DESIGN AND CONSTRUCTION OF GRAVITY DAMS

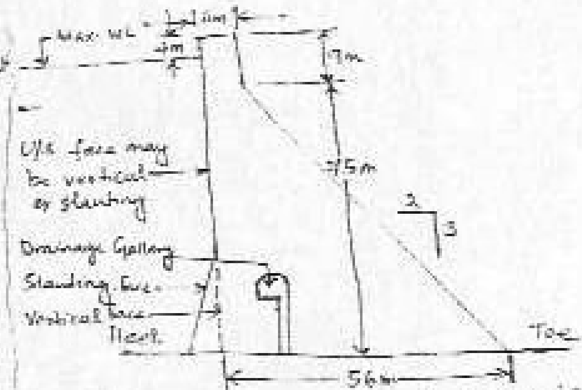
Gravity dam is defined as a dam structure designed in such a way that its own weight resists the external forces coming over it. Most durable & requires low maintenance cost. Gravity dams are constructed with masonry or concrete.

- They can be easily constructed on any dam site having proper natural foundations to bear the weight of the dam.
- Dam is a barrier across flowing water that obstructs, direct or slows down the flow, often creating a reservoir, lake or impoundments.

- Dams are considered "installations containing dangerous forces" under International Humanitarian Law, due to the massive impact of a possible destruction on the civilian population & the environment.

Typical c/s of a Dam

- The u/s face may be kept straight vertical or partly slanting for some of its length.
- A drainage gallery is provided in order to relieve the uplift pressure exerted by the seeping water.



[A typical cross-section of a concrete gravity dam]

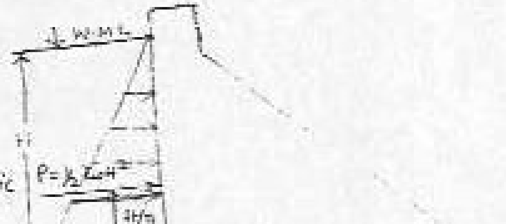
FORCES ACTING ON GRAVITY DAM

The various external forces acting on a gravity dam may be:

(1) Water Pressure
(2) Uplift Pressure
(3) Pressure due to earthquake forces
(4) Silt Pressure
(5) Wave Pressure
(6) Ice Pressure
(7) The stabilising force is the weight of dam itself.

(1) Water Pressure

- Major external force on gravity dams
- Horizontal water pressure, exerted by the weight of the water stored on the u/s side on the dam can be estimated from rule of hydrostatic pressure distribution, which is triangular in shape.



γ_w = unit wt. of water
9.81 kN/m^3 = 1000 kgf/m^3

- When the u/s face is vertical, the intensity is zero at the water surface & equal to $\gamma_w H$ at the base

Resultant force due to this external water

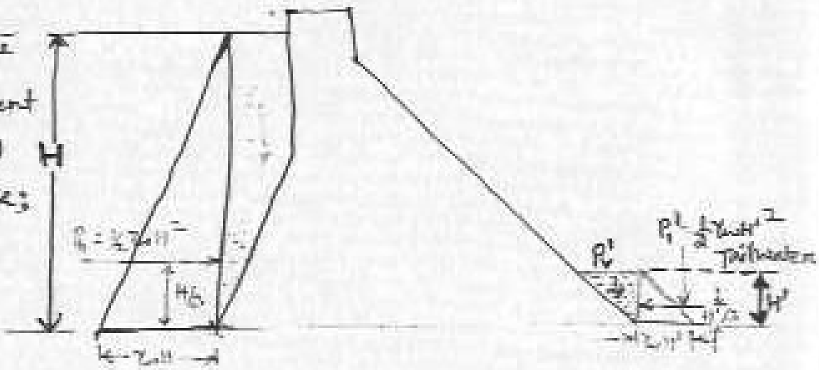
$$= \frac{1}{2}\gamma_w H^2, \text{ acting at } \frac{H}{3} \text{ from base}$$

Case

· When the u/s face is partly vertical & partly inclined, the resulting water force can be resolved into horizontal component (P_h) and vertical component (P_v)

$P_h = \frac{1}{2}\gamma_w H^2$ acts at $H/3$ from base;

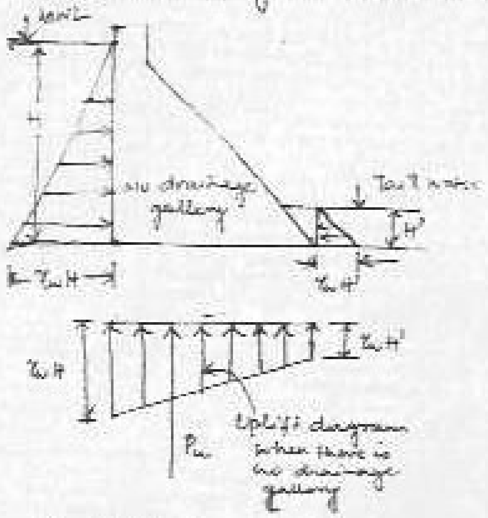
P_v = Weight of the water stored in the column ABCA & acts at the c.g. of the area



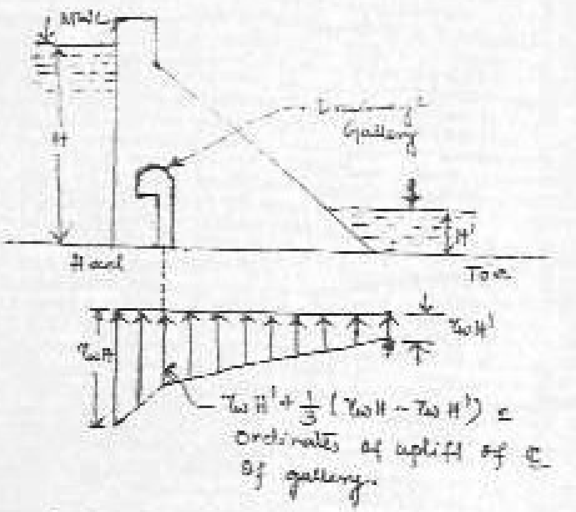
* Similar formulation for tail water on d/s side.

(2) Uplift Pressure

– Uplift force vertically reduces the downward weight of the body of the dam & hence, acts against dam stability.



(a) Uplift pressure (U) diagram, when no drainage gallery is provided



(b) Uplift pressure (U) diagram, when drainage gallery is provided

Uplift pressures can be controlled by

(a) constructing cut-off walls under the u/s face,

(b) constructing drainage channels b/w the dam & its foundations

(c) pressure grouting the foundation

(3) Earthquake Forces

– An earthquake produces waves which are capable of shaking the earth upon which the dam is resting, in every possible direction.

– Earthquake wave may move in any direction, and for design purposes, it has to be resolved in vertical & horizontal components. Hence 2 accelerations, i.e. one horizontal acceleration (α_h) & one vertical acceleration (α_v) are induced by earthquake.

