Module - IV (12 hours)

Time Value of Money- Interest - Simple and compound, nominal and effective rate of interest, Cash flow diagrams

Time Value of Money-

Firms are confronted with opportunities to ***earn positive rates of return on their funds***, either through investment in ***attractive projects*** or ***in interest bearing securities*** or deposits.

Therefore, the timing of cash flows – ***both out flows and inflows*** – has important economic consequences, which finance managers explicitly recognize as **the *time value of money.***

Time Value of Money-

Time value is based on the belief that

A Rupee today is worth more than a Rupee that will be received at some future date.

A rupee today is more valuable than a rupee a year hence.

Reasons of Time Value of Money

- *Individuals, in general,* ***prefer current consumption to future consumption***.

- **An investment of one rupee today would grow to (1 + r) a year hence. (r is the rate of return earned on the investment).**

- *In an inflationary period a rupee today represents a* ***greater real purchasing*** *power than a rupee a year hence*.

Simple Interest

- *The sum of money paid by the* ***borrower*** *to the* ***lender*** *for the use of the borrowed money is called interest.*
- *It is the periodic payment for the use of "capital".*
  - The period for which the money is borrowed or lent is called ***time*****.** Time period may be yearly, half year, quarter or a month.
  - The sum of the principal and the interest at the end of any time is called ***amount***
  - *Directly proportional to time*

Simple Interest

$I = P.n.i$ [Where, $i = r/100$]

Where P = Principal,

n = years,

i (r %) = rate of interest per annum.

Ex.1. If Rs 1000 is borrowed for 3 years at 10% interest rate, the interest earned will be Rs. ------. The Amount (F) will be ……….

Simple Interest

Amount (F) = (principal + Interest)

$$F = P + I$$
$$= P + Pni \quad [\, I = P.n.i \,]$$
$$= P\,(1 + ni)$$

Here the factor (1+ ni) is called the interest factor.

Simple Interest

when "n" is not the full year; simple interest can be calculated in two ways.

I . When ***ordinary simple interest is used***, *the* ***year is divided into <u>twelve 30-days period</u>.***

II. When ***exact simple interest*** is used, year is divided into a **calendar division**.

Ex.2. If Rs 1000 are borrowed for 3 months at 10% interest rate, the interest earned will be Rs. ------. The Amount (F) will be …….

Simple Interest

Ex.3. If Rs 1000 is borrowed for 3 months (Jan, Feb & March 2020), at 10% interest rate, the interest earned will be Rs. ------. The Amount (F) will be …….

Ex.4: If Rs 100000 is borrowed for 3 months (Jan, Feb & March 2020), at 10% interest rate, the interest earned will be Rs. ------. The Amount (F) will be …….

Compound interest

❖When interest due at the end of the period becomes a part of the principal and itself earns interest along with the principal, it is called **"Compound Interest".**

❖ F_1 = $P(1 + i)$ [Where n = 1 year]

[F_1= Compound amount due in one year]

F_2 = Amount borrowed + year 1 interest + (Amount borrowed plus year 1 interest due) (interest rate)

Compound interest

$$F = P + Pi + (P + Pi)\,i$$
$$= P + Pi + Pi + Pi_2$$
$$= P\,(1 + i + i + i^2)$$
$$= P\,(1 + i)^2$$

Generalized for any number of interest periods n, this expression becomes

$$\mathbf{F_n = P\,(1 + i)^n}$$

Where $\mathbf{(1+i)^n}$ is known as the ***Compound Amount Factor* (CAF)**

Compare Simple Interest & Compound Interest

Ex- 5 . With simple interest, if Rs1,000 is loaned for three years at 10% , the interest earned be Rs.1,000 x 3 x 0.10 = Rs.300,
The Amount = Rs.1,300.

Whereas if compounded

The Amount = Rs.1,331.
The difference is Rs 31

Nominal Interest Rates

- Interest rates are normally quoted on an **annual basis.**
- However, interest will be compounded several times per year: ***monthly***, **quarterly**, ***semi-annually***, etc.

Ex.6: 1 year divided into four quarters with interest at 2 % per quarter as ***8 percent compounded quarterly***, the 8 % rate is called a ***nominal annual interest rate.***

Nominal Interest Rates

Import ant to Remember **:**

Compare : Future value of Rs 200 earning interest at 8 % compounded ***quarterly*** with 8 % compounded ***annually***.

The result of the nominal interest rate is to produce a higher future value.

$F_{12\ month}$ = Rs 216.48 **(8 % compounded quarterly)**

$F_{12\ month}$ = Rs 216.00 **(8 % compounded annually)**

Effective Interest Rates

The effective or true annual rate (EAR) is the annual rate of interest actually paid or earned.

The effective annual interest rate is simply the ***ratio of the interest charge for the year 1 to the principal (amount loaned or borrowed).***

Effective annual interest rate = F – P / P [With reference to the principal]

$$i_{eff} = (1 + k)^m - 1$$ [without reference to the principal]

Where , m = frequency of compounding per year,
k = nominal rate of interest

Ex- 7: A bank offers 8 per cent nominal rate of interest with frequently compounding. What is the effective rate of interest?
Ans : 8.24% (Without reference to the Principal Sum)

Cash Flows

Business transactions involve flow of money both cash inflows and out flows.

The actual ***inflows and outflows of money*** are called cash flows.

A cash flow diagram is simply a graphical representation of cash flows drawn on a time scale.

In cash flow diagram t = 0 represents the present, and t = 1 represents the end of time period 1, and so on.

Cash Flow Diagram

- ***Cash flows diagram*** shows both ***time and cash flow magnitude.***

- The ***direction of the arrows*** on the cash flow diagram is important.

- Vertical arrow pointing up indicates ***positive cash flows***, whiling pointing down indicate ***negative cash flows***.

