4rth Semester

Kinematics and Dynamics Of Machine

Module 5

Department of Mechanical Engineering
Government College Of Engineering Kalahandi, Odisha

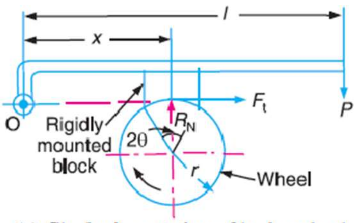
Module-5

BRAKES AND DYNAMOMETERS

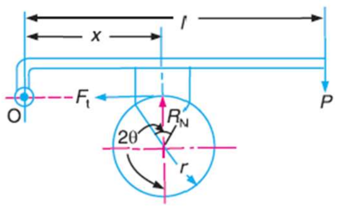
A ***brake*** is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc

Single Block or Shoe Brake

A single block or shoe brake is shown in Fig. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig. 19.1. The other end of the lever is pivoted on a fixed fulcrum *O*.



(*a*) Clockwise rotation of brake wheel



(*b*) Anticlockwise rotation of brake wheel.

Let P = Force applied at the end of the lever

R_N= Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ = Angle of contact surface of the block,

μ = Coefficient of friction, and

F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$$F_t = \mu.R_N \quad ...(i)$$

The braking torque, $T_B = F_t.r = \mu.R_N.r$ ... (*ii*)

Let us now consider the following three cases:

Case1. When the line of action of tangential braking force (Ft) passes through the fulcrum O of the lever, and the brake wheel rotates clockwise as shown in Fig. 19.1(*a*), then for equilibrium, taking moments about the fulcrum O, we have

$$R_N \times x = P \times l \text{ or } R_N = \frac{P \times l}{x}$$

Braking torque,

$$T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$$

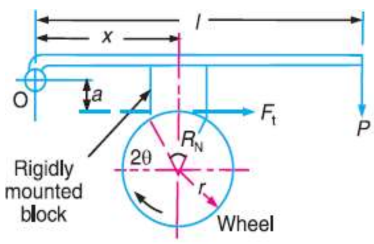
It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 19.1 (*b*), then the braking torque is same, *i.e.*

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

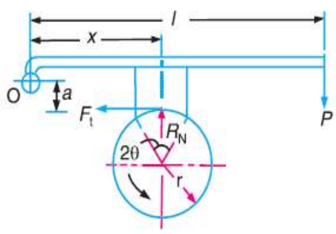
Case2. When the line of action of the tangential braking force (Ft) passes through a distance '*a*' below the fulcrum O, and the brake wheel rotates clockwise as shown in Fig. 19.2 (*a*), then for equilibrium, taking moments about the fulcrum O,

$$R_N \times x + F_t \times a = P.l \quad \text{or} \quad R_N \times x + \mu R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \mu.a}$$

$$T_B = \mu R_N.r = \frac{\mu.p.l.r}{x + \mu.a}$$



(*a*) Clockwise rotation of brake wheel.



(*b*) Anticlockwise rotation of brake wheel.

When the brake wheel rotates anticlockwise, as shown in Fig. 19.2 (*b*), then for equilibrium,

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a$$

$$R_N\,(x - \mu.a) = P.l \qquad \text{or} \qquad R_N = \frac{P.l}{x - \mu.a}$$

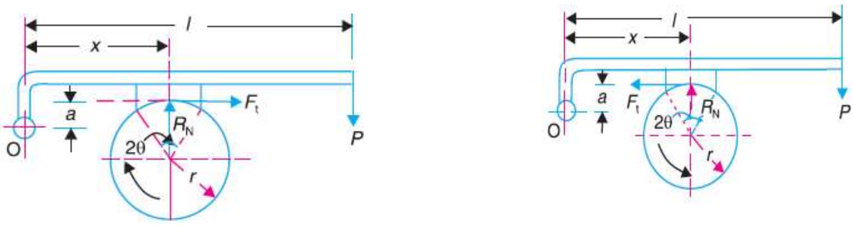
$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

Case 3. When the lie of action of the tangential braking force (*Ft*) passes through a distance '*a*' above the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig. 19.3 (*a*), then fore equilibrium, taking moments about the fulcrum *O*, we have

$$R_N.x = P.l + Ft.a = P.l + \mu.R_N.a$$

$$R_N\,(x - \mu.a) = P.l$$

$$R_N = \frac{P.l}{x - \mu.a}$$



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

When the brake wheel rotates anticlockwise as shown in Fig. 19.3 (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x + Ft \times a = P.l$$
$$R_N \times x + \mu.R_N \times a = P.l$$

$$R_N = \frac{P.l}{x + \mu.a} \qquad T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$

Pivoted Block or Shoe Brake

We have discussed in the previous article that when the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. But when the angle of contact is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre. In such cases, the block or shoe is pivoted to the lever, as shown in Fig.

instead of being rigidly attached to the lever. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque or a pivoted block

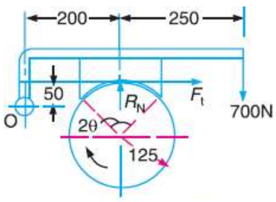
$$T_B = F_t \times r = \mu'.R_N.r$$

$$\mu' = \text{Equivalent coefficient of friction} = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta},$$

μ = Actual coefficient of friction.

