... designed as a high gravity dam ...

Let us divide the height ... into 3 strips:

Depth of I strip = 3.8m

Depth of II strip = 4m

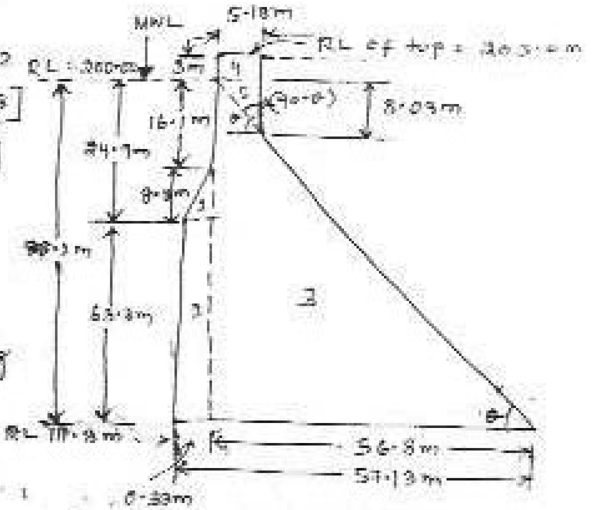
Depth of III strip = 4m

Weight of dam section upto RL 111.8m (i.e., weight of low dam)

$$= 24\times1\left[\tfrac{1}{2}\times8.8\times0.33+63.3\times0.33+\tfrac{1}{2}\times56.8\times88.2+3\times5.18+\tfrac{1}{2}\times8.03\times5.18\right]$$

$$= 24\left[1.5+20.9+2{,}505+15.5+20.8\right]$$

$$= 24\left[2{,}563.7\right] = 61{,}600\ \text{kN}$$



Low dam portion from RL 200 to RL 111.8

$\tan\theta = \dfrac{88.2}{56.8}$

$\theta = \tan^{-1}\left(\dfrac{88.2}{56.8}\right) = 57.21$

$(90-\theta) = 32.79°$

$\tan(90-\theta) = \dfrac{5.18}{x}$

or $x = \dfrac{5.18}{\tan 32.79°} = 8.03\text{m}$

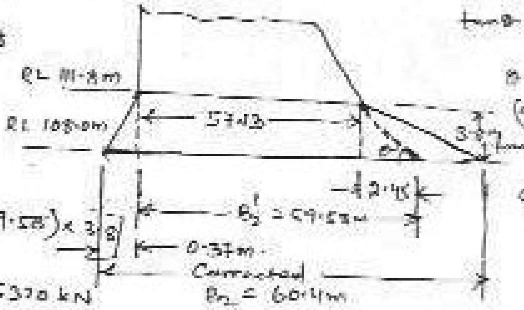
Approximate width of bottom of 1st strip (say B_2') is obtained by drawing a horizontal line at RL 108 m & by producing the already provided d/s face & u/s face of low dam.

B_2' = Approximate width of bottom of 1st strip (say B_2').

$$B_2' = 57.13+\cot\theta\times3.8$$

$$= 57.13+\left(\frac{56.8}{88.2}\right)\times3.8$$

$$= 59.58\text{m}$$



Approximate weight of 1st strip =

$$\left[24\times1\times\tfrac{1}{2}(57.13+59.58)\times3.8\right]$$

$$= 24\times58.35\times3.8 = 5320\ \text{kN}$$

Weight of water resting on u/s face $= 10\times1\left[16.1\times0.33+\tfrac{1}{2}\times8.8\times0.33\right]$

$= 10\times0.33\times20.5 = 67.7$ say 68 kN.

Total weight of dam & water at top of 1st strip i.e., at base of small dam,

$= W_1 = 61{,}600+68 = 61668$ kN; say 61670 kN $= W_1$

Total approximate weight of dam & water at base of 1st strip.

$= (61670+5320) = 66990$ kN $= W_2$

The correct base width B_2, which shall keep the maximum compressive stress within the allowable limits is given by,

$$B_2 = \sqrt{\frac{\gamma_w H_2^2}{f}\left[1+\frac{\gamma_w^2 H_2^4}{4W_2^2}\right]}$$

$H_2 = 88.2+3.8 = 92$ m.

