## UNIT - II

### Concept of waves and Huygen's principle :

A wave is a disturbance that propagates through space and time, usually with the transference of energy from one point to another without any particles of the medium being permanently displaced. The Huygen's theory states that, every point on wavefront is the source of new disturbance. It sends small secondary wavelets propaged in all directions. The wavelets are spherical in shape and move forward in the homogeneous medium with the same velocity with which the original wave travels. At any instant the position and shape of the new wavefront can be obtained by drawing a surface enveloping these secondary wavelets.

Huygen's theory could successfully explain the phenomenon of reflection, refraction, total internal reflection, etc., but fails to explain rectilinear propagation of light, polarization

### Wavefront :

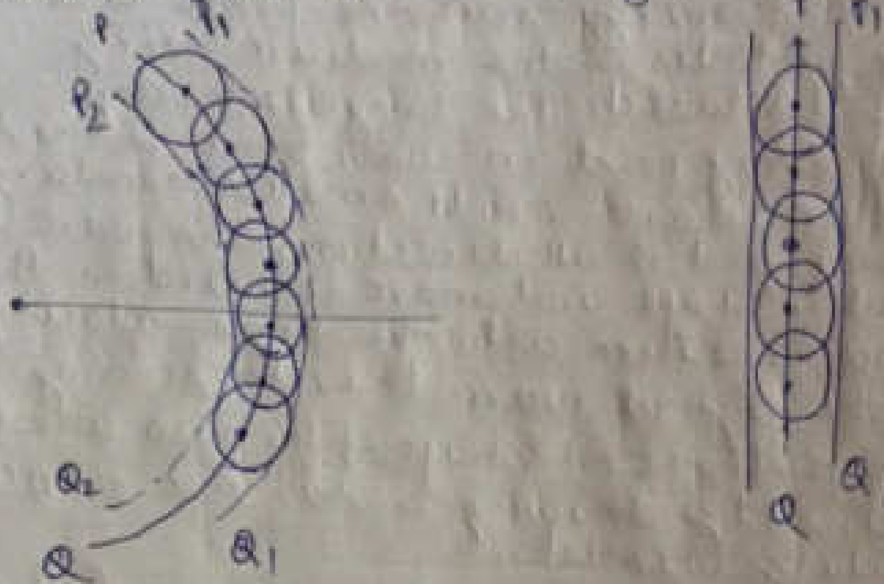
The wavefront at any instant of time is defined as the locus of all the neighbouring particles in the medium, which are being just disturbed at that instant of time and are consequently in the same phase of vibration.

Huygen's principle is based on the following assumptions:

i) Each point on the given wavefront acts as a source of secondary wavelets.

ii) The secondary wavelets from each point travel through space in all the directions with the velocity of light.

iii) A surface touching the secondary wavelets tangentially in the forward direction at any given time construct the new wavefront at that instant. This is known as secondary wavefront.




PQ is a primary wavefront. $P_1Q_1$ is a secondary wavefront.

Phase difference = $\frac{2\pi}{\lambda}$ x path difference

## Interference of Light

When light waves from two monochromatic sources proceed in the same direction and superimpose on a medium on the screen, the displacement of the particle will be the maximum or minimum according to the waves meet the particle in the same phase or in opposite phase respectively. This phenomenon is the interference of light.

# Analytical treatment of two source interference

Let us consider the superposition of two waves of the same frequency $\omega$ and a constant phase difference $\phi$ travelling in the same direction. Their amplitudes are taken as $a_1$ and $a_2$, respectively. The displacement due to one wave at any instant is given by

$$\psi_1 = a_1 \sin \omega t \quad \text{--- (1)}$$

and the displacement due to another wave at any instant is given by

$$\psi_2 = a_2 \sin(\omega t + \phi) \quad \text{--- (2)}$$

According to the principle of superposition, the resultant displacement ($\psi_R$) is given by

$$\psi_R = \psi_1 + \psi_2 \quad \text{--- (3)}$$

$$= a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 \{\sin \omega t \cos\phi + \cos \omega t \sin\phi\}$$

$$= (a_1 + a_2 \cos\phi) \sin \omega t + a_2 \cos \omega t \sin\phi$$

Let

$$a_1 + a_2 \cos\phi = A \cos\theta \quad \text{--- (4)}$$

$$a_2 \sin\phi = A \sin\theta \quad \text{--- (5)}$$

so

$$\psi_R = A \cos\theta \sin \omega t + A \cos \omega t \sin\theta$$

$$\boxed{\psi_R = A \sin(\omega t + \theta)} \quad \text{--- (6)}$$

squaring and adding equation ④ & ⑤

$$(a_1 + a_2\cos\phi)^2 + a_2^2\sin^2\phi = A^2(\cos^2\theta + \sin^2\theta)$$

or $A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$

The resultant intensity is given by

$$\boxed{I = A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi} \quad \text{--- ⑦}$$

Dividing ⑤ by ④

$$\boxed{\tan\theta = \frac{a_2\sin\phi}{a_1 + a_2\cos\phi}} \quad \text{--- ⑧}$$

<u>Condition for constructive Interference:</u>

Equation ⑦ will be maximum, i.e. $I = I_{max}$ when $\cos\phi = +1$ or $\phi = 2n\pi$ with $n = 0, 1, 2 \ldots$

$$I_{max} = (a_1 + a_2)^2$$

if $a_1 = a_2 = a$, $I_{max} = 4a^2$

In terms of wavelength

path diff $(\Delta) = \frac{\lambda}{2\pi} \times \phi$

$$\Delta = \frac{\lambda}{2\pi} \times 2n\pi = n\lambda$$

$$\boxed{\Delta = n\lambda} \quad \text{--- ⑨}$$

## Condition for destructive Interference:

From eq$^n$ (9) $\quad I = I_{min}$ for $\cos\phi = -1$

or $\quad \underline{\phi = (2n+1)\pi}$ for $n = 0, 1, 2, 3 \cdots$

$$I_{min} = (a_1 - a_2)^2$$

If $a_1 = a_2 = a$

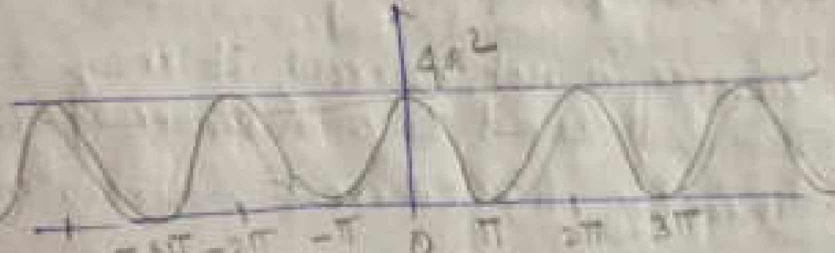
$$I_{min} = 0$$

In terms of wavelength

$$\Delta = \frac{\lambda}{2\pi} \times (2n+1)\pi$$

$$\boxed{\Delta = \left(n + \tfrac{1}{2}\right)\lambda} \quad \text{--- (10)}$$

## Pictorially




## Conservation of Energy

$$I_{max} = (a_1 + a_2)^2 \quad \& \quad I_{min} = (a_1 - a_2)^2$$

If $a_1 = a_2 = a$ then $I_{max} = 4a^2$ & $I_{min} = 0$

So average intensity $I_{av} = 2a^2$

In interference, part of the energy from minima is trasferred to maxima and energy is conserved.

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Condition for sustained interference:

(i) The two sources should emit waves of the same frequency and the two waves should propagate along the same direction.

(ii) The phase difference between the two interfering waves should be zero or it should remain constant.

(iii) The two coherent sources should be very close to each other, otherwise the interfering fringes will be very close to each other due to the large path difference.

(iv) A reasonable distance between the sources and screen should be kept, as the maxima and minima appear quite close if this distance is closer. On the other hand large distance reduces the intensity.

(v) The amplitude of two interferring waves must be equal or nearly equal.

