi+\delta)\sin(\phi-i)}{\sin(\beta-\delta)\sin(\beta+i)}}\right]^2}$$

The line of action of Pa will be at height of H/3 above the base of the wall and it will be inclined at an angle δ to the normal drawn to the back of the wall.

Example:- 19.6

Determine the Rankine passive ~~earth~~ force per unit length of the wall shown in fig. The water table is at the level of B. Take $\gamma_w = 10 kN/m^3$.

Solution:- $K_p = \frac{1+\sin\phi}{1-\sin\phi}$



for top layer-I $(k_p)_1 = \frac{1+\sin 30°}{1-\sin 30°} = 3.00$

for bottom layer II $(k_p)_2 = \frac{1+\sin 24°}{1-\sin 24°} = 2.37$

$\boxed{P_p = K_p \gamma z + 2c\sqrt{k_p}}$ $P_p = k_{p1}\gamma_1 z_1 + 2c\sqrt{k_p}$

Top Layer

At Point A, $z = 0$ $P_p = 2c\sqrt{k_p} = 2 \times 0 \times \sqrt{k_p} = 0$

At point B, $z = 2m$ $P_p = k_p \gamma z = 3.00 \times 16 \times 2m = 96 kN/m^2$

Bottom Layer $\gamma_2' = 19 - 10 = 9 kN/m^3$

$P_p = k_{p2}\gamma_1 z_1 + k_{p2}\gamma_2' z_2 + 2c\sqrt{k_p}$

$= \cancel{2.37 \times 16 \times 2 + 2.37 \times 18 \times 2}$

$= 2.37 \times 16 \times 2 + 2.37 \times 9 \times z_2 + 2 \times 10 \times \sqrt{2.37}$

$= 75.84 + 21.33 z_2 + 30.78$

$= 106.6 + 21.33 z_2$

At B where $z_2 = 0$

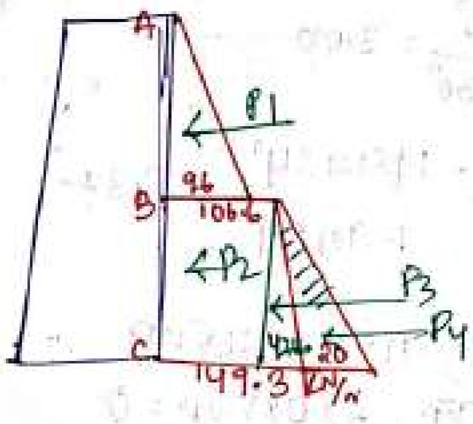
$$P_p = 106.6 \text{ kN/m}^2$$

At C. where $z_2 = 2$

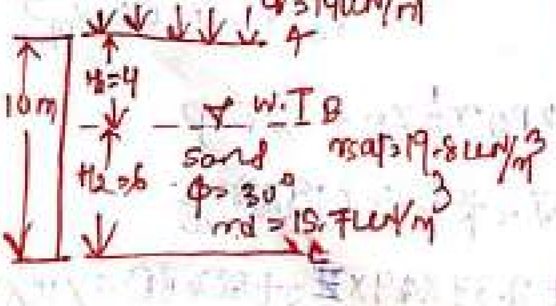
$$P_p = 106.6 + 21.33 \times 2$$
$$= 149.3 \text{ kN/m}^2$$

Pore water pressure = $2 \times 10 = 20 \text{ kN/m}^2$

Total pressure = $\frac{1}{2} \times 96 \times 2 + 106.6 \times 2 + \frac{1}{2} \times 42.7 \times 2 + \frac{1}{2} \times 20 \times 2 = 371.9 \text{ kN}$



Q. Example 20.25



① sketch the earth pressure diagram

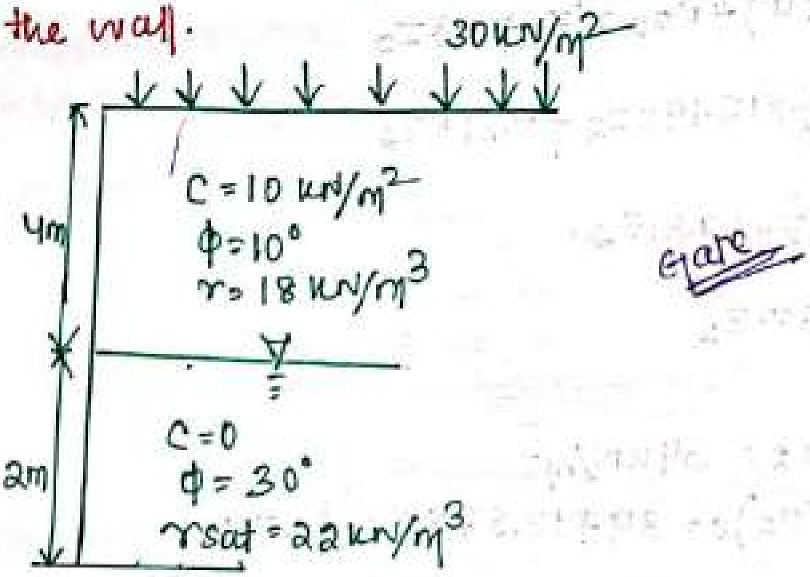
② Total thrust

③ Location of total thrust.

$P = 450.66 \text{ kN/m}$

$\bar{z} = 3.085 \text{m}$

Example 20.32 :- Calculate the total active earth pressure on the retaining wall 6m high shown in fig and also calculate the line of action of lateral force from the base of the wall.



Solution :- Condition-1. Active earth pressure due to c-φ soil

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 10^\circ}{1+\sin 10^\circ} = \frac{1-0.1736}{1+0.1736} = 0.704$$

~~Pa = γ×z×Ka − 2c√Ka~~ $P_a = K_a \times q + \gamma z K_a - 2c\sqrt{K_a}$

$= 0.704 \times 30 + 18 \times \cancel{4} z \times 0.704 - 2 \times 10 \times \sqrt{0.704}$

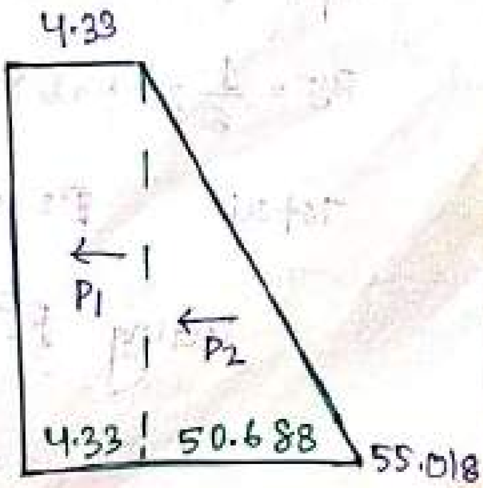
$= 21.12 + 12.672z - 16.78$

$= 12.672z + 4.33$

At depth $z = 0$ $\quad P_a = 4.33$

At depth $z = 4m$ $\quad P_a = 55.018$

So, pressure distribution diagram



Condition-2: Active earth pressure due to cohesion less

$c=0.\ Ka_2 = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3};\ \gamma' = 22-9.81 = 12.19\ kN/m^3$

$Pa_2 = Ka_2(30+18\times4) + Ka_2\gamma' z_2 + \gamma_w z_2$

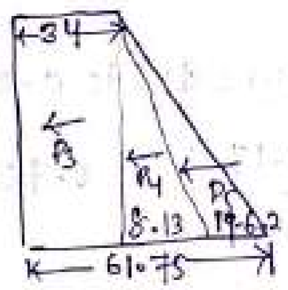
$= \frac{1}{3}\times102 + \frac{1}{3}\times12.19\times z_2 + 9.81\times z_2$

$= 34 + 4.06 z_2 + 9.81 z_2$

$= 34 + 13.87 z_2$

At B, $z_2=0$: $Pa_2 = 34\ kN/m^2$

At C, $z_2 = 2m$; $(Pa_2)_C = 34 + 13.87\times2 = 61.74$



$P_1 = 4.34\times4 = 17.36\ kN/m$ at $z_1 = 2+4/2 = 4m$ above base

$P_2 = \frac{1}{2}\times50.688\times4 = 101.38\ kN/m$ at $z_2 = 2+4/3 = 3.33m$ " "

$P_3 = 34\times2 = 68\ kN/m$ at $z_3 = 2/2 = 1m$ above base

$P_4 = \frac{1}{2}\times8.13\times2 = 8.13\ kN/m$ at $z_4 = 2/3 = 0.667m$ " "

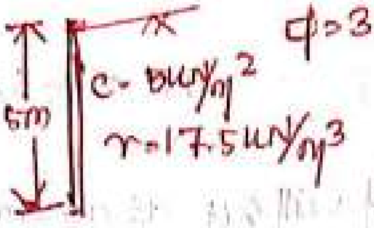
$P_5 = \frac{1}{2}\times19.62\times2 = 19.62\ kN/m$ at $z_5 = 2/3 = 0.667m$ " "

Total $P = 17.36+101.38+68+8.13+19.62 = 214.49\ kN/m$

acting $P\bar{z} = 17.36\times4 + 101.38\times3.3 + 68\times1 + 8.13\times0.667 + 19.62\times0.667$

$\bar{z} = 2.302m$ above base.

Q. A 5m retaining wall is shown in fig. Determine the Rankine active pressure on the wall (a) Before the formation of crack (b) After the formation of the crack.



