4rth Semester

Kinematics and Dynamics Of Machine

Module 3

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Dynamic Force Analysis:

Dynamic forces are associated with accelerating masses. In situations where dynamic forces are dominant or comparable with magnitudes of external forces and operating speeds are high, dynamic analysis has to be carried out.

D' Alembert's Principle: The inertia forces and couples and the external forces and torques on a body together give statical equilibrium.

Inertia is a property of matter by virtue of which a body resists any change in velocity.

$$\text{Inertia force} \quad F_i = -mf_g$$

Where m = mass of body, f_g = acceleration of center of mass of the body.

- Negative sign indicates that the force acts in the opposite direction to that of acceleration.

- The force acts through center of mass of the body.

Similarly, an inertia couple resists any change in the angular velocity.

Inertia couple, $C_i = -I_g\alpha$

Where I_g = moment of inertia about an axis passing through center of mass G and perpendicular to plane of rotation of the body.

A = angular acceleration of the body.

Let $\sum F = F_1 + F_2 + F_3 + \cdots$ = external forces on the body.

and $\sum T = T_1 + T_2 + T_3 + \cdots$ = external torques on the body about the center of mass G. According to D' Alembert's principle,

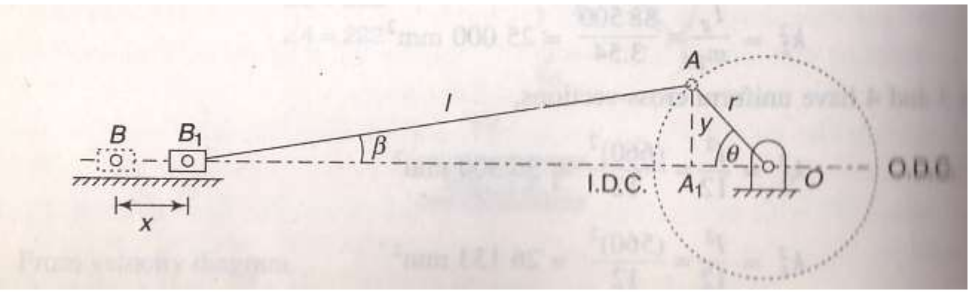
`$\sum F + F_i = 0$ and $\sum T + Ci = 0$

Thus , a dynamic analysis problem is reduced to one static problem.

Dynamic analysis of slider – crank mechanism :

Velocity and Acceleration of Piston:

Figure shows a slider crank mechanism in which the crank OA rotates in the clockwise direction. *l* and *r* are the lengths of the connecting rod and the crank respectively.



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Let x = displacement of piston from inner – dead center. At the moment when the crank has turned through angle θ from the inner – dead center,

$$
\begin{aligned}
x &= B_1B = BO - B_1O \\
&= BO - \left(B_1A_1 + A_1O\right) \\
&= \left(l + r\right) - \left(l\cos\beta + \cos\theta\right) \\
&= \left(nr + r\right) - \left(nr\cos\beta + r\cos\theta\right) \qquad (\text{taking } l/r = n)
\end{aligned}
$$

$$= r\left[(n+1) - (n\cos\beta + \cos\theta)\right]$$

Where $\cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\frac{y^2}{l^2}}$

$$= \sqrt{1-\frac{(r\sin\theta)^2}{l^2}} = \sqrt{1-\frac{\sin^2\theta}{n^2}}$$

$$= \frac{1}{n}\sqrt{n^2-\sin^2\theta}$$

$$x = r\left[(n+1) - \left(\sqrt{n^2-\sin^2\theta} + \cos\theta\right)\right]$$

$$= r\left[(1-\cos\theta) + \left(n - \sqrt{n^2-\sin^2\theta}\right)\right]$$

If the connecting rod is very large as compared to crank, n^2 will be large and the maximum value of $\sin^2\theta$ can be unity. Then $\sqrt{n^2-\sin^2\theta}$ will be approaching $\sqrt{n^2}$ or n, and $x = r(1-\cos\theta)$

This is the expression for simple harmonic motion. Thus the piston executes a simple harmonic motion when the connecting rod is large.