PROBLEMS

Example1. A single block brake is shown in Fig. 19.5.The diameter of the drum is 250 mm and the angle of contact is90°. If the operating force of 700 N is applied at the end of a lever and the coefficient of friction between the drum and the lining is0.35,Determine the torque that may be transmitted by the block brake.



All dimensions in mm.

Fig. 19.5

Solution. Given : d = 250 mm or r = 125 mm ; 2θ = 90° = π / 2 rad ; P = 700 N ; μ = 0.35

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45°}{\pi/2 + \sin 90°} = 0.385$$

R_N = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu'.R_N$

Taking moments about the fulcrum O, we have

$$700(250+200) + F_t \times 50 = R_N \times 200 = \frac{F_t}{\mu'} \times 200 = \frac{F_t}{0.385} \times 200 = 520\,F_t$$

$520\,F_t - 50F_t = 700 \times 450$ or $F_t = 700 \times 450/470 = 670$ N

We know that torque transmitted by the block brake,

$T_B = F_t \times r = 670 \times 125 = 8\,3750$ N-mm = 83.75N-m

Example 2. A bicycle and rider of mass 100 kg are travelling at the rate of 16 km/h on a level road. A brake is applied to the rear wheel which is 0.9 m in diameter and this is the only resistance acting. How far will the bicycle travel and how many turns will it make before it comes to rest? The pressure applied on the brake is 100 N and $\mu = 0.05$.

Solution. Given: $m = 100$ kg, $v = 16$ km / h = 4.44 m / s ; $D = 0.9$ m ; R
N = 100 N; $\mu = 0.05$

Distance travelled by the bicycle before it comes to rest

Let x = Distance travelled (in meters) by the bicycle before it comes to rest.

We know that tangential braking force acting at the point of contact of the brake wheel,

$$F_t = \mu . R_N = 0.05 \times 100 = 5 \text{ N}$$

$$= F_t \times x = 5 \times x = 5x \text{ N-m} \text{ ---------------(i)}$$

We know that kinetic energy of the bicycle

$$= \frac{m.v^2}{2} = \frac{100(4.44)^2}{2}$$

$$= 986 \text{ N-m} \qquad \ldots (ii)$$

In order to bring the bicycle to rest, the work done against friction must be equal to kinetic energy of the bicycle. Therefore equating equations ***(i)*** and ***(ii)***,

$$5x = 986 \text{ or } x = 986/5 = 197.2 \text{ m}$$

Number of revolutions made by the bicycle before it comes to rest

Let N = Required number of revolutions.

We know that distance travelled by the bicycle (x),

$197.2 = \pi DN = \pi \times 0.9N = 2.83N$

$N = 197.2 / 2.83 = 70$

Double Block or Shoe Brake

When a single block brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to the normal force (RN). This produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake, as shown in Fig. 19.9, is used. It consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate or reduces the unbalanced force on the shaft. The brake is set by a spring which pulls the upper ends of the brake arms together. When a force P is applied to the bell crank lever, the spring is compressed and the brake is released. This type of brake is often used on electric cranes and the force P is produced by an electromagnet or solenoid. When the current is switched off, there is no force on the bell crank lever and the brake is engaged automatically due to the spring force and thus there will be no downward movement to the load.

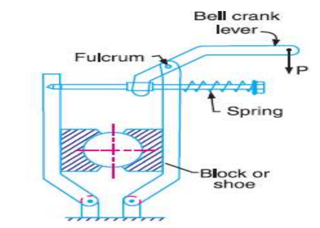


Fig. 19.9. Double block or shoe brake.

In a double block brake, the braking action is doubled by the use of two blocks and these blocks may be operated practically by the same force which will operate one. In case of double block or shoe brake, the braking torque is given by

$$T_B = (F_{t1} + F_{t2})\, r$$

Where F_{t1} and F_{t2} are the braking forces on the two blocks.

Internal Expanding Brake

An internal expanding brake consists of two shoes $S1$ and $S2$ as shown in Fig. 19.24. The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum $O1$ and $O2$ and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are normally held in off position by a spring as shown in Fig. 19.24. The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.

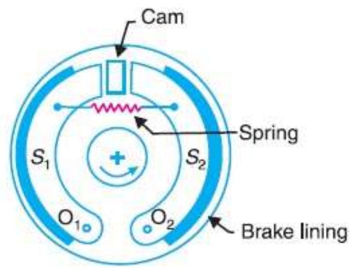


Fig. 19.24. Internal expanding brake.

