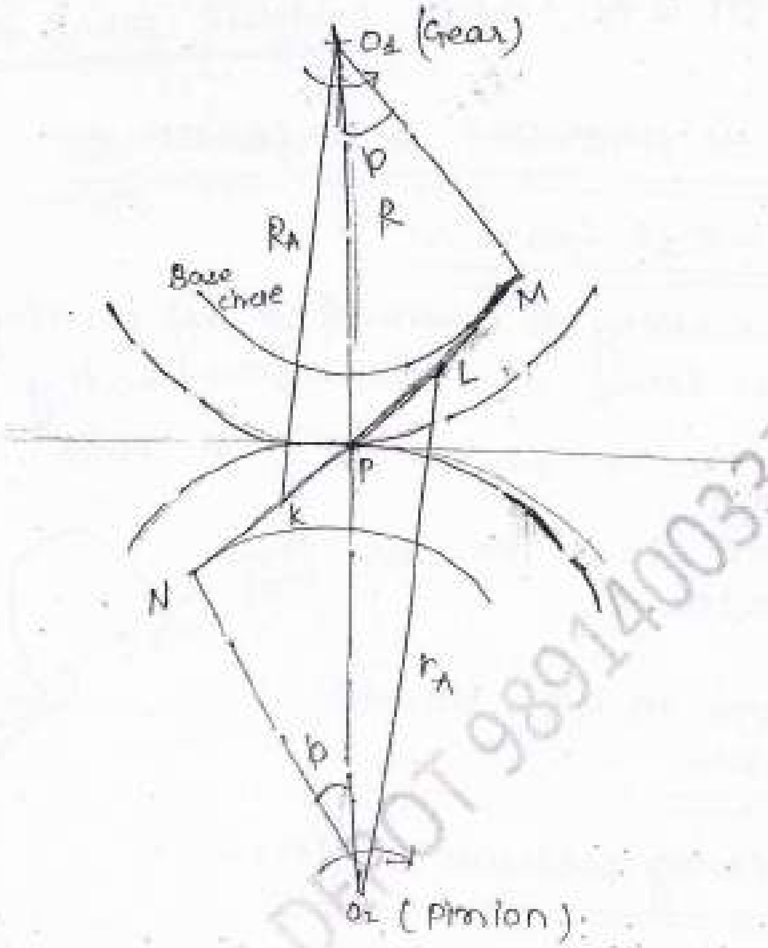
4rth Semester

Kinematics and Dynamics Of Machine

Module 2(a)

Department of Mechanical Engineering
Government College Of Engineering Kalahandi, Odisha

Interference



If $r_A > O_2M$

→ Involute tip of pinion will touch Non-involute flank position of gear.

→ Involute to non-involute connection.

→ Law of Gearing is not satisfied.

→ Involute tip of pinion will remove some material from non-involute flank position of gear.

This removal of material is a process called under cutting.

Similar will be there when $R_A > O_1N$.

- The last safety points of K & L
  ↓ ↓
  N M
- Points M & N are critical pts (Interferance pts.)

Methods to prevent Interference

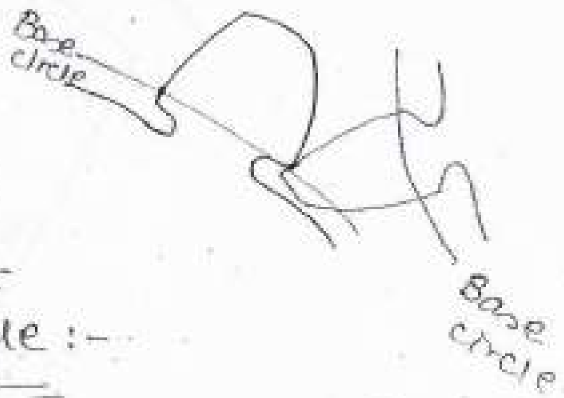
(1) under - cut gears :-

under cutting is provided by cutting tool at the time of manufacturing.

Strength is less at the root.

Limitation

used upto low power transmission.

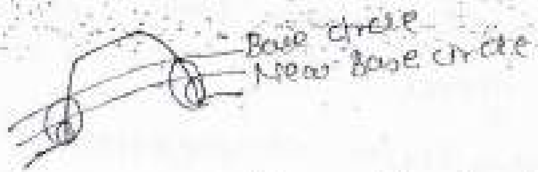


For Medium to High power transmissions :-

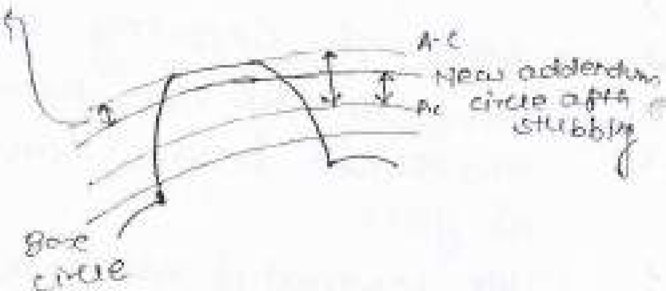
(2) Increasing pressure angle :-

If $\phi \uparrow \Rightarrow r_{Base} \downarrow \Rightarrow$ Non - Involute portion ↓

$\Rightarrow$ Interference ↓↓



(3) By stubbing the teeth



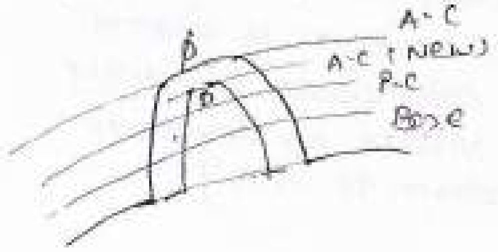
By stubbing :-

- Addendum ↓
- Addendum circle radius ↓.
- Interference ↓↓.

By stubbing :-

- ϕ no change.
- Path of contact ↓↓.
- Arc " " "
- Contact ratio ↓↓ (Min. = 1)

(4) Increasing to the no. of teeth (Best method)



By T↑ ⇒ Add. ↓
⇒ " circle rad. ↓
⇒ Int. ↓↓.

