4rth Semester

Kinematics and Dynamics Of Machine

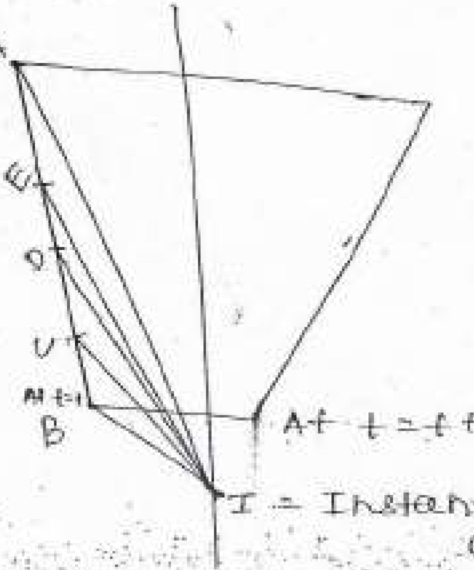
Module 1(b)

Department of Mechanical Engineering
Government College Of Engineering Kalahandi, Odisha

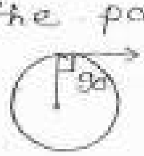
Velocity Mechanism

Instantaneous centre method approach
[SIR ARNHOLD K]

Instantaneous centre of rotation (General motion)



Always in a motion in rotation, is the velocity is perpendicular to the path of tracing that velocity.



In Reality

$AA_1 \to 0$
$BB_1 \to 0$] Differential (At very small diff of time)

The link AB at this instant is in general motion.

But for finding this general motion firstly the motion is assumed in rotatory motion. Thus,

$$\omega_{AB} = \frac{V_A}{AI} = \frac{V_B}{BI} = \frac{V_C}{CI} = \frac{V_D}{DI} = \frac{V_E}{EI} \ldots$$

V_A → given as input

velocity → corresponding to its radius

Meaning of I_{14}

→ Relative motion.

Motion of link 4 w.r.t link 1.

or, " 1 " " 4.

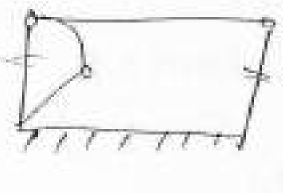
is seen pure rotation about a pt. which is I_{14}

In general which link changes its position its (IC) Instantaneous centre of rotation also changes.

Locus of IC → of rotation for a link during its motion → CENTRODE.

Locus of Instantaneous Axis of rotation for a link during its motion ⇒ Axode.

Motion	Centrode	AXODE
General Motion	Curve	Curved surface
Pure translation	St. line	Plane surface.
Pure rotation.	Point.	St. line.



"In general the motion of a link in a mechanism is neither pure translation nor pure rotation it is a combination of translation and rotation which we normally say the link is in general motion but any link at any instant can be assumed to be in pure rotation w.r.t to the pt. in the space known as instantaneous centre of rotation. This centre is also known as virtual centre".

[20/03/14]

No. of instantaneous centres in a mechanism

If no. of links : l

No. of IC (Instantaneous centre)

$$= {}^{l}C_2$$

$$= \frac{l(l-1)}{2}$$

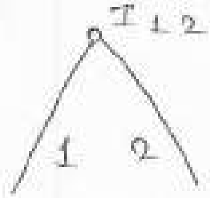
If $l = 5$

$IC = 10$

$$\left.\begin{matrix} I_{12} & I_{13} & I_{14} & I_{15} \\ & I_{23} & I_{24} & I_{25} \\ & & I_{34} & I_{35} \\ & & & I_{45} \end{matrix}\right] \text{Total } 10 \text{ IC.}$$

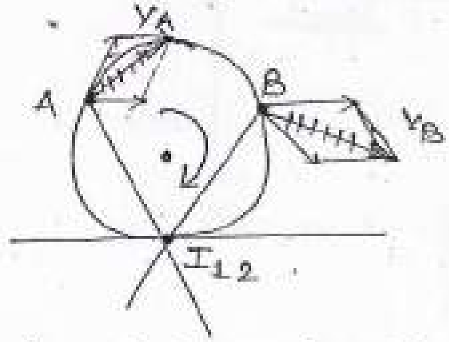
Basic IC in a mechanism

(1) Turning pair



The pt. of contact of two links is IC.
Here, (link 1 , link 2)
thus IC is I_{12}.
if 2,1 then
I_{21} ——

(2) Rolling pair



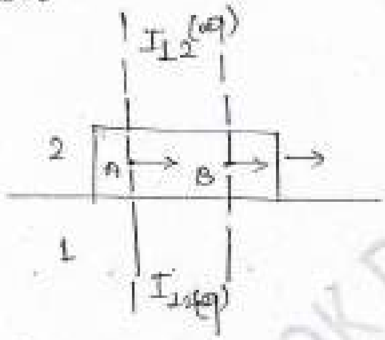
Take the dirn of two motions i.e translational & rotational then obtain the actual dirn of V_A & V_B.

Here, Ic is the pt. of intersection of the two line passing through the two taken pts.

Pt. of intersection of the two pt. is I c.

(3) sliding pair

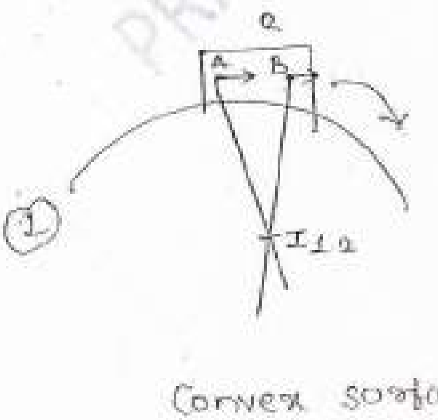
on plane surface.



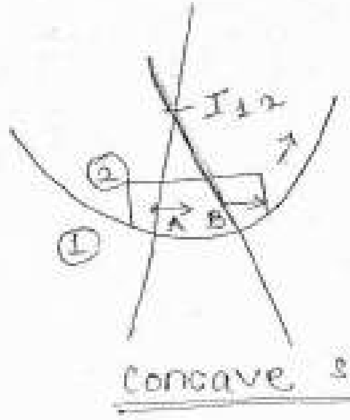
Firstly take two pts.
Then Draw perpendicular to the two pts.
These two lines will meet at infinity and thus Ic is at ∞

Ic will go at ∞ in the dirn ⊥r to the sliding surface.

on curved surface.

