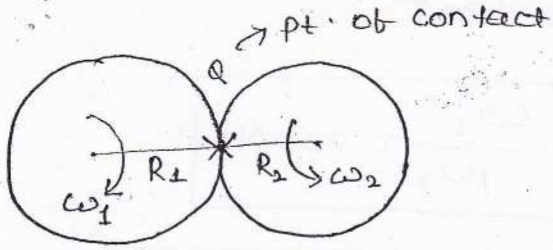
4rth Semester

Kinematics and Dynamics Of Machine

Module 2(a)

Department of Mechanical Engineering
Government College Of Engineering Kalahandi, Odisha

Gears



Point of contact (Q).

Lies also on body I

$v_1 = R_1\omega_1$

lies also on body 2

$v_2 = R_2\omega_2$

$$\boxed{R_1\omega_1 = R_2\omega_2}$$ ⇒ condition for no slipping ⇓ No power loss

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \text{const}}$$

In case of no slipping, the friction is static (f_s)

$$\boxed{0 \le f_s \le \mu N}$$

In condition of slip

Power loss takes place and $\frac{\omega_1}{\omega_2} \neq$ const.

Those drives in which slip is possible is negative drive.

Eg:- Belt drive, rope drive, chain driver.

90% of drive used is ther type of drive.

In some system (only 10%), very high level of accuracy is demanded for the velocity ratio to be constant.

$$\frac{\omega_1}{\omega_2} = \text{const}$$

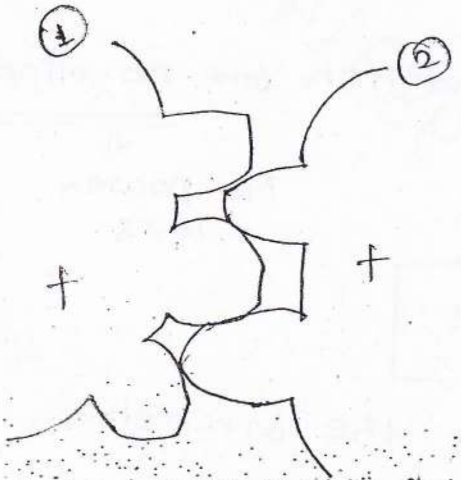
Slip is impossible

↓

Positive drive

↓

Gear drive



$$\frac{\text{Manufacturing cost}}{\text{Material cost}} > 100$$

Classification of gears

(A) On the basis of axes of shaft connected

(i) Both axes are parallel

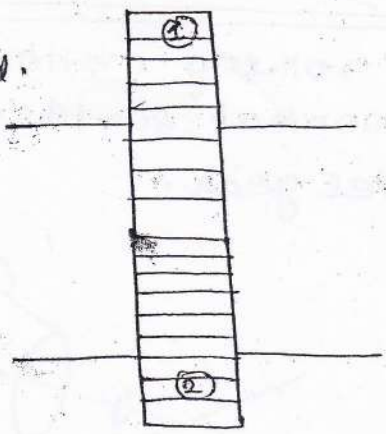
In this case pure rolling motion can be transmitted b/w two cylinder surfaces in contact.

(i) Teeth are straight (top land) and parallel to axis of rotation.] ⇒ spur gear.

Spur gear

99% failed, only 1% use in very low transmission of power at very low speed.

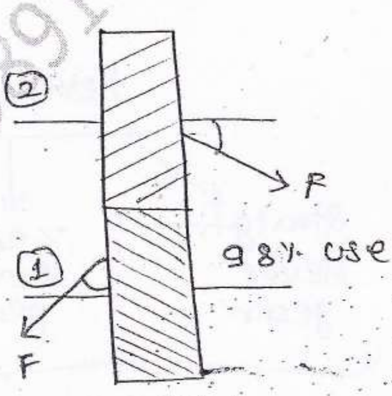
- Here instantaneous engagement and disengagement takes place so impact stresses is developed.
- Here axial thrust is zero, force acts normally.



(ii) Teeth are straight but inclined to the axis of rotation
↓
Helical Gear.

Helical gear
- Right hand helical gear
- left hand helical gear

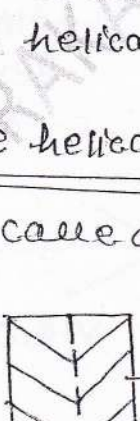
- Gradual engagement takes place because opposite hand helical gears must be in contact.
- Due to this impact stress is eliminated.
- Since here F can't be cancelled so axial force is developed so axial thrust is developed.



(III) To eliminate axial thrust of helical gear] ⇒ Double helical gear.

- Double helical gear

Also called herringbone gear.