Solution:-

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = 0.33$$

equation for c-φ soil $\quad P_a = K_a \gamma z - 2c\sqrt{K_a}$

$$P_a = 0.333 \times 17.5 \times z - 2 \times 5\sqrt{0.333}$$
$$= 5.83z - 5.77$$

At top, $z = 0$. $\quad P_a = -5.77\ \text{kN/m}^2$

At bottom $z = 5\text{m}$. $\quad P_a = 5.83 \times 5 - 5.77 = 23.38\ \text{kN/m}^2$

A point where $P_a = 0$ need that point to draw pressure distribution diagram.

$$5.83z - 5.77 = 0 \Rightarrow z = 0.99\text{m}$$

Before formation of crack

Negative pressure. $\quad P_1 = \frac{1}{2} \times 0.99 \times 5.77 = 2.86\ \text{kN}$

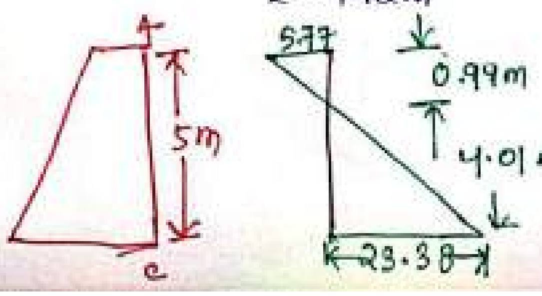
Positive pressure $\quad P_2 = \frac{1}{2} \times 4.01 \times 23.38 = 46.88\ \text{kN}$.

Net $P_a = 46.88 - 2.866 = 44.02\ \text{kN}$.

Line of action of P_a is determined as under, by taking moment about c.

$$44.02 \times \bar{z} = 46.88 \times \frac{4.01}{3} - 2.86 \times (4.01 + 0.67)$$

$\bar{z} = 1.12\text{m}$



$c = 5\ \text{kN/m}^2$
$\phi = 30°$
$\gamma = 17.5\ \text{kN/m}^2$

Scanned by CamScanner

After the formation of crack

After the formation of the crack, the negative pressure is eliminated. The pressure distribution is given by the area bed.

$$Pa = \frac{1}{8} \times 23.38 \times 4.01 = 46.88.$$

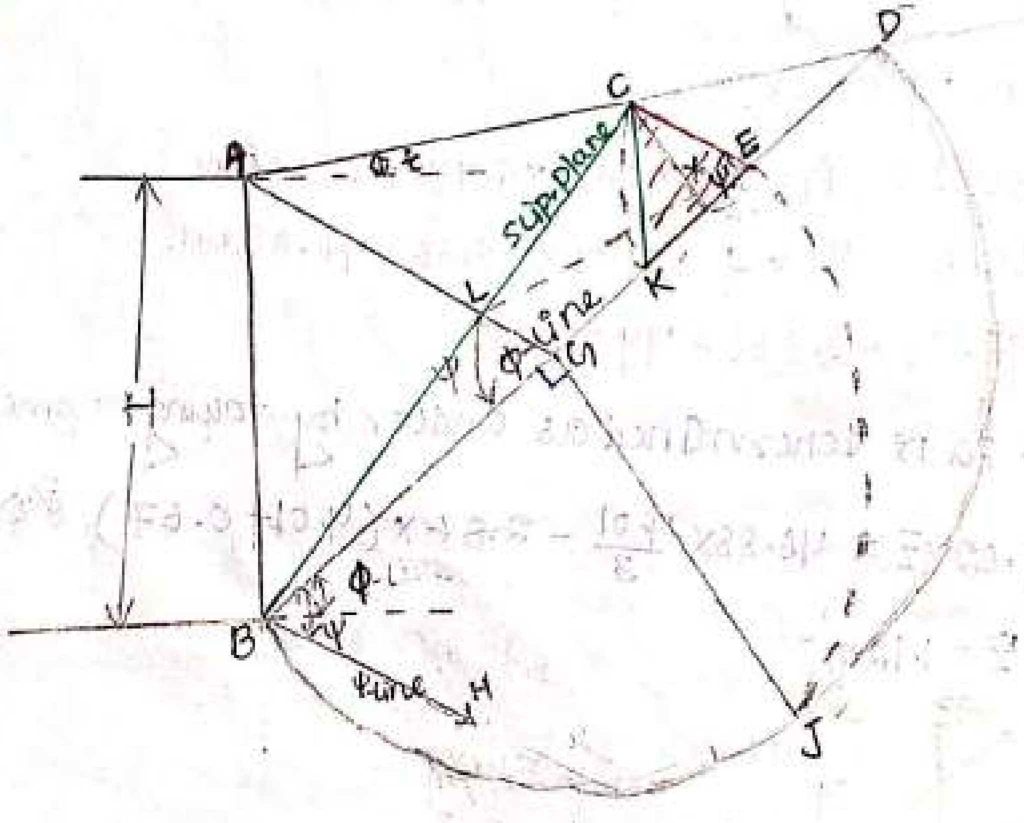
it will act at a height of 4.01/3 m above base

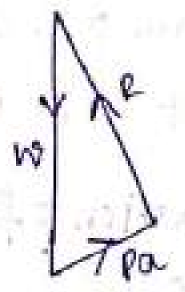
$$Pa = \frac{1}{8} rH^2 ka - 2c'H\sqrt{ka} + \frac{2c^2}{r}$$

$$Pa = 46.85 kN.$$

REBHANN'S GRAPHICAL METHOD FOR ACTIVE PRESSURE

Rebhann (1871) presented a graphical method for the location of the slip plane and the total active earth pressure according to the Coulomb's wedge theory.





Procedure:→

1. Draw the ground line and φ line at an angle i and φ respectively with the horizontal to meet in point D.
2. Draw a semi-circle on BD as diameter.
3. Through B, draw a line BA at an angle ψ with BD.
4. Through A, draw a line AG parallel to the ψ line
5. Draw GJ perpendicular to BD to meet the semi-circle in J.
6. With B as centre and BJ as radius draw an arc to cut BD in E.
7. Through E, draw EC parallel to the ψ-line: BC then represent the slip plane.
8. With E, as the centre and EC as radius, draw an arc to cut BD in K. Join CK
(9) Calculate the total active earth pressure from the relation

$$\boxed{P_a = \gamma(\Delta KCE) = \frac{1}{2} \times \gamma \times (KE) x} \text{ —— (1)}$$

Proof:→ of eqn-(1) } from the properties of a circle

$BG \times GD = GJ^2$ ∴ $BG \times GD + BG^2 = GJ^2 + BG^2$

$BG(BG + GD) = BJ^2 = BE^2$

$\Rightarrow BG \times BD = BE^2$

$$\frac{BG}{BE} = \frac{BE}{BD} \text{ —— (2)}$$

Since AG is parallel to CE, $\frac{BG}{BE} = \frac{BL}{BC}$ —— (3) ∵ ΔBGL & ΔBEC similar Δle.

Hence from (2) and (3), $\frac{BE}{BD} = \frac{BL}{BC}$

LE ∥ to AD and figure ALEC becomes a parallelogram

Hence perpendiculars on diagonal LC, drawn from A and E are of equal length.

In other words, triangle ABC and BCE have equal altitude, as they have common base BC and have equal altitudes they become equal in area. and hence BC represents the critical slip plane. ①

Now the total active pressure is given

$$\frac{Pa}{CE} = \frac{w}{BE} \Rightarrow \frac{Pa}{w} = \frac{CE}{BE} \Rightarrow Pa = w.\frac{CE}{BE}$$

$$= \gamma(\Delta BCE)\cdot\frac{CE}{BE}$$

$$Pa = \gamma\left(\frac{1}{2}\times \cancel{BE}\times x\right)\cdot\frac{CE}{\cancel{BE}}$$

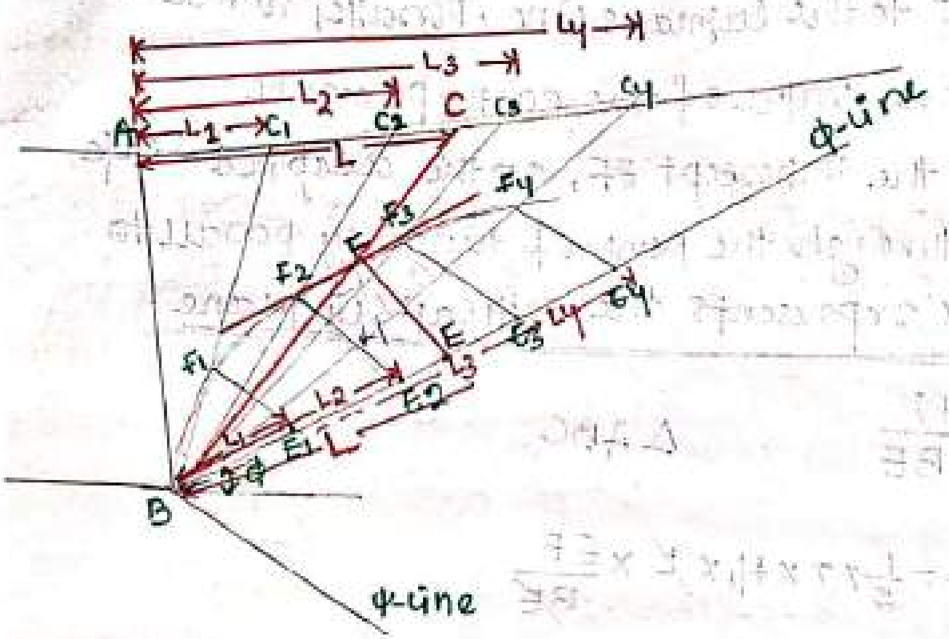
$$= \frac{1}{2}\gamma \overset{KE\ \text{or}}{CE}\cdot x = \gamma(\Delta KCE) \text{ — ③}$$

Also, since $x = CE\sin\psi$ equation ③ can be written as $Pa = \frac{1}{2}\gamma(CE)^2\sin\psi$

$$Ka = \left[\frac{\sec\theta\cdot\cos(\phi-\theta)}{\sqrt{\cos(\theta+\delta)}+\sqrt{\frac{\sin(\phi+\delta)\times\sin(\phi-\beta)}{\cos(\beta-\theta)}}}\right]^2$$

we don't know crit. & slip plane must desire from taken triangle from previous Analysis

Culman's graphical method for Active pressure



Culman also gave a graphical solution to evaluate the active earth pressure by Coulomb's theory. This method is more general than Rebhan's method, and can be conveniently used for ground surface of any shape, for various types of surcharge loads and for a layered backfill of different densities.