– The values of acceleration are generally expressed as some percentage of acceleration due to gravity (g);

$\alpha = 0.1g$ or $0.2g$ etc.

- On an average, a value of α_h = 0.1 to 0.15 g is generally sufficient for high dams in seismic zones.
- For areas not subjected to extreme earthquakes α_h = 0.1 g & α_v = 0.05 g may be used.
- In extremely seismic regions, even a value upto 0.3 g may sometimes be adopted.

EFFECT OF VERTICAL ACCELERATION (α_v)

- Vertical acceleration may either act downward or upward.

Upward direction :- When it is acting in the upward direction, then the foundation of the dam will be lifted upward and becomes closer to the body of the dam, & thus effective weight of the dam will increase & hence, stress developed will increase.

Downward direction :- When vertical acceleration is acting downward, the foundation shall try to move downward away from the dam body; thus reducing the effective weight & the stability of the dams & hence is the worst case for designs.

- Acceleration will therefore exert an inertia force given by,

$$\frac{W}{g}\alpha_v \; ; \text{ where } W \to \text{the total weight of the dam.}$$

$$\text{Net effective weight of the dam} = W - \frac{W}{g}\alpha_v$$

If $\alpha_v = K_v\, g$ [K_v = fraction of gravity adopted for vertical acceleration = 0.1 or 0.2]

$$\therefore \text{Net effective weight of the dam} = W - \frac{W}{\cancel{g}} \cdot K_v \cdot \cancel{g}$$

$$= W\left[1 - K_v\right]$$

In other words, vertical acceleration reduces the unit weight of dam material & that of water to $(1-K_v)$ times their original unit weights.

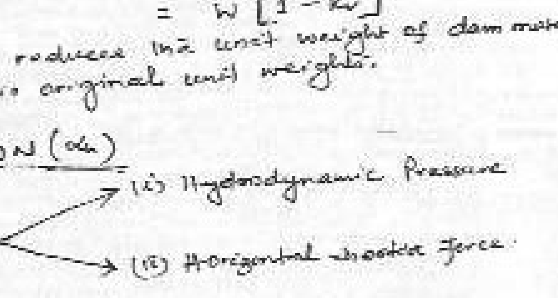
EFFECTS OF HORIZONTAL ACCELERATION (α_h)

Horizontal acceleration may cause 2 forces → (i) Hydrodynamic Pressure; (ii) Horizontal inertia force.

Hydrodynamic Pressure :-

Horizontal acceleration acting towards the reservoir causes a momentary increase in water pressure, as the foundation & dam accelerate towards the reservoir and the water resists the movement owing to its inertia.

The extra pressure exerted by this process is known as hydrodynamic pressure.



$$P_e = 0.555\, K_h\, \gamma_w\, H^2$$; and it acts at a height $\frac{4H}{3\pi}$ above the

Moment of this force about base

$$= M_e = P_e\left(\frac{4H}{3\pi}\right) = 0.424\, P_e\, H$$

(ii) Horizontal Inertia Force-

In addition to exerting the hydrodynamic pressure, the horizontal acceleration produces an inertia force into the body of the dam. This force is generated in order to keep the body & the foundation of the dam together as one piece.

The direction of the produced force will be opposite to the acceleration imparted by earthquake.

$$\text{Horizontal Inertia Force} = \left(\frac{W}{g}\right)\alpha_h = \frac{W}{g}\times k_h\cdot g = W k_h.$$

The force should be acting at the c.g of the mass, regardless of the shape of the cross-section.

(4) Silt Pressure

If h is the height of silt deposited, then the force exerted by this silt in addition to external water pressure, can be represented by Rankine's formula as:

$$P_{silt} = \frac{1}{2}\,\gamma_{sub}\, h^2\, K_a \quad \text{\& it acts at } \frac{h}{3} \text{ from base.}$$

where K_a is the coefficient of active earth pressure of silt $= \dfrac{1-\sin\phi}{1+\sin\phi}$, where ϕ is the angle of internal friction of soil, & cohesion is neglected.

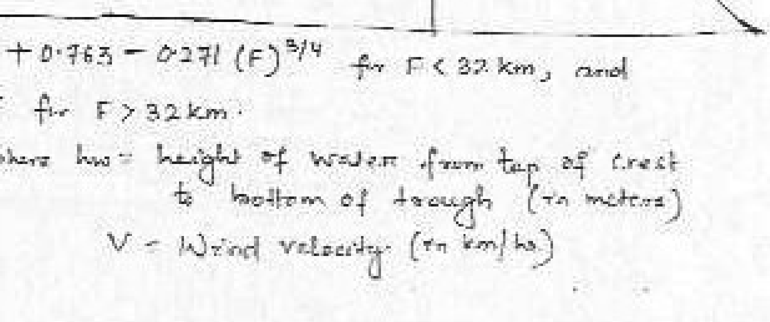
γ_{sub} = Submerged unit weight of silt material

h = height of silt deposited.

- This pressure is neglected while designing high concrete dams. When silt deposits over long period in such dams, the silt consolidates & it act as impervious layer to the entry/seepage of the water b/L the foundation.

(5) Wave Pressure

- Waves are generated on the surface of the reservoir by the blowing winds, which causes a pressure towards the d/s side.
- Wave pressure depends on wave height.



Wave height $= h_w = 0.032\sqrt{V\cdot F} + 0.763 - 0.271\,(F)^{3/4}$ for $F < 32$ km, and

$h_w = 0.032\sqrt{V\cdot F}$ for $F > 32$ km.

where h_w = height of water from top of crest to bottom of trough (in metres)

V = Wind velocity (in km/hr)

The maximum pressure intensity due to wave action may be given by:

$P_m = 2.4\,\gamma_w h_w$ & acts at $\left(\frac{h_w}{8}\right)$ m above the still water surface.

The pressure distribution may be assumed to be triangular, of height $\frac{5h}{3}$

$$P_w = \frac{1}{2} \times \left(2.4\,\gamma_w h_w\right) \times \frac{5}{3} h_w$$

$$= 2\,\gamma_w h_w^2 = 2 \times 9.81 \times h_w^2 \text{ kN/m}$$

$= 19.62\, h_w^2$ kN/m ; P_w acts at a distance $\frac{3}{8}h_w$ above the still water surface (or) above the reservoir surface.

(6) <u>Ice Pressure</u>

- Ice may be formed on the water surface of the reservoir in cold countries, may sometimes melt & expand.
- Dam face has to resist the thrust exerted by expanding ice.
- This force acts linearly along the length of the dam & at reservoir level.
- Magnitude of this force varies from 250 – 1500 kN/m² depending upon temp. variations. On an average, a value of 500 kN/m² may be allowed under ordinary conditions.

(7) <u>Weight of the dam (The stabilising force)</u>

- Weight of the dam and its foundation is the major resisting force.
- In 2-D analysis of a gravity dam, a unit length of dam is considered.
- Dam cross-section can be divided into rectangles & triangles. The weight of each along their c.g's can be determined. The resultant of all these downward forces will represent the total weight of the dam acting at the c.g. of the dam.