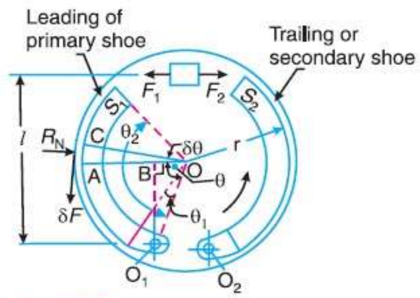


Fig. 19.25. Forces on an internal expanding brake.

We shall now consider the forces acting on such a brake, when the drum rotates in the anticlockwise direction as shown in Fig. 19.25. It may be noted that for the anticlockwise direction, the left hand shoe is known as ***leading*** or ***primary shoe*** while the right hand shoe is known as ***trailing*** or ***secondary shoe.***

Let r = Internal radius of the wheel rim,
b = Width of the brake lining,
p_1 = Maximum intensity of normal pressure,
p_N = Normal pressure,
F_1 = Force exerted by the cam on the leading shoe, and
F_2 = Force exerted by the cam on the trailing shoe.

Consider a small element of the brake lining

AC subtending an angle $\delta\theta$ at the centre. Let OA makes an angle θ with OO_1 as shown in Fig. 19.25. It is assumed that the pressure distribution on the shoe is nearly uniform, however the friction lining wears out more at the free end. Since the shoe turns about$O1$, therefore the rate of wear of the shoe lining at A will be proportional to the radial displacement of that point. The rate of wear of the shoe lining varies directly as the perpendicular distance from $O_1$to OA, *i.e.*
O_1B. From the geometry of the figure,

$$O_1B = OO_1 \sin\theta$$

normal pressure at A,

$$p_N \propto \sin\theta \quad \text{or} \quad p_N = p_1 \sin\theta$$

$\therefore$ Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$
$$= p_N(b.r.\delta\theta) = p_1 \sin\theta(b.r.\delta\theta)$$

and braking or friction force on the element,

$$\delta F = \mu \times \delta R_N = \mu.p_1 \sin\theta(b.r.\delta\theta)$$

$\therefore$ Braking torque due to the element about O,

$$\delta T_B = \delta F \times r = \mu.p_1 \sin\theta(b.r.\delta\theta)r = \mu.\,p_1 b\,r^2(\sin\theta.\delta\theta)$$

and total braking torque about O for whole of one shoe,

$$T_B = \mu\,p_1 b\,r^2 \int_{\theta_1}^{\theta_2} \sin\theta\,d\theta = \mu\,p_1 b\,r^2[-\cos\theta]_{\theta_1}^{\theta_2}$$
$$= \mu\,p_1 b r^2(\cos\theta_1 - \cos\theta_2)$$

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N \times O_1B = \delta R_N(OO_1 \sin\theta)$$
$$= p_1 \sin\theta(b.r.\delta\theta)(OO_1 \sin\theta) = p_1 \sin^2\theta(b.r.\delta\theta)OO_1$$

$\therefore$ Total moment of normal forces about the fulcrum O_1,

$$M_N = \int_{\theta_1}^{\theta_2} p_1 \sin^2\theta(b.r.\delta\theta)OO_1 = p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \sin^2\theta\,d\theta$$
$$= p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \frac{1}{2}(1-\cos 2\theta)d\theta \qquad \ldots\left[\because\ \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)\right]$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[\theta - \frac{\sin 2\theta}{2} \right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2} \right]$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2) \right]$$

Moment of frictional force δF about the fulcrum O_1,

$$\delta M_F = \delta F \times AB = \delta F (r - OO_1 \cos\theta) \qquad \ldots (\because AB = r - OO_1 \cos\theta)$$

$$= \mu\, p_1 \sin\theta (b.r.\delta\theta)(r - OO_1 \cos\theta)$$

$$= \mu.p_1.b.r(r \sin\theta - OO_1 \sin\theta \cos\theta)\delta\theta$$

$$= \mu.p_1.b.r\left(r\sin\theta - \frac{OO_1}{2} \sin 2\theta \right)\delta\theta \qquad \ldots (\because 2\sin\theta\cos\theta = \sin 2\theta$$

$\therefore$ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu\, p_1 b r \int_{\theta_1}^{\theta_2} \left(r\sin\theta - \frac{OO_1}{2} \sin 2\theta \right) d\theta$$

$$= \mu\, p_1 b r \left[-r\cos\theta + \frac{OO_1}{4} \cos 2\theta \right]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1 b r \left[-r\cos\theta_2 + \frac{OO_1}{4} \cos 2\theta_2 + r\cos\theta_1 - \frac{OO_1}{4} \cos 2\theta_1 \right]$$

$$= \mu\, p_1 b r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4} (\cos 2\theta_2 - \cos 2\theta_1) \right]$$

Now for leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

Braking of a Vehicle

In a four wheeled moving vehicle, the brakes may be applied to

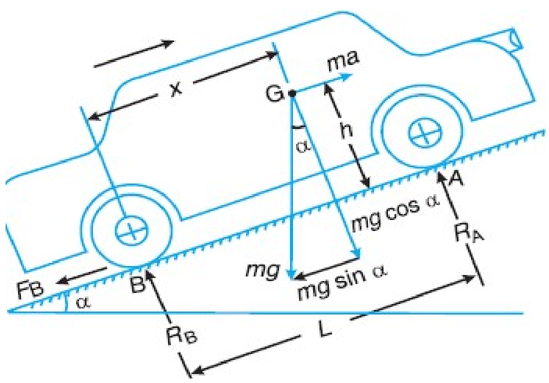
1. the rear wheels only,

2. the front wheels only, and

3. all the four wheels.