By T↑ ⇒ ϕ ⇒ No change
contact ratio ⇒ ↑↑

* No limitation.

T → No. of teeth.

26|03|14

Rack (Biggest Gear)

Gear of infinite pitch circle radii dia.

P → Pinion.
G → Gear.
R → Rack.

A → Fractional addendum for 1mm module in order to avoid interference.

A_P → fractional Addendum of pinion.
A_G → " " Gear.
A_R → " " Rack.

Addendum required in order to avoid interference $= mA$

where $m \rightarrow$ module.

$\left.\begin{matrix} mA_p \\ mA_G \\ mA_R \end{matrix}\right\}$ Represented as Addendum of pinion, gear and Rack resp.

For example:-

Add: 7.5 mm.
m : 8mm

$mA = 7.5$

$A = \frac{7.5}{8}$

• [If module is one then the addendum is taken as A otherwise for any other value of module addendum is mA]

• Involute Gear Systems

Full Depth involute [when full depth involute is given, then it means addendum is 1].

$(14\frac{1}{2}°, 20°)$

Addendum = standard Addendum
= one MODULE VALUE

$\cancel{m}A = 1\cancel{m}$

$\boxed{A = 1}$

Represented as A_P, A_G, A_R for pinion, Gear and rack resp. as its value is equal to 1.

Stub Involute

$(20°, 25°)$

Addendum < standard Add.

$\cancel{m}A < 1\cancel{m}$

$\boxed{A < 1}$

$$\left.\begin{matrix} A_P \\ A_G \\ A_R \end{matrix}\right\} < 1$$

20° stub Involute (Best)

1) Lesser Interference.
2) Min^m no. of teeth requirement is less
3) Cost is less.
4) Strength is more.

Velocity ratio

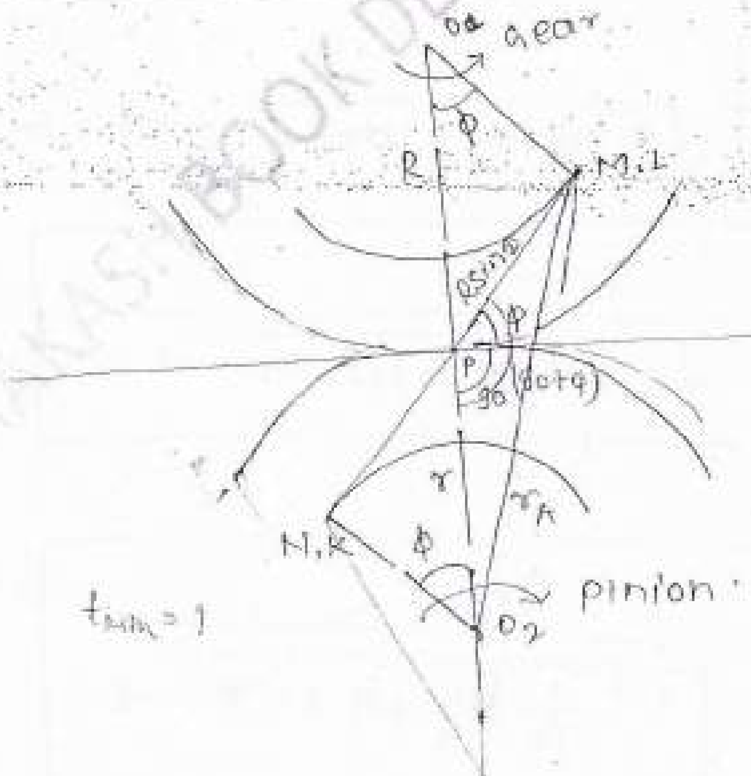
$$\frac{\omega_P}{\omega_G} = \frac{T}{t} > 1$$

$$\frac{\omega_G}{\omega_P} = \frac{t}{T} < 1$$

Gear ratio (G)

$$G = \frac{T}{t} \quad (\text{Always} \geq 1)$$

Minimum no. of teeth required on the pinion / Gear in order to avoid interference



R → Pitch circle radius of Gear.

r → pitch circle radius of pinion

$t_{min} = ?$

In $\Delta O_2 PM$

Cos Rule

$$r_A^2 = r^2 + R^2\sin^2\phi - 2(r)(R\sin\phi)\cos(90+\phi)$$
$$= r^2 + (R^2 + 2rR)\sin^2\phi$$
$$= r^2\left[1 + \frac{(R^2+2rR)}{r^2}\sin^2\phi\right]$$
$$= r^2\left[1 + \left(\underbrace{\frac{R}{r}}_{G} + \frac{2R}{r}\right)\sin^2\phi\right]$$

$$r_A^2 = r^2\left[1 + G(G+2)\sin^2\phi\right]$$

$$r_A = r\sqrt{1 + G(G+2)\sin^2\phi}$$

Add. (pinion) $= r_A - r$

$$= r\left[\sqrt{1 + G(G+2)\sin^2\phi} - 1\right]$$

Add. (pinion)

