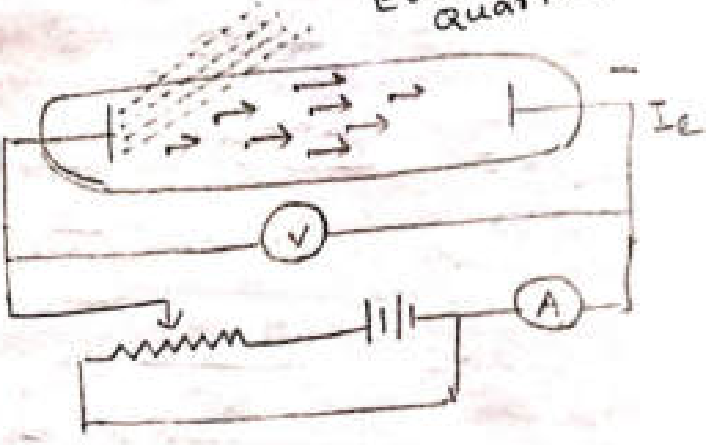
Quantum Physics

(a) Need for quantum physics:

So far as we have studied motion basically governed by classical physics (or more specifically based on Newton's laws of motion) or also known as Newtonian mechanics and Maxwell's equation describes the electromagnetic field in classical electromagnetism. The classical mechanics correctly describes motion of celestial bodies like planets, stars, macroscopic and microscopic terrestrial bodies moving with non-relativistic speeds. However, classical theory fails in the atomic dimensions, ie can not explain the non-relativistic motion of electrons, protons etc. classical physics could not explain the stability of atoms, spectral distribution of black body radiation, discreteness of atomic spectra etc. Also classical mechanics could not explain a large number of experiments like, photo-electric effect, compton effect, Raman effect etc. so these inefeciencies of classical mechanics led to the foundation of Quantum mechanics

The development of quantum mechanics took place in two stages. The first stage began with Max Planck's hypothesis according to which the radiation is emitted or absorbed in discrete packets or quanta of energy. The second stage of quantum mechanics began in 1925 along with two points of view. matrix mechanics was introduced by Heisenberg, in which only observed quantities like frequencies and intensities of spectral lines are taken into account and unobserved quantities like positions velocities etc. in electronic orbits are ~~neglected~~ omitted. The wave mechanics in quantum physics was developed by Schrödinger in 1926. In this mechanics concepts of classical wave theory and deBroglie's wave particle relation are combined.

PHOTOELECTRIC EFFECT



The photoelectric effect refers to the emission or ejection of electrons from the surface of metal in response to incident light. Energy contained within the incident light is absorbed by the electrons within the metal gaining sufficient energy to be knocked out, i.e. emitted from the surface of metal. Experimental setup shown in the figure. It consists of an evacuted quartz tube (or glass tube with quartz window) and consists of an photocathode (alkali metal) and and an anode (metallic plate with positive potential.

Features:

(i) The photocurrent increases with the increasing intensity I of the incident radiation if the frequency is kept constant.

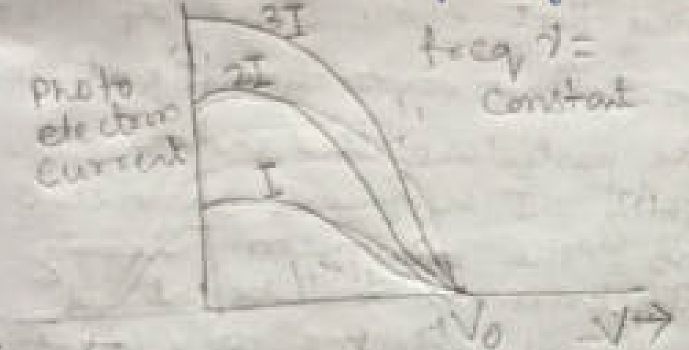
(ii) There is no time lag between illumination of metal surface and the emission of electrons. (Instataneous process)