Cash Flow Diagram

Ex. Cash inflows or receipts = Revenues, operating cost reductions, receipt of loan principal, asset salvage value, income tax savings, receipts from stock and bond sales

Ex. Cash outflows or disbursements = First cost of assets, operating costs, periodic maintenance & and rebuild cost, loan interest and principal payments

Net cash flow = Receipts – Disbursements

= Cash inflow – Cash outflows

Kinds of cash flow transactions

- Single Payment Cash Flow: involves a single present or future cash flow.

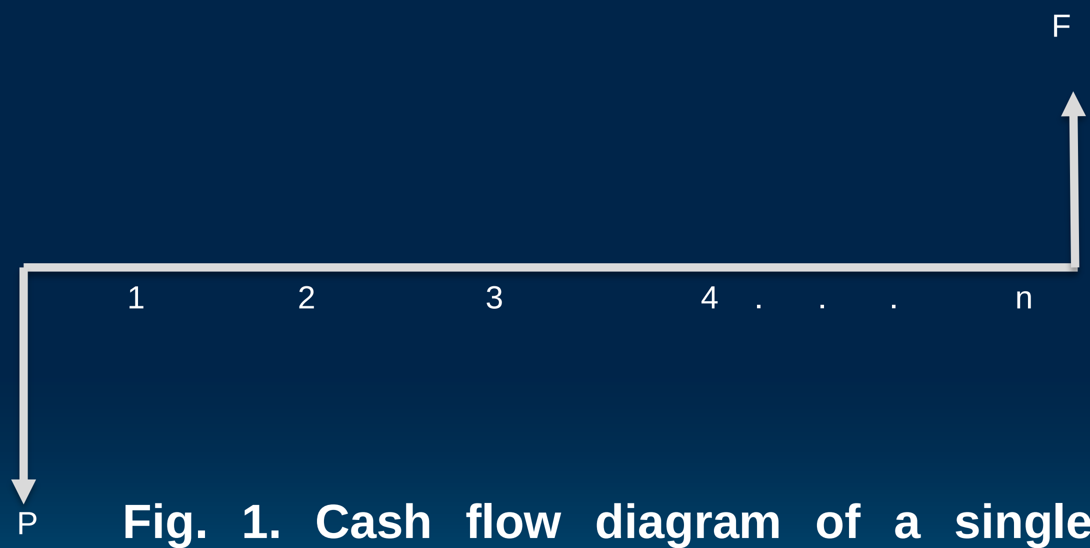


Fig. 1. Cash flow diagram of a single – payment cash flow diagram.

Uniform Payment Series

- Involves a series of flows of equal amounts at regular intervals

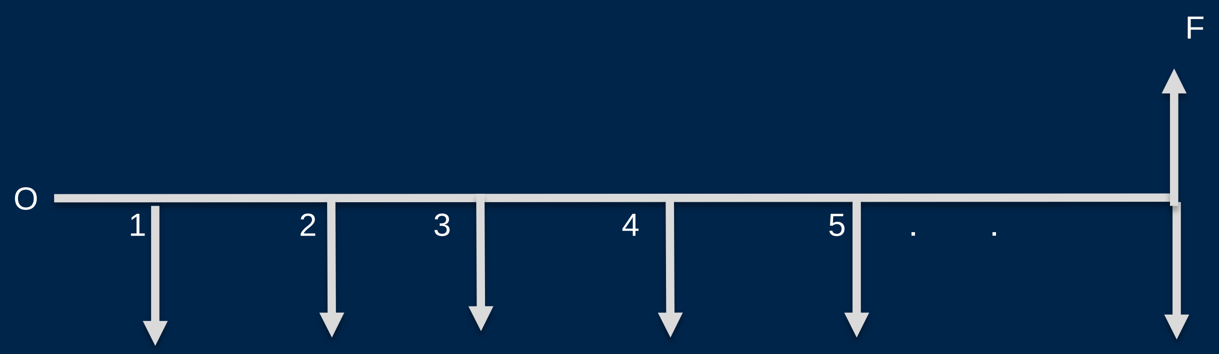
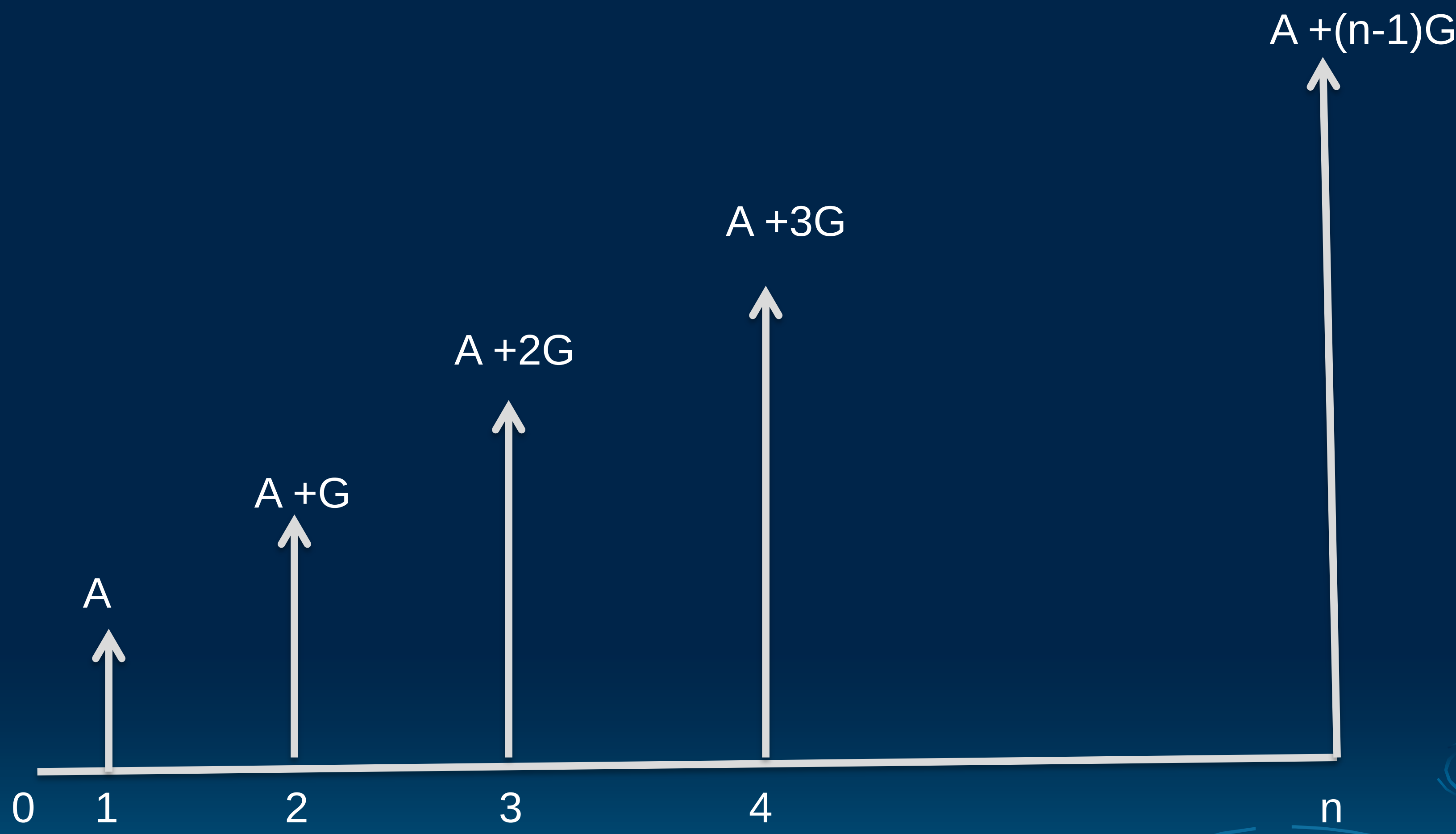