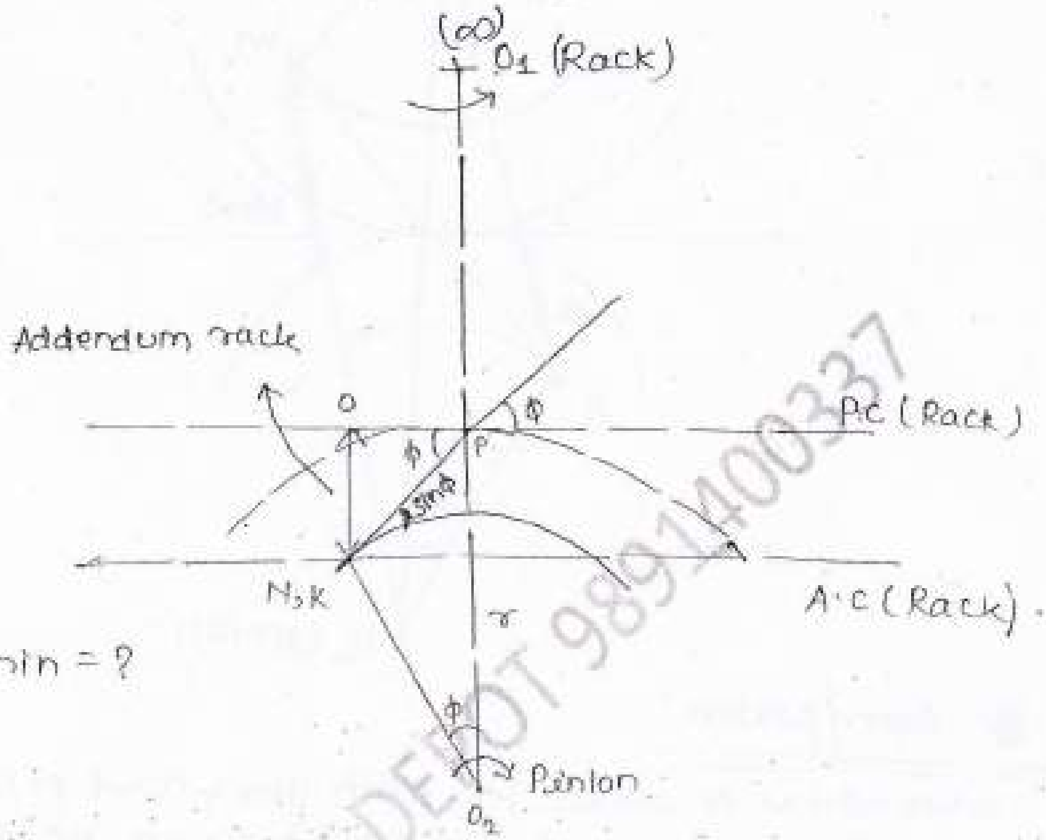
$$= \frac{m t_{min}}{2}\left[\sqrt{1 + G(G+2)\sin^2\phi} - 1\right] = mA_p$$

$$\boxed{t_{min} = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}}$$

for Gear,

$$\boxed{T_{min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}}$$

Minimum no of teeth required on the pinion in order to avoid interference in involute Rack and pinion arrangement



$t_{min} = ?$

In Δ ONP

$$\sin\phi = \frac{Add\,(Rack)}{r\sin\phi}$$

$$Add\,(Rack) = r\sin^2\phi$$

$$= \frac{m t_{min}}{2}\sin^2\phi$$

But we know,

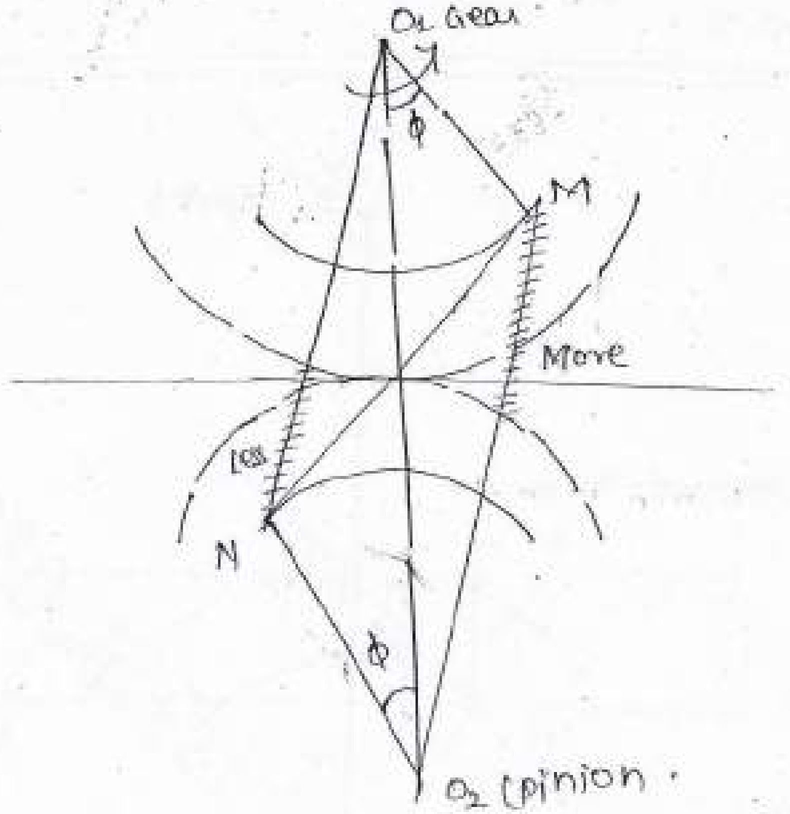
$$Add\,(Rack) = m A_R$$

$$\therefore \frac{\cancel{m} t_{min}}{2}\sin^2\phi = \cancel{m} A_R$$

$$\boxed{t_{min} = \frac{2 A_R}{\sin^2\phi}}$$

{ When full depth involute is given then $A_R = 1$. }

Note :-



If Gear/pinion

Addendum is same for both gear and pinion then the chances of interference of Gear is more because addendum circle of gear is more nearer to the pinion as compared to A.C of pinion near Gear.

- Firstly we have to safe the Gear to avoid interference.

$$\boxed{T_{min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}}$$

- when Gear is safe, pinion is automatically safe as chances of interference of pinion is less. So, in any numerical pb we can safe Gear in order to safe pinion.

$$\boxed{t_{min} = \frac{T_{min}}{G}}$$

In gear / pinion, when addendum is different

- $$T_{min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

- $$t_{min} = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

- $$G = \frac{T_{min}}{t_{min}}$$

Eg:- If $T_{min} = 40 / 42$ ← possible $\left[\text{Here } G = 3\right]$.
$t_{min} = 12 / 13 / 14$
not possible

From these two values check G if equal to 3

ES → 2007 (20 marks)

$G = 3$, $A_P = A_G = 1$ (full depth)
$\phi = 20°$, $t_{min} = ?$

$$T_{min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1} = 45$$

$t_{min} = \frac{45}{3} = 15$

$t_{min} = 15 - 3 = 12.$

$T_{min} = 12 \times 3 = 36.$

$$T_{min} = 36 = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

Putting, $G = 3$, $\phi_1 = 20°$

we get, $A_G = 0.8$.

It means 20% stubbing.