COMBINATION OF FORCES FOR DESIGNS

Design of a gravity dam should be checked for two cases, i.e.,
(i) When Reservoir is full ; and
(ii) When Reservoir is empty.

(a) Case I. Reservoir full case:

- When Reservoir is full, major forces acting are:
(a) weight of the dam (b) external water pressure (c) uplift pressure, and (d) earthquake pressure in various seismic zones.
- Minor forces are: wave pressure, silt pressure & Ice pressure.
- From theoretical point of view, such situation may arise when all forces may act together. But in practice such a situation will [illegible] arise & hence all the forces are not generally taken together.

(a) Normal load Combinations

(i) Water pressure upto normal pool level + normal uplift + silt pressure and silt pressure. This class of loading is taken when the force is serious.

(ii) Water pressure upto normal pool level, normal uplift + earthquake forces and silt pressures.

(iii) Water pressure upto maximum reservoir level (max^{m} pool level), normal uplift and silt pressures.

(b) Extreme load Combinations.

(i) Water pressure due to max^{m} pool level + extreme uplift pressure without any reduction due to drainage + silt pressure.

(2) Case II. Reservoir empty case:

(i) Empty reservoir without earthquake forces to be computed for determining bending diagrams, etc. for reenforcement design, for grouting studies or other purposes.

(ii) Empty reservoir with a horizontal earthquake force produced towards the u/s has to be checked for non-development of tension at toe.

MODES OF FAILURE AND CRITERIA FOR STRUCTURAL STABILITY OF GRAVITY DAMS.

A gravity dam may fail in the following ways:

(1) By Overturning (or rotation) about the toe.
(2) By Crushing
(3) By development of tension, causing ultimate failure by crushing.
(4) By shear failure called sliding

(1) <u>OVERTURNING</u>.

- If the resultant of all forces acting on a dam at any of its sections, passes outside the toe, the dam shall rotate and overturn about the toe.
- Practically, such a condition shall not arise, as the dam will fail much earlier by compression.
- Factor of safety against Overturning = $\dfrac{\text{Resisting (i.e. self wt. of the dam) Righting moments about toe (Anti clockwise)}}{\text{Overturning moments about toe (Clockwise)}}$

Ultimate stress / Working stress

It's value generally varies between 2 - 3.

(2) <u>COMPRESSION or CRUSHING</u>

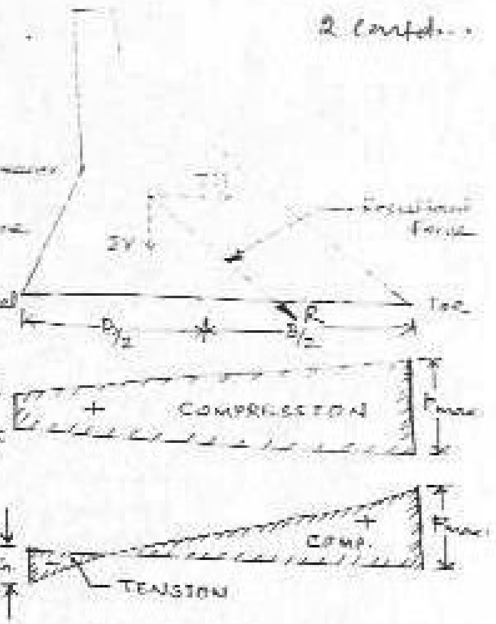
- A dam may fail by the failure of its material, i.e; the compressive stress produced may exceed the allowable stress, and the dam material may get crushed.
- The vertical direct stress distribution at the base is given by the equation:

$$p = \text{Direct stress} + \text{Bending stress}.$$

$$P_{max/min} = \frac{\Sigma V}{B} \pm \frac{M}{I} y = \frac{\Sigma V}{B} \pm \frac{\Sigma V \cdot e}{B^2/6} = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right]$$

$P_{max/min} = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right]$, where e = Eccentricity of the resultant

NOTE: Resultant is nearer the toe & hence maximum compressive stress is produced at the toe (Reservoir full case)



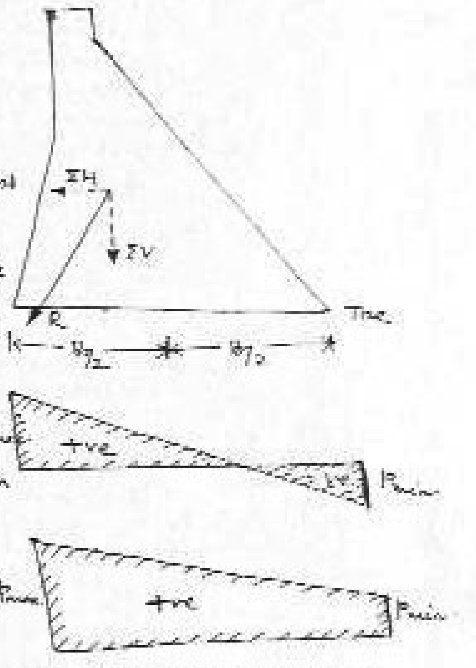
[Vertical stress Distribution for Reservoir full case]

- The maximum stress i.e.; p_{max} will be produced on the end which is nearer to the resultant.

- If P_{min} comes out to be negative, it means that tension shall be produced at the appropriate end.

- If P_{min} exceeds the allowable compressive stress of dam material [generally taken as 3000 KN/m² or 30 kg/cm² for Concrete]; the dam may crush & fail by crushing.

NOTE: The resultant is nearer the heel & hence most compressive stress (+ve Heel stress) is produced at the heel. (Reservoir empty with horizontal earthquake wave moving away from reservoir case)



[Vertical Stress Distribution for Reservoir Empty case]

(3) TENSION

- Masonry and concrete gravity dams are usually designed in such a way that no tension is developed anywhere, because these materials cannot withstand sustained tensile stress.

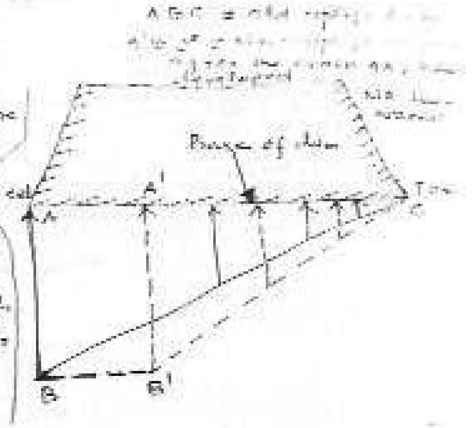
Inorder to ensure that no tension is developed anywhere, we must ensure that P_{min} is at the most equal to zero.

$$P_{min} = \frac{\Sigma V}{B}\left[1 - \frac{6e}{B}\right] = 0$$

$$\text{or } 1 - \frac{6e}{B} = 0$$

Hence a maximum value of eccentricity that can be permitted [illegible] is equal to $\frac{B}{6}$

"The resultant must lie within the middle third"



— Tension, in case of high gravity dams may be permitted under severest loading condition. This may be permitted because of the fact that such worst loading conditions shall occur only momentarily for a little time & would neither last long nor occur frequently.