Velocity of piston:

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}\left[r\left\{(1-\cos\theta) + n - \left(n^2-\sin^2\theta\right)^{1/2}\right\}\right]\frac{d\theta}{dt}$$

$$= r\left[(0+\sin\theta) + 0 - \frac{1}{2}\left(n^2-\sin^2\theta\right)^{1/2}(-2\sin\theta\cos\theta)\right]\omega$$

$$= r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]$$

If n^2 is large compared to $v = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$

If $\frac{\sin 2\theta}{2n}$ can be neglected (when is quite large).

$$v = r\omega\sin\theta$$

Acceleration of Piston:

$$f = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}\left[r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)\right]\omega$$

$$= r\omega\left(\cos\theta + \frac{2\cos 2\theta}{2n}\right)\omega$$

$$= r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

If n is very very large,

$$f = r\omega^2 \cos\theta \text{ as in case of SHM}$$

When $\theta = 0^0$ i.e. at IDC, $f = r\omega^2\left(1 + \frac{1}{n}\right)$

When $\theta = 180^0$, i.e. at ODC, $f = r\omega^2\left(-1 + \frac{1}{n}\right)$

At $\theta = 180^0$, when the direction of motion is reversed, $f = r\omega^2\left(1 - \frac{1}{n}\right)$

Note that this expression of acceleration has been obtained by differentiating the approximate expression for the velocity. It is, usually, very cumbersome to differentiate the exact expression for velocity.

Angular velocity and angular acceleration of connecting rod:

As y = l sinβ = r sinθ

∴ $$\sin\beta = \frac{\sin\theta}{n} \qquad \left(n = {}^{l}\!/\!_{r}\right)$$

Differentiating with respect to time,

$$\cos\beta\frac{d\beta}{dt} = \frac{1}{n}\cos\theta\frac{d\theta}{dt}$$

or $$\omega_c = \omega\frac{\cos\theta}{n\frac{1}{n}\sqrt{n^2 - \sin^2\theta}}$$

where ω_c is the angular velocy of the connecting rod or $$\omega_c = \omega \frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

Let α_c = angular acceleration of the connecting rod

$$= \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta}\frac{d\theta}{dt}$$

$$= \omega \frac{d}{d\theta}\left[\cos\theta\left(n^2 - \sin^2\theta\right)^{-1/2}\right]\omega$$

$$= \omega^2\left[-\cos\theta\frac{1}{2}\left(n^2 - \sin^2\theta\right)^{-3/2}\left(-2\sin\theta\cos\theta\right) + \left(n^2 - \sin^2\theta\right)^{-1/2}\left(-\sin\theta\right)\right]$$

$$= \omega^2 \sin\theta\left[\frac{\cos^2\theta - \left(n^2 - \sin^2\theta\right)}{\left(n^2 - \sin^2\theta\right)^{3/2}}\right]$$

$$= -\omega^2 \sin\theta\left[\frac{n^2 - 1}{\left(n^2 - \sin^2\theta\right)^{3/2}}\right]$$

The negative sign indicates that the sense of angular acceleration of the rod is such that it tends to reduce the angle β.

Engine Force Analysis:

An engine is acted upon by various forces such as weight of reciprocating masses and connecting rod, gas forces, forces due to friction and inertia forces due to acceleration and retardation of engine elements, the last being dynamic in nature. The analysis is made of the forces neglecting the effect of the weight and the inertia effect of the connecting rod.

(i) Piston Effort (effective driving force):

Piston effort is termed as the net or effective force applied on the piston. In reciprocating engines, the reciprocating masses accelerate during the first half of the stroke and the inertia force tends to resist the same. Thus the net force on the piston is decreased. During the later half of the stroke, the reciprocating masses decelerate and the inertia force opposes this deceleration or acts in the direction of the applied gas pressure and thus, increases the effective force on the piston.

In vertical engine, the weight of the reciprocating masses assists the piston during the down stroke, thus, increases the piston effort by an amount equal the weight of the piston. During the upstroke, piston effort is decreased by the same amount.

Let A_1 = area of the cover end

A_2 = area of the piston rod end

p_1 = pressure on the cover end

p_2 = pressure on the rods end

m = mass of the reciprocating parts

Force on the piston due to the gas pressure, $F_p = p_1A_1 - p_2A_2$

Inertia force, $F_b = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$. It is opposite direction to that of the acceleration of the piston.