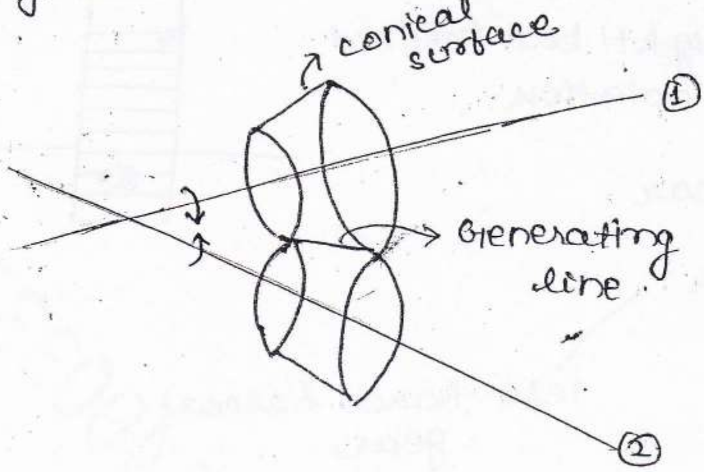
(Ideal)

→ Herringbone suggested this type of gear but it is impossible to construct it, so in real, in b/w tool run out space is provided.

(ii) Axes are non-parallel but intersecting

Pure rolling motion can be transmitted b/w two conical surfaces in contact.

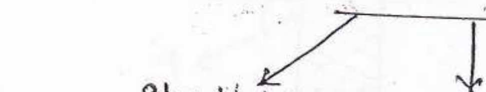
→ Bevel gear.



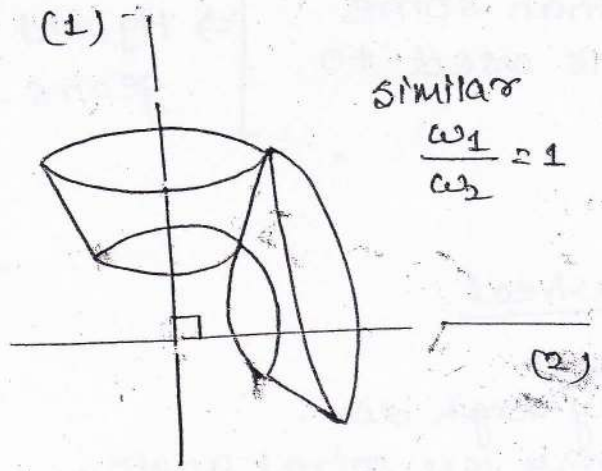
Bevel gear
- Straight Bevel gear
- Zerol Bevel gear
- Helical Bevel gear
  - Right hand helical Bevel gear
  - Left hand helical Bevel gear

(Straight and Zerol bevel gear:)

- Impact stress is there
- Teeth are straight and parallel to the axis of rotation.
- 1% use.

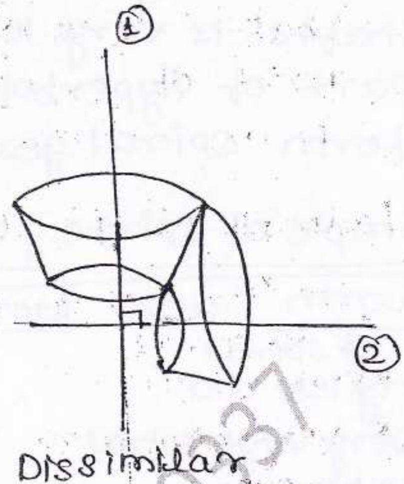


Use of straight Bevel gear → Drill machine.



similar

$\frac{\omega_1}{\omega_2} = 1$



Dissimilar

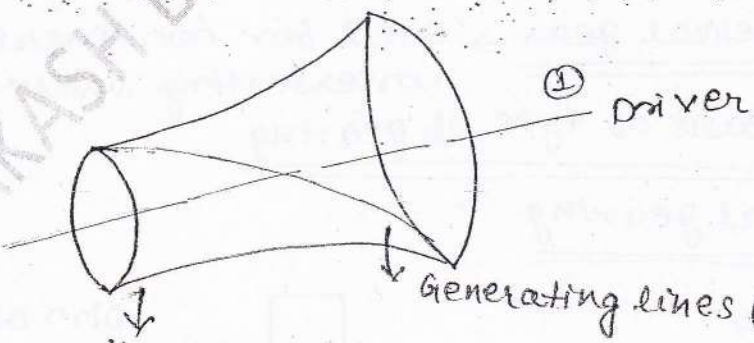
$\frac{\omega_1}{\omega_2} \neq 1$

$\frac{\omega_1}{\omega_2} = const.$

Mitre gear

(III) Axes are neither parallel nor intersecting

- pure rolling is impossible.
- Rolling is possible and it is having rotational + partial sliding motion.



Skew → neither parallel nor intersecting.