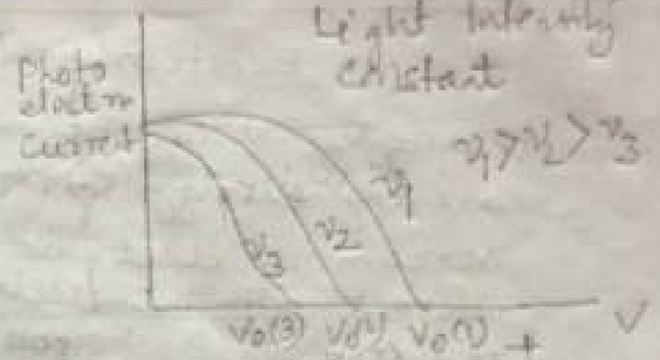
(iii) If the frequency of incident radiation is greater than the threshold frequency ν_0 (Certain minimum frequency), only than the emission of electron takes place.

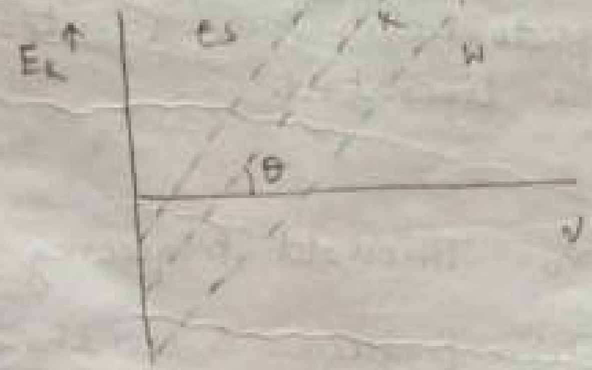
(iv) The maximum kinetic energy of photoelectrons is independent of intensity I, of incident light. stopping potential is the same for light of three different frequencies having intensities having same frequency.

(v) The maximum kinetic energy of the photoelectrons depends on the frequency of the incident radiation. We observe that for different frequencies stopping potentials are also different but the saturation current remains the same.

(vi) There is a linear relation between maximum K.E and frequency







Failure of classical Physics:

(i) If the radiation has sufficient intensity and is incident for a long time, the electron should ~~should~~ be able to receive the required energy and be emitted from the cathode. This should be independent of the ν and hence there need not be any threshold freq ν_0.

(ii) Maximum K.E of photoelectrons should increase with intensity which is contrary to the observation.

(iii) The binding energy of photoelectrons is typically of the order of few eV. The average time required by the electrons from the e.m waves would be seval seconds. So the photo electrons should start after a few second after the incident radiation.

Einstein's explanation:

Einstein assumed that e.m. wave of frequency ν can be regarded as a stream of particles carrying energy $h\nu$. These particles called photons. The stream of electrons hitting the surface of the metal interact with the electrons. The minimum energy required by a bound electron to be liberated from the metal is called the photoelectric work function ϕ_0 of the metal. The minimum energy of photons must be equal to the photo electric work function if a photo electron to be liberated.

$$h\nu_0 = W_0$$

$$\nu_0 = \frac{W_0}{h} \qquad \nu_0 \text{ Threshold frequency}$$

If the incident radiation frequency is less than ν_0 then there will be no photoelectron.

$$\boxed{(K.E)_{max} = h\nu - W_0 = h\nu - h\nu_0}$$

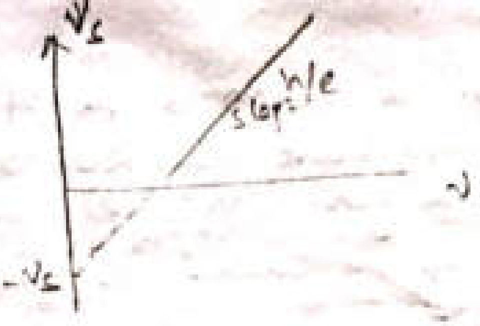
Stopping potential.