Pb
20-Marks

same Q. upto $T_{min} = 36$.

$$T_{min} = 36 = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

(Given, $A_G = 1$, $G = 3$, $\phi = ?$].

Pb
IES 1998 (20-marks)

$\phi = 20°$, $m = 10mm$,

$A_P = A_G = 1$

$\left.\begin{matrix} T = 50 \\ t = 13 \end{matrix}\right]$ $G = \frac{50}{13}$.

Here, $T < T_{min}$ thus interference will occur.

(i) $$T_{min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1} = 60$$

But given

(ii) $$T_{min} = 50 = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

$\Rightarrow \phi = ?$

Pb

$G = 4$, $A_p = A_G = 1$, $\phi = 20°$, $T_{min} = ?$

$$T_{min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1} = \underline{\underline{62}} \Rightarrow \text{not correct}$$

Correct $= 64$

But check t_{min} firstly then decide the ans.

$$t_{min} = \frac{62}{4} = 15.5 \simeq 16.$$

$\therefore$ To obtain $G = 4$, T_{min} must be equal to $\underline{64}$.

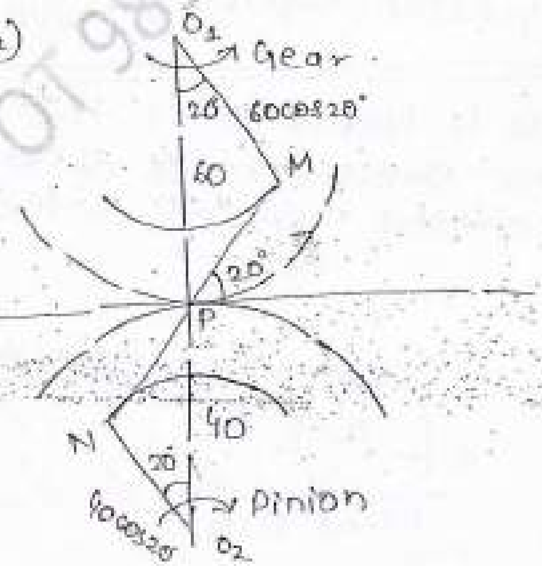
Effect of centre distance variation due to vibration on the performance of Involute gears

Centre distance $= 100\,mm\ (O_1O_2)$

$R = 60\,mm$

$r = 40\,mm$

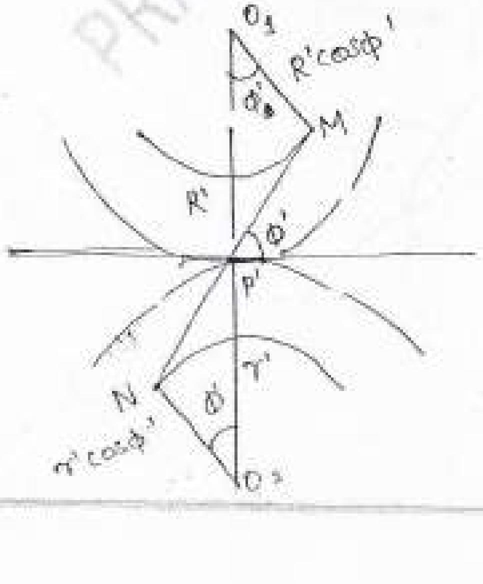
$\phi = 20°$



At a moment

If centre distance is ↑ by 2%.

$\boxed{100 \rightarrow 102}$ mm



$$R'\cos\phi' = 60\cos 20°$$
$$r'\cos\phi' = 40\cos 20°$$

$$(r'+R')\cos\phi' = 100\cos 20°$$

$$\cos\phi' = \frac{100\cos 20°}{102}$$

$\phi' = ?$

Due to vibration

- → centre distance changed
- pitch circle changed
- P' changed
- ϕ changed
- $r_{\text{Base circle}}$ remains const. as it is physically dependent on gear. r_{Base} changes only when gear changes.

In case of Involute gear

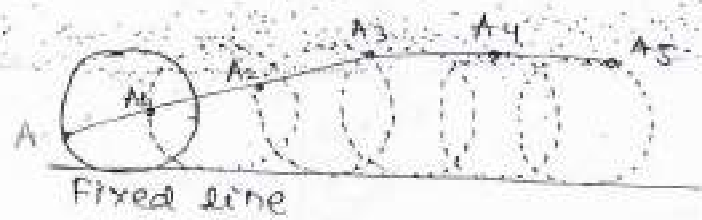
This is the most imp. advantage of Involute Gear and thus most widely used practically.

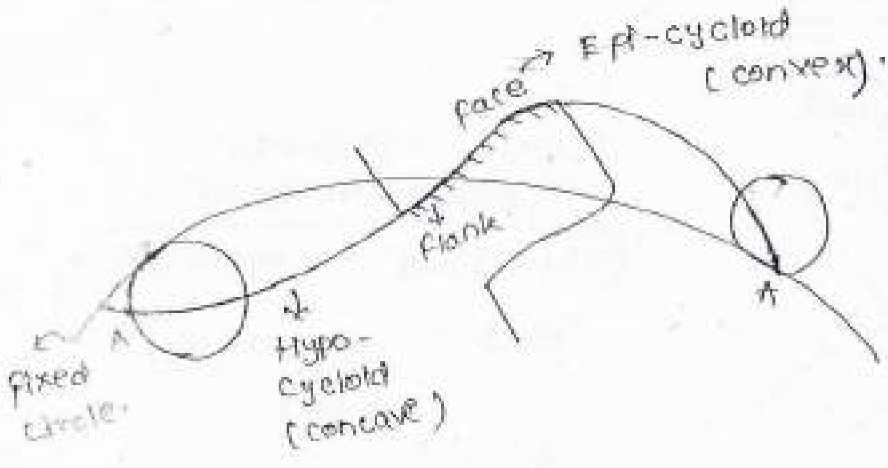
$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \text{const.}}$$

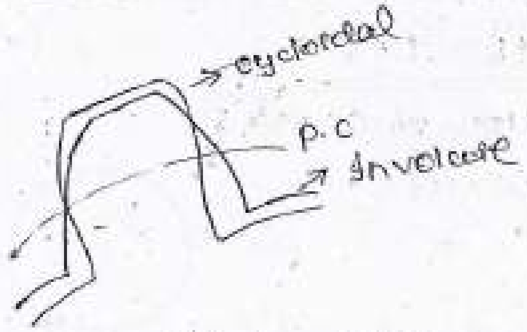
Due to vibration also there is no effect on $\frac{\omega_1}{\omega_2}$ ratio.