Procedure:—

1. Draw the ground line, φ-line and ψ line as usual.
2. Take a slip plane BC_1, calculate the weight of the wedge ABC_1 and plot it as BE_1 to same scale on φ-line.
3. through E_1, draw E_1F_1 parallel to the ψ line to cut the slip plane BC_1 in F_1.
4. Similarly take another slip plane BC_2, calculate the weight of wedge ABC_2 and plot it as BE_2 on the φ-line. Draw E_2F_2 parallel to the ψ line to cut the slip plane BC_2 in F_2.
5. Take a number of such slip plane BC_3, BC_4 etc plot the weight of the corresponding wedge on the ψ line and obtain point F_3, F_4 etc.

6. Draw a smooth curve through points B, F_1, F_2, F_3, F_4 etc. this curve is known as the Culmann's line.

7. Draw a tangent to the Culmann's line, parallel to the ϕ-line. The maximum value of the earth pressure is represented by the intercept EF, on the adopted scale, EF being drawn through the point of tangency parallel to the ψ-line. BFC represents the critical slip plane.

$$\frac{P_a}{EF} = \frac{W}{BE} \qquad \Delta ABC$$

$$P_a = W \cdot \frac{EF}{BE} = \frac{1}{2} \times \gamma \times H_1 \times \cancel{L} \times \frac{EF}{\cancel{BE}}$$

$$= \frac{1}{2} \times \gamma \times H_1 \times EF$$

when the ground line is plane, the weight of the wedge

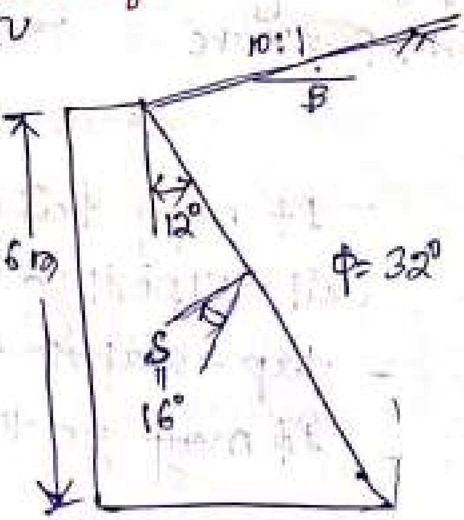
∴ ~~$A_{C1} = L_1$, $A_{C2} = L_2$, $A_{C3} = L_3$, $A_C = L$ etc.~~

ABC₁, ABC₂, ABC₃, ABC etc are proportional to the length $AC_1 (= L_1)$, $AC_2 (= L_2)$, $AC_3 (= L_3)$, $AC (= L)$ etc., since the height of the each soil wedge is constant being equal to H_1.

Hence the weight of these wedges are plotted as their base length L_1, L_2, L_3, L etc on the ϕ-line.

Example 20.14

A masonry retaining wall 6m high is back filled with granular soil having angle of internal friction of 32°. The back face of the wall is inclined to the vertical at the angle of 12° and the back fill is sloping upward from the top of the wall at a slope of 10:1. Assuming the angle of wall friction as 16°, calculate the total active earth pressure on the wall per meter length. The back fill has water content of 16% degree of saturation of 70% and G = 2.68.



$\tan\beta = \frac{P}{b} = \frac{1}{10}$

$\beta = 5.71°$

$w = 0.16,\ S_r = 0.70,\ G = 2.68$

$\gamma_d = \frac{\gamma_w G}{1+e}$

$e = \frac{wG}{S_r} = \frac{0.16 \times 2.68}{0.7} = 0.613$

$\gamma_d = \frac{2.68 \times 9.81}{1+0.613} = 16.3\ kN/m^3$

$\gamma_d = \frac{\gamma}{1+w} \Rightarrow \gamma = \gamma_d(1+w) = 16.3(1+0.16) = 18.91\ kN/m^3$

$$K_a = \left[\frac{\sec\theta \cos(\phi-\theta)}{\sqrt{\cos(\theta+\delta)} + \sqrt{\frac{\sin(\phi+\delta)\sin(\phi-\beta)}{\cos(\beta-\theta)}}} \right]$$

$\theta = 12°,\ \phi = 32°\ \ \delta = 16°\ \ \beta = 5.71°$

$K_a = 0.402$

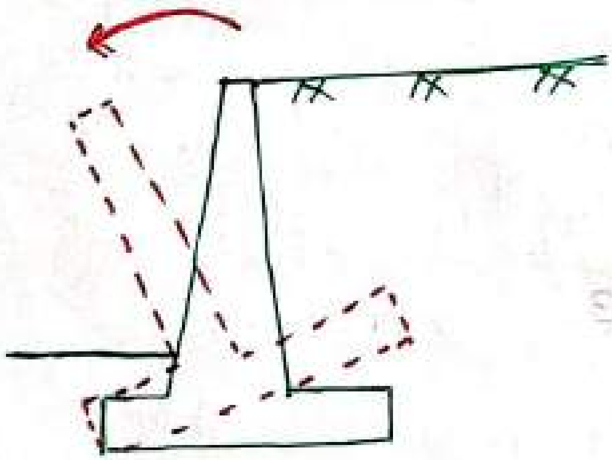
$$P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2} \times 0.402 \times 18.91 \times (6)^2$$

$$= 136.85 \text{ kN/m}$$

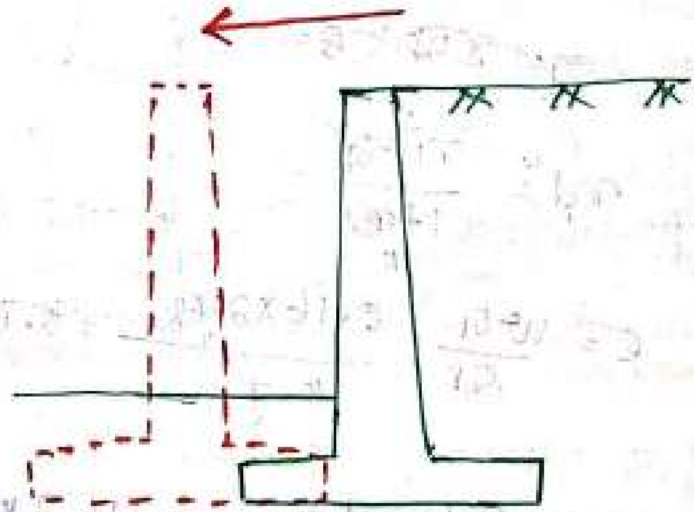
Stability of retaining walls :—

A retaining wall may fails in any of the following ways

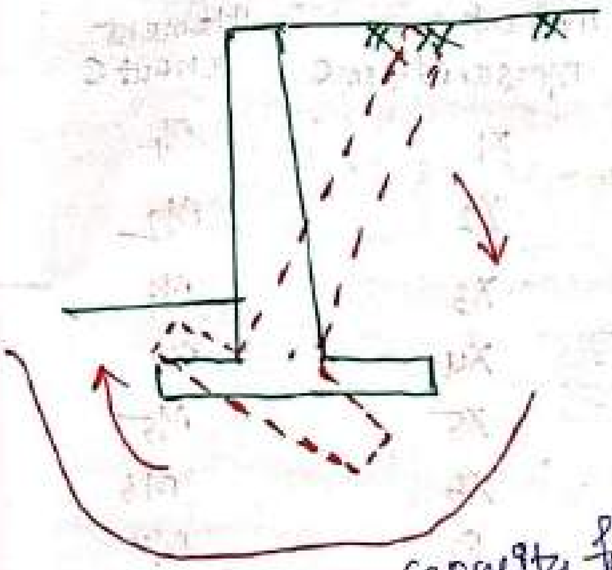
- It may overturn about its toe
- It may slide along its base
- It may fail due to loss of bearing capacity of the soil supporting the base.
- deep - seated shear failure
- It may go through excessive settlement.