If the photo cathode is made ~~(-ve)~~ is made more negative no more photo electron can reach anode ~~the anode is made (-ve) the K.E of the photo electrons~~ and the photo current becomes zero. This extinction voltage ~~will be reduced by an amount eV. V is the retarding potential~~ corresponds to the maximum photo electron KE

$$(K.E)_{max} = eV_0$$

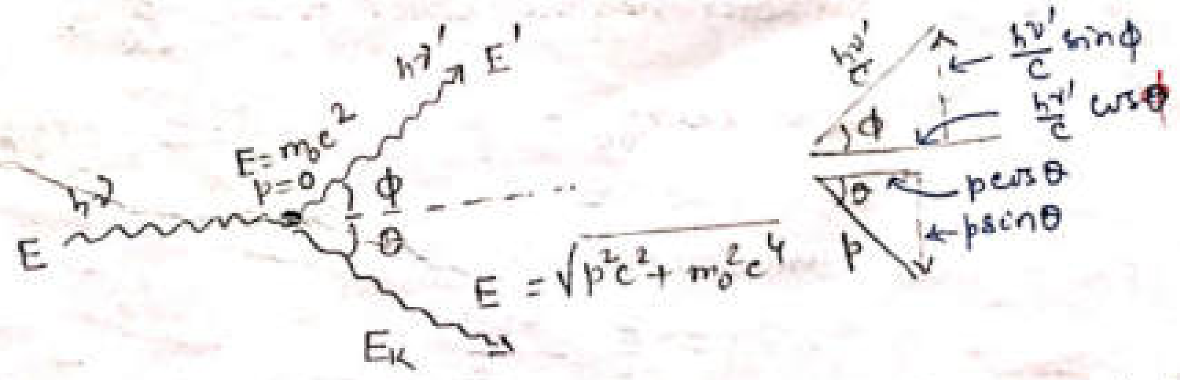
$$\boxed{eV_s = h\nu - h\nu_0 \Rightarrow V_0 = \frac{h}{e}\nu - \frac{h\nu_0}{e}}$$

$V_0 \rightarrow$ Stopping potential (V_s)



COMPTON EFFECT

Compton effect (or scattering) is another experiment that clarifies the particle nature of e.m. radiation. It describes the incoherent scattering of x-rays by an electron. Let us consider a collision: an x-ray ~~photo electric~~ photon strikes an electron (assumed to be initially at rest in the laboratory coordinate system) and is scattered away from its original direction of motion while the electron receives an impulse and begin to move.



According to classical e.m. theory the scattered x-ray photon should have the same frequency (or wavelength) as that of incident radiation but that is not the case. The scattered x-ray photon has an frequency less than that of the incident x-ray (or higher wavelength). The observations are

(i) For a given angle of scattering, the scattered x-rays have two components. A component with wavelength ($\lambda = c/\nu$) same as that of the incident x-ray is called un-modified or Thomson component. Another component called Compton component has wave-length greater than that of the incident x-ray.

(ii) The increase in wave-length in the compton component ($\Delta\lambda$) is called Compton shift. It depends on the angle of scattering.

(iii) The compton shift is independent of the incident wavelength of x-rays.

We can think of the photon as losing an amount of energy in the collision that is the same as the K.E gained by the electron. If the initial photon has the frequency ν associated with it, the scattered photon has the lower frequency ν'.

Loss in photon energy = gain in electron energy

$$h\nu - h\nu' = KE \quad \text{--- (1)}$$

For a mass less particle $E = pc$ --- (2)

$$\text{or } p = \frac{E}{c} = \frac{h\nu}{c} \quad \text{--- (3)}$$

Momentum is a vector quantity and must be conserved. The initial photon momentum is $h\nu/c$, the scattered photon momentum is $\frac{h\nu'}{c}$, and the initial and final electron momentum are respectively 0 and p.