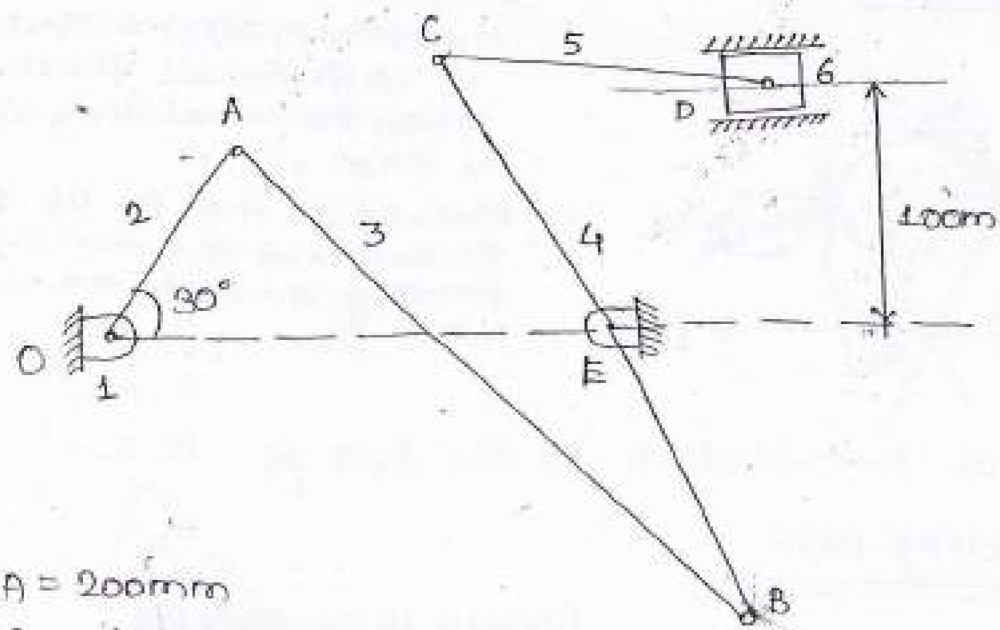


Convex surface



Concave surface

(8)



OA = 200mm

AB = 1.5m

OE = 1.35m

BE = 400mm

BC = 600mm

CD = 500mm

If OA → 120 rpm (clock)

find :- $V_B = ?$, $V_C = ?$, $V_D = ?$

$\omega_{AB} = ?$, $\omega_{BC} = ?$, $\omega_{CD} = ?$

link :- L = 6

IC : 15

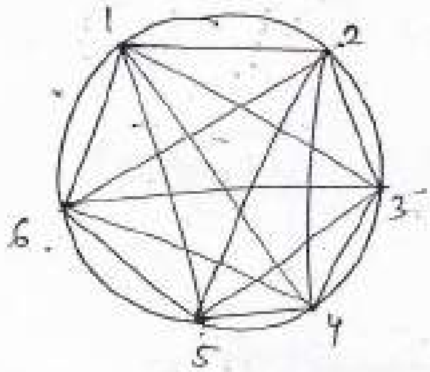
I_{12} I_{13} I_{14} I_{15} I_{16}

I_{23} I_{24} I_{25} I_{26}

I_{34} I_{35} I_{36}

I_{45} I_{46}

I_{56}



- First of all write all ICs.
- Here there are 6 links and thus the ICs are shown in previous page.
- Mark each IC known in the fig. (Next page). At each turning pair one IC and at each sliding pair one IC also
- Depending on no. of links Draw a polygon inside a circle. Here 6 links are available thus the polygon is Hexagon. Join each IC available (if not present along the sides). Now, obtain the each unknown IC obtain the each possible paths through which we can reach. Now Bisect the two pts available and obtain the new IC.

13 — 12, 23; 14, 43

15 — 16, 65; 14, 45

24 — 23, 34; 21, 14

25 — 24, 45; 21, 15

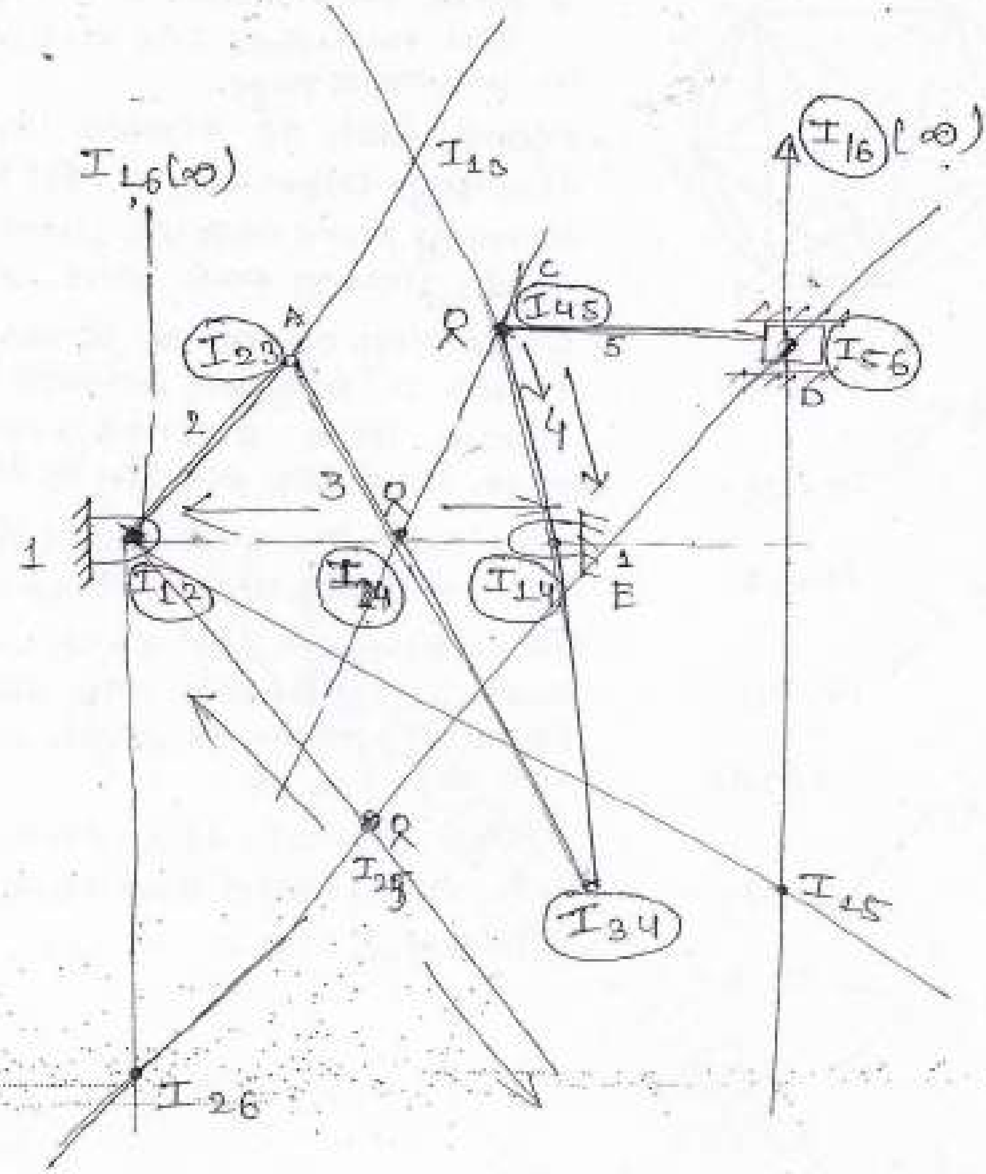
26 — 25, 56; 21, 16

35 — 34, 45; 32, 25; 31, 15 — same.

36 — 35, 56; 32, 26; 31, 16

46 — 41, 16; 42, 26; 43, 36; 45, 56

Graphical Scale



$V_B = 3.2$ m/s.

$V_C = 1.6$ m/s

$V_D = 1.08$ m/s.

$\omega_{AB} = 2.99$ rad/s, $\omega_{BC} = 8$ rad/s,

$\omega_{CD} = 2.16$ rad/s.