In all the above mentioned three types of braking, it is required to determine the retardation of the vehicle when brakes are applied. Since the vehicle retards, therefore it is a problem of dynamics. But it may be reduced to an equivalent problem of statics by including the inertia force in the system of forces actually applied to the vehicle. The inertia force is equal and opposite to the braking force causing retardation.

Now, consider a vehicle moving up an inclined plane, as shown in Fig.



Let α= Angle of inclination of the plane to the horizontal,

m = Mass of the vehicle in kg (such that its weight is $m.g$ newtons),

h = Height of the C.G. of the vehicle above the road surface in metres,

x = Perpendicular distance of C.G. from the rear axle in metres,

L = Distance between the centres of the rear and front wheels of the vehicle in metres,

R_A = Total normal reaction between the ground and the front wheels in newtons,

R_B = Total normal reaction between the ground and the rear wheels in newtons,

μ = Coefficient of friction between the tyres and road surface, and

a = Retardation of the vehicle in m/s2.

We shall now consider the above mentioned three cases of braking, one by one. In all these cases, the braking force acts in the opposite direction to the direction of motion of the vehicle.

1. **When the brakes are applied to the rear wheels only**

It is a common way of braking the vehicle in which the braking force acts at the rear wheels only.

Let F_B = Total braking force (in newtons) acting at the rear wheels due to the application of the brakes. Its maximum value is $\mu.R_B$.

The various forces acting on the vehicle are shown in Fig. For the equilibrium of the vehicle, the forces acting on the vehicle must be in equilibrium.
Resolving the forces parallel to the plane,

$$F_B + m.g.\sin\alpha = m.a \; . . . \; \textbf{(\textit{i})}$$

Resolving the forces perpendicular to the plane,

$$R_A + R_B = m\,g\cos\alpha \ldots\ldots\ldots \textbf{(\textit{ii})}$$

Taking moments about G, the centre of gravity of the vehicle

$$F_B \times h + R_B \times x = R_A (L - x) \ldots\ldots\ldots\ldots\ldots\ldots \textbf{(\textit{iii})}$$

Substituting the value of $F_B = \mu.R_B$, and $R_A = m.g\cos\alpha - R_B$ [from equation **(*ii*)**] in the above expression, we have

$$\mu.R_B \times h + R_B \times x = (m.g\cos\alpha - R_B)(L - x)$$

$$R_B (L + \mu.h) = m.g\cos\alpha(L - x)$$

$$R_B = \frac{m.g\cos\alpha(L-x)}{L+\mu.h}$$

and

$$R_A = m.g\cos\alpha - R_B = m.g\cos\alpha - \frac{m.g\cos\alpha(L-x)}{L+\mu.h}$$

$$= \frac{m.g\cos\alpha(x+\mu.h)}{L+\mu.h}$$

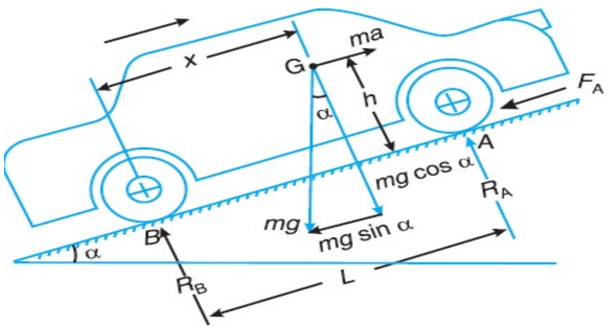
We know from equation (*i*),

$$a = \frac{F_B + m.g\sin\alpha}{m} = \frac{F_B}{m} + g\sin\alpha = \frac{\mu.R_B}{\text{m}} + g\sin\alpha$$

$$= \frac{\mu.g\cos\alpha(L-x)}{L+\mu.h} + g\sin\alpha \qquad \ldots \text{(Substituting the value of } R_B\text{)}$$

2. When the brakes are applied to front wheels only

It is a very rare way of braking the vehicle, in which the braking force acts at the front wheels only.



Let F_A = Total braking force (in newtons) acting at the front wheels due to the application of brakes. Its maximum value is $\mu.R_A$.