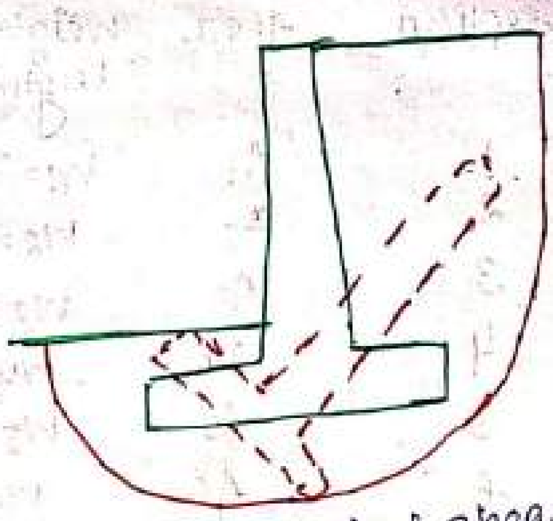
a. by overturning



b. by sliding

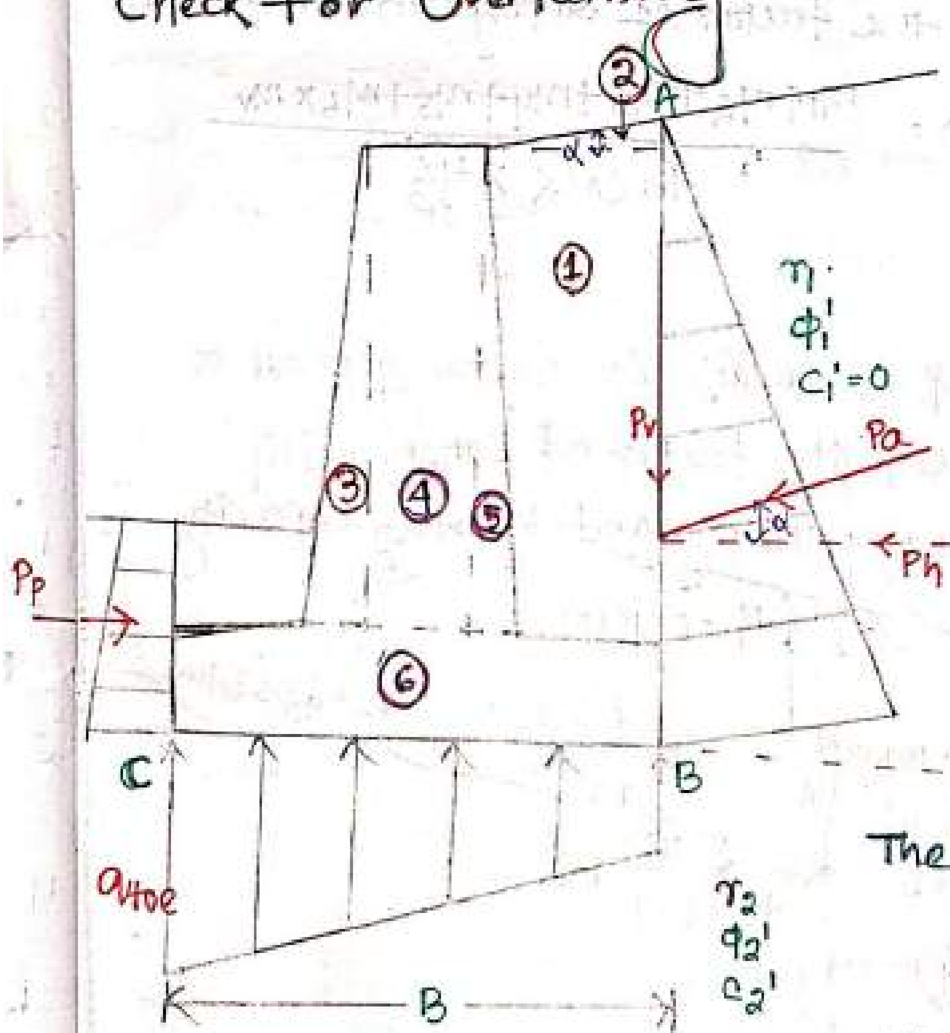


c. by bearing capacity failure.



d. deep-seated shear failure.

Check For Overturning



$$P_p = \frac{1}{2} k_p \gamma H^2 + 2c\sqrt{k_p}H$$

$$FS_{(overturning)} = \frac{\Sigma M_R}{\Sigma M_0}$$

ΣM_R = Sum of the moments of forces tending to resist the overturning about point C.

ΣM_0 = Sum of the moments of forces tending to overturn about point C.

The overturning moment

$$\Sigma M_0 = P_h \left(\frac{H'}{3}\right)$$

where $P_h = P_a \cos\alpha$

$P_v = P_a \sin\alpha$

the moment of the force P_v about C is

$$M_v = P_v \times B = P_a \sin\alpha \, B$$

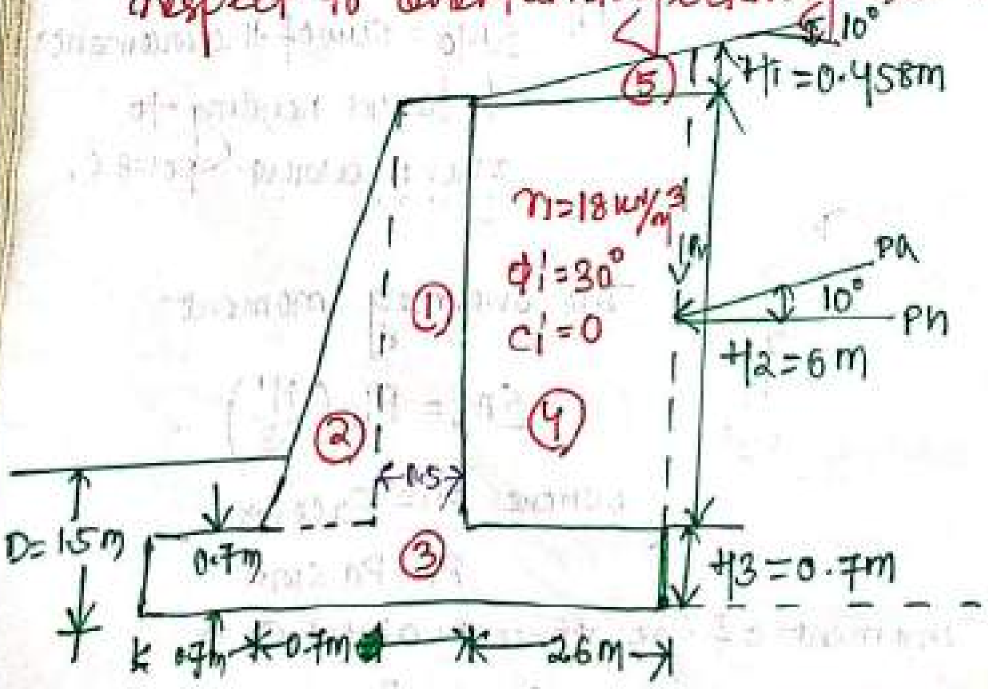
Scanned by CamScanner

Section	Area	weight/unit Length of wall	moment arm measured from C	Moment about C
			x_1	M_1
1	A_1	$W_1 = \gamma_1 \times A_1$	x_2	M_2
2	A_2	$W_2 = \gamma_1 \times A_2$		M_3
3	A_3	$W_3 = \gamma_c \times A_3$	x_3	M_4
4	A_4	$W_4 = \gamma_c \times A_4$	x_4	
5	A_5	$W_5 = \gamma_c \times A_5$	x_5	M_5
6	A_6	$W_6 = \gamma_c \times A_6$	x_6	M_6
		P_v	B	M_v
		ΣV		ΣM_R

once ΣM_R is known, the factor of safety can be calculated as

$$FS_{(overturning)} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 + M_6 \times M_v}{P_a \cos\alpha \times H/3}$$

Q. The cross section of a cantilever retaining wall is shown in fig. calculate the factor of safety with respect to overturning, sliding and bearing capacity.



$$P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2} \times 18 \times 0.35 \times (7.158)^2 = 161.4 \text{ kN/m}$$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1 - 0.5}{1.5} = 0.35$$

$$P_v = P_a \sin 10^\circ = 28.03 \text{ kN/m}$$

$$P_h = P_a \cos 10^\circ = 158.95 \text{ kN/m}$$

$$M_o = P_h \left(\frac{H'}{3}\right) = 158 \times \left(\frac{7.158}{3}\right) = 379.25 \text{ kN-m/m}$$

Section no	Area (m^2)	weight/unit length (kN/m)	moment arm from point C (m)	moment (kN-m/m)
1	6×0.5	70.74	1.15	81.35
2	$\frac{1}{2} \times 0.2 \times 6$	14.15	0.833	11.79
3	0.6×4	66.02	2.0	132.04
4	6×2.6	280.80	2.7	758.16
5	$\frac{1}{2} \times 2.6 \times 0.458$	10.71	3.13	33.52
		$P_v = 28.03$	4	112.12
		$\Sigma V = 470.45$		$\Sigma 1128.98 = \Sigma M_R$

factor of safety against ~~sliding~~ overturning

$$F_s(\text{overturning}) = \frac{\Sigma M_R}{\Sigma M_o} = \frac{1128.98}{379.25}$$

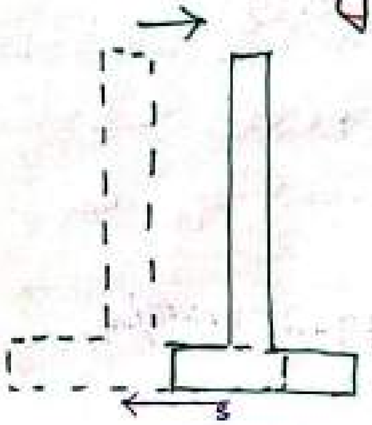
$$= 2.98 > 2.0 \text{ OK}$$

$F_s(\text{sliding}) = 2.73 > 1.5 \text{ OK}$

$F_s(\text{bearing}) = 3.03 > 3.0 \text{ OK}$

Scanned by CamScanner

Check for sliding along the Base :→



Sliding

The factor of safety against sliding may be expressed as

$$f_s = \frac{\Sigma F_R}{\Sigma F_d} > 1.5$$

where

ΣF_R = sum of horizontal resisting force

ΣF_d = sum of the horizontal driving force

Shear strength of the soil immediately below the base slab may be expressed as

$$S = C_a + \sigma \tan\delta$$

where, δ = angle of friction between the base slab and the soil.

C_a = adhesion between the soil and the base slab

Thus, the maximum resisting force that can be derived from the soil per unit length of the wall along the bottom of the base slab is

R = resisting force = $S \times$ (area of cross section of base slab)

$= S \times (B \times 1)$

$= C_a B + B\sigma \tan\delta$.