— Maximum permissible tensile stress for high concrete gravity dams, under worst loadings, may be taken as 500 kN/m² (5 kg/cm²).

Effect produced by tension cracks:

In a dam, when such a tension crack develops, say at the heel, crack width (crack-area) looses contact with the bottom foundations, & thus, becomes ineffective.

Hence the effective width B (considering unit length) of the dam base will be reduced. This will increase P_{max} at toe. Since the uplift increases and the net effective downward force reduces, the resultant will shift more towards the toe & thus further increasing the compressive stress at the toe & further lengthening the crack due to further tension development. The process continues; the effective base width goes on reducing & compressive stress at the toe goes on increasing finally leading to the failure of the toe by direct compression.

"A tension crack by itself does not fail the structure, but it leads to the failure of the structure by producing excessive compressive stress".

(4) SLIDING

Sliding (or shear failure) will occur when the net horizontal force above any plane in the dam or at the base of the dam exceeds the frictional resistance developed at that level.

Friction developed between two surface = $\mu \Sigma V$.

where ΣV = algebraic sum of all vertical forces whether upward or downward.

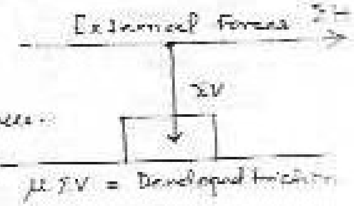
μ = coefficient of friction b/w two surfaces.

If no sliding takes place,

External horizontal force (ΣH) < shear resistance ($\mu \Sigma V$)

or $\Sigma H < \mu \Sigma V$

or $\frac{\mu \Sigma V}{\Sigma H} > 1$ = factor of safety against sliding. (which must be greater than unity)



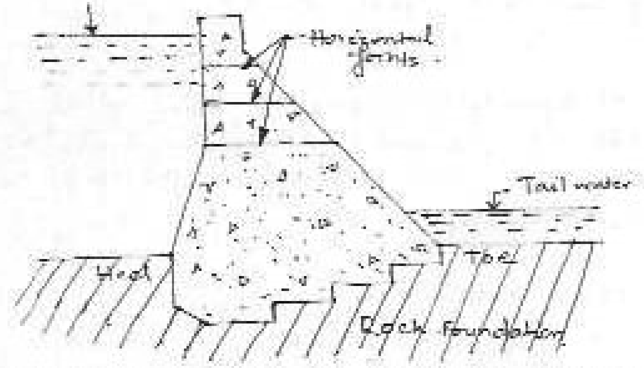
$$F.S.S = \frac{\mu \Sigma V}{\Sigma H}$$

NOTE: In low dams, the safety against sliding should be checked only for

In dams, if [illegible] the joints which is the additional shear resistance must also be considered. If this shear resistance of the joint is also considered, then the equation for factor of safety against sliding which is measured by shear friction factor (S.F.F) becomes,

$$S.F.F = \frac{\mu \Sigma V + B \cdot q}{\Sigma H}$$

where B = width of the dam at the joint



q = Average shear strength of the joint which varies from about 1400 kN/m² (14 kg/cm²) for poor rocks to about 4000 kN/m² (40 kg/cm²) for good rocks.

μ = μ varies from 0.65 – 0.75.

- Attempts are always made to increase this shear strength (q) at the base & at other joints. For this purpose, foundation is stepped at the base & measures are taken to ensure a better bond between the dam base & the rock foundation.

PRINCIPAL AND SHEAR STRESSES

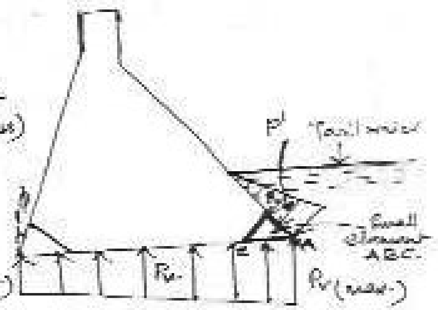
- The vertical stress intensity, P_{max} or P_{min} is not the maximum direct stress produced anywhere in the dam.
- The maximum normal stress will, in fact, be the major principal stress that will be generated on the major principal plane.

$$\sigma = P_v \sec^2\alpha - p' \tan^2\alpha$$

p' is the intensity of water pressure on face AB

P_v is the intensity of vertical pressure on face AC &

σ is the intensity of normal stress (principal stress) on face BC.



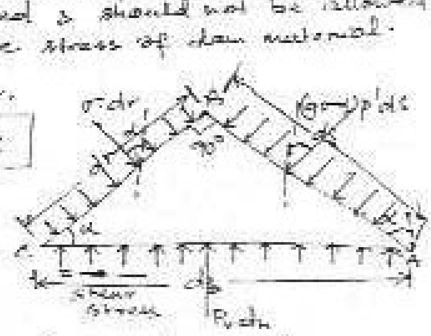
For σ to be maximum, p' should be zero, i.e. when there is no tail water,

$$\sigma = P_v \sec^2\alpha \qquad \sigma > P_v$$

Since $\sec^2\alpha$ is always more than 1, it follows that σ will be more than P_v. This value of normal stress, which is the maximum produced anywhere on the body of the dam, must be calculated & should not be allowed to exceed the maximum allowable compressive stress of dam material.

- The principal stress (σ) can then be given by,

$$\boxed{\sigma_{at\ toe} = P_v \sec^2\alpha - (p' - p_e') \tan^2\alpha}$$



- p_e' → hydrodynamic pressure exerted by the tail water during an earthquake moving towards the reservoir. Then the net pressure on the face AB will be

$\sigma_1 = \sigma_{heel} = p_v \sec^2\phi - (p + p_e') \tan^2\phi$

where ϕ is the angle which the u/s face makes with the vertical

But at the heel, the pressure of water p is always more than σ, and hence p will be the minor principal stress at the heel

<u>SHEAR STRESS ON THE HORIZONTAL PLANE NEAR THE TOE</u>

$$\tau_0 = (p_v - p') \tan\alpha$$

Neglecting tail water, shear stress is given by,

$$\tau_0 = p_v \cdot \tan\alpha$$

- If the effect of hydrodynamic pressure produced by an earthquake moving towards the reservoir, is also considered, the equation for shear stress on a horizontal plane near the toe becomes

$$\tau_0 = [p_v - (p' - p_e')] \tan\alpha$$

Similarly, Shear stress at heel

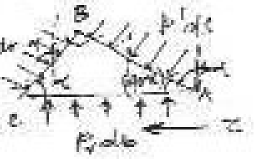
$$= \tau_{0(heel)} = [p_v - (p + p_e)] \tan\phi .$$

–ve sign shows that the direction is reversed.

Considering unit length of the dam, the forces acting on the faces AB, BC and CA are $p' ds$, $\sigma \cdot dr$ and $p_v \cdot db$ resp.