$W_2 = 66{,}990$ kN

$f = 3000\ \text{kN/m}^2$

$\gamma_w = 10\ \text{kN/m}^3$

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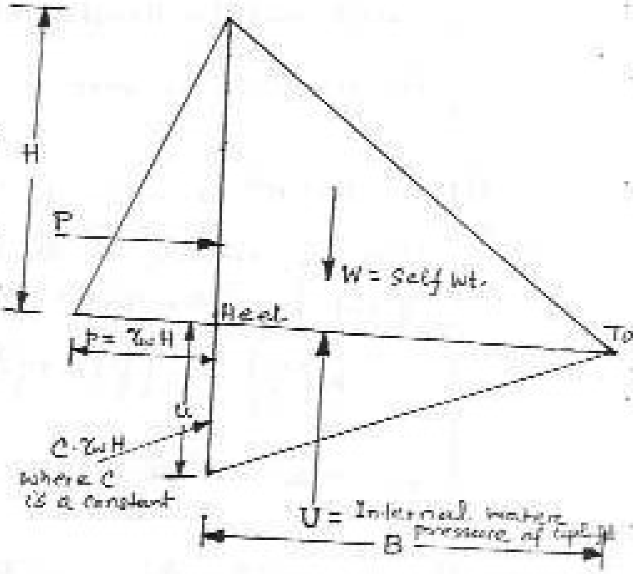
C = a constant, called seepage coefficient.



I When the reservoir is empty.

- Single force acting on it is the self-weight (W) of the dam.
  - It acts at a distance B/3 from the heel.
    (This is the maximum possible innermost position of the resultant for no tension to develop.)
  - Line of action of W is the most ideal, as it gives the maximum possible stabilising moment about toe without causing tension at toe, when reservoir is empty.
- The vertical stress distribution at the base,

$$P_{max/min} = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right]$$

Here $\Sigma V = W$, & $e = \frac{B}{6}$.

$$P_{max/min} = \frac{W}{B}\left[1 \pm \frac{6}{B} \times \frac{B}{6}\right]$$

or $P_{max} = \frac{2W}{B}$ and $P_{min} = 0$.

Hence, the maximum vertical stress $= \frac{2W}{B}$ (will act at the heel)
($\because$ the resultant is nearer the heel.)

the vertical stress at toe will be zero.

II. When the reservoir is full

- The base width is governed by :

(i) Resultant of all the forces, i.e; P, W & U passes through the outer most middle third point (i.e; lower middle third point).

(ii) The dam is safe in sliding.

(i) For the 1st condition to be satisfied.

Taking moments of all the forces about the lower middle third point (i.e. the point through which resultant is passing), we get

$$W\left(\frac{B}{3}\right) - U\left(\frac{B}{3}\right) - P\left(\frac{H}{3}\right) = R \times 0$$

$$\text{or} \quad (W-U)\,\frac{B}{3} - P\left(\frac{H}{3}\right) = 0 \qquad (i)$$

But $W = \frac{1}{2} \times B \times H \times 1 \times S_c \times \gamma_w$

where S_c = Sp. gravity of concrete, i.e; that of material of the dam.

γ_w = Unit wt. of water = 9.81 kN/m

Let uplift at the heel = $C \gamma_w H$,

where, C = Seepage coefficient (a constant)

[According to U.S.B.R recommendation is taken equal to 1.0 in calculation & will be equal to 0 when no uplift is considered.]

$$\therefore \quad U = \left(\frac{1}{2} C \gamma_w H\right) \cdot B$$

and $P = \frac{1}{2} \gamma_w H \cdot H = \frac{\gamma_w H^2}{2}$

Putting the values of W, U & P in eqn (i) we get,

$$\left[\frac{1}{2} \times B \times H \times S_c \times \gamma_w - \frac{1}{2} C \gamma_w H B\right] \frac{B}{3} - \frac{\gamma_w H^2}{2} \cdot \frac{H}{3} = 0$$

$$\text{or} \quad \frac{B}{3} \times \frac{1}{2} \times B \times H \times \gamma_w \left[S_c - C\right] = \frac{\gamma_w H^3}{6}$$

$$\text{or} \quad B^2 \left(S_c - C\right) = H^2$$

$$\text{or} \quad \boxed{B = \frac{H}{\sqrt{S_c - C}}} \qquad (ii)$$

Hence, if B is taken equal to or greater than $\frac{H}{\sqrt{S_c - C}}$, no tension will be developed at the heel with full reservoir, i.e.,

when $C = 1$ $\quad B = \frac{H}{\sqrt{S_c - 1}}$

If uplift is not considered, $B = \frac{H}{\sqrt{S_c}}$ $\quad (\because C = 0)$

(ii) For the II condition (i.e., dam is safe in sliding) to be satisfied.