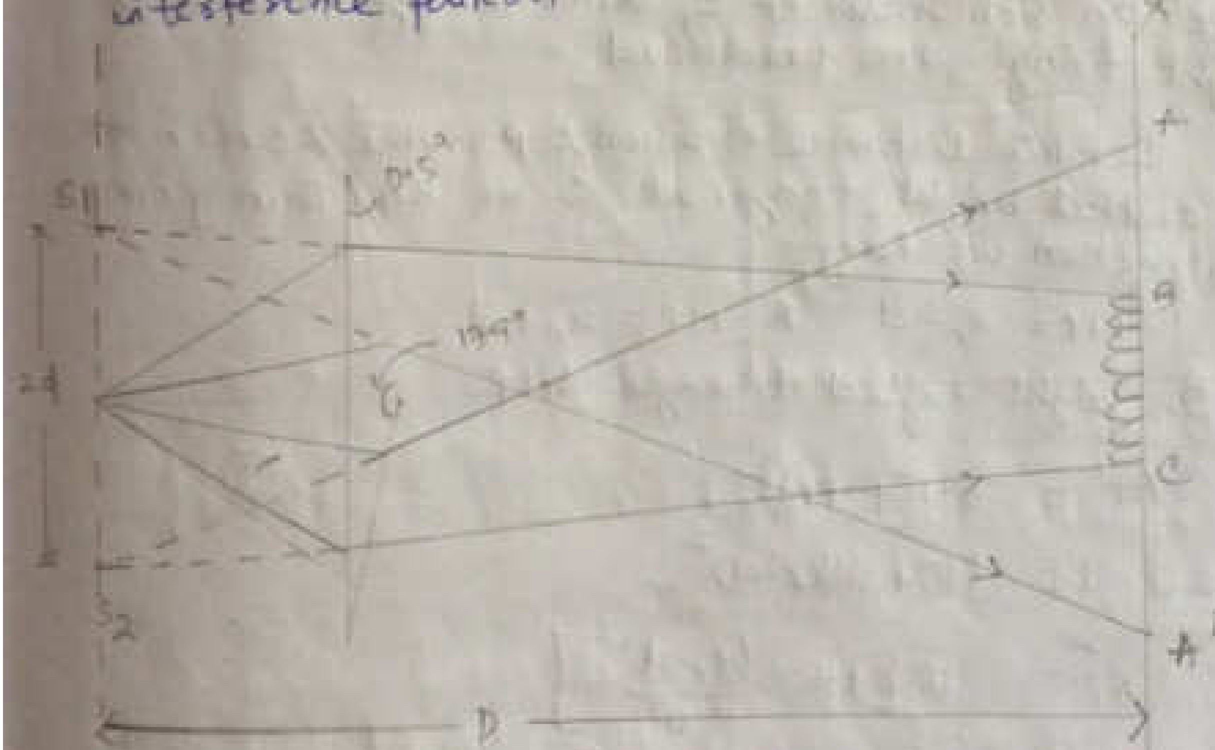
(vi) If the source is not narrow, it may act as multiple source. This will lead to a number of interference patterns. Therefore, the coherent sources must be narrow.

(vii) The sources should be monochromatic and the background should be dark.

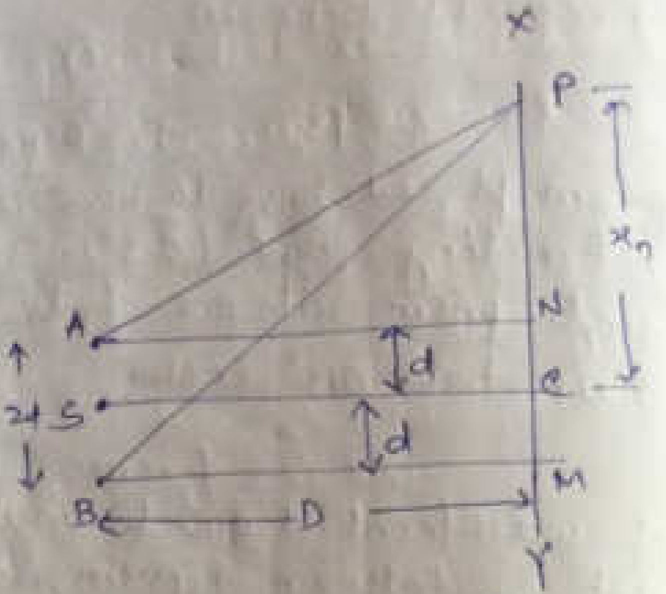
# Fresnel's Biprism

A Fresnel's Biprism is a thin double prism placed base to base and have very small refracting angle (0.5°). This is equivalent to a single prism with one of its angle nearly 179° and other two 0.5° each.

The interference is observed by the division of wavefront. Monochromatic light through a narrow slit 'S' falls on biprism, which divided it into two components. One of the components is refracted from upper portion of biprism and appears to come from $S_1$ whereas the other gets refracted from the lower surface and appears to come from $S_2$. Thus $S_1$ & $S_2$ act as two virtual coherent sources. Light waves coming from $S_1$ & $S_2$ can interfere and give rise to interference pattern.



# Theory of Fringes:




Let A and B two vertical coherent sources of light separated by a distance 2d. The screen XY on which the fringes are obtained is separated by a distance D from two coherent sources. The point C on the screen is equidistant form A & B. Therefore the path difference between the two waves form A & B at C is zero. Thus C will be the centre of the bright fringe. On both sides of C, alternately dark and bright fringes are produced.

Let the distance of a point P on the screen form the central bright fringes at C be $x_n$. Form geometrical construction we have.

$$NP = x_n - d \quad \& \quad MP = x_n + d$$

In right angled triangle ANP

$$AP^2 = AN^2 + PN^2 \quad \text{------} \quad (1)$$

$$= D^2 + (x_n - d)^2$$

$$AP^2 = D^2\left[1 + \frac{(x_n-d)^2}{D^2}\right]$$

$$AP = D\left[1 + \frac{(x_n-d)^2}{D^2}\right]^{1/2}$$

As $(x_n - d) \ll D$, using Binomial expansion.

$$AP = D\left[1 + \frac{1}{2}\frac{(x_n - d)^2}{D^2}\right]$$

$$AP = D + \frac{1}{2}\frac{(x_n - d)^2}{D} \quad \text{---} \quad ②$$

similarly in right angle $\Delta$ BPM

$$BP = D + \frac{1}{2}\frac{(x_n + d)^2}{D} \quad \text{---} \quad ③$$

Hence the path difference

$$\Delta = BP - AP = \frac{4x_n d}{2D} = \frac{2d}{D}x_n \quad - ④$$

<u>Condition for Bright Fringes:</u>

$$\Delta = n\lambda$$

so $\frac{2d}{D}x_n = n\lambda$

$$\boxed{x_n = \frac{n\lambda D}{2d}} \quad \text{----} \quad ⑤$$

$x_n$ is the distance of the $n^{th}$ order fringe from the central point 'c'.

Now $x_{n+1} = \frac{(n+1)\lambda D}{2d}$

The separation between two consequitive bright or dark fringe is called fringe width ($\beta$)

$$\beta = x_{n+1} - x_n$$

$$\boxed{\beta = \frac{\lambda D}{2d}} \quad \text{---} \quad ⑥$$

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Condition for Dark Fringes:

From equation (4), the condition for dark fringes

$$\Delta = \left(n+\frac{1}{2}\right)\lambda$$

or

$$\frac{2d}{D} x_n = \left(n+\frac{1}{2}\right)\lambda$$

$$x_n = (2n+1)\frac{\lambda}{2} \cdot \frac{D}{2d}$$

$$x_n = \frac{(2n+1)\lambda D}{4d} \quad \text{--- (7)}$$

We can obtain the fringe width by similar to previous page

$$\beta = x_{n+1} - x_n$$

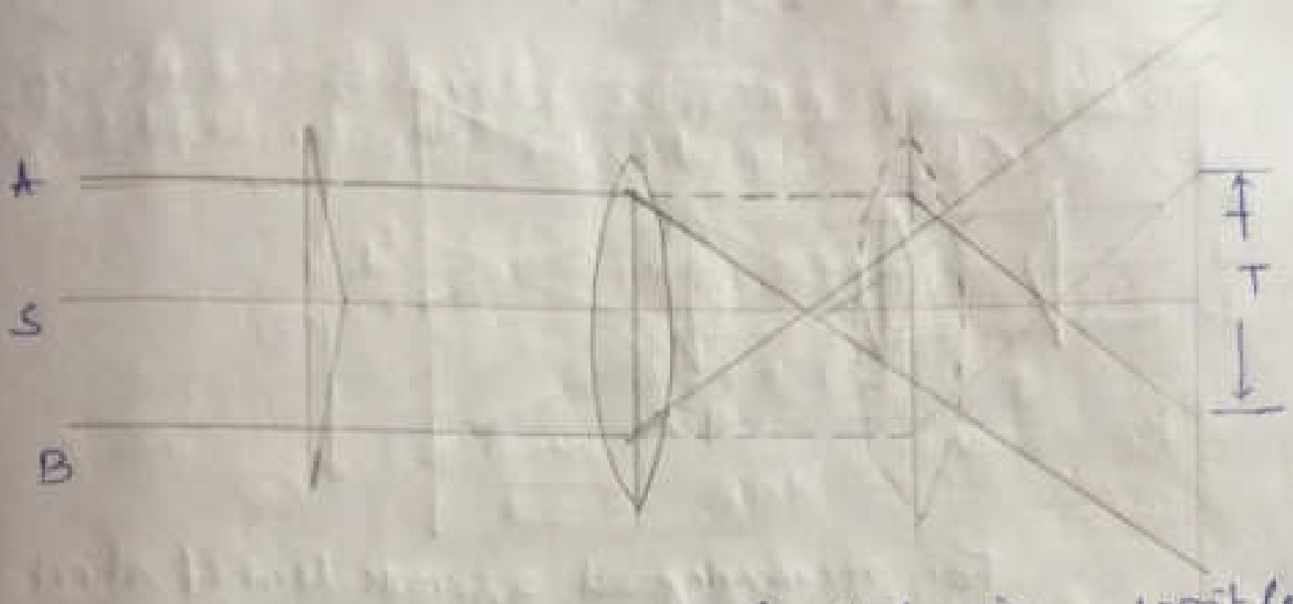
$$= \frac{\{2(n+1)+1\}\lambda D}{4d} - \frac{(2n+1)\lambda D}{4d}$$

$$\boxed{\beta = \frac{\lambda D}{2d}}$$

## Applications:

1. Experimental method for Determination of wavelength of Light:

The experimental setup used for the determination of wavelength of light is consists of a light source and optical bench, stands to hold biprism, convex lens and telescope




(1) optical bench must be levelled using a sprit level

(2) Adjust all uprights to the same hight.