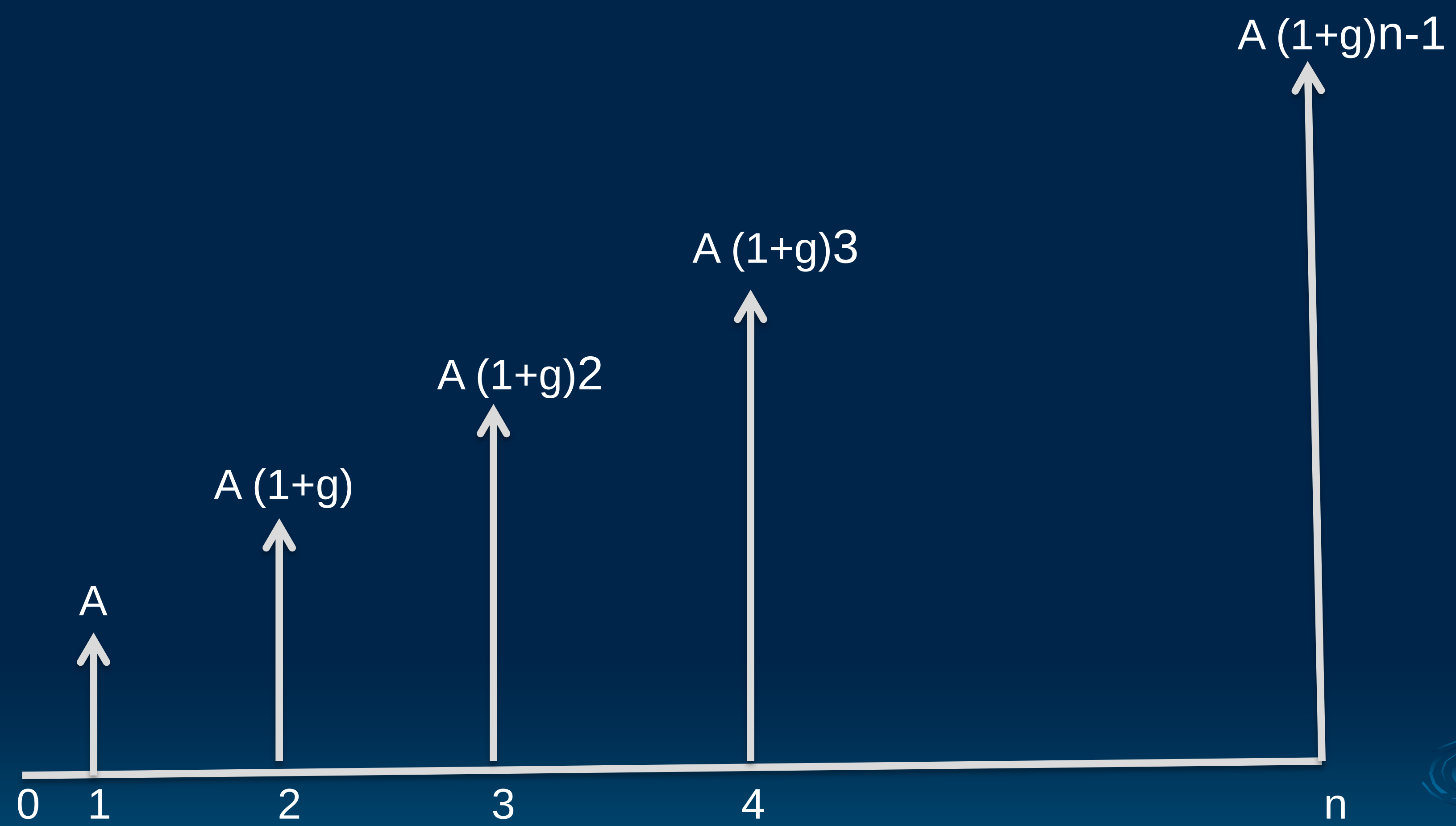
Fig. Uniform Payment series cash flow diagram

Linear Gradient Series



Liner Gradient Cash Flow Series is a series of flows increasing or decreasing by an fixed amount at regular intervals

Geometric Gradient Series



Geometric Gradient Cash Flow Series is a series of flows increasing or decreasing by a fixed percentage at regular intervals.