The frictional resistance $= \mu \Sigma V = \mu (W - U) \geq$ Horizontal Forces $(\Sigma H = P)$

$$\text{or} \quad \mu (W - U) \geq P$$

$$\text{or} \quad \mu \left[\frac{1}{2} B H S_c \gamma_w - \frac{1}{2} C \gamma_w H B \right] \geq \frac{\gamma_w H^2}{2}$$

$$\text{or} \quad \mu (S_c - C) \cancel{\tfrac{1}{2}} B \cancel{H} \cancel{\gamma_w} \geq \frac{\cancel{\gamma_w} H^{\cancel{2}}}{\cancel{2}}$$

$$\text{or} \quad \mu (S_c - C) B \geq H$$

$$\text{or} \quad B \geq \frac{H}{\mu (S_c - C)}$$

If $C = 1$; $\quad B \geq \frac{H}{\mu (S_c - 1)}$

If $C = 0$; i.e., no uplift is considered, then

$$B \geq \frac{H}{\mu S_c}$$

Under limiting condition

$$\boxed{B = \frac{H}{\mu (S_c - C)}} \qquad \text{(iii)}$$

The value of B chosen should be greater of the two values given by equations (ii) & (iii)

➔ Using $S_c = 2.4$ and $\mu = 0.7$ and $C = 0$, we get

$$B \text{ (by eq}^n \text{ (ii))} = \frac{H}{\sqrt{2.4 - 0}} = \frac{H}{\sqrt{2.4}}$$

$$B \text{ (by eq}^n \text{ (iii))} = \frac{H}{0.7(2.4 - 0)} = \frac{H}{1.68}$$

But $\frac{H}{1.68} < \frac{H}{\sqrt{2.4}}$

$\therefore$ For all practical purposes, the base width may be taken as $\frac{H}{\sqrt{S_c}}$

VERTICAL STRESS

The vertical stress distribution when reservoir is full is given as.

$$P_{max/min} = \frac{\Sigma V}{B}\left[1 \pm \frac{6e}{B}\right]$$

where $\Sigma V = W - U$

$$= \left[\frac{1}{2} \cdot B \cdot H \cdot 1 \cdot S_c \cdot \gamma_w - \frac{1}{2} C \cdot \gamma_w \cdot H \cdot B\right]$$

$$= \frac{1}{2} B \gamma_w H \left[S_c - C\right]$$

$$e = B/6.$$

$$\therefore P_{max/min} = \frac{\frac{1}{2} \cdot B \cdot \gamma_w H (S_c - C)}{B}\left[1 \pm \frac{6B}{B6}\right]$$

maximum stress will occur at toe, because the resultant is near the toe.

$$\therefore P_{max} \text{ at toe} = \frac{1}{2} \gamma_w H (S_c - C) \times 2 = \gamma_w H (S_c - C)$$

$$\boxed{P_v \text{ at toe} = \gamma_w H (S_c - C)} \qquad (iv)$$

$$P_{min} \text{ at heel} = 0.$$

PRINCIPAL STRESS.

The principal stress near the toe (σ) which is the maximum normal stress in the dam.

$$\sigma = P_v \sec^2\alpha - p' \tan^2\alpha$$

when there is no tail water i.e; $p' = 0$

$$\sigma = P_v \sec^2\alpha$$

σ at toe, with full reservoir in elementary profile

$$\sigma = \gamma_w H (S_c - C) \sec^2\alpha = \gamma_w H (S_c - C)\left[1 + \tan^2\alpha\right]$$

$$= \gamma_w H (S_c - C)\left[1 + \frac{B^2}{H^2}\right]$$

But $B = \frac{H}{\sqrt{S_c - C}}$ (from eqn (iii))

or $\frac{B^2}{H^2} = \frac{1}{(S_c - C)}$

$$\therefore \quad \sigma = \gamma_w H (S_c - C)\left[1 + \frac{1}{(S_c - C)}\right]$$

or $$\boxed{\sigma = \gamma_w H (S_c - C + 1)} \qquad \text{(v)}$$

When $C = 1$, $\quad S_c = 2.4$

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$$\tau_0 = (P_v - p')\tan\alpha$$

If $p' = 0$ (no tail water)

$$\tau_0 = P_v \tan\alpha$$

But $P_v = \gamma_w H (S_c - C)$

$$\therefore \quad \tau_0 = \gamma_w H (S_c - C)\tan\alpha$$

or $$\tau_0 = \gamma_w H (S_c - C)\frac{B}{H}$$

$$= \gamma_w H (S_c - C)\frac{1}{\sqrt{S_c - C}}$$

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HIGH AND LOW GRAVITY DAMS.