Spiral gear

- It is also called skew-bevel gear;
- when the space (offset) b/w the shafts is very less than some part of hyperboloid is used to form spiral gear. ⇒ Hypoid gears.

Example of spiral gear

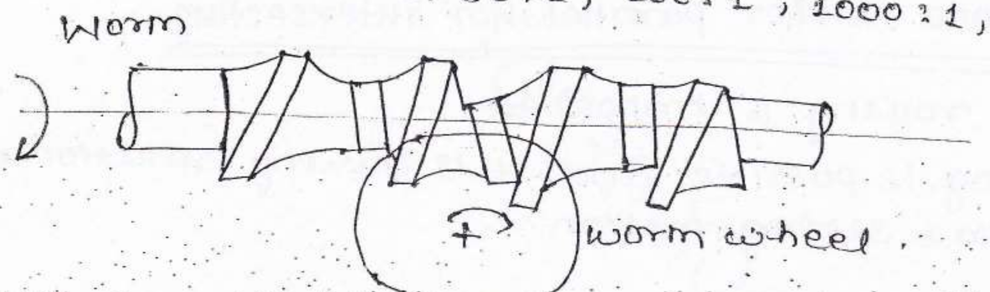
worm and **worm wheel**

worm	worm wheel
⇓ spiral	⇓ spiral
• very less dia.	• very high dia.
• very high spiral angle	• very less spiral angle.

worm and worm wheel gear is used or famous for high speed reduction ratio.

↓

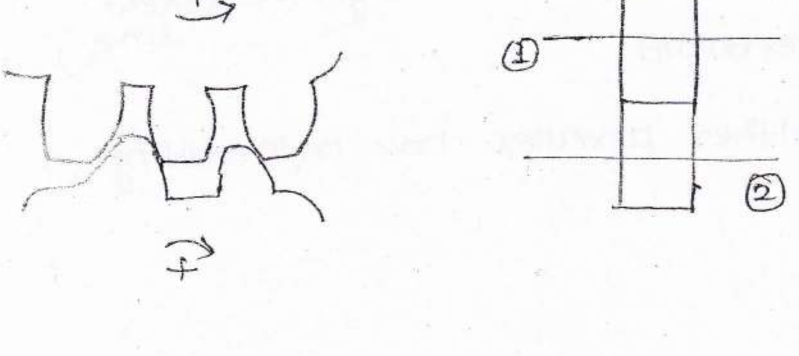
10 : 1, 30 : 1, 100 : 1, 1000 : 1, 1250 : 1.



* Helical gear → known as master of all gear;
* crossed helical gear → used for non-parallel non-intersecting shaft.

(B) on the Basis of type of gearing

(i) External gearing

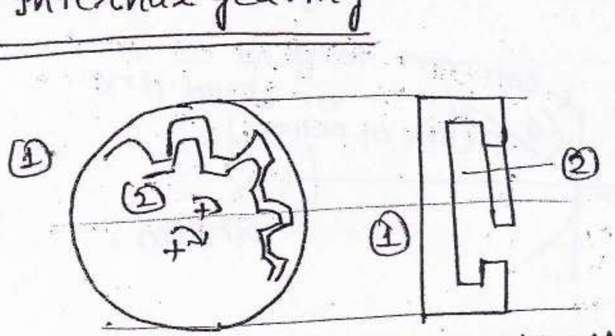


Dirn of rotation of both the gears is opposite.

Bigger :- Gear.