Cycloidal profile (By nature conjugate)

" It is defined as the locus of a point on the circumference of the circle which rolls without slipping on the fixed straight line. "







Advantages of ~~Involute gear~~ or cycloidal gear over Involute gear

- Per tooth cost more but overall cost of gear is almost same and even sometimes less.
- Flank is wider, thus tooth are more stronger.
- Interference is absent
- Life is more as there is less wear and As in case of convex - concave connection wear is always less.
- ϕ is changing.

  $\underline{\phi_{max}}$ → At start of engagement.

  $\underline{Zero}$ → " pitch point.

  $\underline{\phi_{max}}$ → In opposite dirⁿ at the end of engagement.

- Inspite of huge no. of advantages of cycloidal gear, these are not used practically as velocity ratio is not const in this gear due to vibration.

Gear Trains

Gear train is combination of gears.

Why gear trains are required?

(i) large centre distance.

(ii) very high / very low velocity ~~distance~~ ratio.

- Inspite of having less centre distance the smaller gear can't able to drive bigger one;

Any gear train consist of :-

(i) Main Driver

(ii) Main Driven.

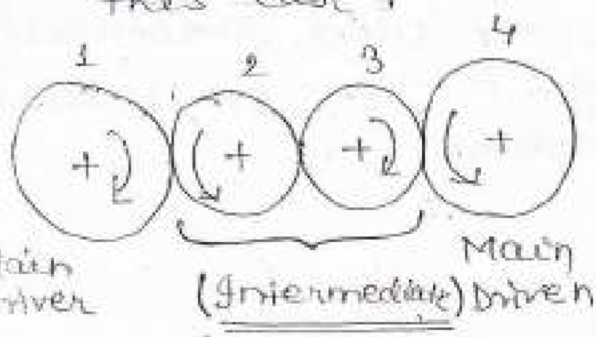
(iii) Intermediate Gears.

$$\boxed{\frac{\omega_{\text{Main Driver}}}{\omega_{\text{Main Driven}}} = \text{speed ratio of Gear train (S.R)}}$$

$$\boxed{\frac{1}{S.R} = \frac{\omega_{\text{main Driven}}}{\omega_{\text{main Driver}}} = \text{Train ratio}}$$

Simple gear train

Every shaft is having only one gear in use. module of all gear is same in this case.



(Also called Idler as there is no contribution of these gear in velocity ratio).

Gear

(1, 2) $\quad \dfrac{\omega_1}{\omega_2} = \dfrac{T_2}{T_1}$ — (1)

(2, 3) $\quad \dfrac{\omega_2}{\omega_3} = \dfrac{T_3}{T_2}$ — (2)

(3, 4) $\quad \dfrac{\omega_3}{\omega_4} = \dfrac{T_4}{T_3}$ — (3)

(1) × (2) × (3)

$$\boxed{\frac{\omega_1}{\omega_4} = \text{speed ratio} = \frac{T_4}{T_1}}$$

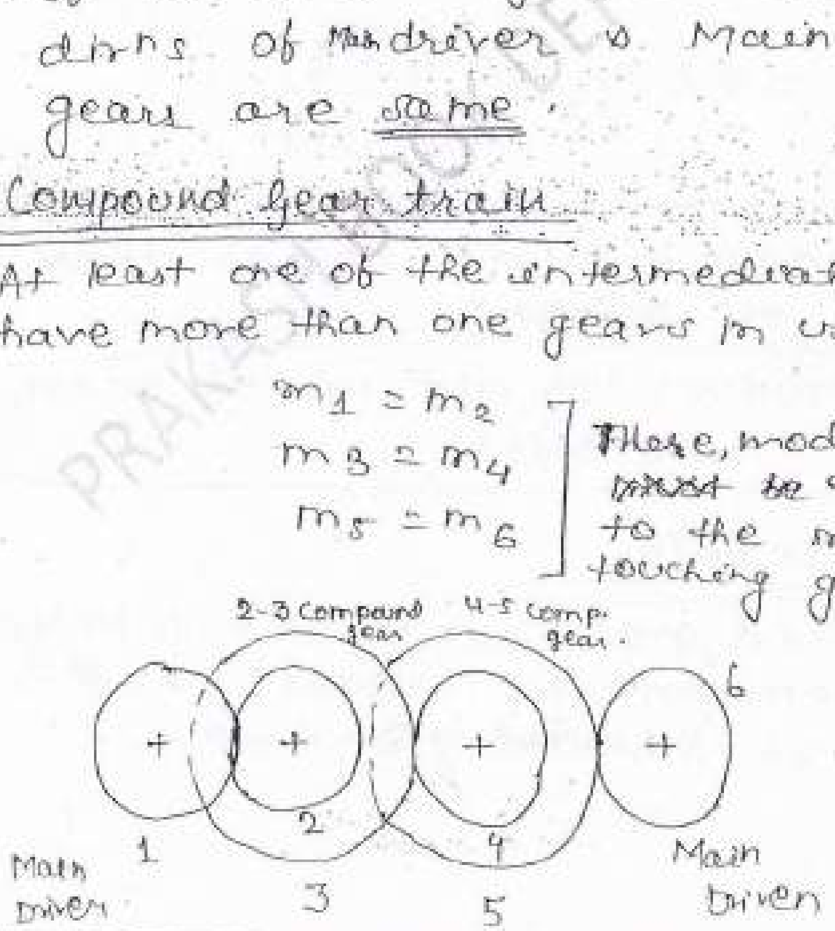
- If no. of Idler gears are <u>even</u> then the dirns of main driver & driven gears are <u>different</u>.
- If no. of Idler gears are <u>odd</u> then the dirns of main driver & main driven gears are <u>same</u>.