COMPOUND INEREST FORMULAS

current period.

The notations which are used in various interest formulae are as follows:

P = principal amount

n = No. of interest periods

i = interest rate (It may be compounded monthly, quarterly, semiannually or annually)

F = future amount at the end of year n

A = equal amount deposited at the end of every interest period

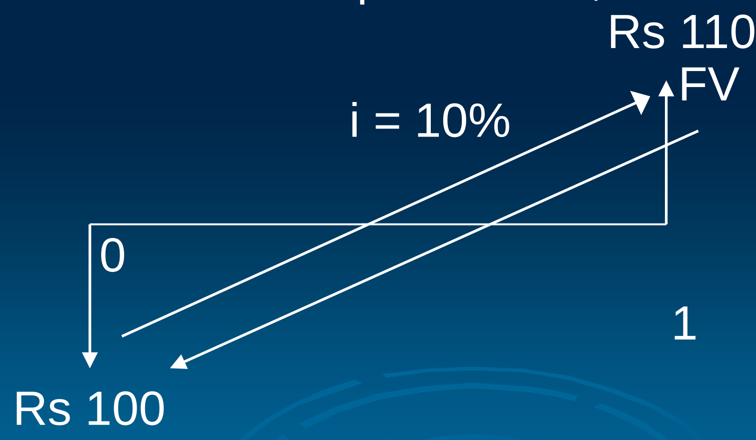
G = uniform amount which will be added/subtracted period after period to/from the amount of deposit A1 at the end of period 1

COMPOUDING & DISCOUNTING

- Compounding is the process of finding the **Future Value (FV) of Present Value (PV)**.
- Discounting is the reverse of compounding. It is a process through which we can find the **Present value (PV) of Future Value (FV)**

Ex: A sum of Rs 100 is deposited today and if it would grow at an compound interest of 10% per annum, what would the future value after an year.

Ans :



$$F_n = P\,(1 + i)^n$$

$$F_1 = P\,(1 + 0.1)^1$$
$$= 100 \times 0.1$$
$$= 110$$

$$P = \frac{F}{(1+i)n}$$
$$= \frac{110}{(1+0.10)1}$$
$$= \frac{110}{1.1} = 100$$

SINGE PLAYMENT COMPUND AMOUNT

- Here the objective is **to find the single future sum (F) of the initial payment (P)** made at time 0 after n periods at an interest rate i compounded every period. The cash flow diagram is as follows

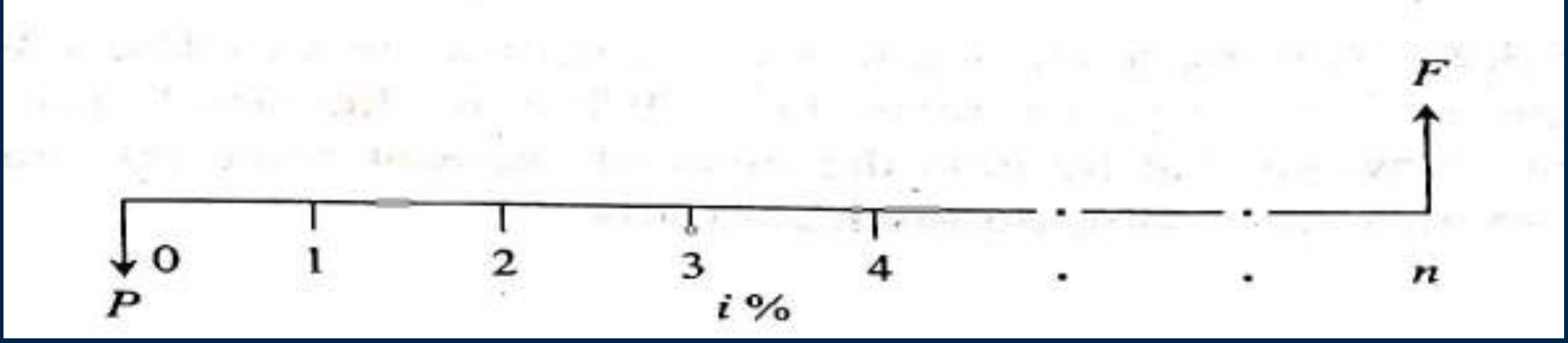


Fig. 1: Single Payment Compound Amount

The formula to obtain the single-payment compound amount is

$$F = P(1 + i)^n = P(F/P, i, n)$$

$(F/P, i, n)$ is called as single-payment compound amount factor.

Example of SINGE PLAYMENT COMPUND AMOUNT

EXAMPLE 3.1 A person deposits a sum of Rs. 20,000 at the interest rate of 18% compounded annually for 10 years. Find the maturity value after 10 years.

Solution

P = Rs. 20,000

i = 18% compounded annually

n = 10 years

$$F = P(1 + i)^n = P(F/P, i, n)$$
$$= 20{,}000\ (F/P,\ 18\%,\ 10)$$
$$= 20{,}000 \times 5.234 = \text{Rs. } 1{,}04{,}680$$

The maturity value of Rs. 20,000 invested now at 18% compounded yearly is equal to Rs. 1,04,680 after 10 years.

Ref : Engineering Economics by Pannerselvam, P-29

Single-Payment Present Worth Amount

Here, the objective is to find the present worth amount (P) of a single future sum (F) which will be received after n periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 3.3.