The various forces acting on the vehicle are shown in Fig. Resolving the forces parallel to the plane,

$$F_A + m.g \sin\alpha = m.a \quad \ldots \textbf{(\textit{i})}$$

Resolving the forces perpendicular to the plane,

$$R_A + R_B = m.g \cos\alpha \quad \ldots \textbf{(\textit{ii})}$$

Taking moments about G, the centre of gravity of the vehicle,

$$F_A \times h + R_B \times x = R_A (L - x)$$

Substituting the value of $F_A = \mu.R_A$ and $R_B = m.g \cos\alpha - R_A$ [from equation **(*ii*)**] in the above expression, we have

$$\mu.R_A \times h + (m.g \cos\alpha - R_A)\, x = R_A (L - x)$$

$$\mu.R_A \times h + m.g \cos\alpha \times x = R_A \times L$$

$$\therefore \qquad R_A = \frac{m.g \cos\alpha \times x}{L - \mu.h}$$

and

$$R_B = m.g \cos\alpha - R_A = m.g \cos\alpha - \frac{m.g \cos\alpha \times x}{L - \mu.h}$$

$$= m.g \cos\alpha \left(1 - \frac{x}{L - \mu.h}\right) = m.g \cos\alpha \left(\frac{L - \mu.h - x}{L - \mu.h}\right)$$

We know from equation (*i*),

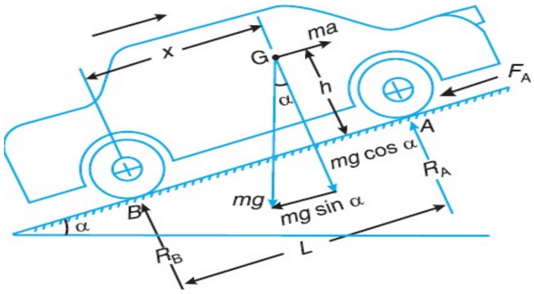
$$a = \frac{F_A + m.g \sin\alpha}{m} = \frac{\mu.R_A + m.g \sin\alpha}{m}$$

$$= \frac{\mu.m.g \cos\alpha \times x}{(L - \mu.h)m} + \frac{m.g \sin\alpha}{m} \qquad \ldots \text{(Substituting the value of } R_A\text{)}$$

$$= \frac{\mu.g \cos\alpha \times x}{L - \mu.h} + g \sin\alpha$$

3. When the brakes are applied to all the four wheels

This is the most common way of braking the vehicle, in which the braking force acts on both the rear and front wheels.



Let F_A = Braking force provided by the front wheels = $\mu.R_A$, and

F_B = Braking force provided by the rear wheels = $\mu.R_B$.

Little consideration will show that when the brakes are applied to all the four wheels, the braking distance (i.e. the distance in which the vehicle is brought to rest after applying the brakes) will be the least. It is due to this reason that the brakes are applied to all the four wheels. The various forces acting on the vehicle are shown in fig.

Resolving the forces parallel to the plane,

$F_A+F_B+m.g\sin\alpha=m.a$…….(i)

Resolving forces vertical to the plane

$R_A + R_B = m.g\cos\alpha$. . . ***(ii)***

Taking moments about *G*, the centre of gravity of the vehicle,

$(F_A + F_B)\,h + R_B \times x = R_A(L - x)$…….(iii)

Substituting the value of $F_A = \mu.R_A$, $F_B = \mu.R_B$ and $R_B = m.g\cos\alpha - R_A$

[From equation ***(ii)***] in the above expression,

$\mu\,(R_A + R_B)\,h + (m\,g\cos\alpha - R_A)\,x = R_A(L - x)$

$\mu\,(R_A + mg\cos\alpha - R_A)h + (mg\cos\alpha - R_A)x = R_A(L - x)$

$\mu.m.g\cos\alpha \times h + m.g\cos\alpha \times x = R_A \times L$

$$R_A = \frac{m.g\cos\alpha(\mu.h + x)}{L}$$

$$R_B = m.g\cos\alpha - R_A = m.g\cos\alpha - \frac{mg\cos\alpha(\mu.h + x)}{L}$$

$$= m.g\cos\alpha\left[1 - \frac{\mu.h + x}{L}\right] = m.g\cos\alpha\left(\frac{L - \mu.h - x}{L}\right)$$

Now from equation (i),

$\mu.R_A + \mu R_B + m.g \sin\alpha = m.a$

$\mu(R_A + R_B) + m.g \sin\alpha = m.a$

$\mu.m.g.\cos\alpha + m.g \sin\alpha = m.a$ . . . [From equation (ii)]

$a = g(\mu.\cos\alpha + \sin\alpha)$

PROBLEMS

Example 1. A car moving on a level road at a speed 50 km/h has a wheel base 2.8metres, distance of C.G. from ground level 600 mm, and the distance of C.G. from rear wheels 1.2metres. Find the distance travelled by the car before coming to rest when brakes are applied,

1. To the rear wheels,
2. To the front wheels, and
3. To all the four wheels. The coefficient of friction between the tyres and the road may be taken as 0.6.

Solution.

Given : u = 50 km/h = 13.89 m/s ; L = 2.8 m ; h = 600 mm = 0.6 m ; x = 1.2 m ; μ = 0.6

Let s = Distance travelled by the car before coming to rest.