Compound gear train

At least one of the intermediate shaft must have more than one gears in use.

$$\left.\begin{aligned} m_1 &= m_2 \\ m_3 &= m_4 \\ m_5 &= m_6 \end{aligned}\right]$$

Here, modules is always ~~must be~~ equal to the module of touching gear.



In case of compound gear train angular velocities are same.

$$\boxed{\omega_2 = \omega_3 \quad \& \quad \omega_4 = \omega_5}$$

Driver : (1, 3, 5)
Driven : (2, 4, 6)

(1, 2)

$$\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} \quad \text{--- (1)}$$

3, 4

$$\frac{\omega_3}{\omega_4} = \frac{T_4}{T_3} \quad \text{--- (2)}$$

(5, 6)

$$\frac{\omega_5}{\omega_6} = \frac{T_6}{T_5} \quad \text{--- (3)}$$

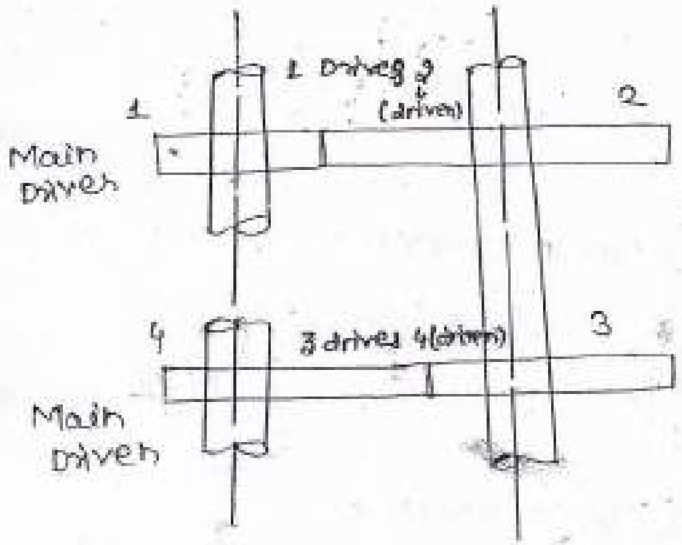
(1) × (2) × (3)

$$\frac{\omega_1}{\omega_6} = S.R = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

$$\boxed{S.R = \frac{\text{Product of the no. of teeth on Driven}}{\text{Product of the no. of teeth on Driver.}}}$$

Reverted Gear train

That compound gear train which is basically used to connect the coaxial shaft is known as Reverted gear train.



Driver :- (1,3)
Driven :- (2,4)

$m_1 = m_2 = m$

$m_3 = m_4 = m'$

$$\boxed{\text{speed ratio} = \frac{\omega_1}{\omega_4} = \frac{T_2 \times T_4}{T_1 \times T_3}}$$

A General concept

[1 & 2 are compound gear thus module is same and same is in the case of 3 & 4]

$$\left\{ \begin{array}{l} m = \frac{D}{T} \\ \therefore \frac{2r}{T} = m \\ \Rightarrow r = \frac{mT}{2} \end{array} \right\} \quad r_1 + r_2 = r_3 + r_4$$

$$\frac{mT_1}{2} + \frac{mT_2}{2} = \frac{m'T_3}{2} + \frac{m'T_4}{2}$$

$$\boxed{m(T_1 + T_2) = m'(T_3 + T_4)}$$

If all gears have same module

$$\boxed{T_1 + T_2 = T_3 + T_4}$$

For example

$r_1 + 2r_2 = r_3$

$$\boxed{T_1 + 2T_2 = T_3}$$ If modules are same.

Driver ⇒ Pinion.

Note :-
If in Reverted gear train problem no. of eqn are less & unknown are more than we can assume -

Assumb. for $$\boxed{\frac{T_2}{T_1} = \frac{T_4}{T_3}}$$

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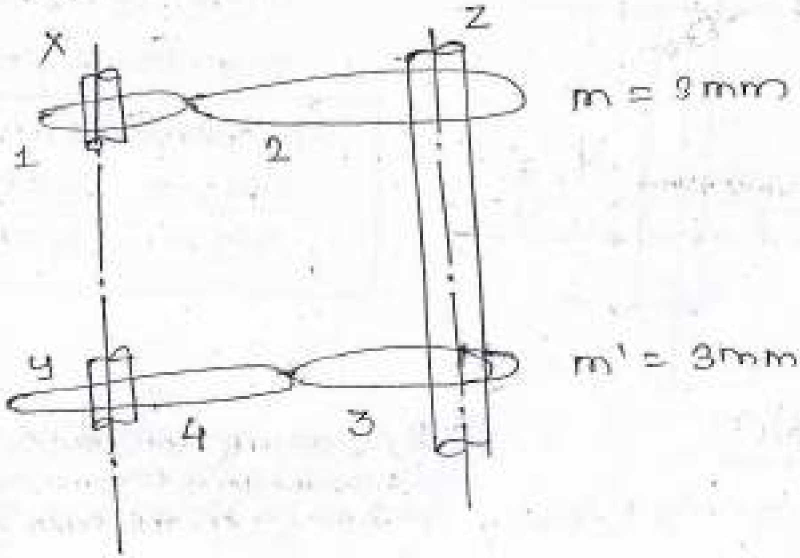
Pb(1) (conventional)



$\omega_4 < \frac{\omega_1}{12}$

$\frac{\omega_1}{\omega_4} > 12$

$\frac{T_2}{T_1} \cdot \frac{T_4}{T_3}$ $T_1 = T_3 = 24$

$2(24 + T_2) = 3(24 + T_4)$

$48 + 2T_2 = 72 + 3T_4$

$3T_4 = 2T_2 - 24$

$T_4 = \frac{2(T_2 - 12)}{3}$ —— (1)