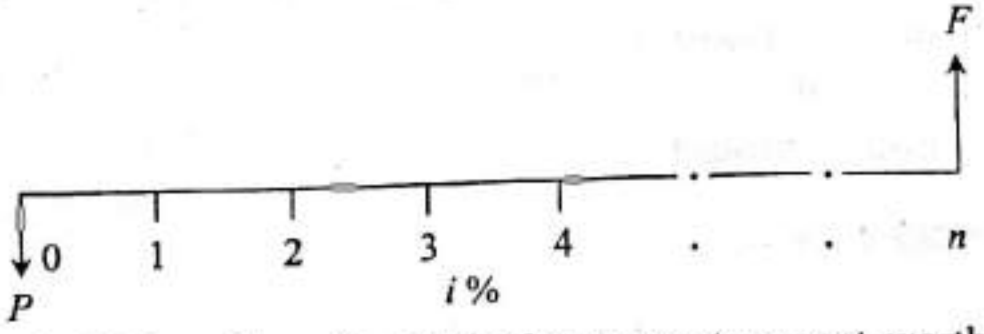


Fig. 3.3 Cash flow diagram of single-payment present worth amount.

The formula to obtain the present worth is

$$P = \frac{F}{(1+i)^n} = F(P/F, i, n)$$

where

$(P/F, i, n)$ is termed as *single-payment present worth factor.*

Single-Payment Present Worth Amount

EXAMPLE 3.2 A person wishes to have a future sum of Rs. 1,00,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 15% interest rate compounded annually.

Solution

F = Rs. 1,00,000

i = 15%, compounded annually

n = 10 years

$$P = F/(1 + i)^n = F(P/F, i, n)$$

$$= 1{,}00{,}000\ (P/F, 15\%, 10)$$

$$= 1{,}00{,}000 \times 0.2472$$

= Rs. 24,720

The person has to invest Rs. 24,720 now so that he will get a sum of Rs. 1,00,000 after 10 years at 15% interest rate compounded annually.

Equal-Payment Series Compound Amount

In this type of investment mode, the objective is to find the future worth of n equal payments which are made at the end of every interest period till the end of the nth interest period at an interest rate of i compounded at the end of each interest period. The corresponding cash flow diagram is shown in Fig. 3.4.

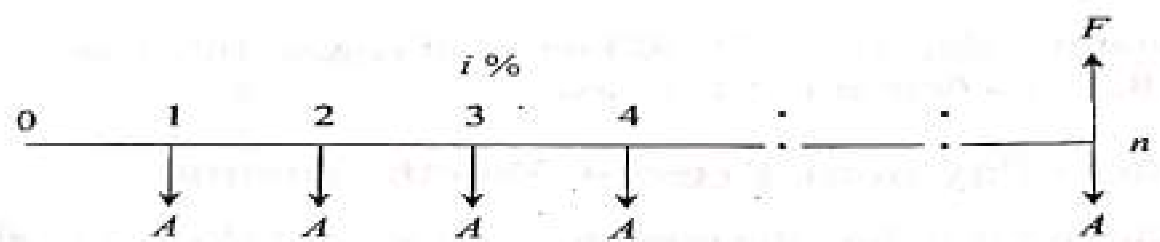


Fig. 3.4 Cash flow diagram of equal-payment series compound amount.

In Fig. 3.4,

A = equal amount deposited at the end of each interest period
n = No. of interest periods
i = rate of interest
F = single future amount

The formula to get F is

$$F = A\,\frac{(1+i)^n - 1}{i} = A(F/A, i, n)$$

where

$(F/A, i, n)$ is termed as *equal-payment series compound amount factor*.

EXAMPLE 3.3 A person who is now 35 years old is planning for his retired life. He plans to invest an equal sum of Rs. 10,000 at the end of every year for the next 25 years starting from the end of the next year. The bank gives 20% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

Solution

A = Rs. 10,000
n = 25 years
i = 20%
F = ?

The corresponding cash flow diagram is shown in Fig. 3.5.

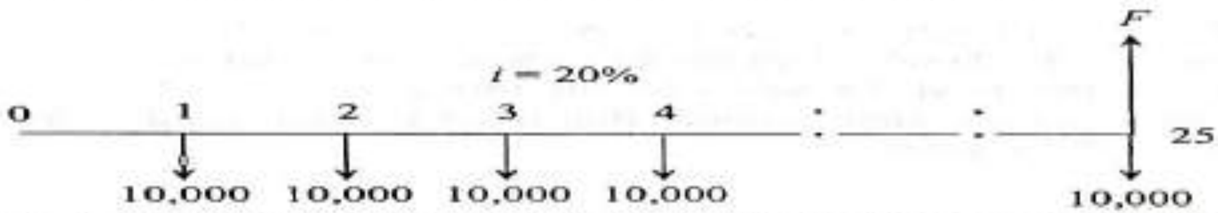


Fig. 3.5 Cash flow diagram of equal-payment series compound amount.

$$F = A\,\frac{(1+i)^n - 1}{i}$$
$$= A(F/A,\ i,\ n)$$
$$= 10{,}000(F/A,\ 20\%,\ 25)$$
$$= 10{,}000 \times 471.981$$
$$= \text{Rs. } 47{,}19{,}810$$

The future sum of the annual equal payments after 25 years is equal to Rs. 47,19,810.

3.3.4 Equal-Payment Series Sinking Fund

In this type of investment mode, the objective is to find the equivalent amount (A) that should be deposited at the end of every interest period for n interest periods to realize a future sum (F) at the end of the nth interest period at an interest rate of i.

The corresponding cash flow diagram is shown in Fig. 3.6.