Resistant force = F_f

Net effective force on t5he piston, $F = F_p - F_b - F_f$

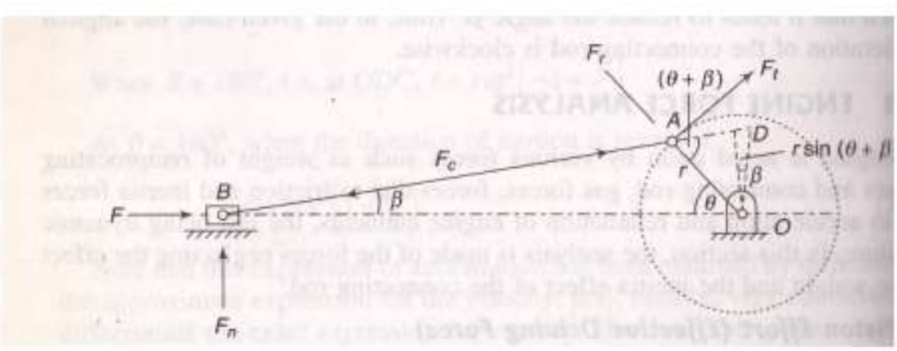
In case vertical engines, the weight of the piston or reciprocating parts also acts.

Force on the piston, $F = F_p + mg - F_b - F_f$

(ii) Force (thrust) along rthe Connecting rod:

Let F_c = Force in the3 connecting rod

Then equating the horizontal components of vforces,



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$$F_c \times \cos\beta = F \text{ or } F_c = \frac{F}{\cos\beta}$$

(iii) Thrust on the Sides of Cylinder

It is the normal reaction on the cylinder walls.

$$F_n = F_c \sin\beta = F \tan\beta$$

(iv) Crank Effort

Force is exerted on the crank pin as a result of the force on the piston. Crank effort is the net effort applied at the crank pin perpendicular to the crank which gives the required turning moment on the crankshaft.

Let F_t = crank effort

As $$F_t \times r = F_c \times r \times sin(\theta + \beta)$$

$$= \frac{F}{cos\,\beta} sin(\theta + \beta)$$

(v) <u>Thrust on the Bearings</u>

The component of F_c along the crank (in the radial direction) produces a thrust on the crankshaft bearings.

$$F_r = F_c\, cos(\theta + \beta) = \frac{F}{cos\,\beta} cos(\theta + \beta)$$

<u>Turning moment on Crankshaft</u>:

$$T = F_t \times r$$

$$= \frac{F}{cos\,\beta} sin(\theta + \beta) \times r = \frac{Fr}{cos\,\beta}(sin\,\theta\, cos\,\beta + cos\,\theta\, sin\,\beta)$$

$$= Fr\left(sin\,\theta + cos\,\theta\, sin\,\beta \frac{1}{cos\,\beta}\right)$$

$$= Fr\left(sin\,\theta + cos\,\theta \frac{sin\,\theta}{n} \frac{1}{\frac{1}{n}\sqrt{n^2 - sin^2\theta}}\right)$$

$$= Fr\left(sin\,\theta + \frac{2\,sin\,\theta\, cos\,\theta}{2\sqrt{n^2 - sin^2\theta}}\right)$$

$$= Fr\left(sin\,\theta + \frac{sin\,2\theta}{2\sqrt{n^2 - sin^2\theta}}\right)$$

Also as $r\, sin(\theta + \beta) = OD\, cos\,\beta$

$$T = F_t \times r$$

$$= \frac{F}{cos\,\beta} r\, sin(\theta + \beta)$$

$$= \frac{F}{cos\,\beta}(OD\,cos\,\beta)$$

$$= F \times OD$$

Dynamically Equivalent System:

As neither the mass of the connecting rod uis uniformly distributed nor the motion is linear, its inertia cannot be found as general manner.

Usually, the inertia of the connecting rod is taken into account by considering a dynamically – equivalent system.

Dynamically–equivalent system means that the rigid link is replaced by a link w2ith tw2o point masses in such a way that it has the same motion as the rigid link when subjected to the same force i.e. the center of mass of the equivalent link has the same linear acceleration and the link has the same angular acceleration.