In the original photon direction

initial momentum = final momentum

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta \quad \text{----} \quad (4)$$

and perpendicular to the direction

initial momentum = final momentum

$$0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta \quad \text{--} \quad (5)$$

ϕ is the angle between the direction of initial and scattered photons, and θ is that between the direction of initial photon and the recoil electron.

multiplying c in equation (4) and eqn (5)

$$h\nu = h\nu'\cos\phi + pc\cos\theta$$

$$pc\cos\theta = h\nu - h\nu'\cos\phi \quad \text{--------} \quad (6)$$

$$pc\sin\theta = h\nu'\sin\phi \quad \text{----} \quad (7)$$

squaring and adding

$$(pc\cos\theta)^2 + (pc\sin\theta)^2 = (h\nu)^2 + (h\nu'\cos\phi)^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2\sin\phi$$

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2 \quad \text{---} \quad (8)$$

Next we equate the expressions for energy

$$E = KE + m_0c^2 \quad \text{----} \quad (9)$$

$$E = \sqrt{m_0^2c^4 + p^2c^2} \quad \text{----} \quad (10)$$

The sum of energy of photon and electron before collision must be the same as the total energy after collision

$$\underbrace{E + m_0c^2}_{\text{Before collision}} = \underbrace{E' + \sqrt{m_0^2c^4 + p^2c^4}}_{\text{After collision.}} \quad \text{---} \; (11)$$

$$\left\{(E - E') + m_0c^2\right\}^2 = m_0^2c^4 + p^2c^4$$

$$(E - E')^2 + m_0^2c^4 + 2(E - E')\,m_0c^2 = m_0^2c^4 + p^2c^2$$

$$(E - E')^2 + 2(E - E')\,m_0c^2 = p^2c^2$$

$$\cancel{h\nu = h\nu' + m_0c^2 =}$$

$$(h\nu - h\nu')^2 + 2m_0c^2(h\nu - h\nu') = p^2c^2$$

$$(h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu') + 2m_0c^2(h\nu - h\nu') = p^2c^2 \quad \text{--} \; (12)$$

equating with equation (8)

$$\cancel{(h\nu)^2} + \cancel{(h\nu')^2} - 2(h\nu)(h\nu') + 2m_0c^2(h\nu - h\nu')$$

$$= \cancel{(h\nu)^2} - 2(h\nu)(h\nu')\cos\phi + \cancel{(h\nu')^2}$$

$$2m_0c^2(h\nu - h\nu') = 2(h\nu)(h\nu') - 2(h\nu)(h\nu')\cos\phi$$

$$m_0c^2\,\cancel{h}(\nu - \nu') = h^2\nu\nu'(1 - \cos\phi)$$

dividing both side by h^2c^2 $\qquad \nu\lambda = c$

$$m_0\cancel{c^2}\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right) = h\frac{\cancel{c}}{\lambda}\frac{\cancel{c}}{\lambda'}(1 - \cos\phi)$$

$$\frac{m_0c}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{(1 - \cos\phi)}{\lambda\lambda'}$$

$$\frac{m_0c}{h}\left(\frac{\lambda' - \lambda}{\lambda\lambda'}\right) = \frac{1 - \cos\phi}{\lambda\lambda'}$$

Compton effect.