Given

$N_{OA} = 120$ r.p.m

$$\omega_{OA} = \frac{2\pi \times 120}{60} = 4\pi \text{ rad/s}.$$

$V_A = (OA) \cdot \omega_{OA} = 0.2 \times 4\pi$

$= 2.5132$ m/s.

link 3

(A, B) (I_3) or (I_{13})

$$\omega_3 = \omega_{AB} = \frac{V_A}{I_{13A}} = \frac{V_B}{I_{13B}} \; ??$$

??

link 4 (B, C) (I_{14})

$$\omega_4 = \omega_{BC} = \frac{V_B}{I_{14B}} = \frac{V_C}{I_{14C}} \; ?$$

??

link 5 (C, D) (I_{15})

$$\omega_5 = \omega_{CD} = \frac{V_C}{I_{15C}} = \frac{V_D}{I_{15D}}$$

??

Kennedy's theorem

" For the relative motion b/w the no. of links in a mechanism any three links there are three instantaneous centre must lie in straight line".

Theorem of Angular velocities

Take Any IC

~~Take~~

I_{mn}

Can be treated on link m

Can also be treated on link n.

$$V_{I_{mn}} = \boxed{\omega_m (I_{mn} I_{1m}) = \omega_n (I_{mn} I_{1n})}$$

$= V_{I_{mn}}$

Relation b/w two I centres

This theorem is applied on I_{mn}

Total IC in use

I_{mn}
I_{1m}
I_{1n}

links 1, m, n.

If I_{1m}, I_{1n} lie at the same side of I_{mn} then dirn is also same.

Take

I_{25}

clockwise.

$$\omega_2 (\underbrace{I_{25} I_{12}}) = \omega_5 (\underbrace{I_{25} I_{15}})$$

Measure the two distance from diagram.

ω_2 is given, find ω_5.

Similarly

I_{24}

$\omega_2 (I_{24} I_{12}) = \omega_4 (I_{24} I_{14})$ → A.C. (from diagram). ↓ If dirn of I_{14} in IC are opp.

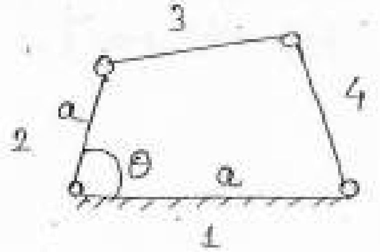
I_{45}

$\omega_4 (I_{45} I_{14}) = \omega_5 (I_{45} I_{15})$ → AC

2 Marks

Problems

(1)

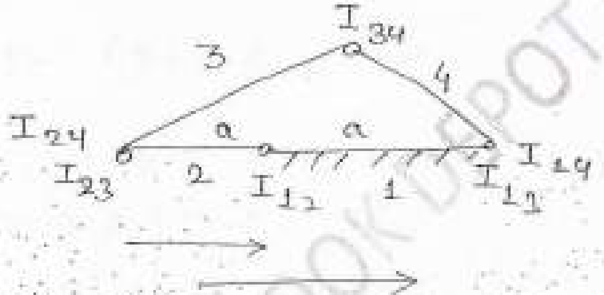


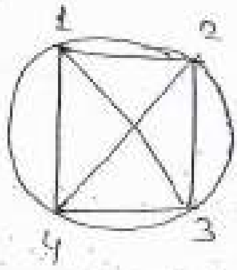
When $\theta = 180°$

$\omega_2 = 5$ rad/s (clock)

$\omega_3 = ?$

Sol :-





Both dirn's of I_{12} & I_{13} are same.

13 < 12, 23 ; 14, 43

24 < 23, 34 ; 21, 14.

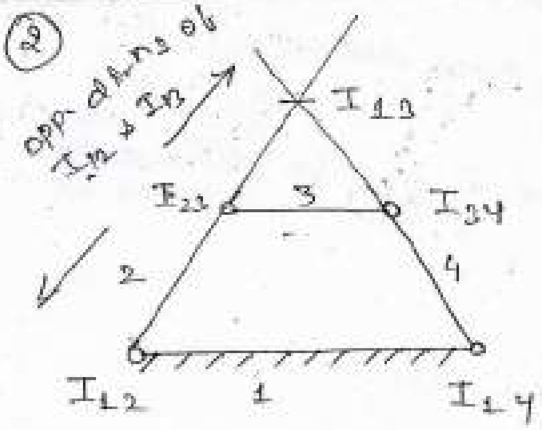
I_{23}

$$\omega_2 (I_{23} I_{12}) = \omega_3 (I_{23} I_{13})$$

$$5a = \omega_3 (2a)$$

$$\Rightarrow \boxed{\omega_3 = 2.5 \text{ rad/s clock}}$$

(2)



Given:- $\omega_2 = 5$ rad/s

$\omega_3 = 14$ rad/s.

Find the angular velocity of link 2 w.r.t link 3.

Sol:-

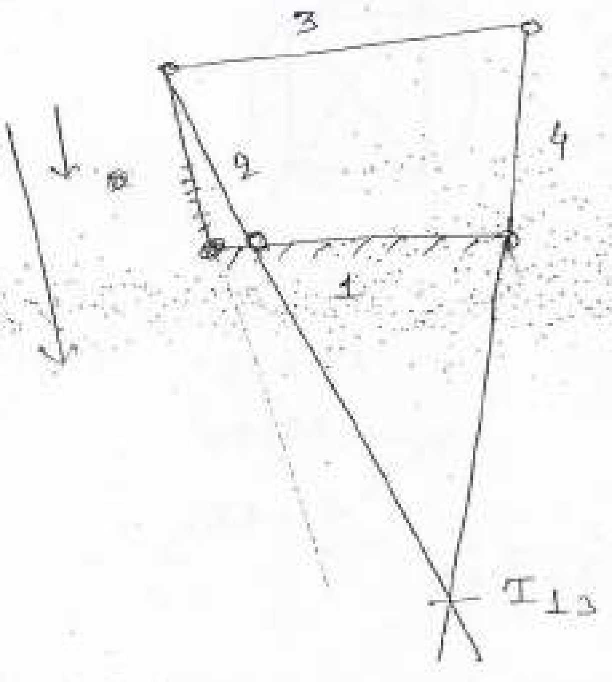
$\vec{\omega}_{23} = \vec{\omega}_2 - \vec{\omega}_3$

$= +5 - (-14)$

$= 19$ (clockwise)

[Dirn very imp. as the quantity is vector].

(3) Same Data



$\vec{\omega}_{23} = (+5) - (+14)$

$= -9$

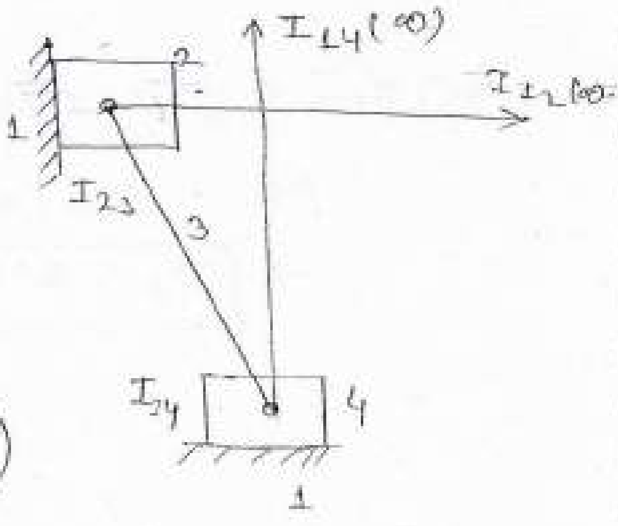
$= 9$ (A C).