(3) Illuminate the vertical slit by monochromatic source of light. Make the slit narrow.

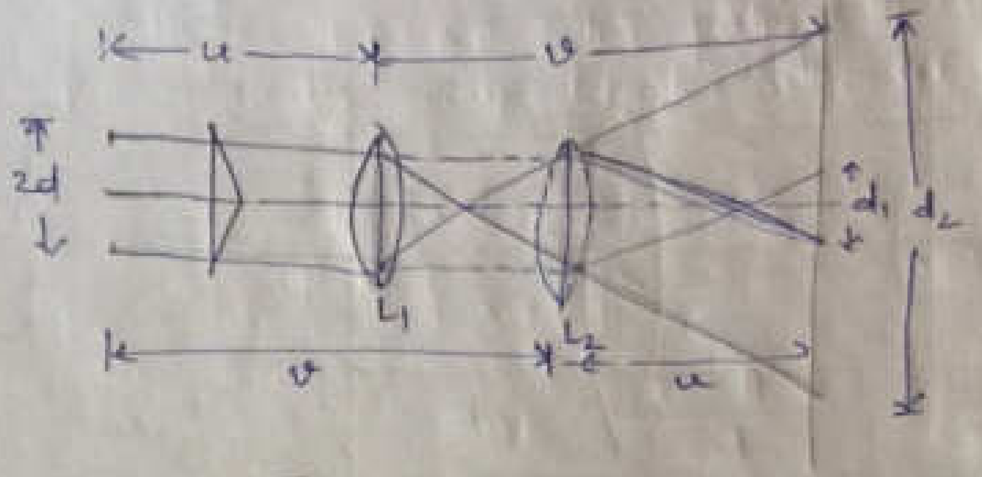
(4) Now place the biprism and adjust its edge parallel to the slit until two equally bright virtual sources A & B are observed.

(5) Adjust the micrometer of telescope so that the fringes appears on the field of view.

(6) To get fine fringes, change the position of the biprism slowly on its own plane such that its edge remains parallel to the slip.

(7) Using this, measure the fringe width, 2d and $\lambda$ can be determined.

## 2. Determination of Distance between two virtual coherent sources: (2d)




For measuring 2d, a convex lens of short focal length is placed between the biprism and the micrometer eye piece. The distance between the biprism and micrometer eye piece is 4 times of the focal length of convex lens. By moving the lens we obtain two positions $L_1$ & $L_2$ of the convex lens such that the separated images $d_1$ & $d_2$ is observed.

For the first position $L_1$, the magnification is given by

$$\frac{v}{u} = \frac{d_2}{2d} \quad --- (1)$$

& for the second position $L_2$,

$$\frac{u}{v} = \frac{d_1}{2d} \quad --- (2)$$

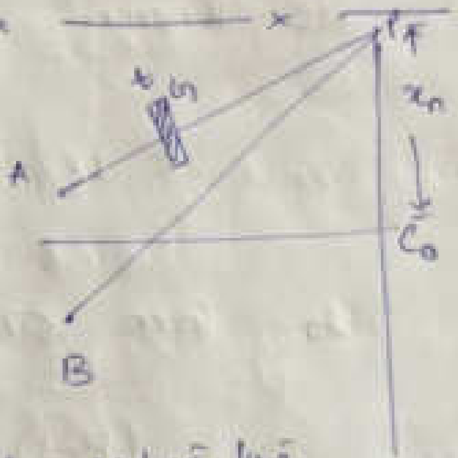
From (1) & (2)

$$\frac{v}{u} \times \frac{u}{v} = \frac{d_1 d_2}{(2d)^2}$$

or $\boxed{2d = \sqrt{d_1 d_2}}$

Therefore, the measurement of positions of images $d_1$ and $d_2$ will determine the distance 2d between the sources. The wavelength $\lambda$ of monochromatic light can be calculated when we substitute the value $\beta$, D & 2d in the formula $\lambda = \beta (2d/D)$

③ Determination of Thickness of thin transparent plate.




Let A & B be two virtual coherent sources of light, The point $C_0$ on the screen is equidistant from both the sources. When a transparent plate G is placed (thickness t and refractive index $\mu$) in the path of the light source wave, we observe that the fringe which was originally at $C_0$ shifts to another position P.

The time taken by the light wave from A to P partially through air and partially through the plate is the same as the time taken by the other light wave from B to P. If 'c' & 'v' be the velocity of light in air and plate respectively

$$\frac{BP}{c} = \frac{AP - t}{c} + \frac{t}{v}$$

or $$\frac{BP}{c} = \frac{AP - t}{c} + \frac{\mu t}{c} \qquad \left[\because \mu = \frac{c}{v}\right]$$

or $$BP = AP - t + \mu t$$

$$BP - AP = t(\mu - 1) \quad \text{--- (1)}$$

Here $BP - AP$ is the path difference $\Delta$. of P is originally occupied by the $n^{th}$ order bright fringe

$$BP - AP = n\lambda$$

$$(\mu - 1)\, t = n\lambda \quad \text{--- (2)}$$

we known $$x_n = \frac{n\lambda D}{2d} \quad \text{--- (3)}$$

or $$n\lambda = \frac{x_n \cdot 2d}{D} \quad \text{--- (4)}$$

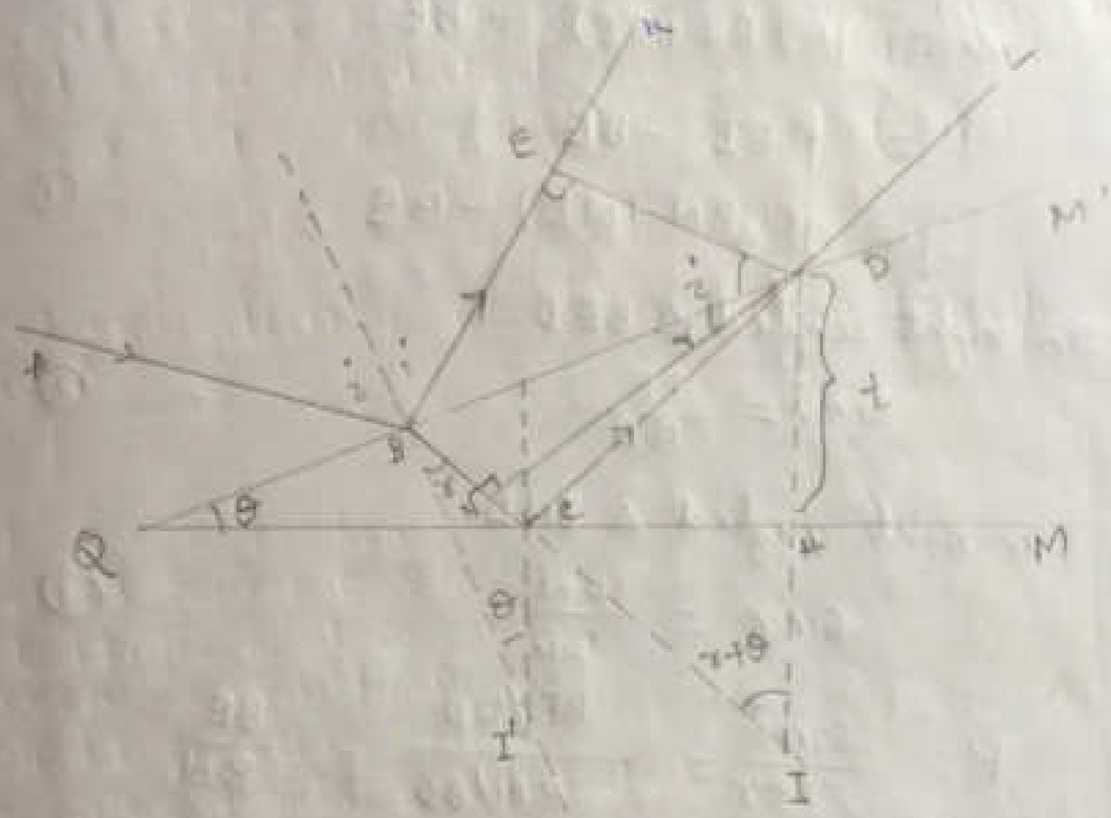
So from (2) & (4)

$$(\mu - 1)\, t = \frac{x_n \cdot 2d}{D}$$

$$\boxed{t = \frac{x_n \cdot 2d}{D(\mu - 1)}}$$

Therefore by knowing $x_n$, $2d$, & $D$ & $\mu$ we can calculate the thickness of the glass plate.