Smaller :- Pinion

(ii) Internal gearing



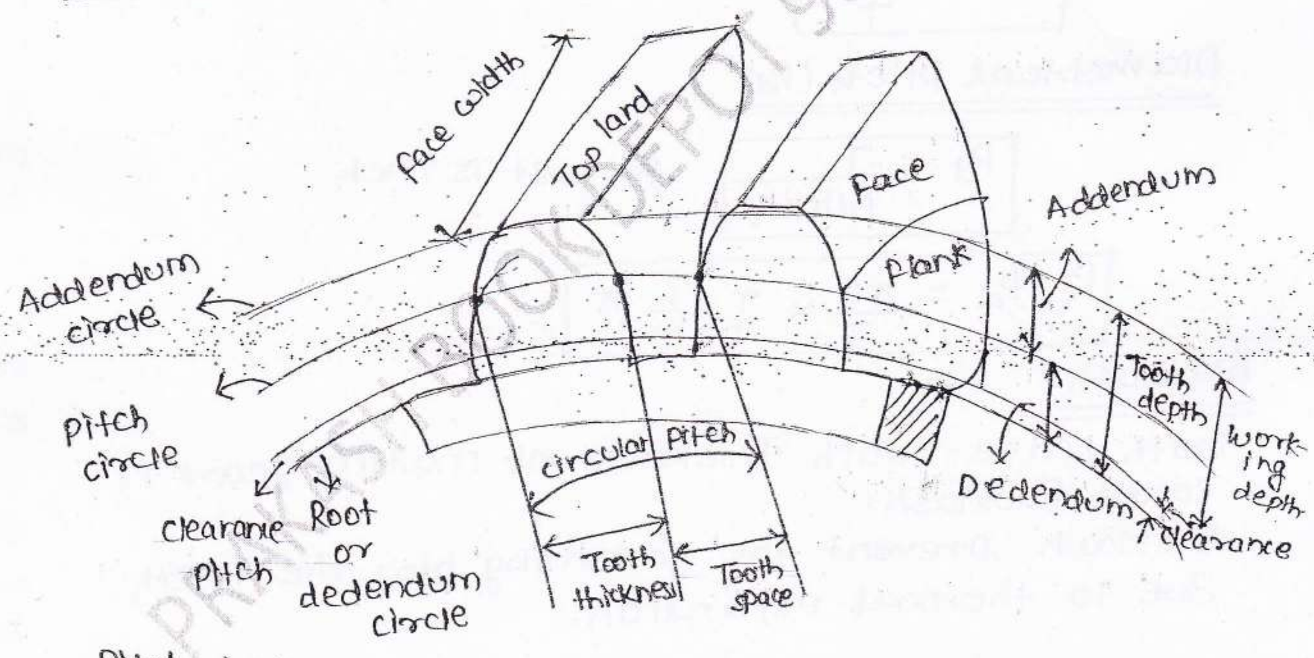
Bigger :- Annular (ring)
Smaller :- pinion.

- Bigger have internal teeth while smaller has external teeth.
- Both rotate in same dirn.

Note

- If more than one gears are mounted on same shaft called compound gears and speed of all are same.
- In general in power transmission, smaller bodies are made as drivers because less torque is required at input due to more speed ($P = T\omega$).

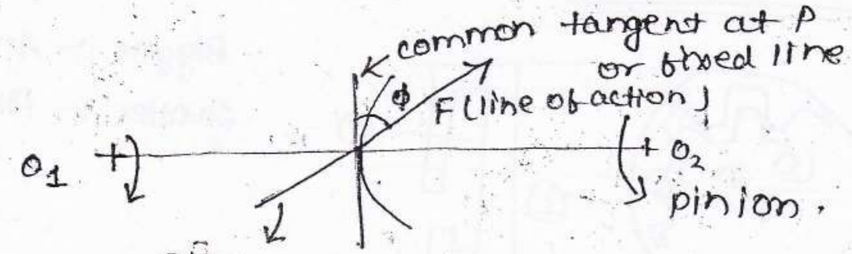
General Gear terminology



Pitch circle

"It is an imaginary circle in the gears where pure rolling motion is observed when mating gears are transmitting powers". Being an imaginary circle, it can't be the physical characteristics of the gear but being the most imp. circle it is one of the biggest specification of the gear.

The size of any gear is specified by the dia. of the circle.



Circular pitch (P_c):-

If pitch circle dia. (P.C.D) = D
no. of teeT.

$$P_c = \frac{\pi D}{T}$$

For two mating gears

$$P_{c_1} = P_{c_2} \Rightarrow \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \Rightarrow \frac{D_1}{T_1} = \frac{D_2}{T_2}$$

Module (m)

$$m = \frac{D(mm)}{T}$$

Its unit is mm

Diametrical pitch (P_d)

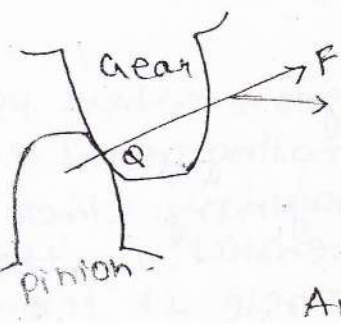
$$P_d = \frac{T}{D(inch)}$$

Its unit is inch.

$$P_c \cdot P_d = \frac{\pi D}{T} \times \frac{T}{D} = \pi$$

Backlash

Tooth space - tooth thickness of mating gears is called Backlash.
Backlash prevent the jamming b/w the teeth due to thermal expansion.