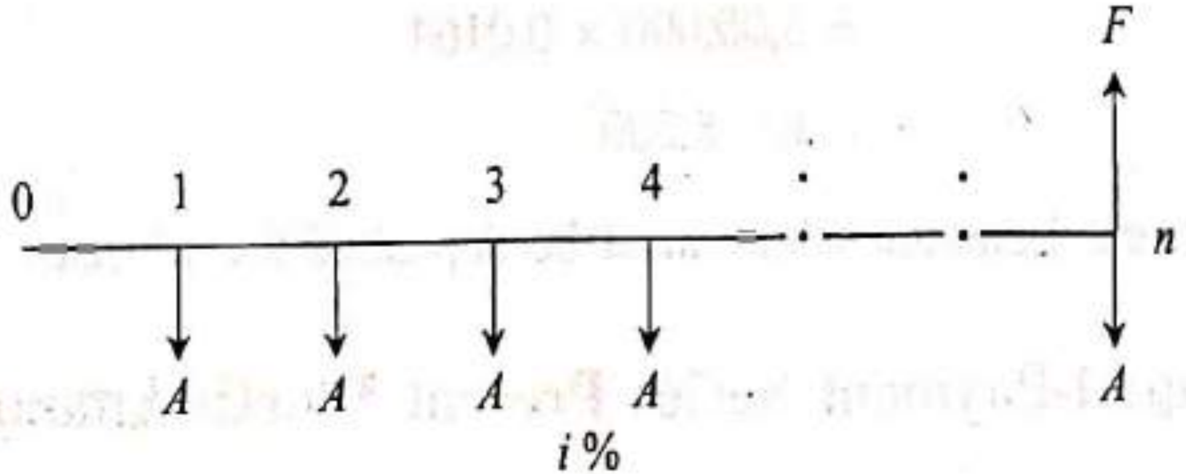


Fig. 3.6 Cash flow diagram of equal-payment series sinking fund.

In Fig. 3.6,

A = equal amount to be deposited at the end of each interest period

n = No. of interest periods

i = rate of interest

F = single future amount at the end of the nth period

The formula to get A is

$$A = F \frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$

where

$(A/F, i, n)$ is called as *equal-payment series sinking fund factor.*

... to replace a present facility after 15 yea

Example : A company has to replace a present facility after 15 years at an outlay of Rs 5,00,000. it plans to deposit an equal amount at the end of the every year for the next 15 years at an interest rate of 18% compounded annually. Find the equivalent amount that must be deposited at the end of the every year for the next 15 years.

Solution

F = Rs. 5,00,000
n = 15 years
i = 18%
A = ?

The corresponding cash flow diagram is shown in Fig. 3.7.

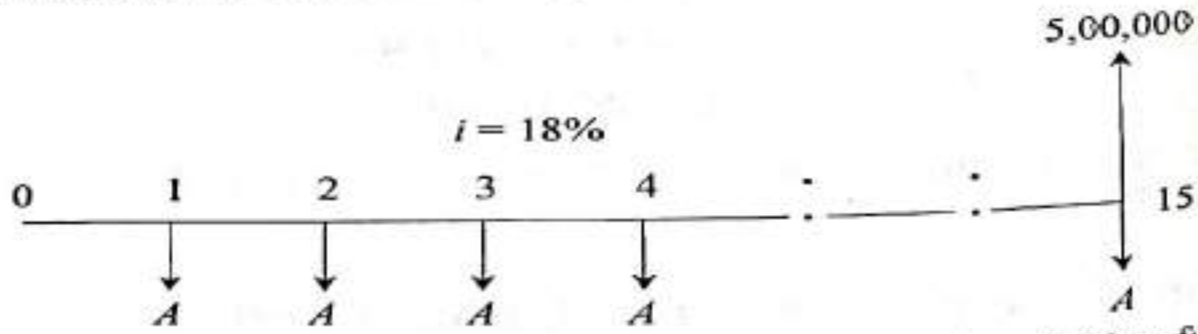


Fig. 3.7 Cash flow diagram of equal-payment series sinking fund.

$$A = F\frac{i}{(1+i)^n - 1} = F(A/F,\ i,\ n)$$
$$= 5{,}00{,}000(A/F,\ 18\%,\ 15)$$
$$= 5{,}00{,}000 \times 0.0164$$
$$= \text{Rs. } 8{,}200$$

The annual equal amount which must be deposited for 15 years is Rs. 8,200

Equal- payment series Present Worth Amount

- The objective is to find the **present worth of an equal payment** made at the end of every interest period for n interest periods at an interest rate of i compounded at the end of every interest period. The cash flow diagram is as follows

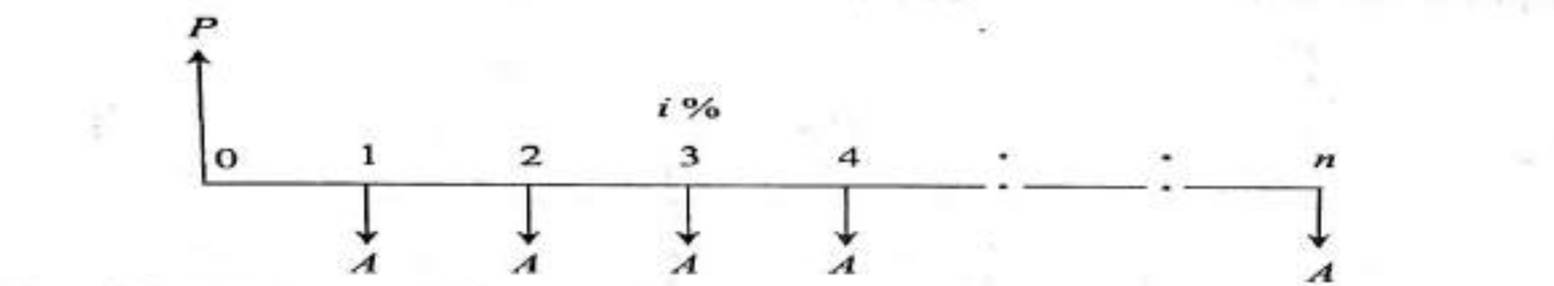


Fig. 3.8 Cash flow diagram of equal-payment series present worth amount.

P = present worth

A = annual equivalent payment

i = interest rate

n = No. of interest periods

Equal- payment series Present Worth Amount

The formula to compute P is

$$P = A\,\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n)$$

where

$(P/A, i, n)$ is called *equal-payment series present worth factor.*

EXAMPLE 3.5 A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs. 10,00,000 for the next 20 years towards its employees welfare measures. The reserve is assumed to grow at the rate of 15% annually. Find the single-payment that must be made now as the reserve amount.

Solution

A = Rs. 10,00,000
i = 15%
n = 20 years
P = ?

The corresponding cash flow diagram is illustrated in Fig. 3.9.