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\phi)} \quad \text{---} \; (13)$$

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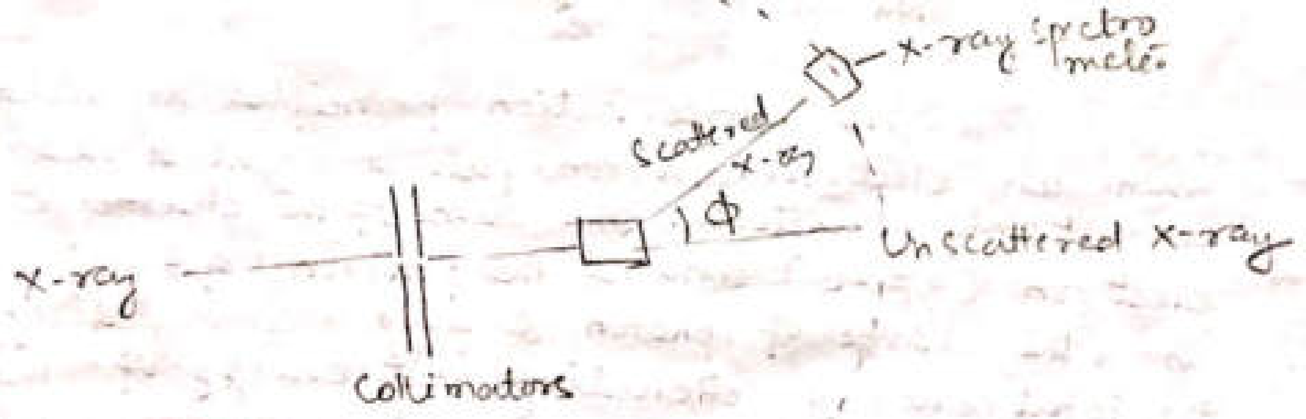
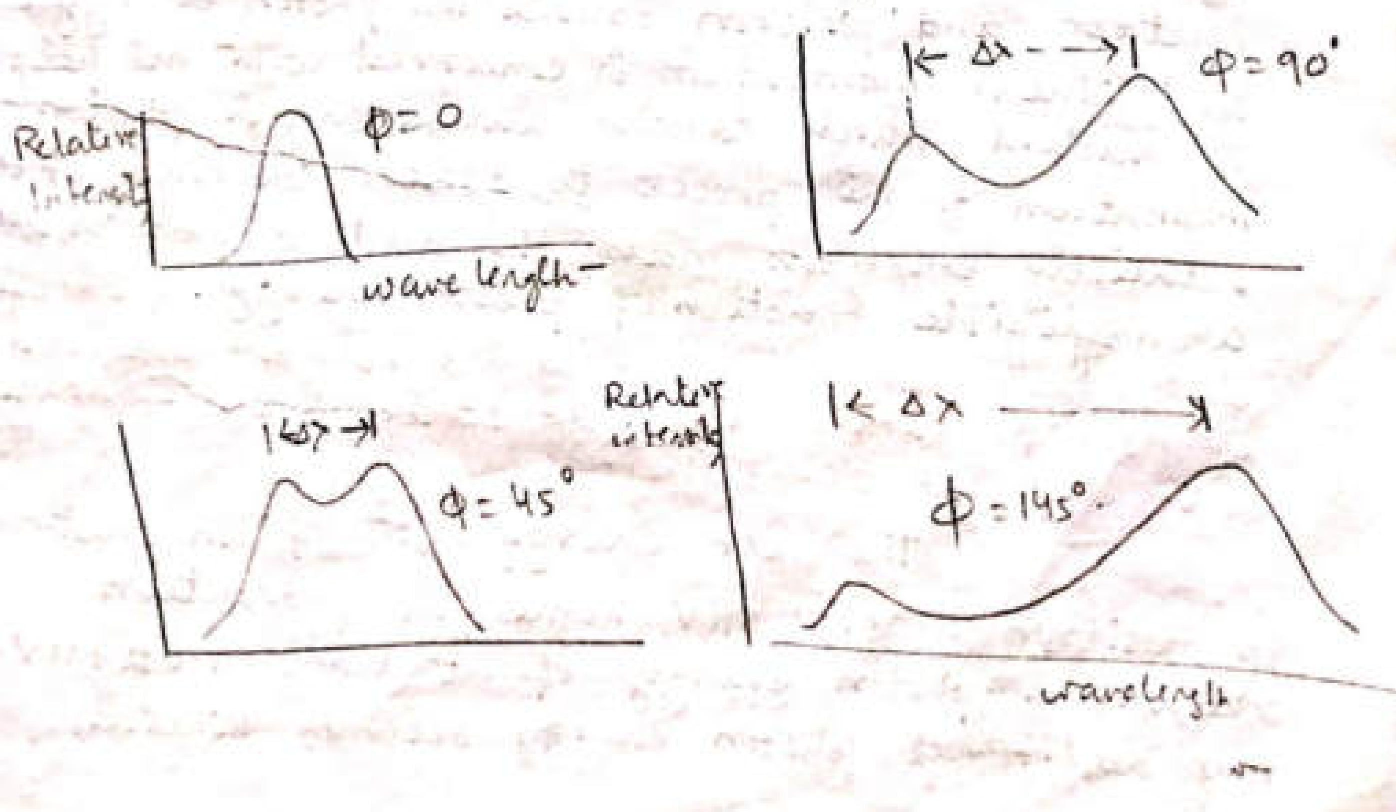
Equation (13) gives the change in wave length expected for a photon that is scattered through the angle ϕ by a particle of rest mass m_0. This change is independent of wave length λ of incident photon