$R' = (\Sigma V)\tan\delta' + B c_a$ —— (a)

Passive force P_p is also a horizontal resisting force

so, $\Sigma F_R = (\Sigma V)\tan\delta' + B c_a + P_p$ —— (1)

the only horizontal force that will tend to cause the wall to slide is the horizontal component of the active force P_a,

$$\Sigma F_d = P_a \cos\alpha \quad \text{—— (2)}$$

So, $$F_{s(sliding)} = \frac{(\Sigma V)\tan\delta' + B c_a + P_p}{P_a \cos\alpha}$$

many cases, the passive force P_p is ignored in calculating the factor of safety with respect to sliding.

In general, we can write $\delta = k_1 \phi_2'$ and $c_a = k_2 c_2'$.

in most of the cases. k_1 and k_2 range $= \frac{1}{2}$ to $\frac{2}{3}$

$$\boxed{F_{s(sliding)} = \frac{(\Sigma V)\tan(k_1\phi_2) + B(k_2 c_2) + P_p}{P_a \cos\alpha}}$$

where,

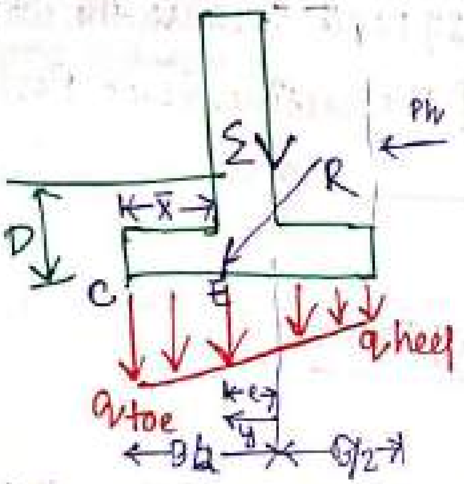
$P_p = \frac{1}{2} k_p \gamma_2 H^2 + 2c\sqrt{K_p} H$

Scanned by CamScanner

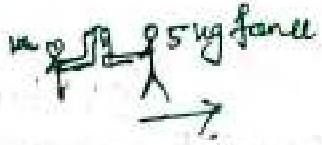
Check for Bearing Capacity failure :→

The vertical pressure transmitted to the soil by the base slab of the retaining wall should be checked against the ultimate bearing capacity of soil.

$$\boxed{F_s(\text{bearing}) = \frac{q_u}{q_{max}}} > 3$$



Ph maximum pressure at toe q_{toe}
minimum " at heel q_{heel}



The sum of the vertical forces acting on the base slab ΣV and the horizontal force Ph is $P_a \cos\alpha$.

Let $R = \Sigma V + P_h$ only to get E the point

be the resultant force.

The net moment of these forces about point C in fig

$$M_{net} = \Sigma M_R - \Sigma M_O$$

Resisting moment then overturning the net moment is responsible for create bending moment

Let the line of action of the resultant R intersect the base slab at E. Then the distance

$$CE = \bar{x} = \frac{M_{net}}{\Sigma V}$$

$$\left[\because M_{net} = \Sigma V \times \bar{x} \atop (\Sigma V + P_h)\bar{x} = P_a \cos\alpha \times H/3\right]$$

at H = 0

Hence, eccentricity of the resultant R may be expressed as $e = B/2 - CE$ or $\bar{x}$

the pressure distribution under the base slab may be determined by using simple principles from the mechanics of materials.

$$\frac{E}{R} = \frac{M}{I} = \frac{f}{y} \quad \leftarrow \text{simple bending equation}$$

$$q = \frac{\Sigma V}{A} \pm \frac{M_{net} \times y}{I}$$

intensity

$$\left[\because \frac{q}{y} = \frac{M_{net}}{I} \quad q = \frac{M_{net} \times y}{I} \right]$$

q due to net moment

$$I = \frac{1}{12} \times 1 \times B^3 \qquad \left[\because I = \frac{bd^3}{12} \right]$$

M_{net} = moment = $(\Sigma V)e$

$$q_{max} = q_{toe} = \frac{\Sigma V}{B \times 1} + \frac{e(\Sigma V) B/2}{\left(\frac{1}{12}\right) B^3} = \frac{\Sigma V}{B}\left(1 + \frac{6e}{B}\right)$$

$$\boxed{q_{\substack{toe \\ heel}} = \frac{\Sigma V}{B}\left(1 \pm \frac{6e}{B}\right)}$$

similarly

$$q_{min} = q_{heel} = \frac{\Sigma V}{B}\left(1 - \frac{6e}{B}\right)$$

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_O}{\Sigma V}$$

$$\boxed{q_u = c_2 N_c F_{cd} F_{ci} + q N_q F_{qd} F_{qi} + \frac{1}{2} \gamma_2 B' N_\gamma F_{\gamma d} F_{\gamma i}}$$

where

$q = \gamma_2 \times D$

$B' = B - 2e$

$F_{cd} = 1 + 0.4 \frac{D}{B'}$

$F_{qd} = 1 + 2 \tan\phi_2' (1 - \sin\phi_2')^2 \frac{D}{B'}$

$F_{\gamma d} = 1$

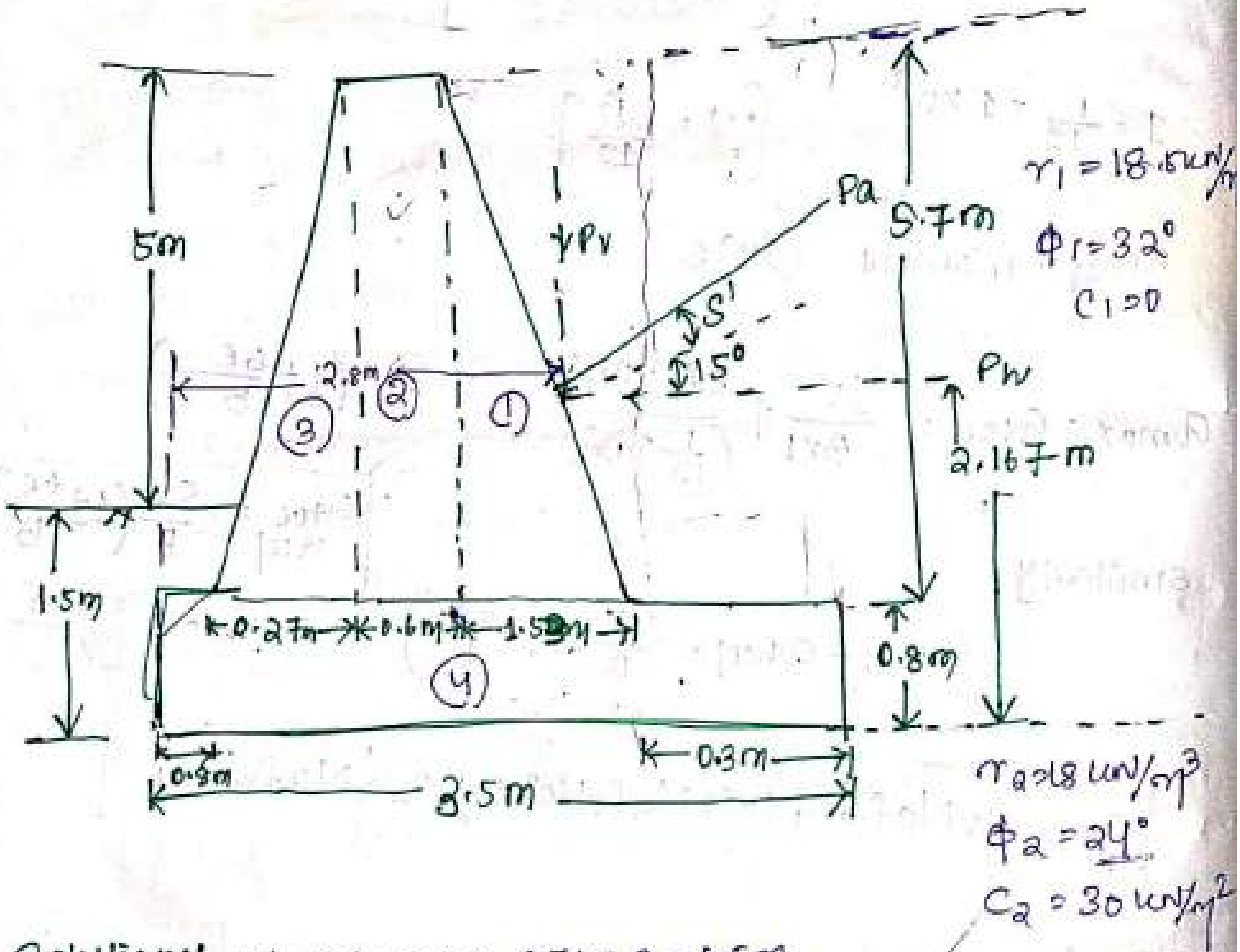
$F_{ci} = F_{qi} = \left(1 - \frac{\psi^\circ}{90^\circ}\right)^2, \quad F_{\gamma i} = \left(1 - \frac{\psi^\circ}{\phi_2^\circ}\right)^2$

$\psi^\circ = \tan^{-1}\left(P_a \cos\alpha / \Sigma V\right)$

Example 5.5

A gravity retaining walls is shown in fig. use $\delta = 2/3\phi$ and coulomb's active earth pressure theory. Determine

a. the factor of safety against overturning
b. the factor of safety against sliding
c. the pressure on the soil at the toe and heel.