(4) Gate

Find I_{24}.

24 — 23, 34

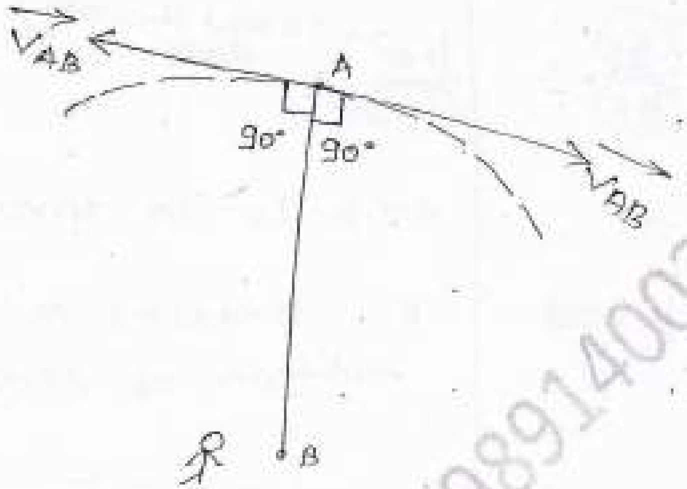
24 — 21, 14.



21 & 14 are infinitely displaced line.
Thus, 24 lie at ∞.

For conventional problems

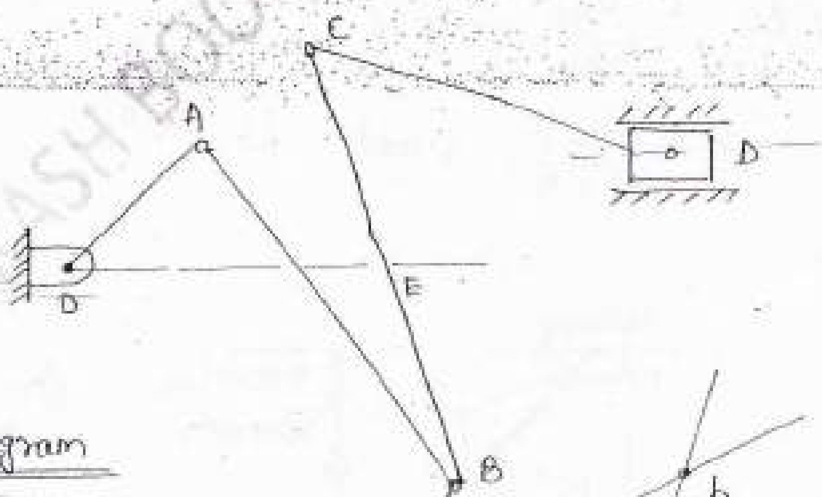
Relative velocity method



Configuration diagram
(Diagram in fair sketch).

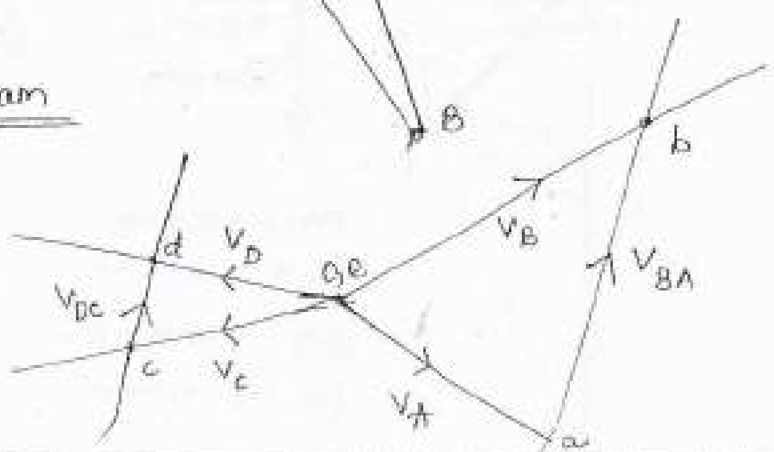
The velocity of point A w.r.t Pt. B will be in the dirn $\perp$r to the link AB.

Graphical scale



Velocity Diagram

$V_A = 2.5132$ m/s
0.4 m/s = 1 cm



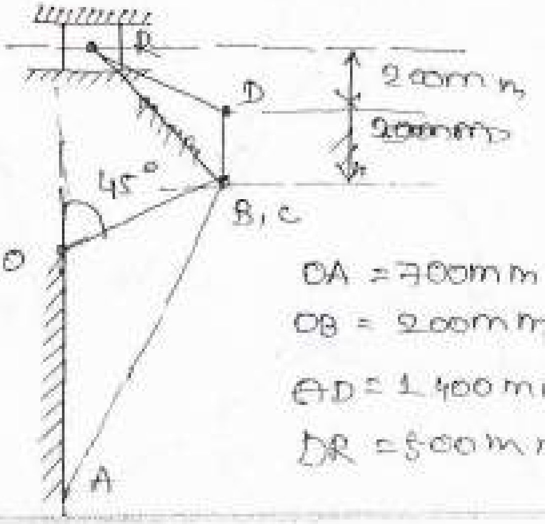
point	w.r.t	Procedure
A	O	line ⊥r to link OA (using set square)
B	A	" " " " AB
B	E	" " " " BE
C	$\frac{BC}{BE} =$	$\frac{bc}{be}$ → ? = find this using other three data.
D	C	line ⊥r to link CD.
D	fixed	" parallel to the motion of slider.

$$\omega_{AB} = \frac{V_{AB}}{AB \text{ is given}} \text{ rad/s (Anticlockwise)}$$

$$\omega_{BC} = \frac{V_{BC}}{BC} \text{ rad/s (")}$$

$$\omega_{CD} = \frac{V_{CD}}{CD} \text{ rad/s (")}$$

Problem



OA = 700mm

OB = 200mm

AD = 1400mm.

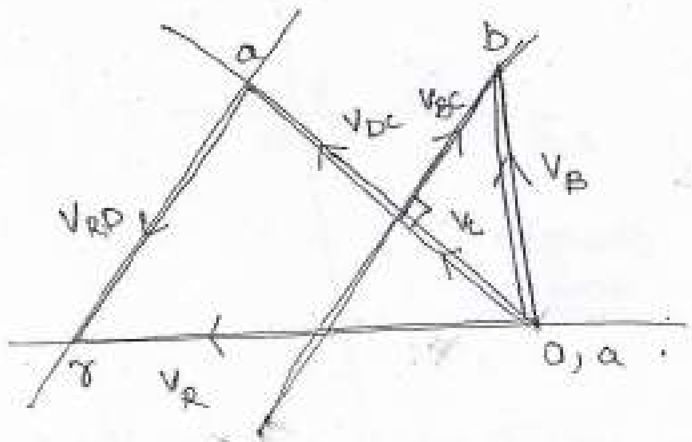
DR = 500mm.