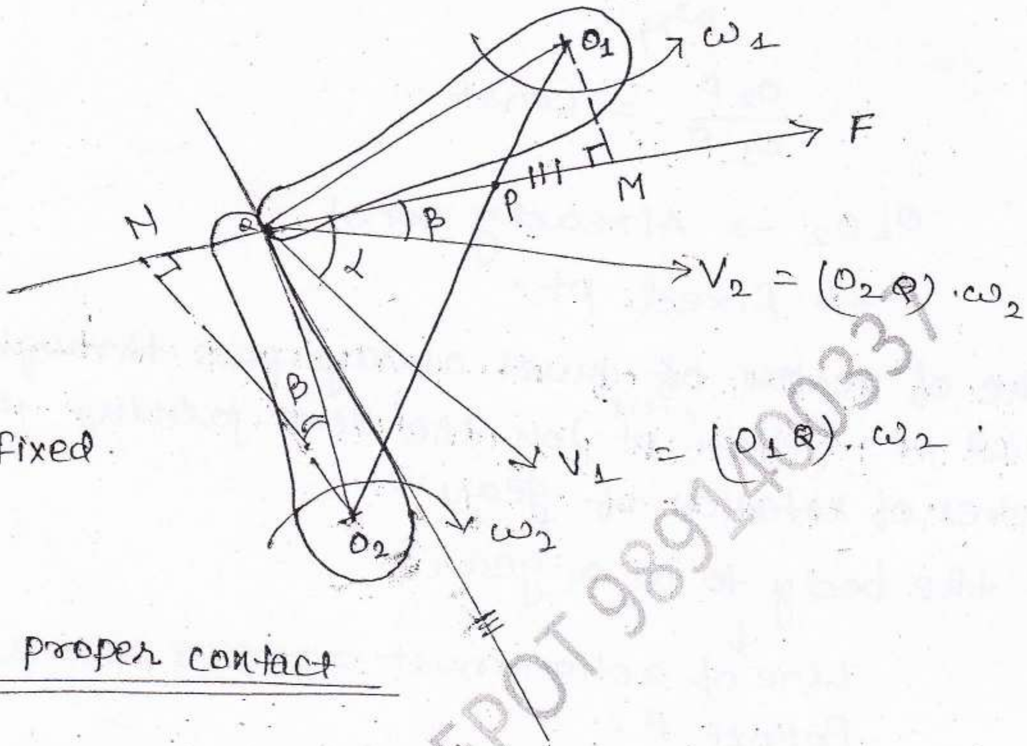
→ common normal at Q (line of action).

ϕ → Pressure angle

Angle b/w line of action and common tangent at P.

25 | 03 | 14

law of Gearing



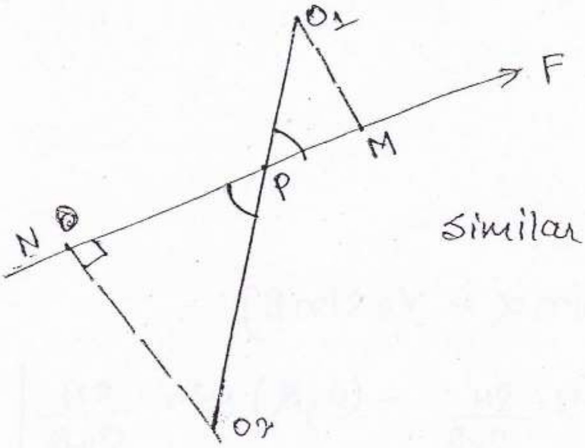
$O_1O_2 \Rightarrow$ Fixed

For proper contact

$$V_1 \cos\alpha = V_2 \cos\beta$$

$$(O_1Q)\,\omega_1 \cdot \frac{O_1M}{O_1Q} = (O_2Q)\,\omega_2 \cdot \frac{O_2N}{O_2Q}$$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}}$$



Similar Δs.

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} = \frac{PN}{PM}$$

If these two bodies are gears:-

$$\frac{\omega_1}{\omega_2} = \text{const}$$

$$\frac{O_2P}{O_1P} = \text{const}$$

$O_1O_2 \rightarrow$ Already const

$P \rightarrow$ fixed pt.

"Line of action of must always pass through the fixed pt. (pitch pt.) on the line joining the centres of rotation of gears".

For the body to be a gear
↓
Line of action must always pass through Point P.

~~Conjog~~

Conjugate profile

"Mating profiles must be designed in such a way such that each and every moment law of gearing is satisfied.

Conjugate profile

(1) Involute

(2) Cycloidal.

Velocity of Sliding

$$V_{sliding} = |V_1 \sin\alpha - V_2 \sin\beta|$$

$$= \left| (O_1Q)\,\omega_1 \cdot \frac{QM}{O_1Q} - (O_2Q)\cdot\omega_2 \cdot \frac{QN}{O_2Q} \right|$$