## Interference due to Wedge shaped Film:



Consider two plane surfaces QM & QM' inclined at an angle θ enclosing a wedge shaped film of increasing thickness. A beam of monochromatic light is incident on the upper surface of the film and the interference occurs between the rays reflected at its upper and lower surfaces. The interference occurs between the reflected rays BK & DL both of which are obtained from the same incident ray of light AB.

The path difference between the reflected rays

$$\Delta = (BC + CD)_{\text{in film}} - BE_{\text{air}}$$

$$= \mu (BC + CD) - BE$$

$\because CD = CI$

$$\Delta = \mu(BC + CI) - BE$$
$$= \mu BI - BE \quad \text{--- (1)}$$
$$= \mu(BN + NI) - BE$$

In right angled $\Delta BED$

$$\sin i = \frac{BE}{BD} \quad \text{--- (2)}$$

In right angled $\Delta BND$

$$\sin r = \frac{BN}{BD} \quad \text{--- (3)}$$

$$\mu = \frac{\sin i}{\sin r} = \frac{BE/BD}{BN/BD} = \frac{BE}{BN}$$

or $\quad BE = \mu BN \quad$ --- (4)

Putting in (1)

$$\Delta = \mu(BN + NI) - \mu BN = \mu NI \quad \text{--- (5)}$$

In right angle $\Delta DNI$

$$\cos(r+\theta) = \frac{NI}{DI}$$

$$\because DI = DH + HI = t + t = 2t$$

$$\cos(r+\theta) = \frac{NI}{2t}$$

or $\quad NI = 2t \cos(r+\theta)$

so (5) becomes

$$\Delta = 2t\cos(r+\theta) \cdot \mu$$
$$\Delta = 2\mu t \cos(r+\theta) \quad \text{--- (6)}$$

The reflected wave BK introduces a phase change of $\pi$ because it gets reflected from the denser medium.

Thus

$$\boxed{\Delta = 2\mu t \cos(r+\theta) + \frac{\lambda}{2}} \quad -(7) \qquad \left[\begin{array}{l}\text{phase } \phi \\ \text{path} = \lambda/2\end{array}\right]$$

This shows that the path difference $\Delta$ depends on the thickness $t$. However '$t$' is not uniform and it is different at different positions.

At $t=0$

$\Delta = \frac{\lambda}{2}$ which basically represents the condition for minima. Therefore the edge of the film appears to be black (dark). This is called zero order band.

For normal incidence $i=0$ & $r=0$, thus

$$\boxed{\Delta = 2\mu t \cos\theta + \frac{\lambda}{2}} \quad -(8)$$

Condition for Maxima:

$$\Delta = n\lambda$$

ie
$$2\mu t \cos\theta + \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos\theta = n\lambda - \frac{\lambda}{2}$$

$$2\mu t \cos\theta = (2n-1)\frac{\lambda}{2} \quad \text{where } n = 0, 1 \ldots \quad -(9)$$

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Condition for Minima

for destructive interference

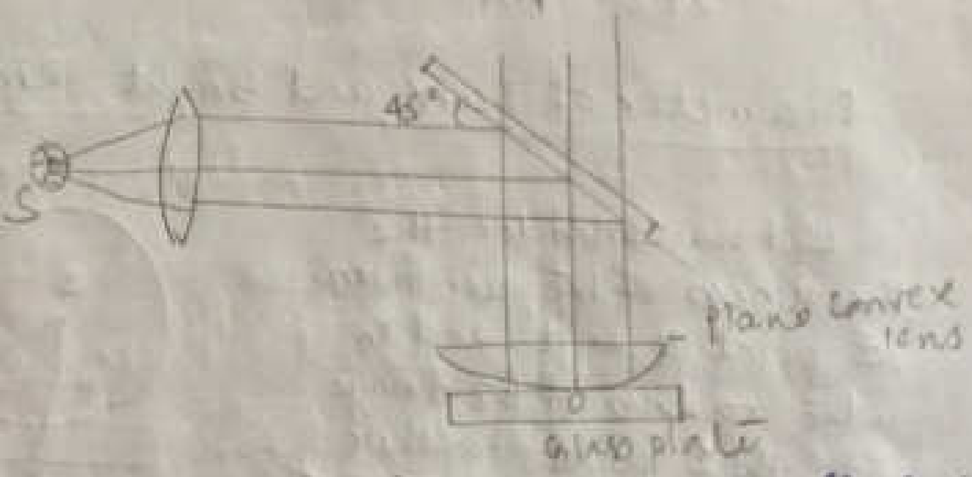
$$\Delta = (n+\tfrac{1}{2})\lambda$$

$$2\mu t \cos\theta + \frac{\lambda}{2} = (n+\tfrac{1}{2})\lambda$$

$$\boxed{2\mu t \cos\theta = n\lambda} \quad n = 0,1,2 \ldots \quad -(10)$$

# Newton's Rings

If a plano-convex lens is placed such that its curved surface lies on a glass plate, then an air film of gradually increasing thickness is formed between the two surfaces. When a beam of monochromatic light is allowed to fall normally on this film and viewed vertically, alternate dark & bright circular fringes are observed. These circular fringes are formed because of the interference between the reflected waves from the top and bottom surface of air film. The fringes are circular, since the air film has a circular symmetry and the thickness of the film corresponding to each fringe is same throughout the circle. The interference fringes so formed were first investigated by Newton and hence known as Newton's rings.




The path difference between the two reflected rays, can be obtained as in the previous section

$$\Delta = 2\mu t \cos(r+\theta) + \frac{\lambda}{2}$$

For normal incidence and air film, $i = r = 0$, $\mu = 1$

If $\theta$ is a very small then $\cos\theta = 1$

$$\Delta = 2t + \frac{\lambda}{2}$$

Here $t$ is the thickness of the air film at a particular point. At the point of contact $t = 0$

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$\Delta = \frac{\lambda}{2}$ which is the condition for minimum intensity and hence, the central spot of the ring will be dark.

## Condition for Maxima :

for bright (constructive interference) fringe

$$\Delta = n\lambda$$

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n-1)\frac{\lambda}{2} \quad \text{for } n = 0, 1, 2 \ldots \quad (1)$$

## Condition for Minima -

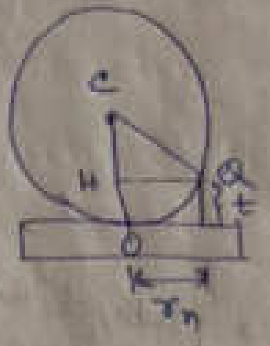
$$\Delta = \left(n+\frac{1}{2}\right)\lambda$$

$$2t + \frac{\lambda}{2} = n\lambda + \frac{\lambda}{2}$$

$$2t = n\lambda \quad \text{when } n = 0, 1, 2 \ldots \quad (2)$$

## Diameter of Dark and Bright Rings

Let us consider the thickness of the air film at point Q as 't' and $r_n$ is the radius of the fringe at that point, together with R as the radius of curvature of the lens.