If crank OB → 40 rpm (ACW)

Find V_R = ?

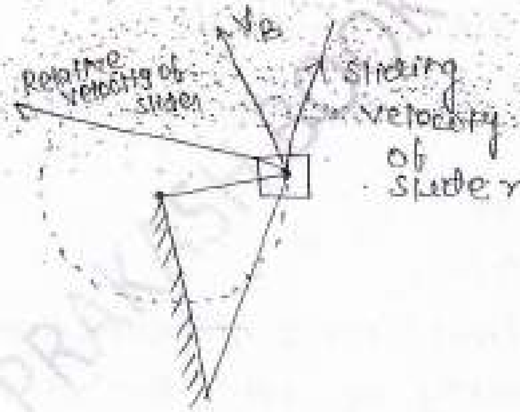
Graphical

$V_B = (OB)\,\omega_{OB} = 0.2 \times \frac{2\pi \times 40}{60}$ m/s = scale of velocity



Location of Pt. B & C

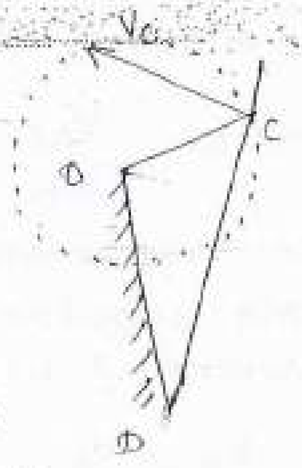
point B (slider)



Point C

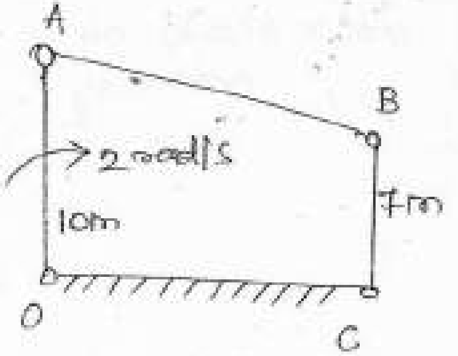
coincident pt. of slider but on slotted Bar.

$\boxed{\text{Slotted Bar - ACD}}$



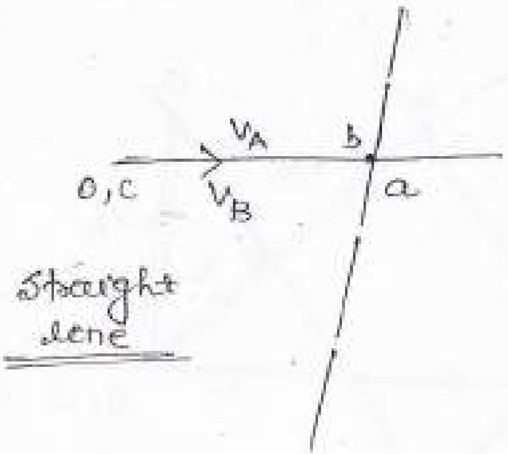
Point	w.r.t	Procedure
C	A	line ⊥r to Link AC.
C	B	line ∥ to link AC.

21/03/24



Pb. 1

The velocity diagram of given configuration at this instant will be :-



Pb 2) The velocity of pt. B will be :- $V_B = ?$

Sol :- $V_B = V_A \Rightarrow V_B = V_A = 2 \times 10$
$= 20 m/s$.

Pb 3) The relative velocity of pt. B w.r.t pt. A will be ?

Sol :- $\overrightarrow{V_{BA}} = 0$ ($\because$ Both pts coincides).

Pb 4) The motion of line AB at this instant will be :-

Sol :- $\omega_{AB} = \frac{V_{AB}}{AB} = \frac{0}{AB} = 0$.

No rotation, thus pure translation.

Pb 1-5) The angular velocity of link BC at this moment will be,

Sol :- $V_B = V_A$

$\Rightarrow 7 \times \omega_{BC} = 10 \times 2$

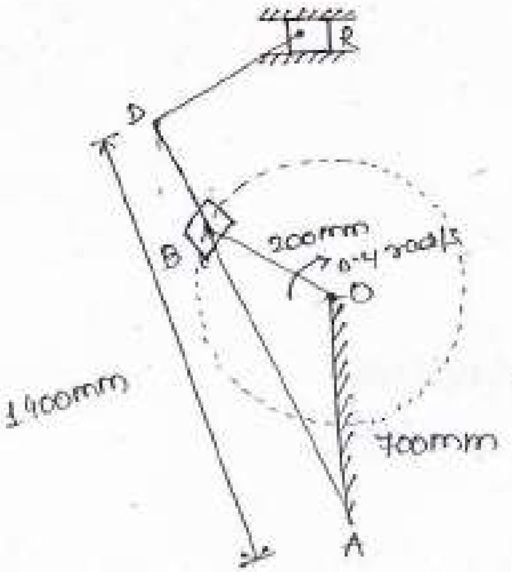
$\Rightarrow \omega_{BC} = \frac{20}{7}$ rad/s.

Pb 5)
2 Marks

1. ______ 2. ______ 3. ______ 4. ______
5. ______ 6. ______ .

(a) 1, 2 (b) 2, 3, 4 (c) 1, 3, 4 (d) None of these .

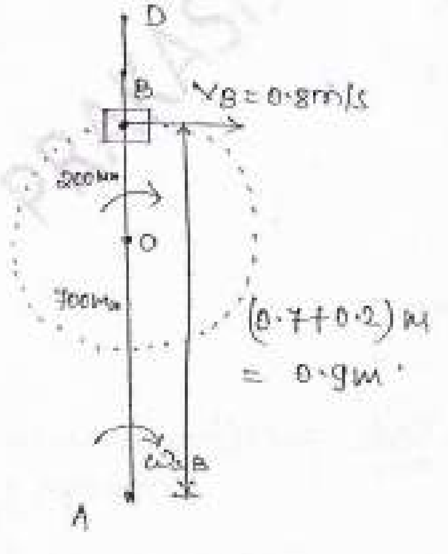
Pb)
2 Marks



Find the angular velocity of slotted bar when the velocity of bar is maxm .

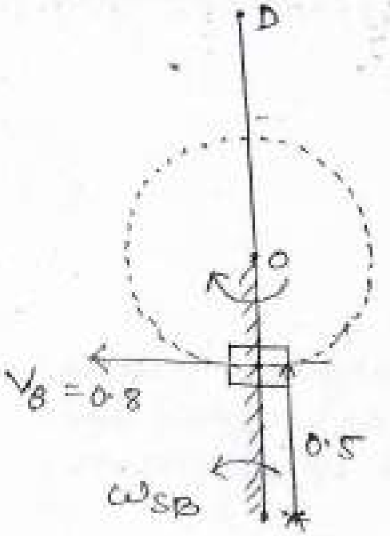
Sol:- $V_B = 0.2 \times 4 = 0.8\,m/s$ (uniform) .