1. **When brakes are applied to the rear wheels**

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = \frac{\mu.g(L - x)}{L + \mu.h} = \frac{0.6 \times 9.81(2.8 - 1.2)}{2.8 + 0.6 \times 0.6} = 2.98 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2 \times 2.98} = 32.4 \text{ m}$$

2. **When brakes are applied to the front wheels**

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = \frac{\mu.g.x}{L - \mu.h} = \frac{0.6 \times 9.18 \times 1.2}{2.8 - 0.6 \times 0.6} = 2.9 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2\times2.9} = 33.26 \text{ m}$$

3. **When the brakes are applied to all the four wheels**

Since the vehicle moves on a level road, therefore retardation of the car,
$a = g.\mu = 9.81\times0.6 = 5.886$ m/s2

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2\times5.886} = 16.4 \text{ m}$$

Example2. A vehicle moving on a rough plane inclined at 10° with the horizontal at a speed of 36 km/h has a wheel base 1.8 metres. The centre of gravity of the vehicle is 0.8 metre from the rear wheels and 0.9 metre above the inclined plane. Find the distance travelled by the vehicle before coming to rest and the time taken to do so when

1. The vehicle moves up the plane, and
2. The vehicle moves down the plane.

The brakes are applied to all the four wheels and the coefficient of friction is 0.5.

Solution.

Given : $\alpha = 10°$; u = 36 km / h = 10 m / s ; L = 1.8 m ; x = 0.8 m ; h = 0.9 m ; $\mu = 0.5$

Let s = Distance travelled by the vehicle before coming to rest, and

t = Time taken by the vehicle in coming to rest.

1. **When the vehicle moves up the plane and brakes are applied to all the four wheel**

Since the vehicle moves up the inclined plane, therefore retardation of the vehicle,

$a = g\,(\mu \cos\alpha + \sin\alpha)$

$= 9.81\,(0.5\cos10° + \sin10°) = 9.81(0.5\times0.9848 + 0.1736) = 6.53$ m/s^2

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2\times6.53} = 7.657 \text{ m}$$

and final velocity of the vehicle (v),

$0 = u + a.t = 10 - 6.53\,t$ ....(Minus sign due to retardation)

$t = 10 / 6.53 = 1.53$

2. **When the vehicle moves down the plane and brakes are applied to all the four wheels**

   Since the vehicle moves down the inclined plane, therefore retardation of the vehicle,

   $a = g\,(\mu \cos\alpha - \sin\alpha)$

   $= 9.81(0.5\cos 10° - \sin 10°) = 9.81(0.5 \times 0.9848 - 0.1736) = 3.13\ \text{m/s}^2$

   We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2 \times 3.13} = 16\ \text{m}$$

and final velocity of the vehicle (v),

$0 = u + a.t = 10 - 3.13\,t$ . . . (Minus sign due to retardation)

$t = 10/3.13 = 3.2$ s

DYNAMOMETER

A dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

Types of Dynamometers

1. Absorption dynamometers,
2. Transmission dynamometers.

In the *absorption* ***dynamometers*****,** the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and is transformed into heat, during the process of measurement. But in the ***transmission dynamometers*****,** the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through the dynamometer to some other machines where the power developed is suitably measured.

Classification of Absorption Dynamometers

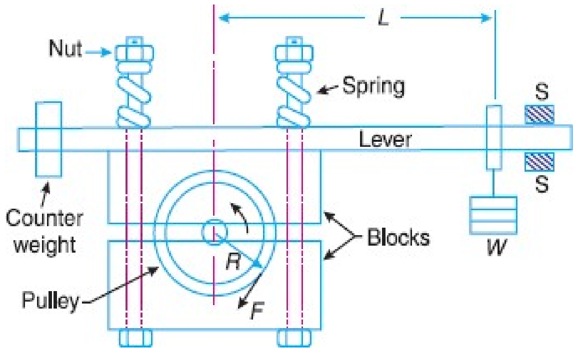
1. Prony brake dynamometer,

2.Rope brake dynamometer.

1. Prony brake dynamometer

A simplest form of an absorption type dynamometer is a prony brake dynamometer, as shown in Fig. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two bolts and nuts, as shown in Fig.

A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it and carries a weight *W* at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of the lever.



When the brake is to be put in operation, the long end of the lever is loaded with suitable weights *W* and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight *W* must balance the moment of the frictional resistance between the blocks and the pulley.

Let W = Weight at the outer end of the lever in newtons,
L = Horizontal distance of the weight W from the centre of the pulley in metres,
F = Frictional resistance between the blocks and the pulley in newtons,
R = Radius of the pulley in metres, and
N = Speed of the shaft in r.p.m.