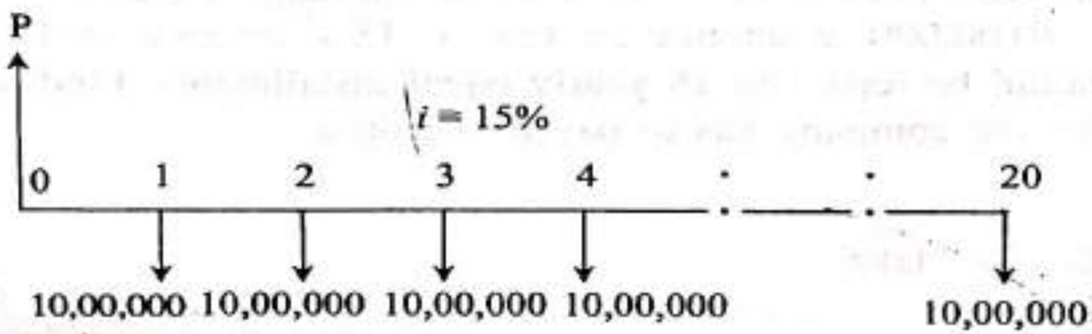


Fig. 3.9 Cash flow diagram of equal-payment series present worth amount.

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n)$$

$$= 10,00,000 \times (P/A, 15\%, 20)$$

$$= 10,00,000 \times 6.2593$$

$$= \text{Rs. } 62,59,300$$

The amount of reserve which must be set-up now is equal to Rs. 62,59,300

3.3.6 Equal-Payment Series Capital Recovery Amount

The objective of this mode of investment is to find the annual equivalen amount (A) which is to be recovered at the end of every interest period for interest periods for a loan (P) which is sanctioned now at an interest rate of compounded at the end of every interest period (see Fig. 3.10).

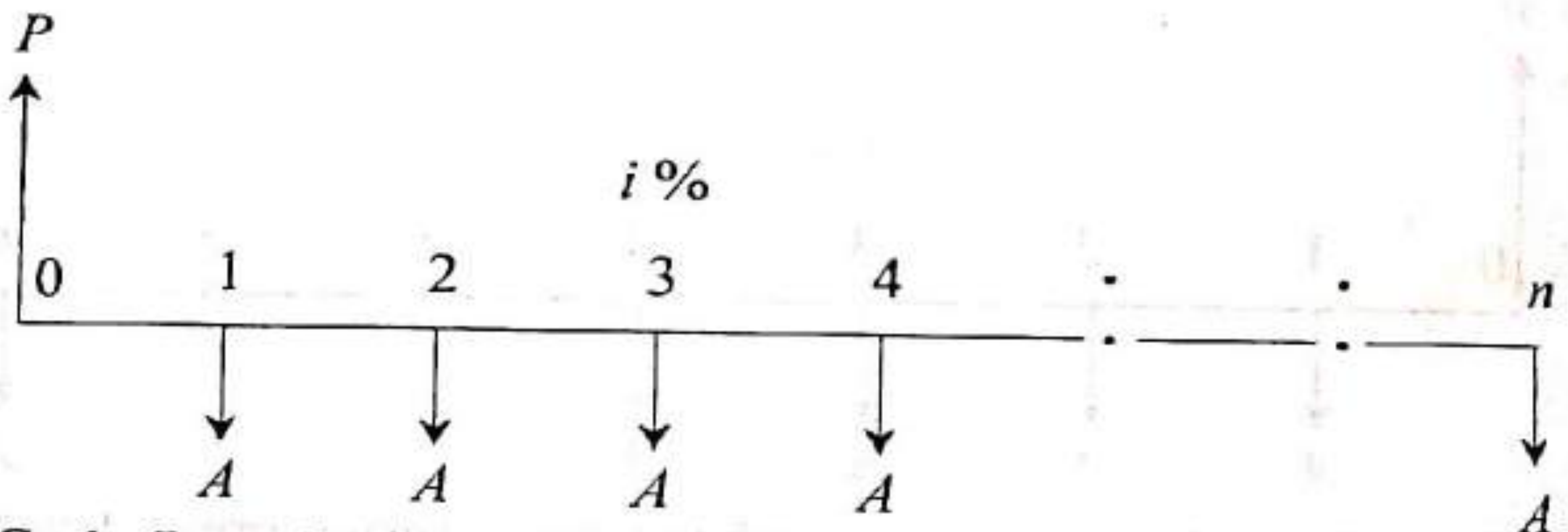


Fig. 3.10 : Cash flow diagram of equal payment series capital recovery factor

Example on Equal Payment Series Capital Recovery Amount

Ex. A bank gives a loan to a company to purchase an equipment worth of Rs 10,00,000 at an interest rate of 18% compounded annually. This amount should be repaid in 15 yearly equal instalments. Find the instalment amount that the company has to pay to the bank.

Solution

$$P = \text{Rs. } 10{,}00{,}000$$
$$i = 18\%$$
$$n = 15 \text{ years}$$
$$A = ?$$

The corresponding cash flow diagram is shown in Fig. 3.11.

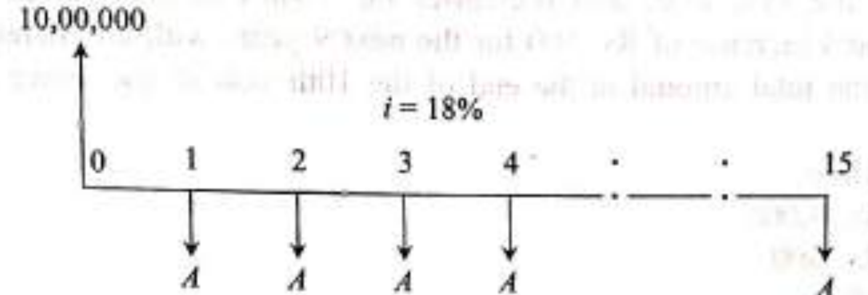


Fig. 3.11 Cash flow diagram of equal-payment series capital recovery amount.