Let the d/s face AB makes an angle α with the vertical.

Resolving all the forces in the vertical direction,

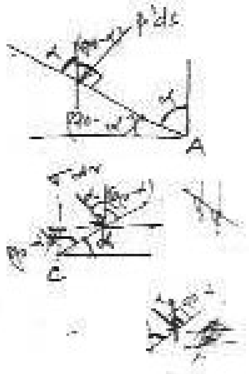


$$p' ds \cos(90 - \alpha) + \sigma \cdot dr \cos\alpha = p_v db$$

or $p' ds \sin\alpha + \sigma \cdot dr \cos\alpha = p_v db$

Now, $\sin\alpha = \frac{ds}{db}$ or $ds = db \sin\alpha$

or $\cos\alpha = \frac{dr}{db}$ or $dr = db \cos\alpha$



or $p'(db \sin\alpha) \cdot \sin\alpha + \sigma(db \cos\alpha) \cdot \cos\alpha = p_v db$

or $p' \sin^2\alpha + \sigma \cos^2\alpha = p_v$

or $$\sigma = \frac{p_v - p' \sin^2\alpha}{\cos^2\alpha} = p_v \sec^2\alpha - p' \tan^2\alpha .$$

$$\boxed{\sigma = p_v \sec^2\alpha - p' \tan^2\alpha}$$

<u>for shear stress</u> → Resolving all the forces in the horizontal direction, we get

→ +ve
← -ve

$$\sigma \cdot dr \sin\alpha - p' ds \sin(90 - \alpha) = \tau_0 db .$$

or $\sigma \cdot dr \sin\alpha - p' ds \cos\alpha = \tau_0 db .$

or $\sigma (db) \sin\alpha \cdot \cos\alpha - p' (db) \sin\alpha \cos\alpha = \tau_0 db .$

or $\tau_0 = (\sigma - p') \sin\alpha \cos\alpha$

or $\tau_0 = [p_v \sec^2\alpha - p' \tan^2\alpha - p'] \sin\alpha \cos\alpha .$

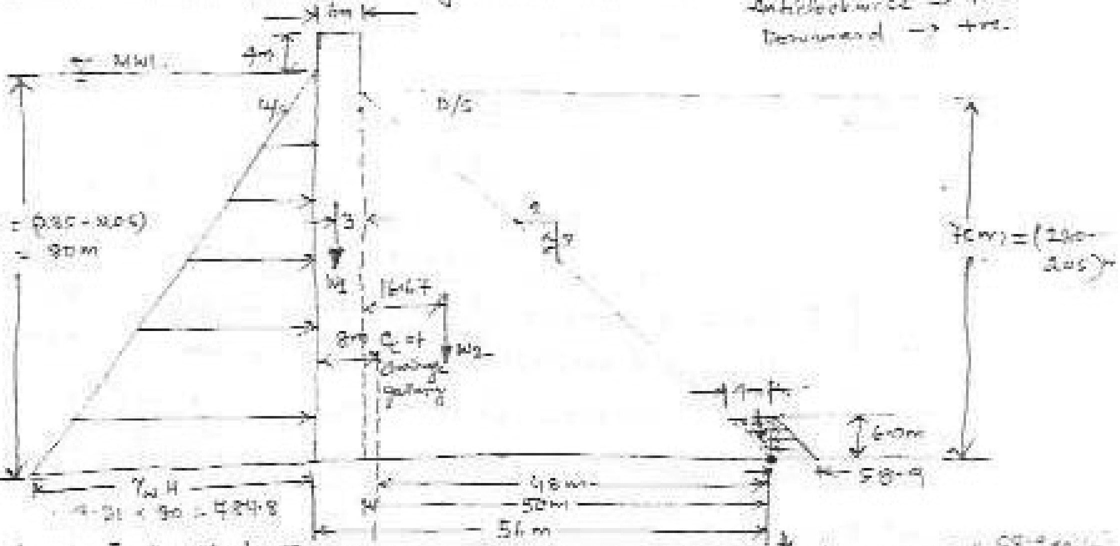
or $\tau_0 = [p_v \sec^2\alpha - p'(1 + \tan^2\alpha)] \sin\alpha \cdot \cos\alpha = [(p_v - p') \sec^2\alpha] \sin\alpha \cos\alpha$

or $\tau_0 = (p_v - p') \sec \frac{1}{\ } \cdot \sin\alpha \cos\alpha = (p_v - p') \tan\alpha$

Assume weight of concrete = 23.5 kN/m³; and unit length of dam.
Allowable stress in concrete = 2500 kN/m²; γ_w = 9.81 kN/m³.

Solution: The various forces acting on the dam are shown below -

Anticlockwise → +ve.
Downward → +ve.



Consider 1m length of dam.

$\gamma_w H'$ = 9.81 × 6 = 58.9 kN/m²

$\gamma_w H$ = 9.81 × 80 = 784.8 kN/m²

$\gamma_w H' + \frac{1}{3}[\gamma_w H - \gamma_w H']$ = 9.81 [6 + 1/3 (80 − 6)] = 300.8 kN/m²

Name of the force	Designation of force	Magnitude in kN	Lever arm in m	Moment about toe in kN-m
Vertical Forces				
1. Downward weight of the dam	W_1	(+) 84 × 6 × 1 × 23.5 = 11,844	53.0	6,27,732
	W_2	(+) ½ × 50 × 75 × 1 × 23.5 = 44,063	⅔ × 50 = 33.33	14,68,620
2. Weight of water supported on u/s face	—	(+) ½ × 4 × 6 × 1 × 9.81 = 118	4/4 × 4 = 133	157
		ΣV = 56,025		ΣM = 20,96,509
3. Uplift Pressures	U_1	(−) 300.8 × 8 × 1 = 2406	48 + 8/2 = 52	(−) 125112
	U_2	(−) ½ × 8 × 484 × 1 = 1936	[48 + ⅔ × 8] = 53.33	(−) 1,03,247
	U_3	(−) 48 × 58.9 × 1 = 2827	24.0	(−) 67,848
	U_4	(−) ½ × 48 × 241.9 = 5806	⅔ × 48 = 32	(−) 1,85,792

Name of the Force	Designation	Magnitude in kN	Lever arm	Moments about toe in kN-m
Horizontal water Pressure				
On u/s face	P	(−) ½ × 784·8 × 80 × 1 = 31,392	$\frac{80}{3}$ = 26.67	(−) 8,37,225
On d/s face	P'	(+) ½ × 58·9 × 6 × 1 = 177	2.0	(+) 354
		ΣH = 31,215 (towards d/s)		Σ = (−) 8,36,871

ΣM = Net (+) Moment = (+) 20,96,509 − 4,81,999 − 8,36,871 = 7,77,639 kN-m

~~∴ Maximum Vertical Stress Produced~~

Distance of resultant from the toe $(\bar{x}) = \frac{\Sigma M}{\Sigma V}$

$$= \frac{7,77,639}{43050} = 18.06 \text{ m}.$$

$e = \left(\frac{B}{2} - \bar{x}\right)$

Eccentricity $= e = \frac{56}{2} - 18.06 = 28 - 18.06 = 9.94$ m.