Compton wave length $\boxed{\lambda_c = \frac{h}{m_0 c}}$

for electron $\lambda_c = 2.426 \times 10^{-12}$ m $= 2.426$ pm

$$\boxed{\lambda' - \lambda = \lambda_c (1 - \cos\phi)}$$





Heisenberg Uncertainity principle.

According to Heisenberg's uncertainity principle it is impossible to determine simultaneously the exact position and momentum (or velocity) of a small moving particle like electron.

$$\Delta x \cdot \Delta p \geq \hbar/2$$

Δx = the uncertainity in measurement in position of the particle

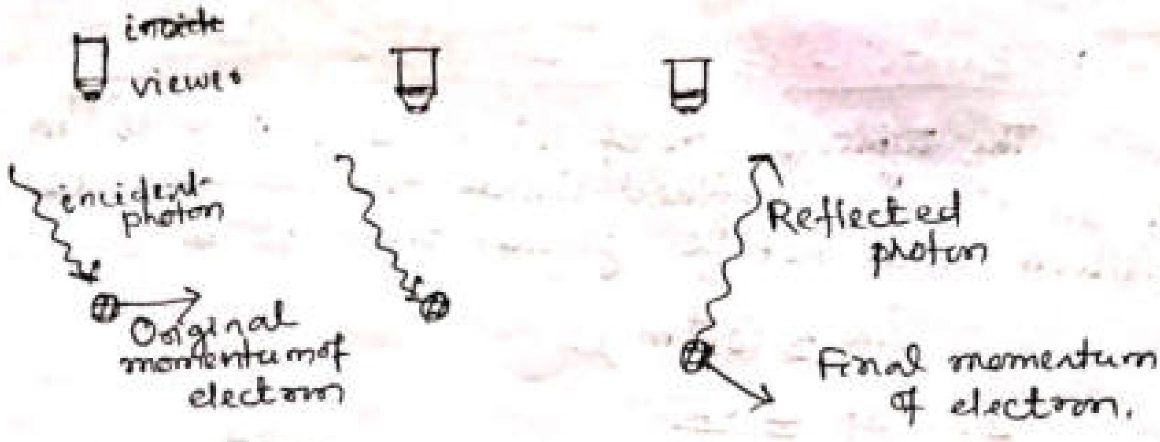
Δp = the uncertainity in measurement in momentum of particle.

$\Delta t \cdot \Delta E \geq \hbar/2$. uncertainity principle for energy and time

The uncertainity principle can be arrived at from the point of view of the particle properties of waves as well as from the point of view of the wave properties of particles.