In forward stroke



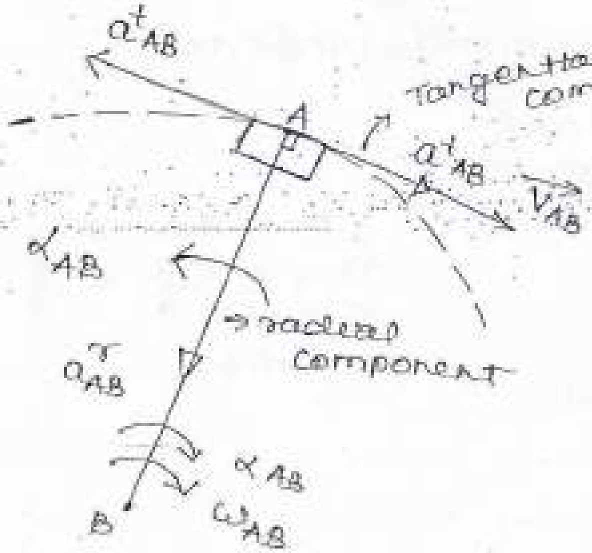
$\omega_{SB} = \frac{0.8}{0.9} = \frac{8}{9}$ rad/s

Slotted bar .



$\omega_{SB} = \frac{0.8}{0.5} = \frac{8}{5}$ rad/s. (Maxm velocity).

Acceleration Analysis



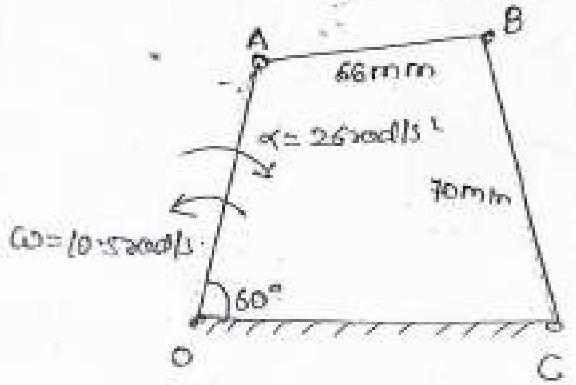
configuration diagram

$\vec{a}_{AB} =$ a^r_{AB} $(A \xrightarrow{dirn} B)$ $= \frac{V_{AB}^2}{AB} \neq 0$ (in circular motion).

$a^t_{AB} = \underbrace{(AB)\,\alpha_{AB}}$ [⊥r to Radial] → Known for input link).

May or May not be zero.

(9)



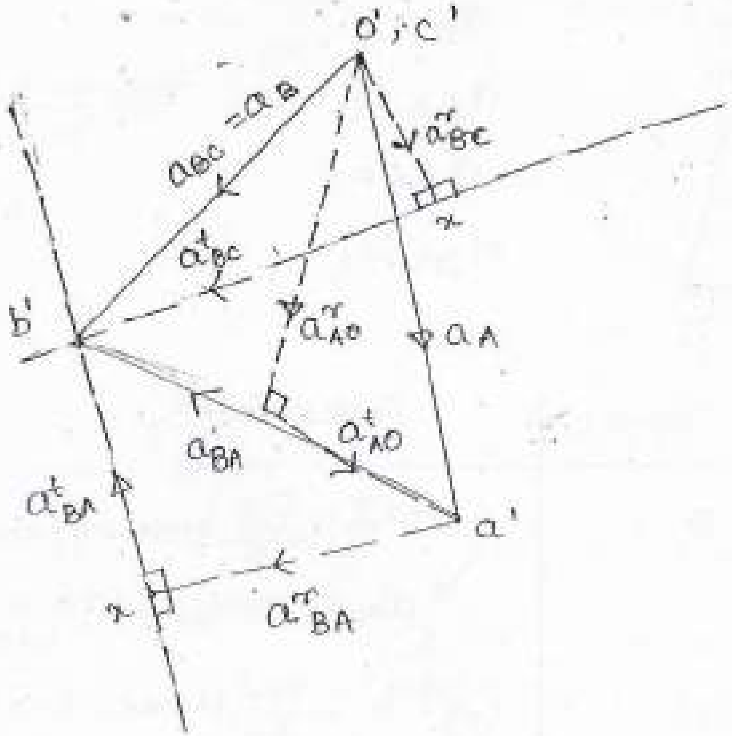
Find
$a_B = ?$
$a_{BA} = ?$
$\alpha_{BA} = ?$
$\alpha_{BC} = ?$

Sol :-

Point	w.r.t	Procedure
A	O	$a^r_{AO} = \frac{V_{AO}^2}{AO}$ (known) $(A \to O)$ $a^t_{AO} = (AO)\alpha_{AO}$ (?) (⊥r to Radial)
B	A	$a^r_{BA} = \frac{V_{BA}^2}{BA}$ (known) $(B \to A)$ $a^t_{BA} = BA\,\alpha_{BA}$ (unknown) (⊥r to radial)
B	C	$a^r_{BC} = \frac{V_{BC}^2}{BC}$ $(B \to C)$ (known) $a^t_{BC} = BC\,\alpha_{BC}$ (unknown) (⊥r to Radial)

Graphical

Scale



Klein's Construction

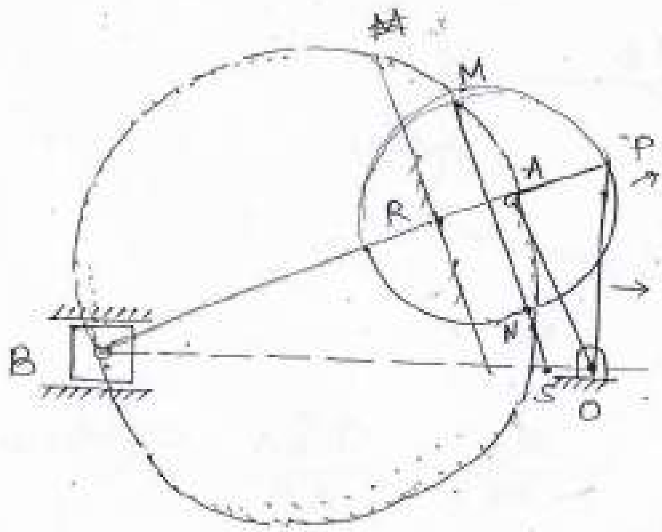
It is only applied in basic single slider-crank mechanism.

When $\alpha_{crank} = 0$ & $\omega_{crank} \rightarrow$ given

Then the diagram represent klein's construction.

Graphical [Scale →]

Graphical Scale

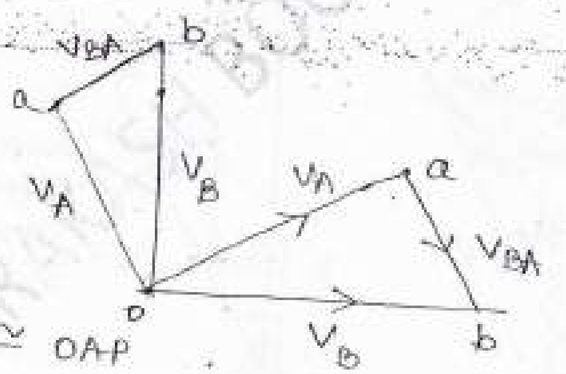


→ velocity triangle [extend the C.R line & draw a ⊥r from slider line]

OAP

Velocity Δ

Velocity diagram (Rough Diagram.