$OC = CQ = R$, $HQ = r_n$

$HC = R - t$

In right angle $\Delta$ CHQ

$$CQ^2 = (CH)^2 + (HQ)^2$$

$$= (R-t)^2 + r_n^2$$

or $r_n^2 = 2Rt - t^2$

In real practice, R is quite large & t is very small. Therefore $t^2 << R^2$ & can be neglected

$$\therefore r_n^2 = 2Rt = R \times 2t \quad — ③$$

For Bright Rings:

Form eq$^n$ ①

$$2t = (2n-1)\frac{\lambda}{2}$$

So $r_n^2 = R \times (2n-1)\frac{\lambda}{2}$

$$\left(\frac{D_n}{2}\right)^2 = R \times (2n-1)\frac{\lambda}{2} \qquad \left[\begin{array}{l} D_n = \text{Diameter} \\ = 2r_n \end{array}\right]$$

$$D_n^2 = \overset{2}{\cancel{4}}\lambda R(2n-1) \times \frac{1}{2}$$

$$D_n^2 = 2\lambda R(2n-1)$$

$$\therefore \boxed{D_n = \sqrt{2\lambda R(2n-1)}} \quad — ④$$

$$D_n \propto \sqrt{(2n-1)}$$

For Dark Rings -

Form eq$^n$ ②

$$2t = n\lambda$$

So $r_n^2 = R \times n\lambda$

$$\text{or} \quad \boxed{D_n^2 = 4n\lambda R} \quad — ⑤$$

or $D_n^2 \propto \sqrt{n}$

## Determination of wavelength of light:

We have seen that the diameter of $n^{th}$ order dark fringe in Newton's ring method :

$$D_n^2 = 4n\lambda R \quad \text{--- (1)}$$

Hence diameter of $(n+p)^{th}$ order dark fringe

$$D_{n+p}^2 = 4(n+p)\lambda R \quad \text{--- (2)}$$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

## Determination of refractive index of liquid:

The liquid whose refractive index is to be determined is placed between the lens and the plane glass plate and then we evaluate the diameter of the dark fringes.

The diameter of the $n^{th}$ order dark fringe of thin air film is given by

$$[D_n^2]_{air} = 4n\lambda R$$

similarly, the diameter of the $n^{th}$ order dark fringe in liquid film would be

$$[D_n]^2_{liquid} = \frac{4n\lambda R}{\mu}$$

when $\mu$ is the refractive index of the liquid

$D_{liquid} < D_{air}$.

So $$\boxed{\mu = \frac{[D_n^2]_{air}}{[D_n^2]_{liquid}}}$$

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## Diffraction:

The phenomenon of bending of light waves around obstacles or aperatures of sizes comparable with the wavelengths of light and resulting thereby in their spreading into the geometrical shadows of the object is called diffraction.

The amount of bending depends on (a) the size of the obstacle (b) the wavelength of the wave.

## Classification of Diffraction:

The diffraction phenomenon can be divided into two main classes, (i) Fresnel (ii) Fraunhofer diffraction

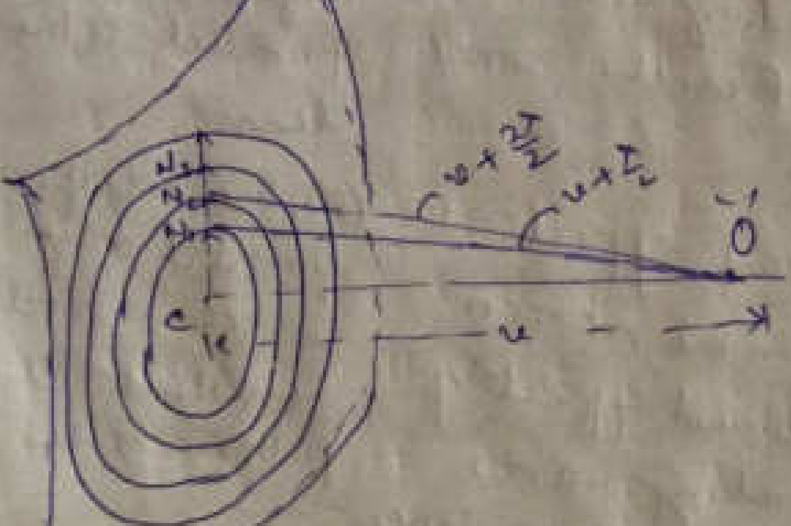
In Fresnel class of diffraction, either the source of light or the screen or both are at ~~finite~~ finite distance from the diffraction aperature or obstacle. The wavefront in Fresnel class is either spherical or cylindrical.

In Fraunhofer class of diffraction, the source of light and the screen are effectively at infinite ~~dista~~ distance from the diffracting aperature and the wavefront is plane.

| Fresnel diffraction | Fraunhofer diffraction |
| --- | --- |
| ① The distance of the source or the screen or both from the diffracting elements are finite. | ① The distance of the source and the screen from the diffracting elements are effectively infinite. |
| ② The wavefronts are not plane but either spherical or cylindrical | ② The wavefronts are plane. |
| ③ No mirrors or lenses are used for study | ③ Converging lens is used to focus parallel rays. |
| ④ In the plane of aperature the phase of the secondary wavelets is not the same at all points. | ④ The secondary wavelets are in the same phase of at every point in the plane of aperature. |
| ⑤ The centre of diffraction pattern may be bright or dark depending upon the number of fresnel's zones. | ⑤ The centre of the diffraction pattern is always bright for all paths parallel to the axis of the lens. |
| ⑥ The resultant effect at any point on the screen is the combined effect of all the secondary waves originating from various zones | ⑥ The interference of all parallel rays brought to focus produces the resultant diffraction pattern on the screen. |

## Half period zones:

The concept of half period zone is given by Fresnel, based on which he could find the effect produced by a slightly divergent spherical wave or wavefront at a point ahead of the wave. Since every point on the wavefront is treated as a secondary source of wavelets, the method of half period zone is used to calculate the effect of all these wavelets at a point O (see figure). For this we divide the wavefront into large number of zones, called Fresnel's half period zones. For constructing these zones, we consider PQRS as a spherical wavefront of a monochromatic light of wavelength 'λ' travelling towards the screen. We draw a perpendicular from the point 'O' to the wavefront. This meets the wavefront at the point C at a distance 'u'. We draw series of circles around 'C' such that their distance



from C are $CN_1, CN_2, CN_3 \cdots CN_n$ and each circle is a half wavelength farther from O. Therefore the circles will be at a distance $u+\lambda/2$, $u+\frac{2\lambda}{2}$, $\cdots$ $u+\frac{n\lambda}{2}$ from 'O'. This way the areas of zones, i.e. areas of the rings between successive circles are equal. The area enclosed between $CN_1$, $N_1N_2$, $N_2N_3$ etc. are called first, second, third half-period zones respectively. Actually the difference of half a period in the vibration from successive zones is the origin of the name half period zones.

## Radii and Areas of Half Period Zones

In most of the cases the wavelength of light $\lambda$ remain much smaller compared with the distance u. Under the condition, i.e $\lambda \ll u$, we can calculate the radii and the areas of the zones. The radii of the first half period zone

$$CN_1 \simeq \sqrt{ON_1^2 - OC^2} = \sqrt{\left(u+\frac{\lambda}{2}\right)^2 - u^2} \simeq \sqrt{u\lambda}$$

we have neglected a term $\frac{\lambda^2}{4}$ in view of $\lambda \ll u$

Similarly

$$CN_2 = \sqrt{ON_2^2 - OC^2} \simeq \sqrt{2u\lambda}$$

$$\vdots$$

$$CN_n = \sqrt{ON_n^2 - OC^2} \simeq \sqrt{nu\lambda}$$

Based on the relation for the radii of the half-period zones, we can find the area of the first, second, third half period zones.

The area of first half period zone

$$a_1 = \pi\, CN_1^2 = \pi r \lambda$$

Similarly the area of second half period zone

$$a_2 = \pi\,(CN_2^2 - CN_1^2) = \pi r \lambda$$

This is clear from the above that areas of all zons are the same under the approximation $\lambda << r$ and are independent of $n$. However, a more exact evaluation shows that the area gets increased very ~~slow~~ slowly with $n$.

## Resultant Amplitude due to whole Wavefront

As per Huygen's principle every point of the wavefront sends secondary wavelets in the same phase. However, these will not reach the point 'O' in the same phase as they travel different distances. Since each zone is $\lambda/2$ farther from the point 'O', the successive zone will produce resultant at 'O' differing by a phase difference of '$\pi$'. If $A_n$ be the

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resultant amplitude of the zone light due to $n^{th}$ half period zone, then its successive value will have alternate signs because of the phase change of '$\pi$' in moving from one zone to the next. If the amplitude of the waves emerging from the first, second third half period zones, etc. are respectively $A_1, A_2, A_3$. etc. Then the resultant amplitude $A$

$$A = A_1 - A_2 + A_3 - A_4 \text{ ---- } - A_n \quad (n \text{ even})$$

$$A = A_1 - A_2 + A_3 - \text{ — } + A_n \quad ('n' \text{ odd})$$

or $$A = A_1 - A_2 + A_3 - - \cdot (-1)^{n-1} A_n$$

Now as the magnitudes of successive amplitudes goes on decreasing with the higher order of zones due to the increased average distance of the zone from 'O' and the larger obliquity, the amplitude $A_2$ is slightly smaller than $A_1$ but slightly greater than $A_3$. Therefore, for computational simplicity we can assume

$$A_2 = \frac{A_1 + A_3}{2}, \quad A_4 = \frac{A_3 + A_5}{2} \quad - -$$

Then

$$\cancel{A = \frac{A_1}{2}\left(\frac{A_1 + A_3}{2}\right)}$$

$$A = \frac{A_1}{2} + \left(-A_2 + \frac{A_1 + A_3}{2}\right) + \left(-A_4 + \frac{A_3 + A_5}{2}\right)$$

$$\ldots + \frac{A_{n-1}}{2} - A_n \quad (n \text{ even})$$

$$- -$$

$$A = \frac{A_1}{2} + \left(-A_2 + \frac{A_1 + A_3}{2}\right) + \left(-A_4 + \frac{A_3 + A_5}{2}\right) + \cdots + \frac{A_n}{2}$$

(if n is odd)

If the number n becomes sufficiently large, then the effect due to nth zone would become insignificant and the resultant amplitude due to the whole wavefront can be approximated as $A = \frac{A_1}{2}$ and hence the intensity $I = \frac{A_1^2}{4}$.