Solution: $H' = 5+1.5$ or $5.7+0.8 = 6.5m$

coulomb's active force is $Pa = \frac{1}{2}\gamma H'^2 Ka$

$Ka = \cancel{0.4023}$ 0.3072

$Pa = \frac{1}{2} \times 18 \times (6.5)^2 (0.4023) = 157.22 \, kN/m$ 120 kN/m

$Ph = Pa \cos\alpha = Pa \cos(15 + 2/3\phi_1) = 157.22 \cos 36.33$
$= 126.65 \, kN/m$

and $P_v = P_a \sin\alpha = P_a \sin(15 + 2/3\,\phi_1) = 157.22 \sin 36.33 = 93\,\text{kN/m}$

Area No.	Area (m^2)	Weight (kN/m)	Moment arm from C	Moment (kN-m/m)
1	$\frac{1}{2} \times 5.7 \times 1.53 = 4.36$	102.81	2.18	224.13
2	$0.6 \times 5.7 = 3.42$	80.64	1.37	110.48
3	$\frac{1}{2} \times 0.27 \times 5.7 = 0.77$	18.16	0.98	17.80
4	$3.5 \times 0.8 = 2.8$	66.02	1.75	115.54
		$P_v = 93.14$	2.83	263.59
		$\Sigma V = 360.77\,\text{kN/m}$		$\Sigma M_R = 731.54\,\text{kN·m/m}$

$\gamma_{concrete} = 23.58\,\text{kN/m}^3$

Overturning moment $= M_0 = P_h (H/3) = 126.65\,(2.167) = 274.45\,\text{kN·m/m}$

Hence $FS_{(overturning)} = \dfrac{\Sigma M_R}{\Sigma M_0} = \dfrac{731.54}{274.45} = 2.67 > 2$ – OK.

$$FS_{(sliding)} = \frac{(\Sigma V)\tan(k_1\phi_2) + B(k_2 c_2) + P_p}{P_a \cos\alpha}$$

$$P_p = \frac{1}{2} k_p \gamma_2 D^2 + 2 c_2 \sqrt{k_p}\, D$$

$k_p = 2.37$

Hence, $P_p = 186.59\,\text{kN/m}$.

$FS_{(sliding)} = 2.84$

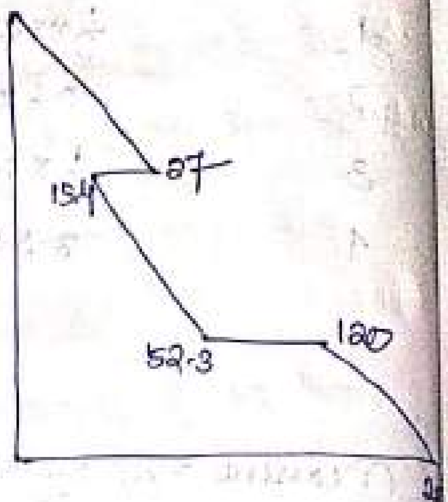
Pressure on soil at toe and heel

$$e = \frac{B}{2} - \frac{\Sigma M_R - \Sigma M_0}{\Sigma V} = \frac{3.5}{2} - \frac{731.54 - 274.45}{360.77} = 0.483$$

$$q_{toe/heel} = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right] = \begin{matrix} 188.43\,\text{kN/m}^2 \\ 17.73\,\text{kN/m}^2 \end{matrix}$$

Example:- fig shows a 3-layered backfill behind a 15m high retaining wall with a smooth vertical back. Draw the active earth pressure distribution. and total active earth pressure

	Layer	γ (kN/m³)	c (kN/m²)	ϕ
$H_1 = 5m$	1	20	0	35
$H_2 = 5m$	2	18	20	25
$H_3 = 5m$	3	16	35	0



Layer-1 (c=0 soil) $\quad K_{a_1} = \frac{1-\sin 35°}{1+\sin 35°} = 0.27$

$P_A = K_a \gamma H_1 - 2c\sqrt{K_{a_1}}$

$= 0.27 \times H_1 \times 20 - 2\times 0 \times \sqrt{0.27}$

$= 5.4 H_1$

at $H_1 = 0 \quad P_A = 0$

at $H_1 = 5m \quad P_A = 27$

Layer-2 (c-ϕ soil) $\quad K_{a_2} = 0.41$

$P_A = (K_{a_2} \gamma H_2 - 2c\sqrt{K_{a_2}}) + K_{a_2}(20\times 5)$

$= 0.41 \times 18 \times H_2 - 2\times 20 \times \sqrt{0.41} + 41$

$= 7.38 H_2 - 25.61 + 41$

at $H_2 = 0 \quad P_A = -25.61 + 41 = 15.39$

at $H_2 = 5 \quad P_A = 11.29 + 41 = 52.29$

Layer-3 (C soil) $K_4 = 1$

$P_4 = K_{a3} \gamma H_3 - 2C\sqrt{K_{a3}} + (\gamma_1 H_1 + \gamma_2 H_2) K_{a3}$

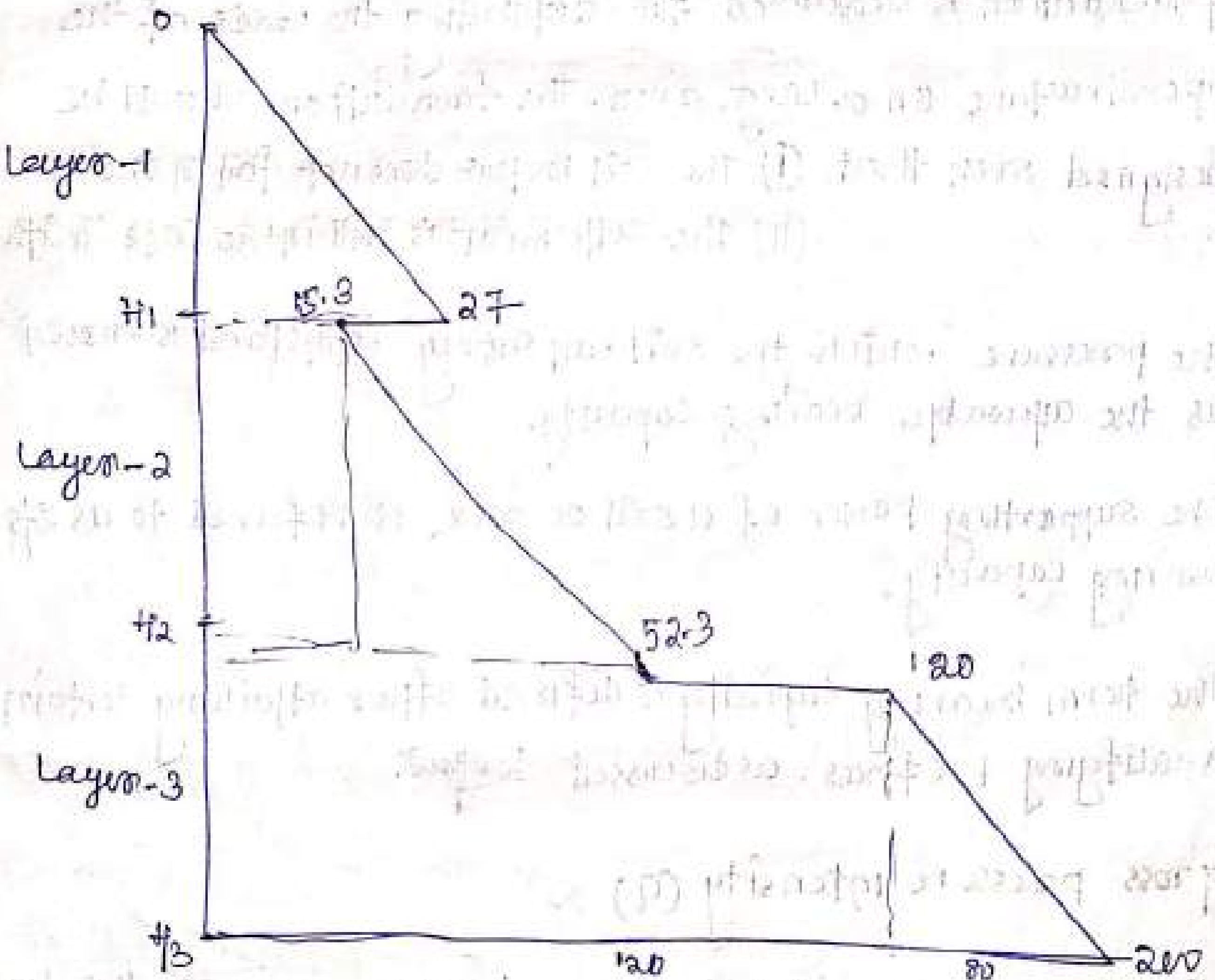
$= 1 \times 16 \times H_3 - 2 \times 35 \times \sqrt{1} + (100 + 90)$

$= 16H_3 - 70 + 190$

at $H_3 = 0$ $P_4 = -70 + 190 = 120$

at $H_3 = 5$ $P_4 = 16 \times 5 - 70 + 190$

$= 80 - 70 = 10 + 190 = 200$



Bearing Capacity

A foundation is the part of a structure which transmits the weight of the structure to the ground. All structures constructed on land are supported on foundations. A foundation is therefore a connecting zone between the structure proper and the ground which supports.

A foundation is required for distributing the loads of the superstructure on a large area. The foundation should be designed such that (i) The soil below does not fail in shear (ii) the settlement is within the safe limits.

The pressure which the soil can safely withstand is known as the allowable bearing capacity.

The supporting power of a soil or rock is referred to as its bearing capacity.

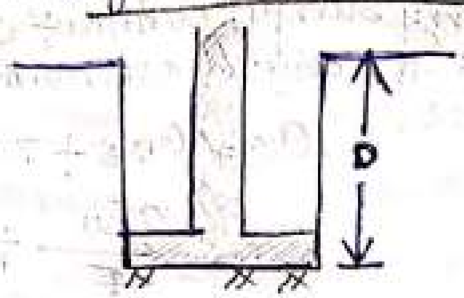
The term bearing capacity is defined after attaching certain qualifying prefixes, as discussed below.