$\frac{\omega_1}{\omega_4} > 12$

$\frac{T_2 \cdot T_4}{24 \times 24} > 12$

$T_2 \cdot T_4 > 12 \times 24 \times 24$

$$T_2 \cdot T_4 > 6912 \quad \text{——} (2)$$

$$\frac{2(T_2-12)}{3} \cdot T_2 > 6912$$

$$T_2^2 - 12T_2 - \frac{6912 \times 3}{2} > 0$$

$$T_2 = 109, \quad T_4 = 64.67$$

$$T_2 = 109.5, \quad T_4 = 65$$

$$T_2 = 110 \Rightarrow T_4 = 65.33$$

$$\boxed{T_2 = 111, \quad T_4 = 66}$$

Speed Ratio

$$\frac{\omega_1}{\omega_4} = \frac{111 \times 66}{24 \times 24} = 12.718$$

centre distance $= r_1 + r_2$

$$= \frac{mT_1}{2} + \frac{mT_2}{2}$$

$$= \frac{2}{2}(24 + 111)$$

$$= 135 \text{ mm}$$

Epicyclic gear train

" Apart from the rotation of the gears if any gear axis is also rotating w.r.t to ~~an~~ some other axis, then the train will be known as Epi-cyclic gear train. It may be

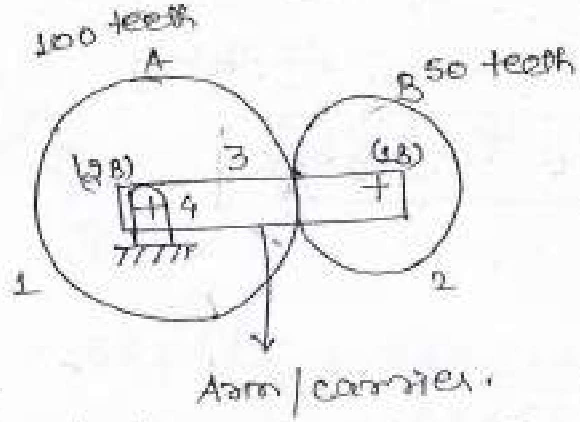
- simple epicyclic
- compound epicyclic
- Reverted epicyclic
- Bevel epicyclic & so on.

Epi - cyclic

⇓ Axis ⇓ Rotation.

In this type of gear a link is required to rotate the axis of gear.

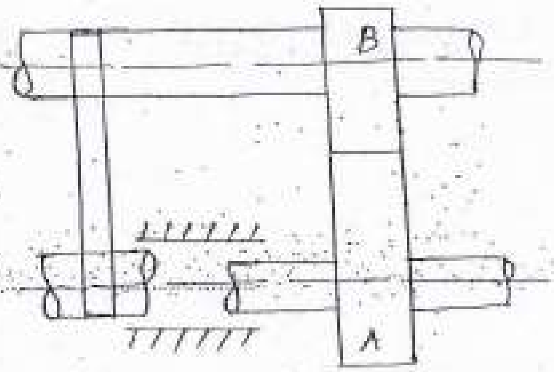
This ~~axes~~ link is called Arm or carrier (this is not a gear)



Degree of freedom

Links :- 4
j :- 3 (2B+1B)
h = 1

$F = 3(4-1) - 2\times3 - 1$
$= 9 - 6 - 1 = 2$.



For example

Gear A

Rotating at 100 r.p.m (CW)

Gear B

$T_A = 50$, $T_B = 100$

$\frac{T_A}{T_B} = \frac{\omega_B}{\omega_A} \Rightarrow \omega_A/\omega_B = \frac{50}{100} \times$

∴ B is rotating at 200 r.p.m (A.C)

Let Arm is rotating at 100 r.p.m (A.C)

∴ Net effect = (200 + 100) A.C

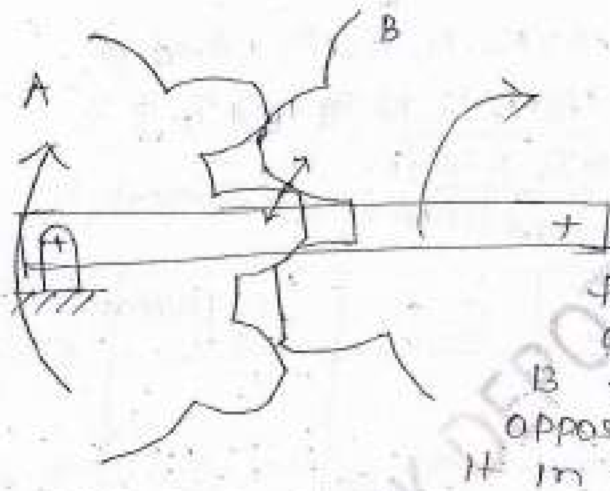
= 300 rpm.

If Arm is rotating at 200 rpm clock.

∴ Net zero.

Arm 300 r.p.m (CW).

∴ Net = 100 r.p.m (CW).



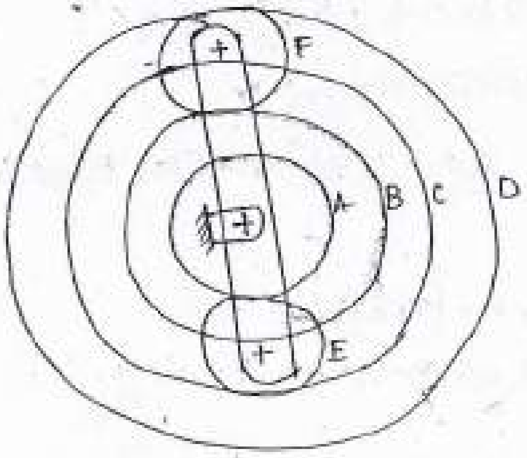
Arm is pulled downward thus A is rotates in clockwise then on gear B the effect is opposite and it tries to rotate it in upward dir^n i.e, CW.

Thus, the advantages that we obtain

(i) We can obtain no of velocity ratios.

(ii) Both Gears can rotate in same dir^n inspite of being external gears

(P6)



A/B → compound gear train.

All gears have same module.

$T_A = 20$, $T_B = 30$.

$T_E = T_F = 10$.

Gear D → fixed.

Arm → 100 (r.p.m) (A.C) (−ve)

Speed of other five?