## Zone Plate

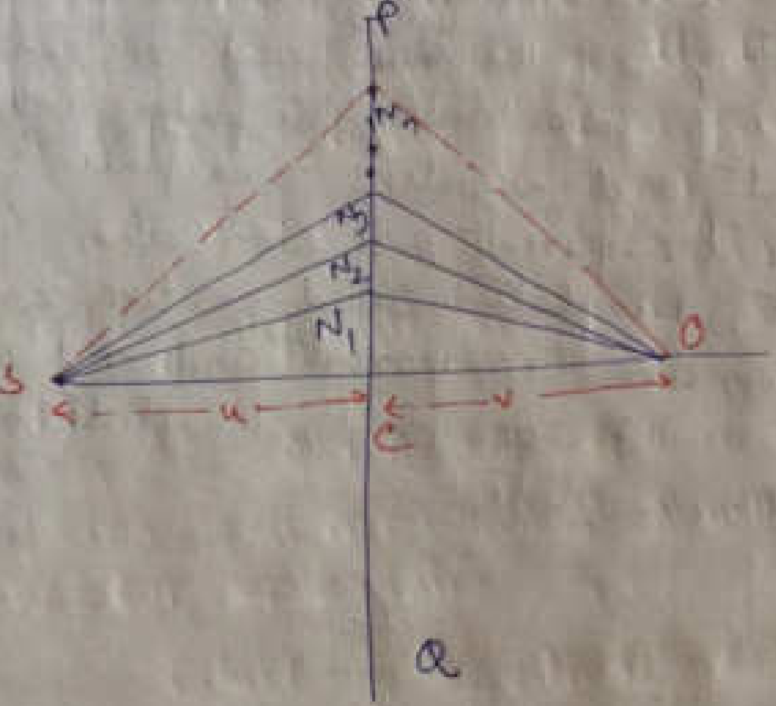
A zone plate is an optical device that is used to verify the correctness of Fresnel's method of dividing a wavefront into half period zones. It is a transparent plate on which a series of concentric circles are drawn with their radii proportional to the square root of natural numbers. This way the formed alternate annular zones are blocked. This type of plate behaves like a convex lens and produces an image of a source of light on the screen placed at a suitable distance.

## Construction of Zone Plate:

On a sheet of white paper we draw concentric circles with their radii proportional to the square root of natural numbers. The alternate zones are painted black and a significantly reduced photograph of the drawing is obtained on a glass plate. The glass plate, when kept in the light path from a distant point source, produces a large intensity at a point on its axis at a distance determined by the size of the zone and the wavelength of the light used. We can construct two types of zone plates, the plates called positive zone plate and negative zone plate. If the odd zones are transparent and even zones opaque on the zone plate, the plate is called positive zone plate. If even zones are transparent and odd zones are opaque on the zone plate, the plate is called negative zone plate.

## Theory of zone Plate

Let 'S' be a point source of monochromatic light sending out spherical waves of wavelength 'λ' and 'O' is the position of the screen for a bright image. Consider an imaginary transparent plate PQ at a right angle to SO passing through the point 'C'. Our goal is to determine the intensity of light at 'O' due to the wavefront emerging from the source 'S'. In order to find this we divide the wavefront into half period zones bounded by circles having centres at C and radii equal to $CN_1, CN_2, CN_3 \cdots CN_n$ as $r_1, r_2 \cdots r_n$. These radii divide the plate PQ into half period zones such that from one zone to the next there is an increasing path difference of $\lambda/2$. So




$$SN_1 + N_1O = SC + CO + \lambda/2$$

$$SN_2 + N_2O = SC + CO + 2\lambda/2$$

$$SN_n + N_nO = SC + CO + n\lambda/2$$

To find the radius $r_n$ of the $n^{th}$ circle or the $n^{th}$ zone, we have

$$SN_n + N_nO = SC + CO + n\lambda/2 \quad -(1)$$

Let $SC = u$ & $CO = v$

$$SN_n = \sqrt{SC^2 + CN^2} = \sqrt{u^2 + r_n^2}$$

$$= \left\{u^2\left(1 + \frac{r_n^2}{u^2}\right)\right\}^{1/2}$$

$$\simeq u\left(1 + \frac{r_n^2}{u^2}\right)^{1/2}$$

$$\simeq u\left(1 + \frac{r_n^2}{2u^2} + \cdots\right)$$

$$\simeq u + \frac{r_n^2}{2u} \quad (r_n << u)$$

Similarly $N_nO = v + \frac{r_n^2}{2v}$

from (1)

$$u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} = u + v + \frac{n\lambda}{2}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{n\lambda}{r_n^2}$$

Applying sign convention

$$\frac{1}{v} - \frac{1}{u} = \frac{n\lambda}{r_n^2}$$

or $$r_n^2 = \frac{n\lambda uv}{u - v}$$

It is clear that $r_n \propto \sqrt{n}$, radii of the zones are proportional to the square root of the natural numbers.

The area of the $n^{th}$ zones

$$a_n = \pi(r_n^2 - r_{n-1}^2) = \pi\left[\frac{n\lambda uv}{u-v} - \frac{(n-1)\lambda uv}{u-v}\right]$$

$$= \frac{\pi\lambda uv}{u-v} \quad -(2)$$

This relation shows that the area of the $n^{th}$ zone is independent of $n$. It means for a given object and image the area of all zone remain the same. From previous discussion, the resultant amplitude A due to the whole wavefront at point A

$$A = A_1 - A_2 + A_3 - - + (-1)^{n-1} A_n \quad -(3)$$

Considering a positive zone plate (even zons are blocked) $A = A_1 + A_3 + A_5 - -$ (4) i.e. the amplitude's positive. Whereas A will be negative for negative zone plates

Considering equation (3) & (4), it is evident that the resultant amplitude produced by a zone plate is greater than due to wholly unobstructed wavefront. Hence, the intensity is very much enhanced, i.e the point O' is extremely bright and can be said to be image of S. This concentration of light at an axial point shows that the zone-plates operates as a lens with O' as focal point.

In order to find the focal length of the zone plate, we compare with the lens formula.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{--- (4)}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{n\lambda}{r_n^2} \quad \text{--- (5)}$$

$$\boxed{f_n = \frac{r_n^2}{n\lambda}} \quad \text{--- (6)}$$

## Multi focus Behaviour of the zone plate :

For an object at infinity, i.e when $u=\infty$, the radius of nth zone can be obtained from

$$r_n = \sqrt{n\lambda v} \quad \text{--- (7)}$$

Comparing (6) & (7)

$$f_1 = \frac{r_1^2}{\lambda}$$

From equation (7) it is clear that for fixed $r_n$, the number n gets increased if we reduce the distance v. It means as the field point O is brought towards the zone plate along the axis, the same zonal area of radius $r_1$ will include more half period zones. If the field point O is brought at a distance b/2 n=2 satisfies the equation (7) for the same zonal radius $r_1$. Therefore, each of the original zone in this case will now contain two half period zones.

For each original zone these two half period zones contribute light at the focal point $v = f_1/2$ out of phase by $\pi$ with each other. So they cancel and no light is focused by the zone plate at the focal point $f_1/2$. If we keep on moving the field point $O'$ towards the zone plate, we will find $n=3$ when $v = f_1/3$, for the same zonal radius $r_1$. In the case now three half period zones are contained in each of the original zones. Out of these three zones, the effect of two will ~~cancel~~ cancelled due to a phase difference of $\pi$ between them and the light will be focused at point $O(f_1/3)$ due to only one half period zone. For further movement of point $O$, we will find no light at $v = f_1/4$, light at $v = f_1/5$, etc. Therefore we can conclude that a zone plate has multiple foci of focal lengths $f_1, f_1/3, f_1/5$. For $v = f_1/3$ the contribution of each original zone is subdivided into three half period zones at the observation point $O'$.

$$A = \underbrace{(A_1 - A_2 + A_3)}_{\text{first zone}} + \underbrace{(-A_4 + A_5 - A_6)}_{\text{2nd zone}} + \underbrace{(A_7 - A_8 + A_9)}_{\text{3rd zone}}$$

For the zones that are reproduced on a smaller scale, the obliquity factor is not very important and we may estimate $A_j = A_1$ where $j = 2, 3, 4 \ldots$ Hence the resultant amplitude at $v = f_1/3$ would be simply equal to $A_1$. However at $v = f_1$ the amplitude will be three times of this amplitude. Thus, the amplitude at $v = f_1/3$, zone by zone, is reduced by a factor of $1/3$ and hence, the intensity at the point is $1/9$ that at $v = f_1$.