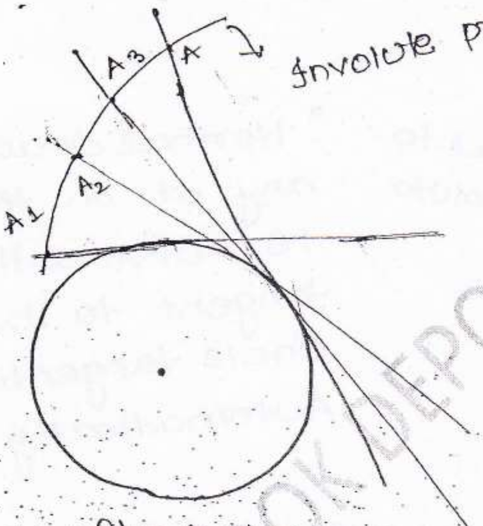
$$\Rightarrow Q_1' = |\omega_1 \cdot (QP + PM) - \omega_2 (PN - QP)|$$

$$= |\omega_1 \cdot QP + \omega_1 \cdot PM - \omega_2 PN + \omega_2 \cdot QP|$$

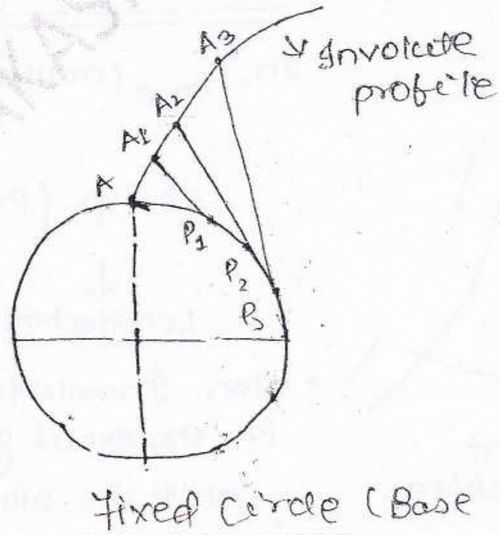
$$\boxed{V_{sliding} = (\omega_1 + \omega_2) QP}$$

Involute profile (By nature conjugate)

"It is defined as the locus of the pt. on the line which rolls without slipping on the fixed circle".



Fixed circle
(Base circle)



Fixed circle (Base circle).

In Reality

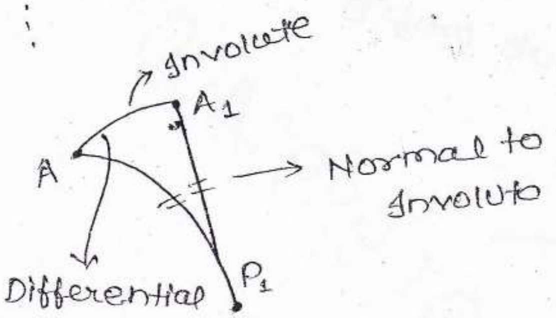
$AP_1 \rightarrow 0$
$P_1 P_2 \rightarrow 0$
$P_2 P_3 \rightarrow 0$

Negligible (Differential)

$Arc\ (AP_1) = P_1 A_1$

$Arc\ (AP_2) = P_2 A_2$

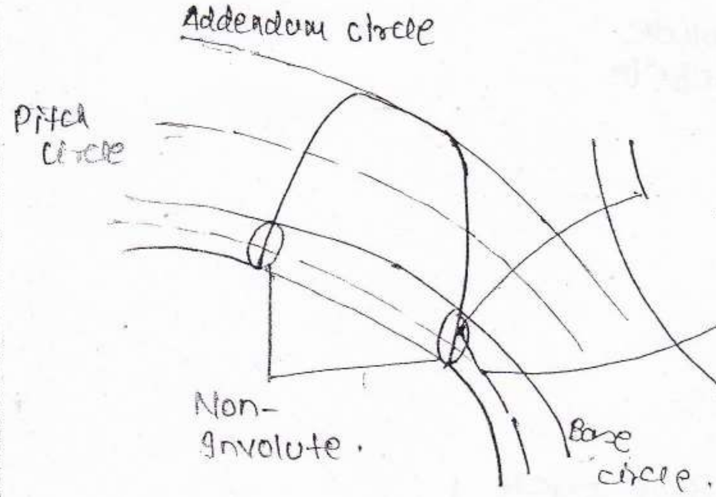
$Arc\ (AP_3) = P_3 A_3$



Portion of circle having centre P_1 & Radius $P_1 A_1$

"Normal drawn at any pt. on the involute curve will become tangent to the base circle tangentially Automatically"