Vertical stress P_v is given as,

$$P_v = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right]$$

$$\text{or } P_v = \frac{43050}{56}\left[1 \pm \frac{6 \times 9.94}{56}\right]$$

or $P_v = 768.8\,(1 \pm 1.065)$

∴ Maximum Vertical Stress = P_{max} at toe = 768.8 × 2.065 = 1587.6 kN/m² < 3000 kN/m² (max)

Minimum Vertical Stress = P_{min} at heel = − 768.8 × 0.065 = (−) 49.97 kN/m² < 500 (min)

Ans.

(ii) Major Principal stress at toe (σ) is given by

$$\sigma_{(toe)} = P_{v(toe)} \sec^2\alpha - p' \cdot \tan^2\alpha$$

$p' \rightarrow$ Tail water pressure at d/s end.

$P_v = 1587.6$ kN/m²

$$\tan\alpha = \frac{2}{3}.$$

$$\sec^2\alpha = 1 + \tan^2\alpha = 1 + \left(\frac{2}{3}\right)^2 = 1 + \frac{4}{9} = \frac{13}{9}.$$

$$\text{or } \sigma = 1587.6 \times \frac{13}{9} - 58.9 \times \frac{4}{9} = 2267 \text{ kN/m}^2 < 2500 \text{ kN/m}^2$$

(OK) Ans.

(iii) Intensity of shear stress on a horizontal plane near toe is given by,

$$\tau_0 = \left[P_{v(toe)} - p'\right] \tan\alpha$$

$$= \left[1587.6 - 58.9\right] \times 2/3$$

$$= 1019.1 \text{ kN/m}^2. \quad \text{Ans.}$$

This is the maximum possible ... position of the resultant (e) ... to develop.

Hence ... $B = \frac{H}{\sqrt{S_c - C}}$ ... the most ideal ... the most stabilising moment about the toe ... will be without causing tension ... full reservoir. Reservoir empty.

Let the [illegible] at the heel with ... developed ... P = External water pressure or Hydrostatic water pressure.

— Vertical stress distribution at the base when the reservoir is empty, ... $c = 1$

$$B = \frac{H}{\sqrt{\frac{S_c}{B}}}, \quad P_{max/min} = \left[1 \pm \frac{6e}{B}\right]$$ Here $\Sigma V = W$, $e = \frac{B}{6}$

If uplift is not considered,

$$P_{max/min} = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right] \quad (\because e = 0) = \frac{2W}{B}$$

(ii) For the II condition (i.e., dam is safe in sliding) to be satisfied, the frictional resistance $\mu \Sigma V$ or $\mu(W - U) \geq P = \frac{\gamma_w H^2}{2}$ (horizontal forces)

Hence, the maximum vertical stress $= \frac{2W}{B}$ will act at heel, since the resultant is nearer the heel.

or $\mu\left(\frac{1}{2} B H S_c \gamma_w - \frac{1}{2} C \gamma_w H B\right) \geq \frac{\gamma_w H^2}{2}$

When the reservoir is full, the base width is governed by:

(i) The resultant (i.e., all the forces, P, W and U) passes through the outer most middle third point (i.e., lower middle third point)

or

(ii) The dam is safe in sliding.

(i) For first condition: $\boxed{B \geq \frac{H}{\mu(S_c - C)}}$

Taking moments of all forces about the lower middle third point through which resultant is passing, we get:

$$W\left(\frac{B}{3}\right) - U\left(\frac{B}{3}\right) - P\left(\frac{H}{3}\right) \quad \boxed{B = \frac{H}{\sqrt{S_c - C}}}$$

If $C = 1$, $B = \frac{H}{\mu(S_c - 1)}$

If $C = 0$, i.e., no uplift is considered,

$$B \geq \frac{H}{\mu S_c}$$ where S_c = sp. gr. of concrete ... of the material of the dam.

→ The vertical stress distribution when reservoir is full ... $= 9.81\ kN/m^3$.

Let the uplift ... where C is a constant which according to U.S.B.R. recommendation is taken equal to 1 ... calculation will be equal to zero when uplift is considered.

$$U = \left(\frac{1}{2} \cdot B \cdot H \cdot 1 \cdot S_c \gamma_w - \frac{1}{2} C \cdot \gamma_w \cdot H \cdot B\right)$$

[illegible]

$$\Rightarrow B^2 (S_c - C) = H^2$$

$$\Rightarrow \boxed{B = \frac{H}{\sqrt{S_c - C}}}$$

$C = 1$, ... $C = 0$, when no uplift is considered

$B \geq \frac{H}{\sqrt{S - C}}$, no tension will be developed at the heel with full reservoir.

when $C = 1$

$$B = \frac{H}{\sqrt{S - 1}}$$

If uplift is not considered,

$$B = \frac{H}{\sqrt{S}} \quad (\because C = 0)$$

(ii) For the II condition (i.e., dam is safe in sliding) to be satisfied, the frictional resistance $\mu \Sigma V$ or $\mu (W - U) \geq P$ = ΣH (horizontal forces)

or $\mu (W - U) \geq P$.

or $\mu \left(\frac{1}{2} B H S_c \gamma_w - \frac{1}{2} C \gamma_w H B \right) \geq \frac{\gamma_w H^2}{2}$.

or $\mu (S_c - C) \frac{1}{2} B H \gamma_w \geq \frac{\gamma_w H^2}{2}$

or $\mu (S_c - C) B \geq H$

$$\boxed{B \geq \frac{H}{\mu (S_c - C)}}$$

Under limiting condition $\boxed{B = \frac{H}{\mu (S_c - C)}}$

If $C = 1$, $B = \frac{H}{\mu (S_c - 1)}$

If $C = 0$, i.e., no uplift is considered then:

$$B \geq \frac{H}{\mu S_c}$$

$\Rightarrow$ The vertical stress distribution when reservoir is full is given as:

$$P_{max/min} = \frac{\Sigma V}{B} \left[1 \pm \frac{6e}{B} \right]$$

where $\Sigma V = W - U$

$$\Sigma V = \left(\frac{1}{2} B \cdot H \cdot 1 \times S_c \gamma_w - \frac{1}{2} C \cdot \gamma_w \cdot H \cdot B \right)$$

$$= \frac{1}{2} B \cdot H \gamma_w (S_c - C)$$

$$\boxed{P_{v} \text{ at toe} = \gamma_w H (S_c - C)}$$

P_v at heel $= 0$.