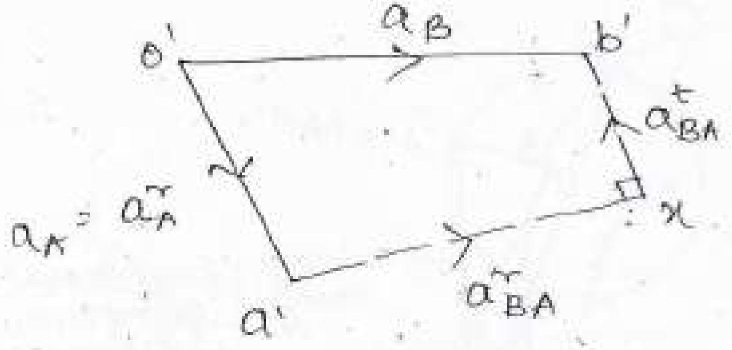
ΔOab ≃ OAP

$$\frac{V_A}{OA} = \frac{V_B}{OP} = \frac{V_{BA}}{AP} = \omega_{crank} \text{ (given)}.$$

Acceleration Analysis

- Quadrilateral OARS ⟶ Acc^n Quad.

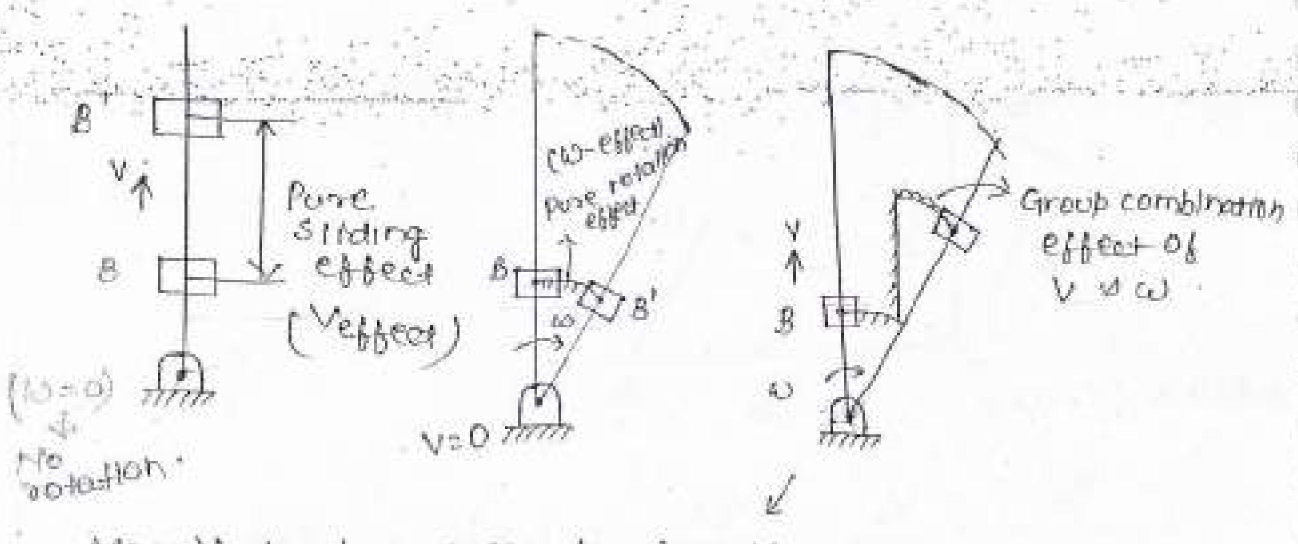
Acc^n Diagram (Rough).



$$\frac{a_A}{OA} = \frac{a^t_{BA}}{R_1 BA} = \frac{a_B}{OS} = \frac{a^r_{BA}}{AR} = (\omega_{crank})^2.$$

Coriolis Acceleration (a^c)

The acceleration will always be associated with slider when slider is sliding on the rotating body.

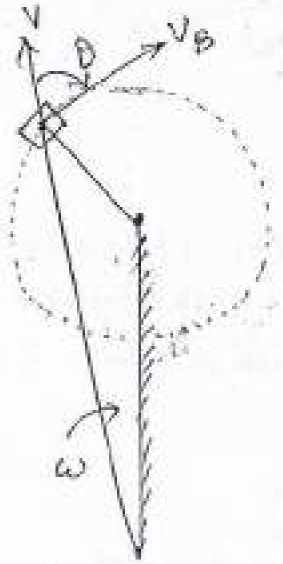


Magnitude of Coriolis Acceleration

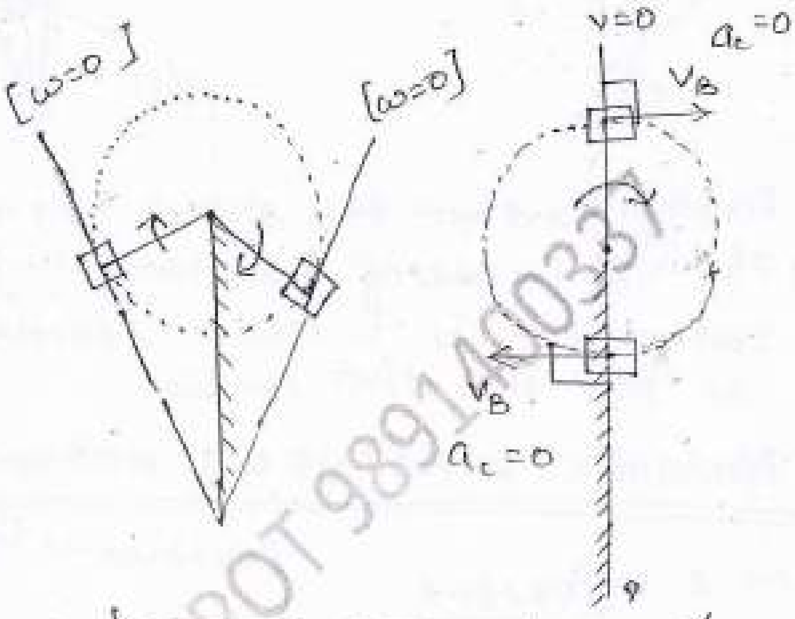
$$\boxed{a^c = 2V\omega}$$

$V \rightarrow$ Sliding velocity of slider.

$\omega \rightarrow$ Angular velocity of body on which slider is sliding.



a^c will exist



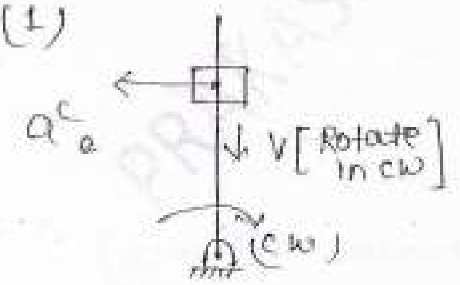
a^c does not exist

Dirn of a^c :-

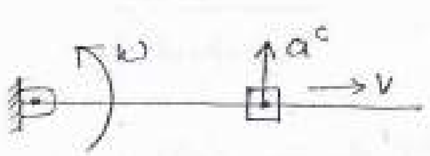
(i) Take the sec sense of ω.

(ii) Rotate " $\vec{V}$ in that sense by 90°.