Position of base circle in an Involute gear



In external gears

In r_{Base} (radius of base circle) ↓

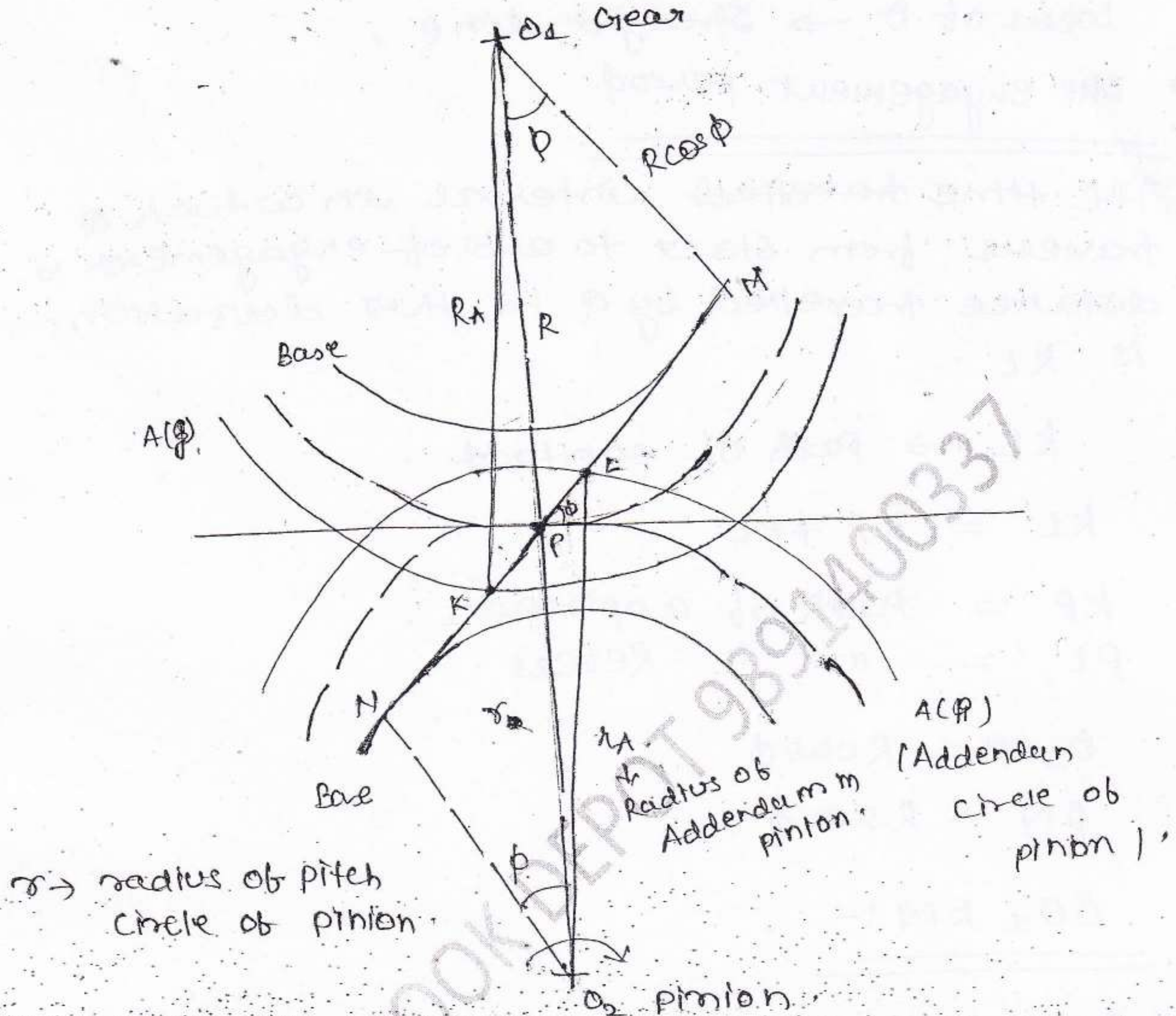
⇒ ϕ (Pressure angle) ↑

↓

Limitation (20-25°)

- Non-Involute portion in external gears can't be eliminated.

Analysis of Involute gears :-



Start of engagement ⇒ K
End " " ⇒ L

Line of Action

(i) Pass through pitch pt. (P).
(ii) Tangent to both the base circle.

- Here, point of contact is changing but line of Action is not Changing.

Thus, ϕ is constant.

- Pt. of contact is travelling along line of Action.

  Locus of Q → Straight line.

- One engagement period

The time ~~travelled~~ interval in which Q travels from start to end of engagement & distance travelled by Q in this duration is KL.

KL → Path of contact.

$KL = KP + PL$

KP = Path of approach.

PL = " " Recess.

$O_1M = R\cos\phi$

$PM = R\sin\phi$.