Total 7 bodies (A, B, C, D, E, F, Arm)

$$\boxed{N_2 = N_1 \frac{T_1}{T_2}}$$

$r_B + r_F = r_C \Rightarrow T_B + T_F = T_C$

$\Rightarrow \boxed{T_C = 40}$

As compound (thus speed same)

Motion	Arm	A/B 20 30	E 10	C 40	F (f/B) 10	D 50 → No effect
Arm fixed (let Gear A rotates +x rpm clock	[As fixed] 0	+x	$-x \cdot \frac{20}{10}$	$-x \cdot \frac{20}{10} \cdot \frac{10}{40}$	$-x \cdot \frac{30}{10}$ (T_B)	$-x \cdot \frac{30}{10} \cdot \frac{10}{50}$
Arm free	y	y+x	y−2x	$y - \frac{x}{2}$	y − 3x	$y - \frac{3x}{5}$

$y - \frac{3x}{5} = 0$ — (1)

$y = -100$ (2)

$x = ?$, $y = ?$

$r_C + r_F = r_D$

$T_C + T_F = T_D$

$\Rightarrow T_D = 40 + 10$

$\boxed{T_D = 50}$

For C (Internally & attached with Gear E).

1. $N_C = N_E \frac{T_E}{T_C} = (-)\left(x \cdot \frac{20}{10}\right) \cdot \frac{10}{40}$

∴ same dirn as E.

∴ −ve.

sign is decided through the type of gearing whether external or internal.

For external Gear dirn is opp.

" Internal " " " same.

First of all choose one Gear other than the Gears which are affected by the Arm (Here, choose other than E & F)

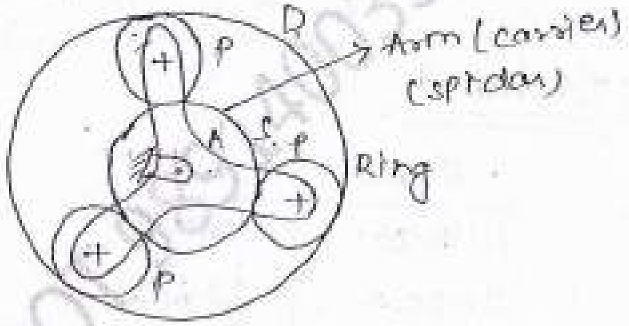
Planetary Gear train

GATE
ES 2000

$T_D = \frac{252}{3.5} \Rightarrow T_D = 72$

$T_S + 2T_P = 72$

$N_D = 0, N_S = +5N_A$.



Arm (A)	S (T_s)	T_P (P)	P(72)
0	$+x$	$-x\frac{T_s}{T_p}$	$-x\frac{T_s}{T_p} \cdot \frac{T_p}{72}$
y	$y+x$	$y - x\frac{T_s}{T_p}$	$y - x\frac{T_s}{72}$

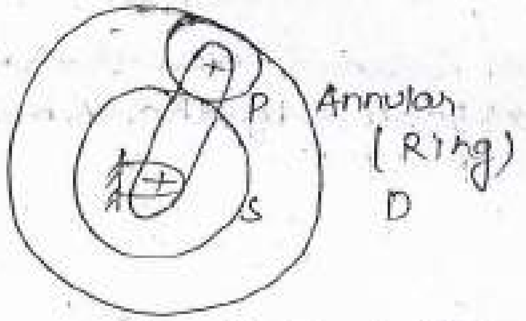
$y + x = 5y \Rightarrow x = 4y$

$N_D = 0 \Rightarrow y - x\frac{T_s}{72} = 0$

$y\left(1 - \frac{T_s}{18}\right) = 0 \Rightarrow T_s = 18$.

$T_p = \frac{72 - T_s}{2}$

Planetary gear train (epi-cyclic)



module → same

$$\boxed{T_S + 2T_P = T_D}$$

(I) Input

Sun	RING
Fixed	Input
Input	fixed.

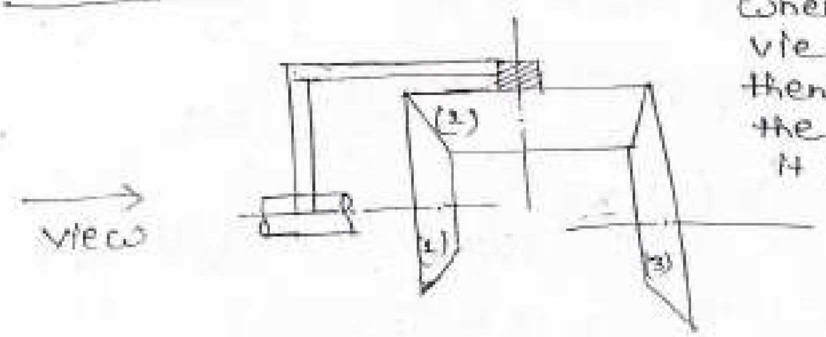
(II) Input → ARM.

Generally no. of planets in an epicyclic Gear train are more than one Because :-

(i) For Balancing.

(ii) High power transmission.

Direction considerations in Bevel epicyclic gear trains



When this gear is viewed from left then we can't determine the dirn of Gear-2, it can be +ve or -ve.

Arm	1(T_1)	2(T_2)	3(T_3)
0	$+x$	$\pm x \cdot \frac{T_1}{T_2}$	$-x \frac{T_1}{T_2} \cdot \frac{T_2}{T_3}$
y	$-y+x$	$y \pm x \cdot \frac{T_1}{T_2}$	$\left(y - x \frac{T_1}{T_3}\right)$

Fixing or Holding Torque in an epi-cyclic Gear train

Total torque in an epi-cyclic Gear train

$$\boxed{\Sigma T = T_{input} + T_{output} + T_{fixing} = 0} \quad -(1)$$

The net torque is equal to zero as total torque in gear train is zero, only movement occurs in inside gears.

Power conservation

$$\boxed{T_{input} \cdot \omega_{input} + T_{output} \cdot \omega_{output} = 0}$$

Applying ① & ② we can find T_{fixing}.

(Pb)
ES-2002

$N_{input} = +100$

$N_{output} = +250$, $T_{input} = +50$

$T_{fixing} = ?$

$$(50)(100) + T_{output} \times (250) = 0$$

$$\boxed{T_{output} = -20}$$

$$T_{fixing} + 50 + (-20) = 0$$

$$\Rightarrow T_{fixing} = -30$$

$$= 30 \text{ kN-m (AC)}.$$