We know that the moment of the frictional resistance or torque on the shaft,

$$T = W.L = F.R \text{ N-m}$$

Work done in one revolution

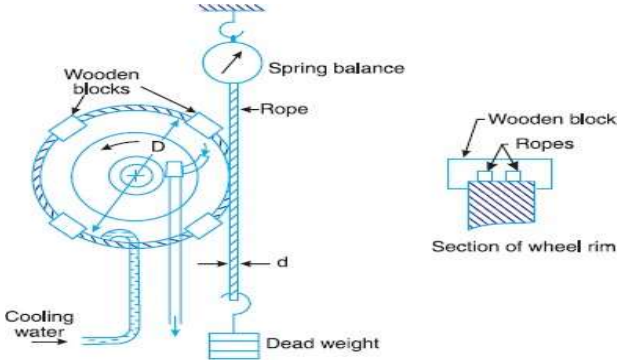
Work done in one revolution
= Torque × Angle turned in radians
= $T \times 2\pi$ N-m
_ Work done per minute

= $T \times 2\pi N$ N-m
We know that brake power of the engine

$$B.P. = \frac{\text{Work done per min.}}{60} = \frac{T \times 2\pi N}{60} = \frac{W.L \times 2\pi N}{60} \text{ watts}$$

Rope Brake Dynamometer

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig.19.32. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.



In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

Let W = Dead load in newtons,
S = Spring balance reading in newtons,
D = Diameter of the wheel in metres,
d = diameter of rope in metres, and
N = Speed of the engine shaft in r.p.m.

Net load on the brake= $(W – S)$ N

We know that distance moved in one revolution= $\pi (D+ d)$ m

Work done per revolution= $(W − S)$ _ $(D + d)$ N-m

Work done per minute= $(W − S) \pi (D + d) N$ N-m

Brake power of the engine,

$$B.P = \frac{\text{Work done per min}}{60} = \frac{(W - S)\,\pi\,(D + d) N}{60} \text{ watts}$$

,

If the diameter of the rope (d) is neglected, then brake power of the engine

$$B.P. = \frac{(W - S)\,\pi D\, N}{60} \text{ watts}$$

Example 1. In a laboratory experiment, the following data were recorded with rope brake: Diameter of the flywheel 1.2 m; diameter of the rope12.5 mm; speed of the engine 200 r.p.m.; dead load on the brake 600 N; spring balance reading 150 N. Calculate the brake power of the engine.

Solution. Given : D = 1.2 m ; d = 12.5 mm
= 0.0125 m ; N = 200 r.p.m ; W = 600 N ; S = 150 N
We know that brake power of the engine,

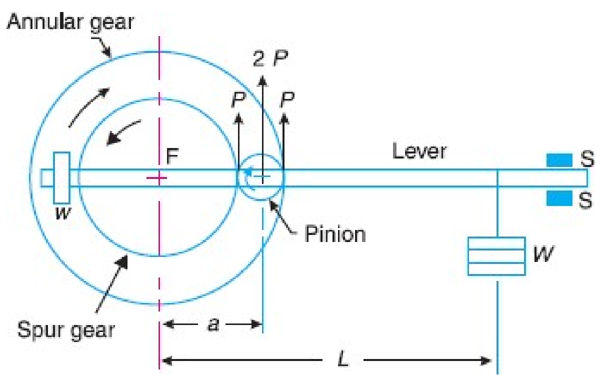
$$\text{B.P.} = \frac{(W-S)\,\pi\,(D+d)N}{60} = \frac{(600-150)\,\pi\,(1.2+0.0125)200}{60} = 5715\text{ W}$$

Classification of Transmission Dynamometers

1. Epicyclic-train dynamometer,
2. Belt transmission dynamometer, and
3. Torsion dynamometer

Epicyclic-train Dynamometer

An epicyclic-train dynamometer, as shown in Fig. 19.33, consists of a simple epicyclic train of gears, *i.e.* a spur gear, an annular gear (a gear having internal teeth) and a pinion. The spur gear is keyed to the engine shaft (*i.e.* driving shaft) and rotates in anticlockwise direction. The annular gear is also keyed to the driving shaft and rotates in clockwise direction. The pinion or the intermediate gear meshes with both the spur and annular gears. The pinion revolves freely on a lever which is pivoted to the common axis of the driving and driven shafts. A weight *w* is placed at the smaller end of the lever in order to keep it in position. A little consideration will show that if the friction of the pinon which the pinion rotates is neglected, then the tangential effort *P* exerted by the spur gear on the pinion and the tangential reaction of the annular gear on the pinion are equal.



Since these efforts act in the upward direction as shown, therefore total upward force on the lever acting through the axis of the pinion is 2*P*. This force tends to rotate the lever about its fulcrum and it is balanced by a dead weight *W* at the end of the lever. The stops *S*, *S* are provided to control the movement of the lever.

For equilibrium of the lever, taking moments about the fulcrum *F*,

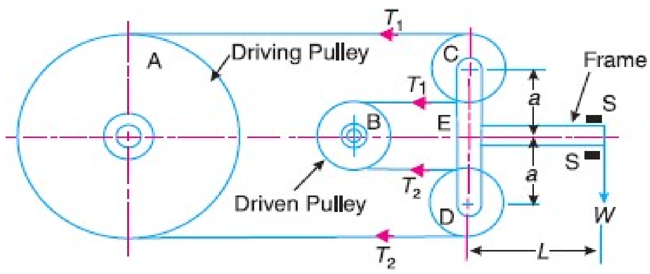
$2P \times a = W.L$ or $P = W.L/2a$
R = Pitch circle radius of the spur gear in metres, and
N = Speed of the engine shaft in r.p.m.