The principal stress near the toe (σ) which is the maximum compressive stress for the dam,

$$\sigma = P_v \sec^2\alpha - p' \tan^2\alpha$$

when there is no tail water i.e; $p' = 0$

$$\sigma = P_v \sec^2\alpha$$

At toe, with full reservoir in elementary profile

$$= \gamma_w H (S_c - C) \sec^2\alpha$$

$$= \gamma_w H (S_c - C) \left[1 + \tan^2\alpha\right]$$

$$= \gamma_w H (S_c - C) \left[1 + \frac{B^2}{H^2}\right]$$

$\tan\alpha = \frac{B}{H}$

But $B = \dfrac{H}{\sqrt{S_c - C}}$

$$\frac{B^2}{H^2} = \frac{1}{S_c - C}$$

$$\sigma = \gamma_w H (S_c - C) \left[1 + \frac{1}{S_c - C}\right]$$

$$\boxed{\sigma = \gamma_w H (S_c - C + 1)}$$

The shear stress τ_0 at a horizontal plane near the toe is given by

$$\tau_0 = (P_v - p') \tan\alpha$$

if $p' = 0$ (tail water $= 0$)

$$\tau_0 = P_v \tan\alpha$$

But $P_v = \gamma_w H (S_c - C)$

$$\tau_0 = \gamma_w H (S_c - C) \tan\alpha$$

or $\tau_0 = \gamma_w H (S_c - C) \dfrac{B}{H}$

$$\tau_0 = \gamma_w H (S_c - C) \times \frac{1}{\sqrt{S_c - C}} \qquad \left[\because B = \frac{H}{\sqrt{S_c - C}}\right]$$

$$\boxed{\tau_0 = \gamma_w H \sqrt{S_c - C}}$$

— To ... failure by crushing, the value of $\sigma \leq$ max^m allowable compressive stress.

$$f = \gamma_w H (S_c - C + 1)$$

$$H = \frac{f}{\gamma_w (S_c - C + 1)}$$

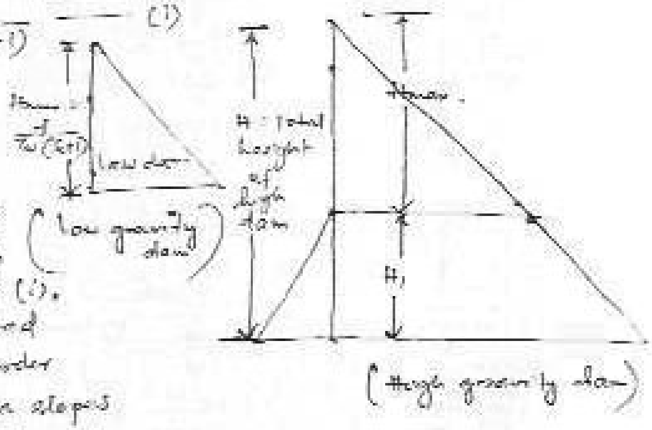
$f \rightarrow$ Allowable stress of dam material.

— The lowest value of H will be obtained when $C = 0$, i.e., when uplift is neglected. Hence, for determining the limiting height & to be on a safer side, uplift is neglected.

Hmax. i.e., max^m possible height is given as:

$$H_{max} = \frac{f}{\gamma_w (S_c + 1)} \quad \text{--- (1)}$$

Hmax is the max^m possible height, for which the dam is to be designed as a low gravity dam.



— Hence if the height of a dam having an elementary profile of a triangle, is more than that given by the eq^n (1), the max^m compressive stress generated will exceed the allowable value. In order to keep it safe within limits, extra slopes on the u/s as well as on the d/s, below the limiting height.

— Limiting height (Hmax) is given by $\frac{f}{\gamma_w (S_c + 1)}$; draws a dividing line between a low gravity dam & a high gravity dam, which are purely technical terms to differentiated between them.

— A low gravity dam is the one whose height is less than that given by eq^n (1). If the height of the dam is more than this, it is known as high gravity dam.

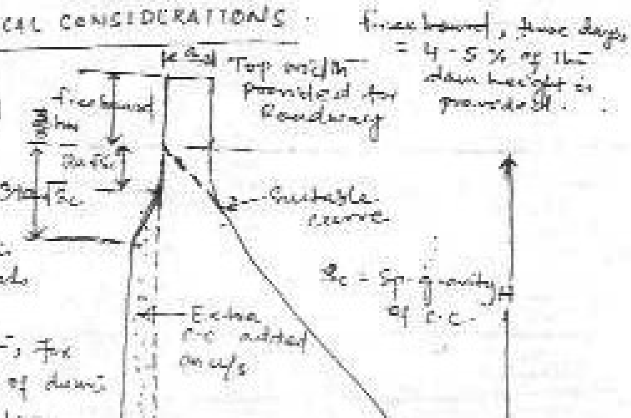
PROFILE OF A DAM FROM PRACTICAL CONSIDERATIONS.

— The elementary profile of a gravity dam (i.e., a triangle with max^m water surface at apex) is only a theoretical profile.

— Certain changes to be made in this profile to cater to the practical needs. These needs are:

(i) Providing a straight top width, for a road construction over the top of dam;

(ii) Providing a free board above the top water surface, so that water may not spill over the ...



– The [illegible] of force from pressure, will cause the resultant force to shift towards the heel. The resultant force when the reservoir is empty, may [illegible] passing through a point beyond the middle third point. This will [illegible] the shift more towards the heel, increasing the [illegible] Consequently, tension will be developed at the toe. In order to avoid the development of this tension, some masonry or concrete will have to be added to the u/s side.

<u>Design of high gravity dams.</u>

1. Design a concrete gravity dam for the following data:

Maximum allowable compressive stress in concrete $= 3000\ kN/m^2$
Maximum reservoir level $= 200.0\ m$
R.L of bottom of dam $= 100.0\ m$
Specific gravity of concrete $= 2.4$
Unit wt. of water $= 10\ kN/m^3$

<u>Solution.</u>

The free board cannot be calculated as the wave height etc. are not given. So for practical profile of dam assume a free board = 3–4% of dam height (RL 200 – RL 100 = 100m)

i.e. 3% of 100 m $= 0.03 \times 100 = 3m$.

R.L of the top of the dam $= 200.0 + 3.0 = 203.0\ m$.

Height of low gravity dam $= H_1 = \dfrac{f}{\gamma_w (S_c + 1)}$

$$H_1 = \frac{3000}{10(2.4+1)} = 88.2 < \text{Height of the dam (i.e., 103 mts.)}$$

Therefore, it is a high gravity dam.

Hence the dam from RL 200 m to RL $(200 - 88.2)$m = RL 111.8 m shall be designed as a low gravity dam, and the remaining bottom height of the dam from RL 111.8 m to RL 100 m shall be designed on the principles of high gravity dam.

Design of low dam between RL 200.0 m to R.L 111.8

Top width required $= a = \sqrt{\dfrac{H_1}{3.28}} = \sqrt{\dfrac{88.2}{3.28}} = \sqrt{26.9} = 5.18m$.