To avoid dam failure by crushing,

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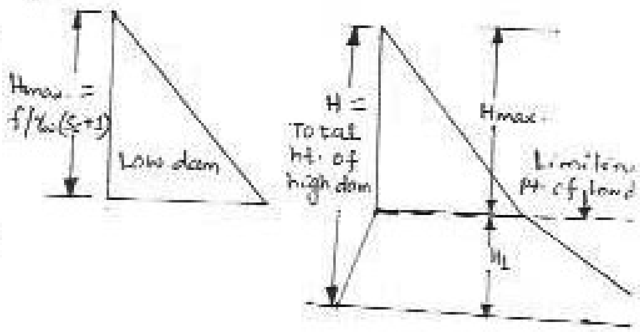
or $$H = \frac{f}{\gamma_w (S_c - C + 1)}$$



Low gravity dam $\qquad$ High gravity dam

The lowest value of H will be obtained when $c = 0$, i.e., when uplift is neglected.

Hence, for determining the limiting height and to be on a safer side, uplift is neglected.

H_{max} i.e; maximum possible height is given as:

$$H_{max} = \frac{f}{\gamma_w(S_c+1)} \qquad (vii)$$

- If the height of a dam having an elementary profile of a triangle, is more than that given by the eq^n (vii), the maximum compressive stress generated will exceed the allowable value. In order to keep it safe within limits, extra slopes on the u/s as well as on the d/s, below the limiting height will have to be given.

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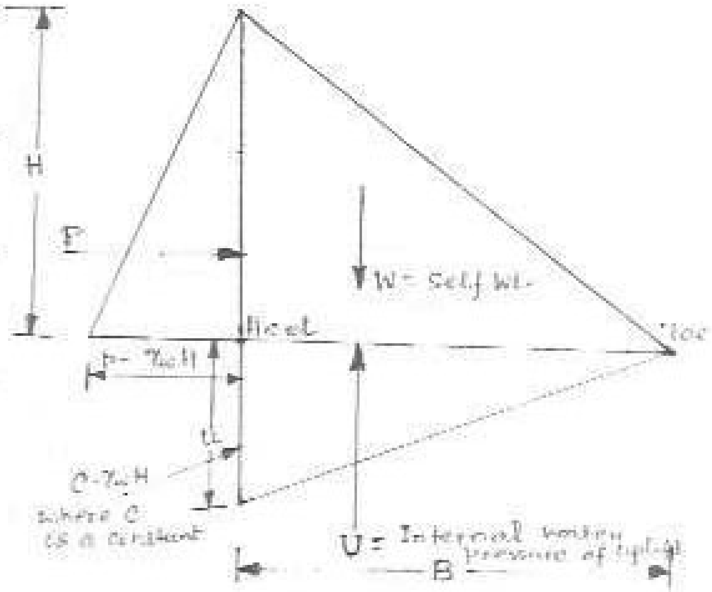

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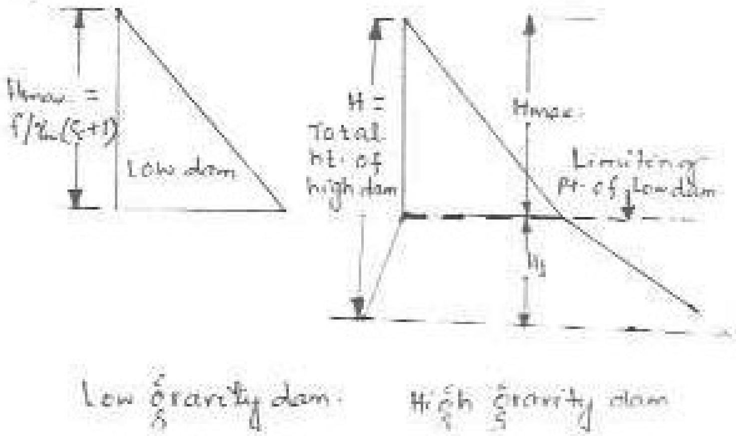

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