## Comparison between Zone Plate and Convex lens

① In case of zone-plate the image is formed by diffraction whereas the rays in case of lens are brought to focus by refraction.

② The image due to convex lens is more intense than due to zone plate.

③ A convex lens has only one focus, whereas a zone plate has n number of foci of reduced intensity between the point $f$ & $c'$.

④ The focal length of a lens is given by the relation $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, where $\mu$ is the refractive index of the material of the lens and $R_1$ & $R_2$ are the radius of curvatures. However, the focal length of the zone plate is given by $\frac{1}{f} = \frac{n\lambda}{r_n^2}$

⑤ The focal length of a lens is directly proportional to the wavelength $\lambda$ as $f \propto \frac{1}{\mu} \Rightarrow f \propto \frac{v}{c} \Rightarrow f \propto \lambda$. However the focal length of a zone plate is inversely proportional to $\lambda$.

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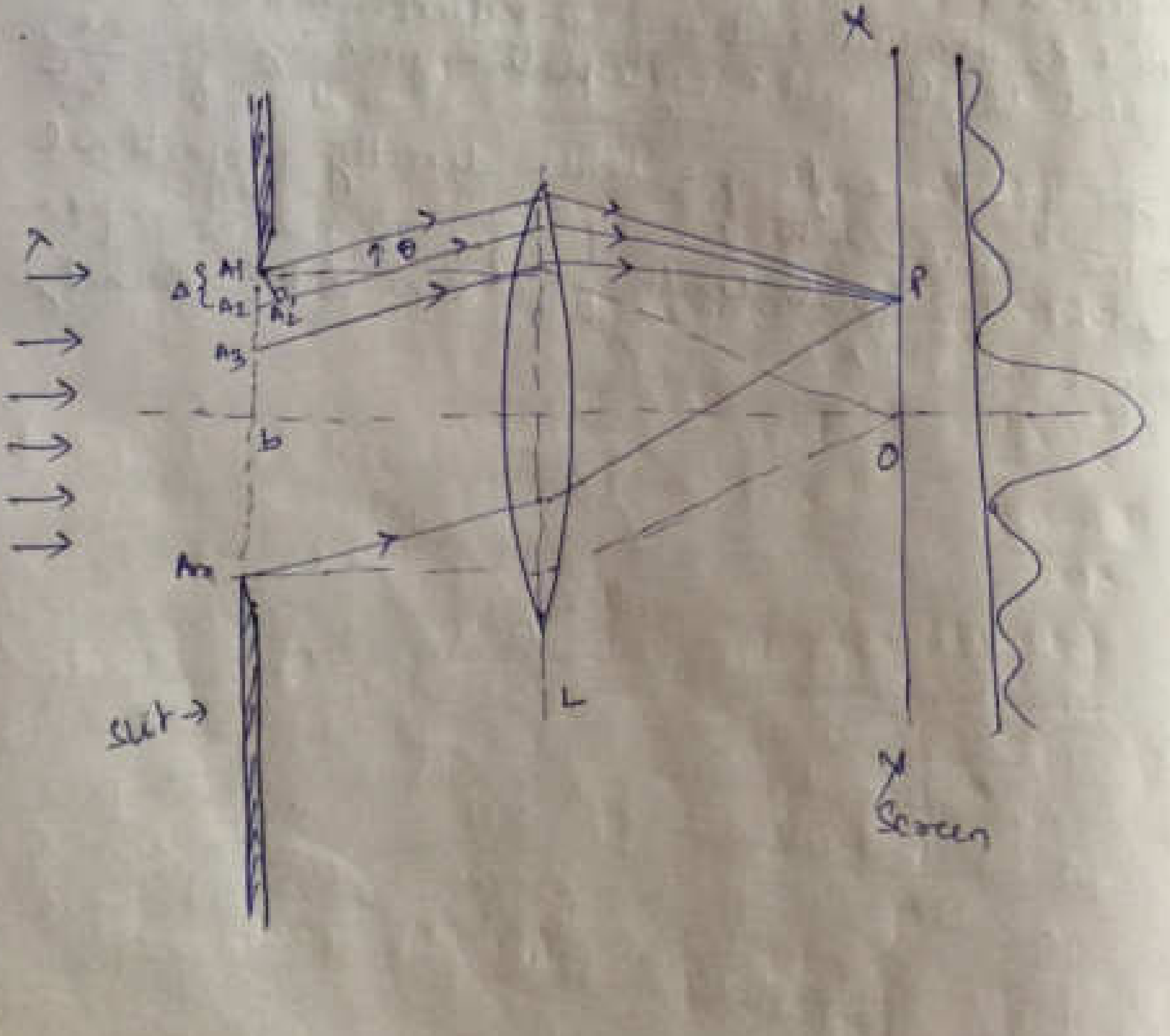
# Fraunhofer Diffraction at a Single-Slit

Let a plane wavefront of monochromatic light of wavelength $\lambda$ be incident normally on a narrow slit AB of width 'b' placed perpendicular to the plane of the paper. We have to calculate the intensity distribution due to diffraction on the screen XY placed at the focal point of the lens L. We assume that (i) each point on the slit is a source of secondary wavelets. Interference takes place between the wavelets originating from two such secondary sources. (ii) The slit consists of a large number of secondary point sources, which are equi-spaced.



If $\Delta$ be the separation between any two point-sources $A_1, A_2, A_3$ - - - then

$b = (n-1)\Delta$, where $n$ is the number of point-sources.

As the point $P$ on the screen is at a large distance from the slit, the amplitudes of the secondary waves from $A_1$ and $A_2$ will be nearly the same. If the diffracted rays from $A_1$ and $A_2$ makes an angle '$\theta$' with the normal to the plane of the slit, then the path difference between them is

$A_2A_2' = \Delta \sin\theta$, where $A_1A_2 = \Delta$

The corresponding phase difference

$$\phi = \frac{2\pi}{\lambda}(\Delta \sin\theta)$$

So if the field point $P$ due to disturbance coming from $A_1$ be represented by

$$E_1 = a\cos\omega t$$

where '$a$' is the amplitude, $\omega$ is the angular frequency, then the field at $P$ due to the disturbance from $A_2$ is

$$E_2 = a\cos(\omega t - \phi)$$

Similarly, the fields at $P$ due to the disturbances from $A_3, A_4, - - A_n$ are

$$E_3 = a\cos(\omega t - 2\phi)$$

$$E_4 = a\cos(\omega t - 3\phi)$$

- - - - - -

$$E_n = a\cos[\omega t - (n-1)\phi]$$

Thus, the resultant field at P is

$$E = E_1 + E_2 + \cdots + E_n$$

$$= a\left[\cos\omega t + \cos(\omega t - \phi) + \cos(\omega t - 2\phi) + \cdots \cos\{\omega t - (n-1)\phi\}\right] \quad \text{--- (1)}$$

Now,

$$\cos\omega t + \cos(\omega t - \phi) + \cdots \cos[\omega t - (n-1)\phi]$$

$$= \frac{\sin\frac{n\phi}{2}}{\sin\phi/2} \cos\left[\omega t - \tfrac{1}{2}(n-1)\phi\right] \quad \text{--- (2)}$$

$$E = a\,\frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}} \cos\left[\omega t - \tfrac{1}{2}(n-1)\phi\right]$$

$$= E_\theta \cos\left[\omega t - \tfrac{1}{2}(n-1)\phi\right] \quad \text{--- (3)}$$

$$E_\theta = a\,\frac{\sin\frac{n\phi}{2}}{\sin\phi/2}$$

Now, if $n \to \infty$, $\Delta \to 0$, then $n\Delta \Rightarrow b$

$$\therefore \frac{n\phi}{2} = \frac{n}{2} \times \frac{2\pi}{\lambda}\Delta\sin\theta = \frac{n\pi\Delta}{\lambda}\sin\theta$$

$$= \frac{b\pi\sin\theta}{\lambda} \text{ and } \frac{\phi}{2} = \frac{1}{2} \times \frac{2\pi}{\lambda}\Delta\sin\theta$$

$$= \frac{\pi b\sin\theta}{n\lambda} \to 0$$

$$\therefore E_\theta \simeq \frac{a\sin\frac{n\phi}{2}}{\phi/2} = \frac{na\sin\left(\frac{\pi b\sin\theta}{\lambda}\right)}{\frac{\pi b\sin\theta}{\lambda}}$$

$$= A\,\frac{\sin\beta}{\beta} \quad \text{--- (4)}$$

where $A = na$ & $\beta = \frac{\pi b\sin\theta}{\lambda}$

$$\therefore \quad E = A\frac{\sin\beta}{\beta}\cos(\omega t-\beta) \quad \text{--- (5)} \qquad \left[\because \frac{(n-1)\phi}{2} \simeq \frac{n\phi}{2} \text{ for } n\to\infty\right]$$

The intensity distribution on the screen due to the diffraction at the single slit is

$$I = I_0 \frac{\sin^2\beta}{\beta^2} \quad \text{--- (6)}$$

when $I_0 = A^2 = n^2a^2$ is the intensity at $\theta \to 0$

Equation (6) shows that the intensity at any point on the screen XY is a function of $\beta$ and hence $\theta$. This shows that a series of alternate maxima and minima of intensity will be obtained on the screen.