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P,\ i,\ n)$$
$$= 10{,}00{,}000 \times (A/P,\ 18\%,\ 15)$$
$$= 10{,}00{,}000 \times (0.1964)$$
$$= \text{Rs. } 1{,}96{,}400$$

The annual equivalent installment to be paid by the company to the bank is Rs. 1,96,400.

3.3.7 Uniform Gradient Series Annual Equivalent Amount

The objective of this mode of investment is to find the annual equivalent amount of a series with an amount $A1$ at the end of the first year and with an equal increment (G) at the end of each of the following $n - 1$ years with an interest rate i compounded annually.

The corresponding cash flow diagram is shown in Fig. 3.12.

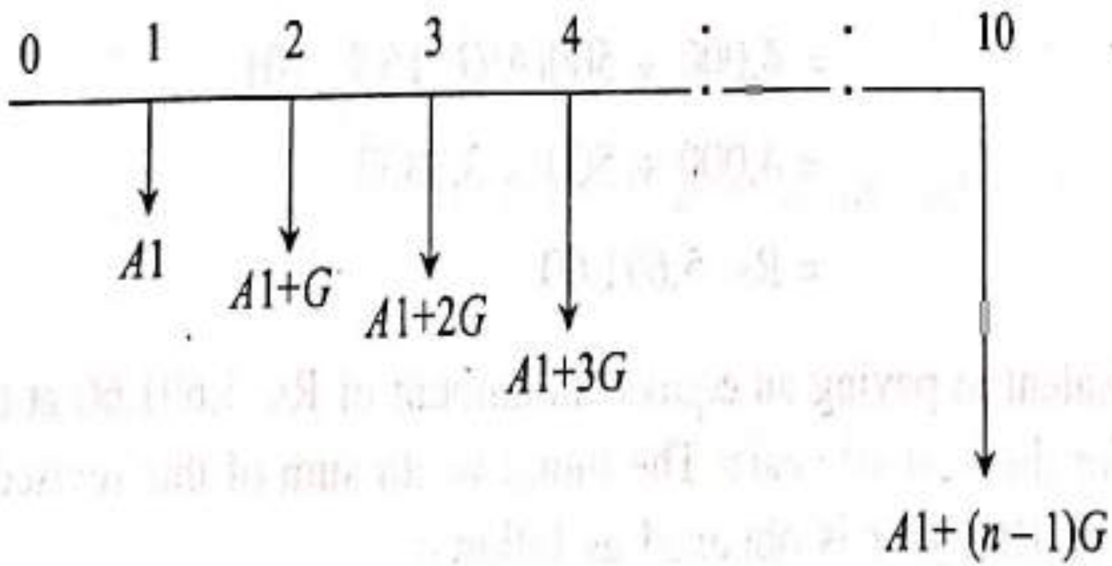


Fig. 3.12 Cash flow diagram of uniform gradient series annual equivalent amount.

The formula to compute A under this situation is

$$A = A1 + G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$
$$= A1 + G\,(A/G,\ i,\ n)$$

where

$(A/G,\ i,\ n)$ is called *uniform gradient series factor.*

Ex. A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs 4000, at the end of the 1st year, and thereafter he wishes to deposit the amount with an annual increase of Rs 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of 10th year.

Solution Here,

$A1$ = Rs. 4,000

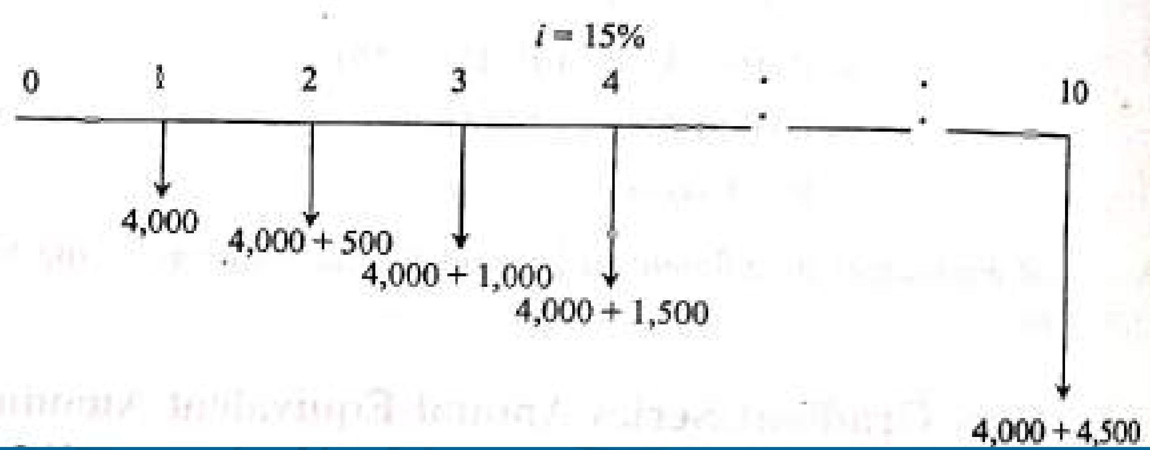
G = Rs. 500

i = 15%

n = 10 years

A = ? & F = ?

The cash flow diagram is shown in Fig. 3.13.



$$A = A1 + G\,\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$

$$= A1 + G(A/G, i, n)$$

$$= 4{,}000 + 500(A/G, 15\%, 10)$$

$$= 4{,}000 + 500 \times 3.3832$$

$$= \text{Rs. } 5{,}691.60$$

This is equivalent to paying an equivalent amount of Rs. 5,691.60 at the end of every year for the next 10 years. The future worth sum of this revised series at the end of the 10th year is obtained as follows:

$$F = A(F/A, i, n)$$

$$= A(F/A, 15\%, 10)$$

$$= 5{,}691.60(20.304)$$

$$= \text{Rs. } 1{,}15{,}562.25$$

At the end of the 10th year, the compound amount of all his payments will be Rs. 1,15,562.25.