Gross pressure intensity (q)

The gross pressure intensity q is the total pressure at the base of the footing due to the weight of the (i) super structure (ii) self-weight of footing & (iii) weight of the earth fill.

Net pressure intensity

It is the difference in intensities of the gross pressure after the construction of the structure and the original overburden pressure.



Thus, if D is the depth of footing

$$q_n = q - \bar{\sigma} = q - \gamma D$$

Ultimate bearing capacity (q_u)

The minimum gross intensity pressure intensity at the base of the foundation at which the soil fails in shear. (q_u)

[when the term bearing capacity is used without any prefix it is consider or understood to be ultimate bearing capacity.]

Net ultimate bearing capacity (q_{nu})

The minimum net pressure intensity at the base of the foundation at which the soil fail in shear.

$$q_{nu} = q_u - \gamma D$$

where, γD = effective surcharge at the base level of the foundation.

Net safe bearing capacity (q_{ns})

It is the net pressure intensity which can be safely applied to the soil considering only shear failure, which is the net ultimate bearing capacity divided by factor of safety. F.

$$q_{ns} = \frac{q_{nu}}{F}$$ where, F = factor of safety = 3.0

Gross Safe Bearing capcity / safe Bearing capacity

It is the maximum gross pressure which the soil can carry safely without shear failure. It is the net safe B.C plus the original overburden pressure.

$$q_s = q_{ns} + \gamma D$$

$$= \frac{q_{nu}}{F} + \gamma D$$

Some authors define the gross safe bearing capacity q_s as the ultimate B.C divided by a factor of safety (F).

that is $$q_s = q_u/F = \frac{q_{nu} + \gamma D}{F} = \frac{q_{nu}}{F} + \frac{\gamma D}{F}$$

As the added strength due to γD is available in full, it doesnot seem logical to apply factor of safety to this term.

Net safe settlement pressure (q_{np})

It is the net pressure which the soil can carry without exceeding the allowable settlement.

The maximum allowable settlement generally varies between 25mm to 40mm for individual footing.

Allowable bearing capacity (q_a)

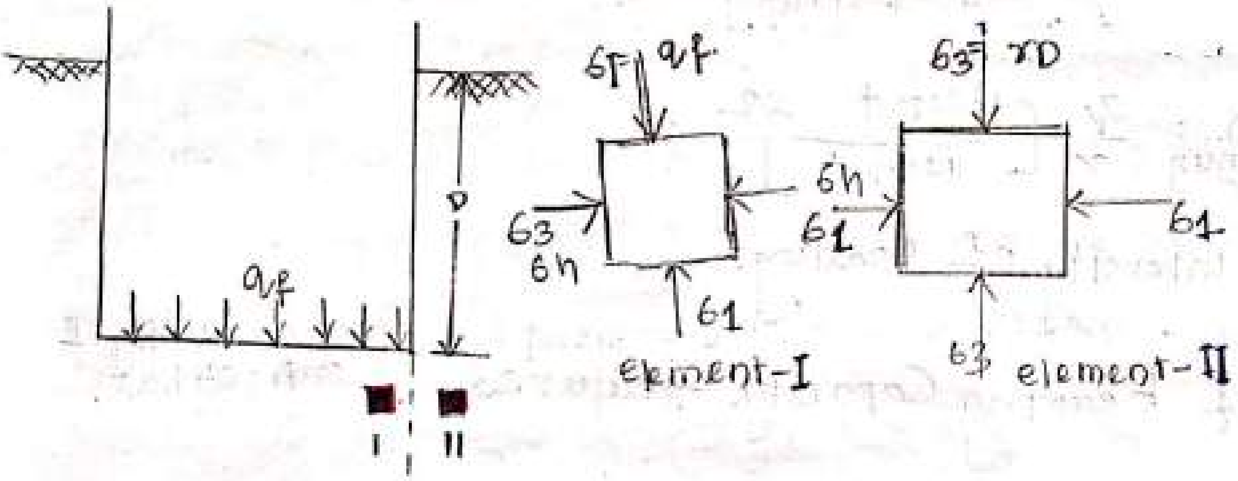
It is the net loading intensity at which neither the soil fails in shear nor there is excessive settlement.

It is also known as the allowable soil pressure or allowable bearing pressure or allowable bearing capacity.

Scanned by CamScanner

Minimum Depth of foundation : Rankine Analysis:

Rankine considered the equilibrium of two soil elements, one immediately below the foundation (element I). and another just beyond the edge of the footing (element II).



When the load on the footing increases and approaches a value q_f, a state of plastic equilibrium is reached under the footing.

I. Shear failure of element I,
II. element II must also fail by lateral thrust from element-I.

During the stage of shear failure (plastic equilibrium). the following principal stress relationship exists.

$\sigma_1 = \sigma_3 \tan^2\alpha + 2c\tan\alpha$ $\tan^2\alpha = \dfrac{1+\sin\phi}{1-\sin\phi}$

for cohesionless soil $\sigma_1 = \sigma_3 \tan^2\alpha$

For active case $\sigma_h = K_a \sigma_v$ for II $\sigma_3 = \sigma_v = \gamma d$

$\sigma_3 = \tan^2(45-\phi/2)\,\sigma_1$ $\sigma_1 = \sigma_h = K_a \sigma_v = K_a \tan^2\alpha$ — (1)

for element-I, substituting

$$\sigma_3 = \tan^2(45-\phi/2)\, q_f \quad \text{— (a)}$$

for element-II, $\sigma_3 = \gamma D$ $\sigma_h = K_a \sigma_v$

therefore $\sigma_1 = \dfrac{\gamma D}{\tan^2(45-\phi/2)}$ — (b)

Scanned by CamScanner

As σ_3 of element I is equal to σ_1 of element II,

from equation (a) and (b)

$$\tan^2(45-\phi/2)\, q_u = \frac{\gamma D_f}{\tan^2(45-\phi/2)}$$

$$q_u = \frac{\gamma D_f}{\tan^4(45-\phi/2)}$$

$$q_u = \gamma D_f \tan^4(45+\phi/2)$$

$$q_u = \gamma D_f \left[\frac{1+\sin\phi}{1-\sin\phi}\right]^2$$

the theory gives the B.C of soil is zero if $D_f = 0$. so min depth of foundation $D_{min} = \frac{q}{\gamma}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2$, where q = intensity of loading at base.

Types of Bearing Capacity failures

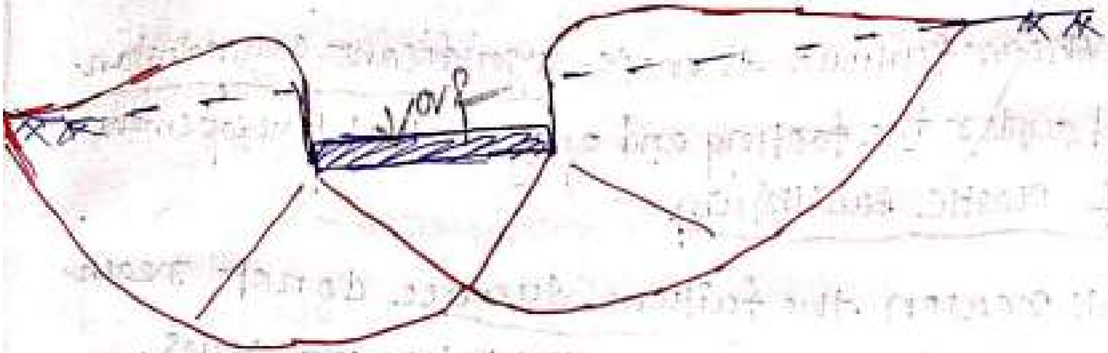
Experimental investigation have indicated that when a footing fails due to insufficient Bearing Capacity, distinct failure patterns are developed, depending upon type of failure mechanism.

Vesic (1963) observed three types of B.C failures

1. General shear failure - ϕ - Low compr./dense/stiff - $D_r > 70\%$
2. Local shear failure - significant compr. partial plastic - ϕS dist. resist - High compr. D_r 35-70 - large settlement
3. Punching shear failure.

General shear failure

In case of general shear failure, continuous failure surface develop between the edge of the footing and the ground surface, as shown in fig

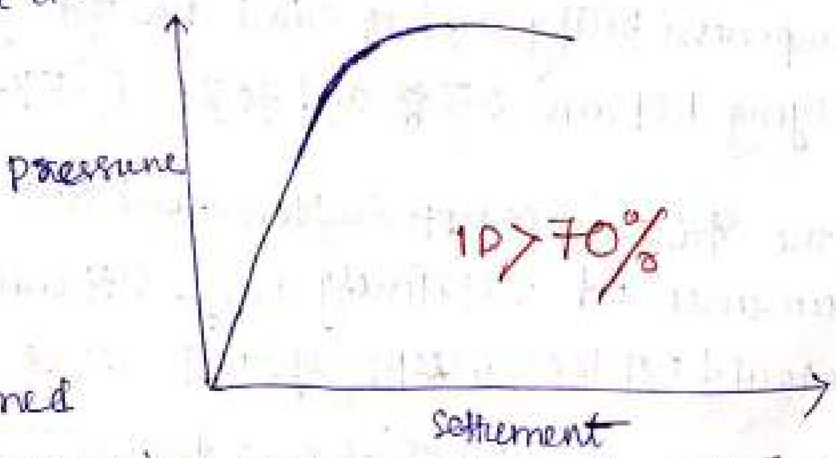


When pressure approaches the value of qf, the state of plastic equilibrium is reached initially in the soil around the edges of the footing, and it then gradually spread downwards and outwards.