(1) Position of the principal maxima: when $\theta \to 0$

$$\beta = \frac{\pi b\sin\theta}{\lambda} \to 0 \quad \& \lim_{\beta\to 0}\frac{\sin\beta}{\beta} \to 1$$

$$I = I_0\left(\lim_{\beta\to 0}\frac{\sin\beta}{\beta}\right)^2 = I_0 \quad \text{(7)}$$

This corresponds to the principal maximum, which occurs in the region of the screen, when $\beta \to 0$, i.e. $\theta \to 0$, i.e. around the point O.

(2) Position of minima:

From equation (6), it is clear that $I = 0$ when $\sin\beta = 0$ i.e. $\beta = \pm m\pi$. when $m = 1, 2, 3 \ldots$

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or $\frac{\pi b \sin\theta}{\lambda} = \pm m\pi$

$b \sin\theta = \pm m\lambda$ ——— (8)

Equation (8) gives the condition for diffraction minima. The value for $\theta = 0$ is not admissible because this corresponds to the position of the principal maximum.

The first minimum occurs at $\theta = \pm \sin^{-1}(\lambda/b)$, the second minimum at $\theta = \pm \sin^{-1}(2\lambda/b)$, etc.

Since $\sin\theta$ can not exceed unity, the maximum value of the order number ~~at~~ of the ~~$\theta = \pm\sin^{-1}(m\lambda/b)$~~ minimum $m \le \frac{b}{\lambda}$

[$\because \sin\theta \le 1$, $b\sin\theta = m\lambda$ gives $\sin\theta = \frac{m\lambda}{b}$ or $\frac{m\lambda}{b} \le 1$ or $m \le \frac{b}{\lambda}$ ]

The minima are situated on either side of the principal maxima.

Since $\sin\theta = \pm m\frac{\lambda}{b}$, the position of first, second, third, etc. minima are given by $\theta_1 = \pm\frac{\lambda}{b}$, $\theta_2 = \pm\frac{2\lambda}{b}$, $\theta_3 = \pm\frac{3\lambda}{b}$, etc which shows that minima are of equal width.

(3) Position of secondary maxima:

$$I = I_0 \frac{\sin^2\beta}{\beta^2} \quad \therefore \frac{dI}{d\beta} = I_0\left[\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3}\right]$$

For I to be minimum

$$\frac{dI}{d\beta} = 0 \quad \therefore 2\sin\beta\left[\frac{\cos\beta}{\beta^2} - \frac{\sin\beta}{\beta^3}\right] = 0$$

i.e. either $\sin\beta = 0$ or $\beta = \pm m\pi$ corresponds to the position of minima

or $$\frac{\cos\beta}{\beta^2} - \frac{\sin\beta}{\beta^3} = 0 \quad \text{--- (9)}$$

$$\beta - \tan\beta = 0 \quad \text{or} \quad \tan\beta = \beta \quad \text{--- (10)}$$

The root $\sin\beta = 0$, i.e. $\theta = 0$ corresponds to the position of the central maximum. The other roots of equation (10) can be found by plotting $y = \beta$ & $y = \tan\beta$ on the same graph paper and then finding the coordinates of the point of intersections. The intersections occurs at $\beta = 1.43\pi$, $2.46\pi$ etc. Hence the secondary maxima occurs at $\beta = 1.43\pi$, $2.4\pi6$ etc.

(i) For the principal maximum $\theta = 0$, $\therefore \beta = 0$ & $I = I_0$

(ii) For the first secondary maximum $\beta = 3\pi/2$

$$I_1 = I_0\left(\frac{\sin 3\pi/2}{3\pi/2}\right)^2 = I_0\left(\frac{-1}{3\pi/2}\right)^2 = \frac{4I_0}{9\pi^2} = \frac{I_0}{22}$$

(iii) For the second secondary maximum $\beta = 5\pi/2$

$$I_2 = I_0\left(\frac{\sin 5\pi/2}{5\pi/2}\right)^2 = I_0\left(\frac{1}{5\pi/2}\right)^2 = \frac{4I_0}{25\pi^2} = \frac{I_0}{61}$$

This indicates that the intensity of the principal maxima is the highest and the rest decreases on either side of $I_0$.

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Effect of white light:

For white light, the secondary maxima will be coloured but the central maxima will be white.

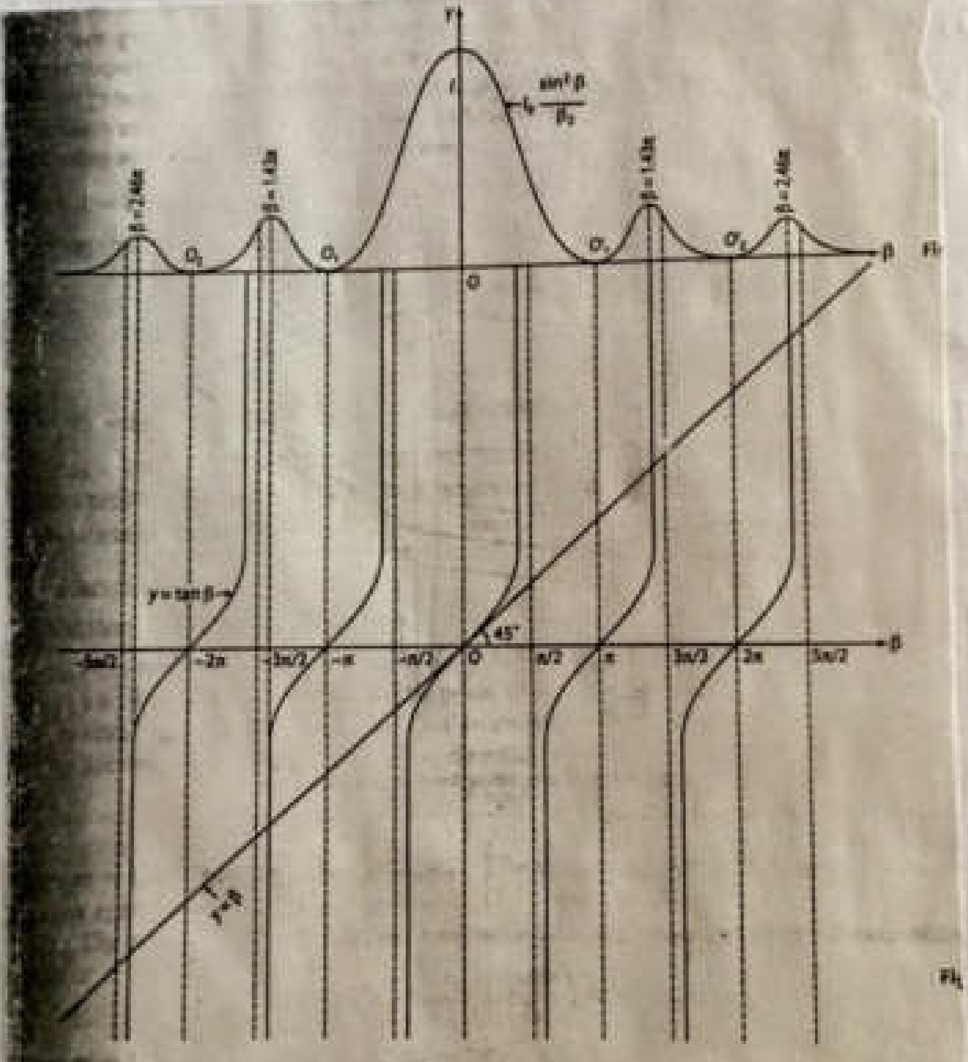


Fig. 5.3 Intensity distribution in Fraunhofer diffraction due to a single-slit.