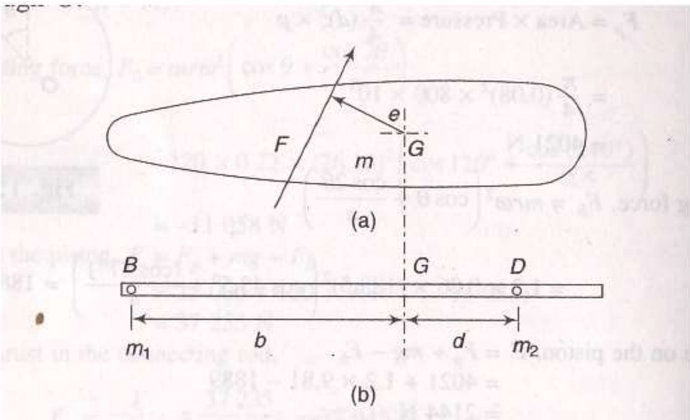

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Figure 39 (a) shows a rigid bnody of mass ‘m’ with center of mass at G. Let a force F is acting on body and the line of action is e distance from the C.G..

As we know $\vec{F} = m \cdot \vec{f}$

and $\vec{F} \cdot e = I \cdot \vec{\alpha}$

Acc. of G, $\vec{f} = \frac{\vec{F}}{m}$

Angular acc. , $\vec{\alpha} = \frac{\vec{F} \cdot e}{I}$

Where, e = perpendicular distance of F from G.

I = moment of inertia of the body about perpendicular axis

through G.

Now to have the dynamically equivalent system, let the replaced massless link has two point masses (m_1 at B and m_2 at D) at distances b and d respectively from the center of mass G.

1. To satisfy same acceleration, the sum of the equivalent masses m_1 amd m_2 has to be equal to m.

   $\Rightarrow m = m_1 + m_2$

2. To satisfy same angular acceleration $\frac{\vec{F} \cdot e}{I}$ should be same.

(i) $\vec{F}$ is already taken same, thus e has to be same which means that the combined center of mass of the equivalent system remains at G. This is possible if $m_1 \cdot b = m_2 \cdot d$

(ii) To have the same moment of inertia of the equivalent system about perpendicular axis through their combined center of mass G, we must have

$$I = m_1 \cdot b^2 + m_2 \cdot d^2$$

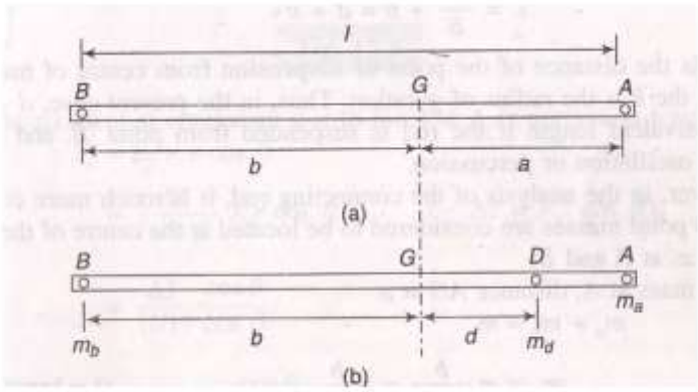
Thus any distributed mass can be replaced by two point masses to have the same dynamical properties if the following conditions fulfilled.

(a) The sum of two masses is equal to the total mass.
(b) The combined center of mass coincides with that of rod.
(c) The moment of inertia of two point masses about perpendicular axis through their combined center of mass is equal to that of the rod.

Inertia of the connecting rod:

Let the connecting rod be replaced by an equivalent massless link with two point masses as shown.

Let m be the total mass of the connecting rod and one of the masses be located at the small end B. Let the second mass be placed at D.