Torque transmitted, $T = P.R$

power transmitted $= \dfrac{T \times 2\pi N}{60} = \dfrac{P.R \times 2\pi N}{60}$ watts

Belt Transmission Dynamometer-Froude or Throneycroft Transmission Dynamometer

When the belt is transmitting power from one pulley to another, the tangential effort on the driven pulley is equal to the difference between the tensions in the tight and slack sides of the belt. A belt dynamometer is introduced to measure directly the difference between the tensions of the belt, while it is running.



A belt transmission dynamometer, as shown in Fig, is called a Froude or Throneycroft transmission dynamometer. It consists of a pulley *A* (called driving pulley) which is rigidly fixed to the shaft of an engine whose power is required to be measured. There is another pulley *B* (called driven pulley) mounted on another shaft to which the power from pulley *A* is transmitted. The pulleys *A* and *B* are connected by means of a continuous belt passing round the two loose pulleys *C* and *D* which are mounted on a *T*-shaped frame. The frame is pivoted at *E* and its movement is controlled by two stops *S*,*S*. Since the tension in the tight side of the belt ($T1$) is greater than the tension in the slack side of the belt ($T2$), therefore the total force acting on the pulley *C* (*i.e.* $2T1$) is greater than the total force acting on the pulley *D* (*i.e.* $2T2$). It is thus obvious that the frame causes movement about *E* in the anticlockwise direction. In order to balance it, a weight *W* is applied at a distance *L* from *E* on the frame as shown in Fig.

Now taking moments about the pivot *E*, neglecting friction,

$2\,T_1 \times a = 2T_2 \times a + W\,L$ $\qquad T_1 - T_2 = \dfrac{W.L}{2a}$

Let D = diameter of the pulley A in metres,
N = Speed of the engine shaft in r.p.m.
Work done in one revolution = $(T_1 - T_2)\pi\, D$ N-m

work done per minute = $(T_1 - T_2)\pi\, DN$ N-m

$\therefore$ Brake power of the engine, $\text{B.P.} = \dfrac{(T_1 - T_2)\pi\, DN}{60}$ watts

Torsion Dynamometer

A torsion dynamometer is used for measuring large powers particularly the power transmitted along the propeller shaft of a turbine or motor vessel. A little consideration will show that when the power is being transmitted, then the driving end of the shaft twists through a small angle relative to the driven end of the shaft. The amount of twist depends upon many factors such as torque acting on the shaft (*T*), length of the shaft (*l*), diameter of the shaft (*D*) and modulus of rigidity (*C*) of the material of the shaft. We know that the torsion equation is

$$\frac{T}{J} = \frac{C.\theta}{l}$$

where θ = Angle of twist in radians, and

J = Polar moment of inertia of the shaft.

For a solid shaft of diameter *D*, the polar moment of inertia

$$J = \frac{\pi}{32} \times D^4$$

and for a hollow shaft of external diameter *D* and internal diameter *d*, the polar moment of inertia,

$$J = \frac{\pi}{32}(D^4 - d^4)$$

From the above torsion equation,

$$T = \frac{C.J}{l} \times \theta = k.\theta$$

where $k = C.J/l$ is a constant for a particular shaft. Thus, the torque acting on the shaft is proportional to the angle of twist. This means that if the angle of twist is measured by some means, then the torque and hence the power transmitted may be determined.

We know that the power transmitted

P= 2πNT/60watts,
where *N* is the speed in r.p.m.

PROBLEMS

Example 1. A torsion dynamometer is fitted to a propeller shaft of a marine engine. It is found that the shaft twists 2° in a length of 20 metres at 120 r.p.m. If the shaft is hollow with 400 mm external diameter and 300 mm internal diameter, find the power of the engine. Take modulus of rigidity for the shaft material as 80 GPa.

Solution.
Given : _ = 2° = 2 × _ /180 = 0.035 rad ; *l* = 20 m ; *N* = 120 r.p.m. ; *D* = 400 mm= 0.4 m ; *d* = 300 mm = 0.3 m ; *C* = 80 GPa = 80 × 109 N/m^2
We know that polar moment of inertia of the shaft

$$J = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}\left[(0.4)^4 - (0.3)^4\right] = 0.0017\text{m}^4$$

and torque applied to the shaft,

$$T = \frac{C.J}{l} \times \theta = \frac{80 \times 10^9 \times 0.0017}{20} \times 0.035 = 238 \times 10^3 \text{ N-m}$$

We know that power of the engine,

$$P = \frac{T \times 2\pi N}{60} = \frac{238 \times 10^3 \times 2\pi \times 120}{60} = 2990 \times 10^3 \text{ W} = 2990 \text{ kW}$$