$\Delta O_1 KM$:-

$$R_A^2 = R^2\cos^2\phi + (KP + R\sin\phi)^2$$

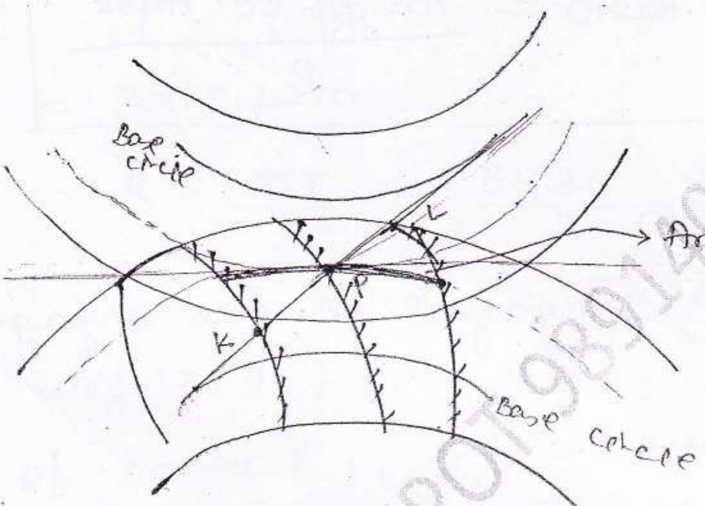
$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

KP + PL = Path of contact (KL)

Arc of contact

"when the pt. of contact is travelling from the start of engagement to the end of engagement the distance travelled by the pinion and gear along their pitch circle in this duration is known as arc of contact".



$$\text{Arc of Approach} = \frac{\text{Path of Approach}}{\cos\phi}$$

$$\text{Arc of Recess} = \frac{\text{Path of Recess}}{\cos\phi}$$

Arc of Recess + Arc of Approach = Arc of contact

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi}$$

Arc of contact

Travel of pinion / Gear along their pitch circles in one engagement period.

$$\text{Contact Ratio} = \frac{\text{Arc of contact}}{P_c}$$

Contact Ratio

No. of pairs engaged in one engagement period.

Minimum = 1.

Generally ∈ [1.2, 1.8].

For example:-

Contact Ratio = 1.33

Means one pair of is engaged in full engagement ~~pair~~ period. But 33% time of the engagement period is like that in which along with this pair one more pair i.e, total two pairs are engaged. Therefore, no. of pairs engaged in one engagement pair period, its avg value comes out to be 1.33.

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(6) $t = 24$

$T = 36$

$m = 8\ mm$

$\phi = 20°$

Add. (each) = 7.5 mm.

$N_p = 450$ r.p.m.

$$\frac{N_G}{N_p} = \frac{t}{T} = \frac{24}{36}$$

$N_G = 300$ r.p.m.

Gear :- $R = \frac{mT}{2} = \frac{8 \times 36}{2} = 144 mm$.

Pinion :- $r = \frac{mt}{2} = \frac{8 \times 24}{2} = 96 mm$.

$r_A = 96 + 7.5 = 103.5\ mm$

Path of contact :-

$KL = KP + PL$

$= \left[\sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi\right]$

$+ \left[\sqrt{r_A^2 - r^2\cos^2\phi} - r.\sin\phi\right]$.

= mm + mm

KL = mm.

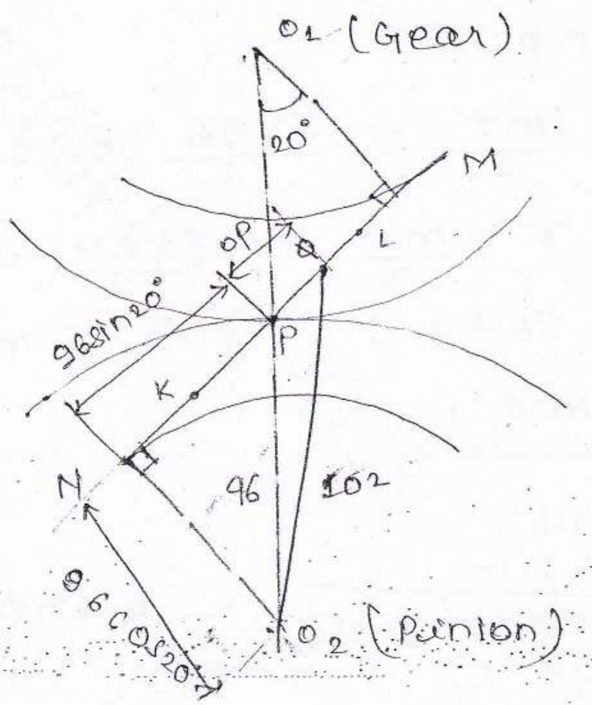
Arc of contact = $\frac{KL}{\cos 20°}$ = ——

(i) (Angle)$_{pinion}$ $= \dfrac{\text{Arc of contact}}{96} \times \dfrac{18}{\pi} =$ °

(ii) velocity of sliding

$$V_{sliding} = (\omega_1 + \omega_2)\, QP$$

$$= \frac{2\pi}{60}\,(450 + 300) \cdot QP$$



$$(102)^2 = (96\cos 20°)^2 + (96\sin 20° + QP)^2$$

$$\Rightarrow QP =$$