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m_b = mass at B, m_d = mass at D

Take BG = b and DG = d

Then $m_b + m_d = m$

$$m_b \cdot b = m_d \cdot d$$

From above these two equations we can get

$$m_b + \left(m_b \frac{b}{d} \right) = m$$

$$\Rightarrow \quad m_b \left(1 + \frac{b}{d} \right) = m$$

$$\Rightarrow \quad m_b \left(\frac{b+d}{d} \right) = m$$

$$\Rightarrow \quad m_b = m \frac{d}{b+d}$$

Similarly $m_d = m \frac{b}{b+d}$

Hence $I = m_b b^2 + m_d d^2$

$$= m \frac{d}{b+d} b^2 + m \frac{b}{b+d} d^2$$

$$= mbd \left(\frac{b+d}{b+d} \right)$$

$$= mbd$$

Let k = radius of gyration of the connecting rod about an axis through center of mass G perpendicular to the plane of motion.

$$mk^2 = mbd$$

$$\Rightarrow \qquad k^2 = bd$$

This result can be compared with that of an equivalent length of a simple pendulum in the following manner :

The equivalent length of a simple pendulum is given by

$$L = \frac{k^2}{b} + b = d + b$$

Where b is the distance of the point of suspension from center of mass of the body and the k is the radius of gyration. Thus , in the present case , $d + b(= L)$ is the equivalent length if the rod is suspended from point B, and d is the center of oscillation or percussion.

However, in the analysis of the connecting rod, it is much more convenient if the two point masses are considered to be located at the center of the two end bearings i.e. at A and B.

Let m_a = mass at A, distance AG = a

$$\Rightarrow \qquad m_a + m_b = m$$

$$\Rightarrow \qquad m_a = m\frac{b}{a+b} = m\frac{b}{l}$$

Similarly $$m_b = m\frac{a}{a+b} = m\frac{a}{l}$$

$$I' = mab$$

Assuming , $a > d$, $\Rightarrow I' > I$

This means that by considering the two masses at A and B instead of at D and B, the inertia torque is increased from the actual value T ($= I\alpha$). The error is corrected by incorporating a correction couple.

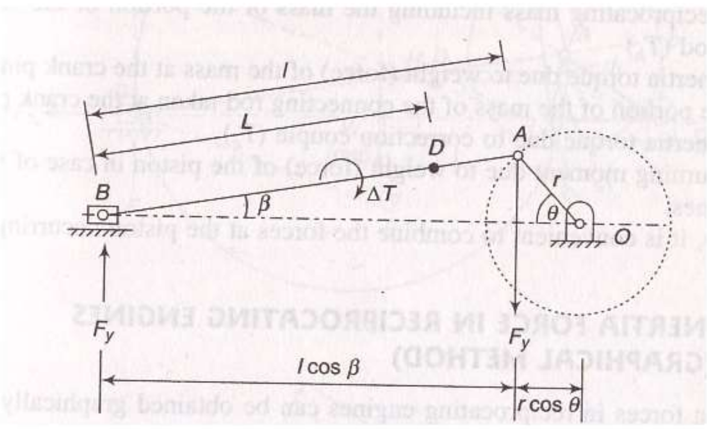
Then correction couple is

$$\Delta T = \alpha_c\left(mab - mbd\right)$$

$$= mb\alpha_c\left(a - d\right)$$

$$= mb\alpha_c\left[(a+b)-(b+d)\right]$$

$$= mb\alpha_c(l-L) \qquad \text{(taking } b+d=L\text{)}$$



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This correction couple must be applied in the opposite direction to that of the applied inertia torque. As the direction of the applied inertia torque is always opposite to the direction of the angular acceleration, the direction of the correction couple will be the same as that of angular acceleration i.e. in the direction of decreasing angle β.

Pivots and Collars:

When a rotating shaft is subjected to an axial load the thrust (axial force) is taken either by a pivot or a collar.

Collar Bearing: A collar bearing or simply a collar is provided at any position along the shaft and bears the axial load on a mating surface. The surface of the collar may be plane (flat) normal to the shaft or of conical shape.

Pivot bearing: When the axial load is taken by the end of the shaft which is inserted in a recess to bear the thrust, it is called a pivot bearing or simply a pivot. It is also known as foot step bearing. The surface of the pivot can also be flat or of conical shape.

Uniform pressure and uniform wear:

Friction torque of a collar or a pivot bearing is calculated, usually on the basis of two assumptions. In one case it is assumed that the intensity of pressure on the bearing surface is constant, where as in 2nd case, it is the uniform wearing of the bearing surface.

For uniform pressure :-