Ultimately the state of plastic equilibrium is fully developed throughout the soil above the failure surface.

This type of failure occurs in soil of low compressibility i.e. dense or stiff soil.

The pressure - settlement curve is of the general form as shown is curve a.



= Characteristics

i - It has well defined failure surfaces, reaching upto ground surface.

ii - There is considerable bulging of sheared mass of soil adjacent to the footing.

iii - failure accompained by tilting of the footing

iv - failure is sudden, with pronounced peak resistance.

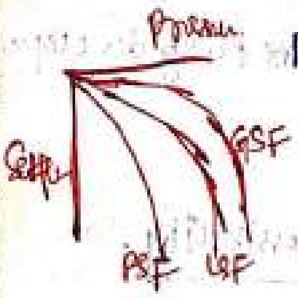
v - The Ultimate B.C is well defined.

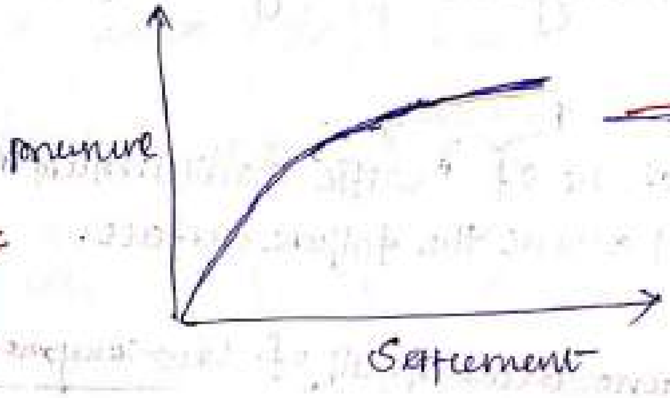
Local shear failure

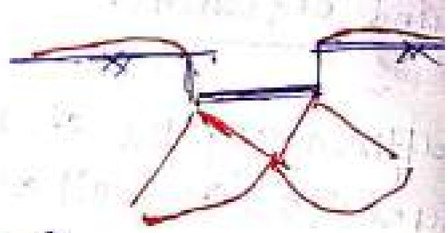
→ In local shear failure, there is significant compression of the soil under the footing and only partial development of state of plastic equilibrium.

→ Due to this reason, the failure surface do not reach the ground surface and only slight heaving occurs.

→ The pressure settlement curve is represented by curve







- In such failure tilting of foundation is not expected.
- Local shear failure is associated with soil of high compressibility and in sand having relative density lying between 35 and 70%. (35% - 70%)
- The failure is not sudden, and it is characterised by occurrence of relatively large settlements which would not be acceptable in practice.
- Also ultimate B.C in such a failure is not well defined.

Characteristics

1. Failure pattern is clearly defined only immediately below the footing.

2. the failure surface do not reach to ground surface.
3. there is only slight bulging of soil around the footing.
4. failure is not sudden and there is not tilting of footing
5. failure is defined by large settlement.
6. Ultimate B.C is not well defined.

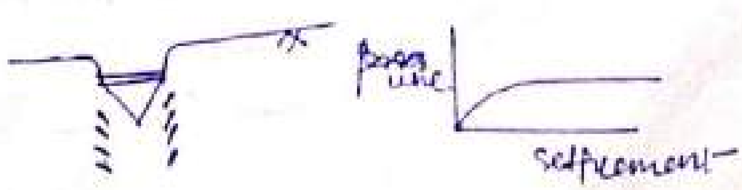
punching shear failure

· punching shear failure occurs where there is relatively high compression of soil under the footing. punching shear may occur in relatively loose sand with relative density less than 35%. (<35%)

punching shear failure may also occur in a soil of low compressibility if the foundation is located at considerable depth.

characteristics

1. No failure pattern observed
2. the failure surface, which is vertical or slightly inclined.
3. there is no bulging of soil around the footing
4. there is not tilting of footing.
5. failure is characterised in terms of very large settlements.
6. the ultimate B.C is not well defined.



Scanned by CamScanner

Terzaghi's Analysis

Terzaghi designed a general bearing capacity equation from a modification of equations proposed by Prandtl.

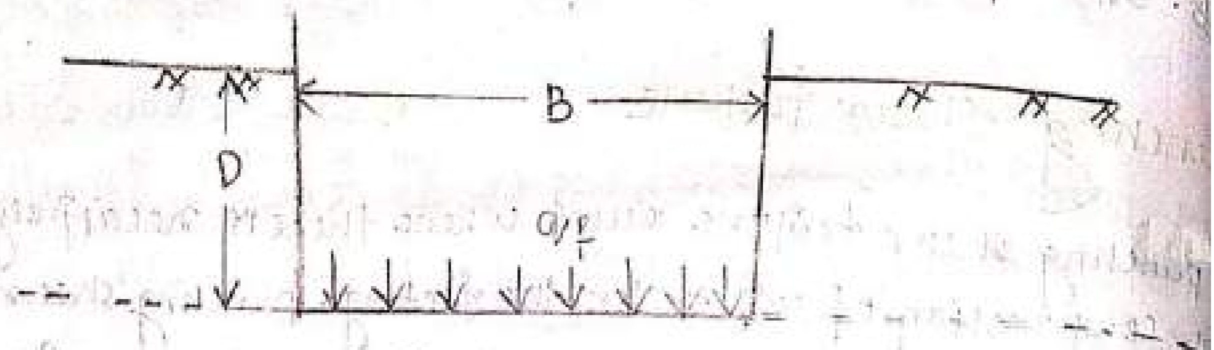
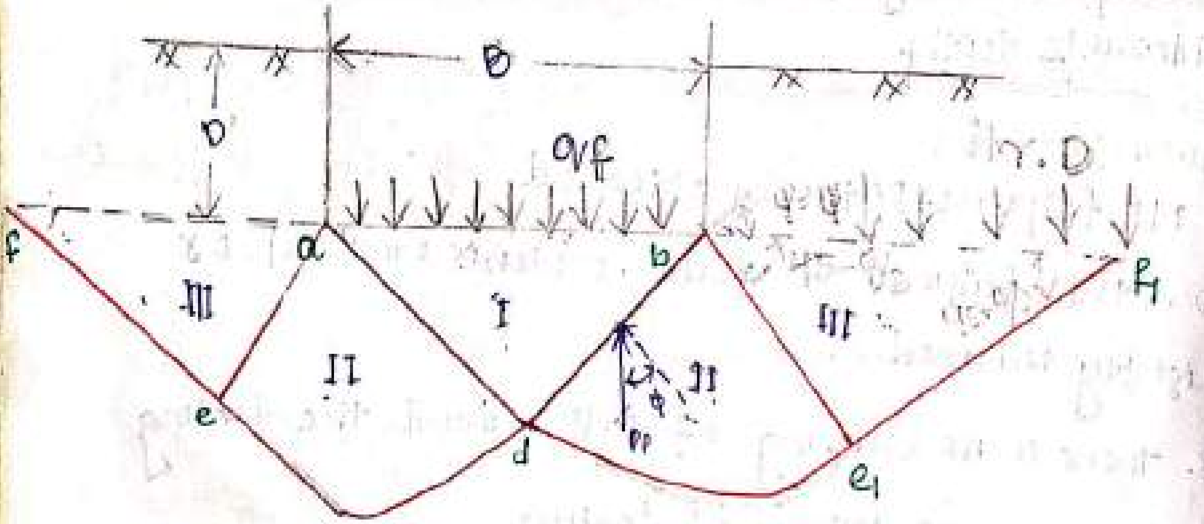


fig shows of footing width B,

Loading intensity q_f to cause failure

the footing is shallow $D \leq B$, footing is continuous



the loaded soil fails along the composite surface $fede_1f_1$.

the region can be divided into five zones: i. zone I

ii. Two pairs of zone II

iii. Two pairs of zone III

→ when the base of the footing ab sink into the ground.
→ when the base of the footing zone I (soil wedge abd) immediately beneath the footing is prevented from undergoing any lateral yield by the friction and adhesion between the soil and the base of the footing.

→ thus zone-I remains in a state of elastic equilibrium.

→ its boundaries db and da are assumed as plane surfaces, rising at an angle $\psi = \phi$ with the horizontal.

→ zone-II is called the zone of radial shear.

→ as the lines that constitute one set in the shear pattern radiate from the outer edge of the base of the footing.

→ zone-III is called the zone of linear shear and it is identical with that for passive Rankine state.

→ the boundaries of zone-III rise at $45° - \phi/2$ with the horizontal.

→ The failure zones are assumed not to extend above the horizontal plane through the base ab of the footing.

→ This implies that the shear resistance of the soil above the horizontal plane through the base of the footing is neglected and the soil above this horizontal plane is replaced with a surcharge $q = \sigma = \gamma D$

the application of qf load intensity on the footing tends to push the wedge of the soil abd into the ground with lateral displacement of zone II and III, but this lateral displacement is resisted by force on the on the plane db and da.

these forces are

(I) the resultant of the passive ~~force~~ pressure P_p

(II) the cohesion $\underline{c}$ acting along the surface ~~d~~ a and db

the passive pressure resultant makes an angle ϕ with the normal to the surfaces. da and db.

if it is assumed that surfaces db and da intersect the horizontal line at an angle ϕ. the passive pressure act vertically

At instant of failure, the downward and upward forces on the wedge adb of unit length must balance.

the downward forces are (i) $q_f B$

(ii) the weight $\frac{1}{4} \gamma B^2 \tan\phi$

the upward force are (i) the resultant passive pressure P_p on each of the surface db and da

(ii) the vertical component of cohesion acting along the lengths ad and bd.