Base width required $= \dfrac{H}{\sqrt{S}} = \dfrac{88.2}{\sqrt{2.4}} = 56.8m$

The u/s projection from the vertical face required,

$$= \frac{a}{16} = \frac{5.18}{16} = 0.33m$$

Total base width (B_1) provided $= (56.8 + 0.33) = 57.13m$

The u/s batter starts at a depth $= 2a\sqrt{S_c}$

$= 2 \times 5.18 \sqrt{2.4} = 16.1m$; from MWL

and u/s batter ends at a depth $= 3.1\ a \sqrt{S_c}$

$= 3.1 \times 5.18 \sqrt{2.4} = 24.9m$ below MWL

[illegible]

Total height of the dam [illegible] = [illegible]
to be designed as a high gravity dam

Let us now divide the [illegible] length into 3 strips.

Depth of I strip = 3.8 m

Depth of II strip = 4 m

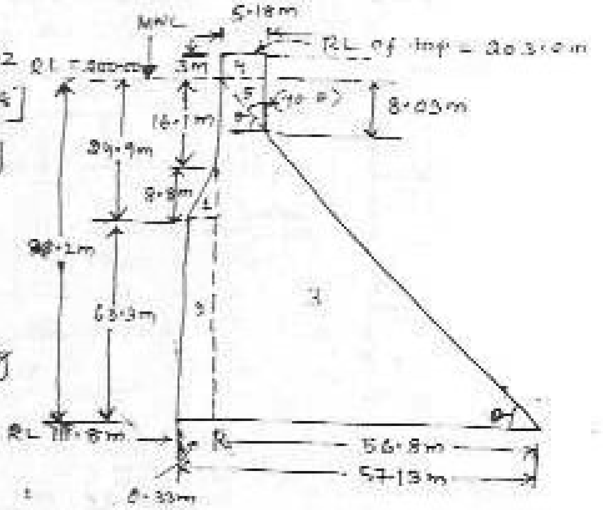
Depth of III strip = 4 m

Weight of dam section upto RL 111.8 m (i.e., weight of low dam)

$$= 24 \times 1 \left[\tfrac{1}{2} \times 3.8 \times 0.33 + 63.3 \times 0.33 + \tfrac{1}{2} \times 56.8 \times 88.2 + 3 \times 5.18 + \tfrac{1}{2} \times 8.03 \times 5.18 \right]$$

$$= 24 \left[1.5 + 20.9 + 2{,}505 + 15.5 + 20.8 \right]$$

$$= 24 \left[2{,}563.7 \right] = 61{,}600 \text{ kN}$$



Low dam portion from RL 200 to RL 111.8

Approximate width of bottom of 1st strip (say B_2') is obtained by drawing a horizontal line at RL 108 m & by producing the already provided d/s face & u/s face of low dam.

B_2' = Approximate width of bottom of 1st strip (say B_2').

$$B_2' = 57.13 + \cot\theta \times 3.8$$

$$= 57.13 + \left(\frac{56.8}{88.2}\right) \times 3.8$$

$$= 59.58 \text{ m}$$

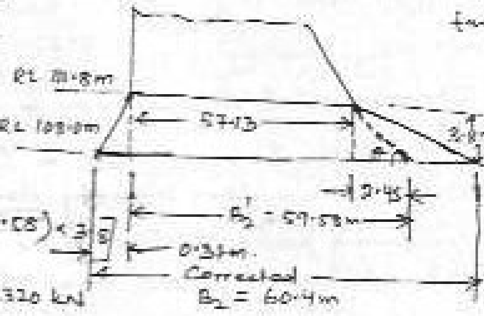
$$\tan\theta = \frac{88.2}{56.8}$$

$$\theta = \tan^{-1}\left(\frac{88.2}{56.8}\right) \quad 57.21$$

$$(90-\theta) = 32.78°$$

$$\tan(90-\theta) = \frac{5.18}{x}$$

$$\text{or } x = \frac{5.18}{\tan 32.78°} = 8.03 \text{ m}$$



Approximate weight of 1st strip =

$$\left[24 \times 1 \times \tfrac{1}{2} (57.13 + 59.58) \times 3.8 \right]$$

$$= 24 \times 58.35 \times 3.8 = 5320 \text{ kN}$$

Weight of water resting on u/s face $= 10 \times 1 \left[16.1 \times 0.33 + \tfrac{1}{2} \times 8.8 \times 0.33 \right]$

$$= 10 \times 0.33 \times 20.5 = 67.7 \text{ say } 68 \text{ kN}$$

Total weight of dam & water at top of 1st strip i.e., at base of small dam,

$$= W_1 = 61{,}600 + 68 = 61668 \text{ kN, say } 61670 \text{ kN} = W_1$$

Total approximate weight of dam & water at base of 1st strip.

$$= (61670 + 5320) = 66990 \text{ kN} = W_2$$

The correct base width B_2, which shall keep the maximum compressive stress within the allowable limits is given by,

$$B_2 = \sqrt{\frac{\gamma_w H_2^3}{f} \left[1 + \frac{\gamma_w^2 H_2^4}{4 W_2^2} \right]}$$

$H_2 = 88.2 + 3.8 = 92$ m.

$W_2 = 66{,}990$ kN

$f = 3000$ kN/m²

$\gamma_w = 10$ kN/m³

- The addition of these two pressures will cause the resultant force to shift towards the face. The resultant then [illegible] ... [illegible] middle third [illegible] ... tension will be developed at the toe. In order to avoid the development of this tension, some masonry or concrete will have to be added to the u/s side.

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[illegible]

Total height of the dam [illegible]
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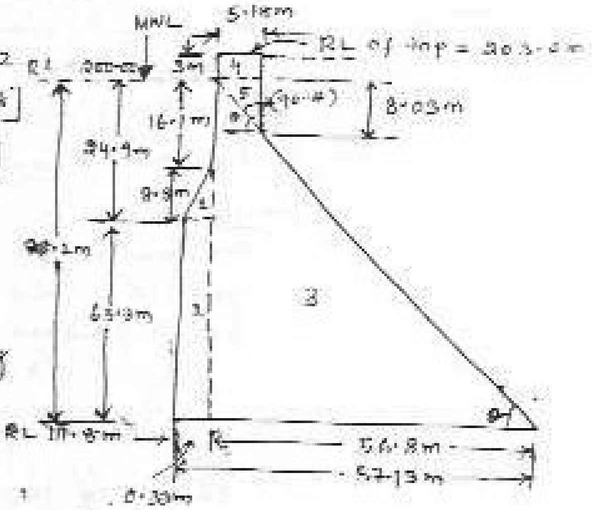

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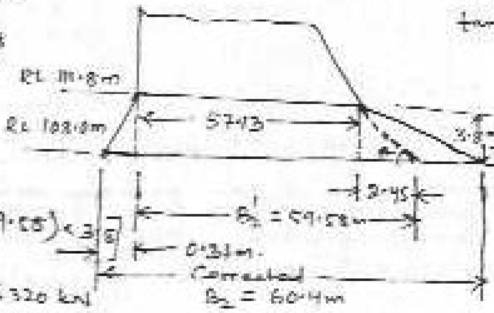

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$\gamma_w = 10$ kN/m3

[illegible]