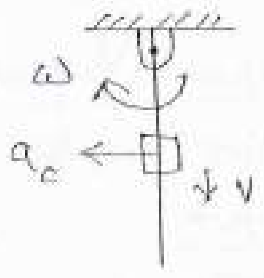
(1)

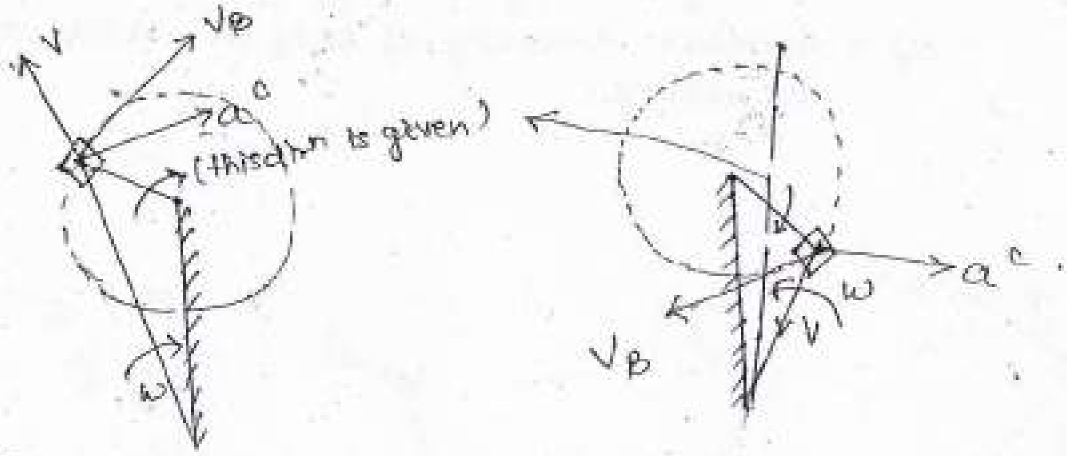


(2)



(3)





Firstly find out the dirn of ω, then the dirn of V_B (along the tangent). Take the component V, then decide the dirn a^c as per the dirn of ω.

Mechanical advantage of a mechanism

$F_{output} \rightarrow$ force output

$$M.A = \frac{F_{output}}{F_{input}}$$

or,
$$M.A = \frac{V_{input}}{V_{output}} \times \eta_{mechanism} \quad [\text{from (1)}]$$

(or)

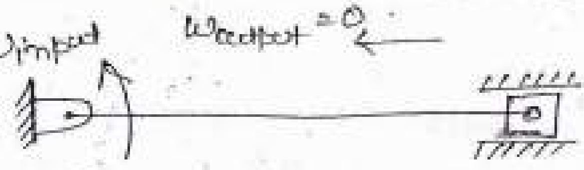
$T_{output} \rightarrow$ Torque output

$$M.A = \frac{T_{output}}{T_{input}}$$

or,
$$M.A = \frac{\omega_{input}}{\omega_{output}} \times \eta_{mechanism} \quad [\text{from (1)}]$$

$$\boxed{\eta_{mechanism} = \frac{P_{output}}{P_{input}} = \frac{F_{output} \cdot V_{output}}{F_{input} \cdot V_{input}} = \frac{T_{output} \cdot \omega_{output}}{T_{input} \cdot \omega_{input}}} \quad - (1)$$

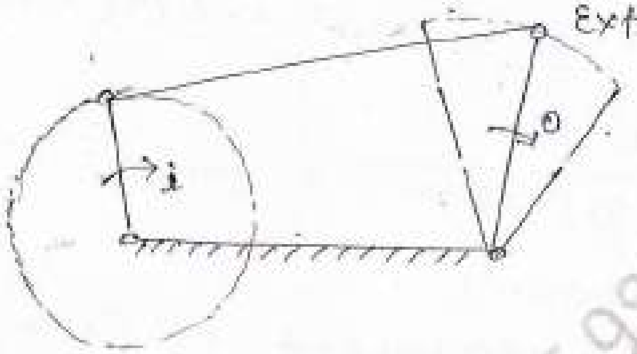
Pb



$$M \cdot A = \frac{W_{input}}{0} = \infty .$$

Toggle position

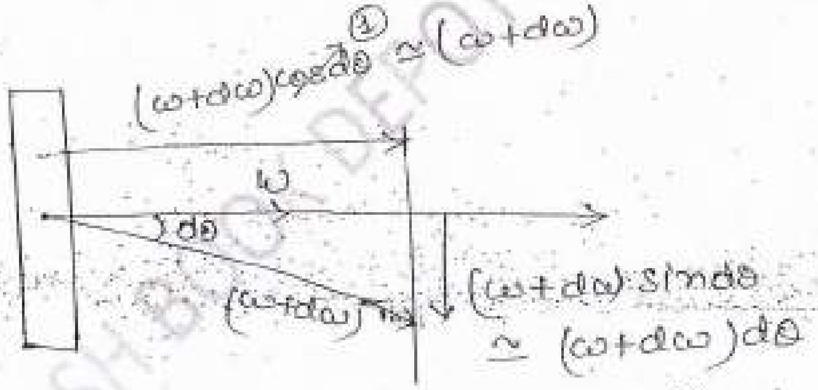
Extreme position of output link as Rocker in 4-Bar Mechanism.



$W_{output} = 0.$

$M \cdot A = \infty .$

Pb



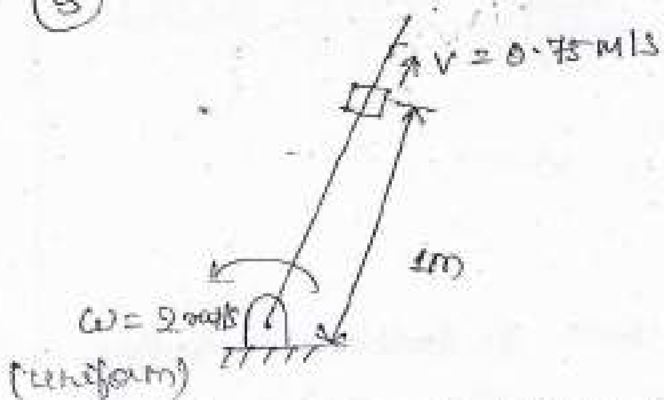
Find Ac^n along OA & $\perp r$ to OA ?

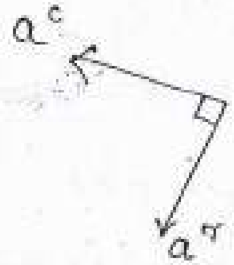
Sol :-

$$(Acc^n)_{\text{along } OA} = \frac{(\omega+d\omega)-\omega}{dt} = \frac{d\omega}{dt}.$$

$$(Acc^n)_{\perp r \text{ to } OA} = \frac{\omega d\theta - 0}{dt} = \left(\omega \cdot \frac{d\theta}{dt}\right)$$

(3)





$a^c = 2v\omega = 2 \times 0.75 \times 2 = 3\ m/s^2.$

$a^r = \frac{v_a^2}{r} = r\omega^2 = 1(2)^2 = 4\ m/s^2.$

$\therefore$ a (Resultant)

$= \sqrt{(3)^2 + (4)^2} = 5\ m/s^2.$

If ω is not uniform.
then one more component of acc^n $(r\dot{\omega})$.