Let us suppose we want to measure the position and momentum of an object at a certain moment. To do so, we must touch it with something that will carry the required information back to us. That is, we must poke it with a stick, shine light on it or do similar act. The measurement process itself thus requires that the object be interfered with some way. If we consider such interferers in details, we are led to some amount of uncertainity.

Suppose we look at an electron using light of wavelength λ. Thus each photon of this light has the momentum h/λ. When one of this photon bounces off the electron, the electron's original momentum will be changed. The exact



amount of change Δp can not be predicted, but it will be of the same order of magnitude as the photon momentum h/λ.

$$\Delta p \approx h/\lambda$$

The longer the wave length smaller the uncertainity in electron's momentum.

Because light is a wave phenomenon as well as particle phenomenon, we can not expect to determine the electron's location with perfectly accuracy

$$\Delta x \geq \lambda$$

The shorter the wavelength, the smaller the uncertainity in location. However, if we use light of short wavelength to increase the accuracy of the position measurement, there will be a corresponding decrease in the accuracy of the momentum measurement because the higher photon momentum will disturb the electron's motion to a greater extent.

$$\Delta x \cdot \Delta p \geq h$$

Applications:-

1. Non-Existence of electron in the Nucleus:

The typical size of the nucleus is $\simeq 10^{-14}$ m
If an electron confined inside the nucleus its position must not be greater than 10^{-14} m

$$\Delta x \simeq 10^{-14}\,m \times 2$$

$$\Delta x \cdot \Delta p \simeq \frac{h}{2\pi}$$

$$\therefore \Delta p = \frac{h}{2\Delta x\,\pi} \doteq \frac{6.625\times10^{-34}\,J\text{-}s}{2\times3.14\times2\times10^{-14}\,m}$$

$$\Delta p = 5.275\times10^{-21}\ kg\,m/sec.$$

This is the uncertainty in momentum of electron. This means momentum of electron would not be less than Δp, rather comparable to Δp

$$p = 5.275\times10^{-21}\ kg\,m/sec$$

$$K.E \Rightarrow T = \frac{1}{2}mv^2 = \frac{1}{2}\frac{mv^2\cdot m}{m} = \frac{p^2}{2m}$$

$$= \frac{(5.275\times10^{-21})^2}{2\times9.1\times10^{-31}}\ J.$$

$$= \frac{(5.275\times10^{-21})^2}{2\times9.1\times10^{-31}\times1.6\times10^{-19}}$$

$$\doteq 95.55\times10^{6}\ ev.$$

$$\simeq 96\ Mev.$$

From the above result, it is clear that, the electron inside the nucleus may exist only when it posses the energy of the order of 96 Mev. However practically the maximum possible K.E of an ~~electron~~ α-particle emitted by a radioactive nucleus is about 4 Mev. Hence electron can not reside inside the nucleus.

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2. Energy of a particle in a box of Infinite Potential well

Let us consider a particle having mass 'm' in infinite potential well of width L. The maximum uncertainity in the position of the particle may be

$$(\Delta x)_{max} = L$$

$$\Delta x \cdot \Delta p \geq \hbar$$

or $$\Delta p = \frac{\hbar}{\Delta x} = \frac{\hbar}{L}$$

$$\boxed{T = \frac{p^2}{2m} = \frac{\hbar^2}{2mL^2}}$$

3. Ground state energy of Linear Harmonic Oscillator:

The total energy E of a linear Harmonic oscillator

$$E = K \cdot E + P \cdot E$$

$$= \frac{p^2}{2m} + \frac{1}{2} K x^2 \qquad [K = m\omega^2]$$

Let a particle of mass m executes a S.H.M along x-axis the maximum uncertainity in the determination of its position can be taken as Δx.

$$\Delta \cdot p \geq \frac{\hbar}{2\Delta x} \qquad \left[\because \Delta p \cdot \Delta x \geq \frac{\hbar}{2}\right]$$

$$E = \frac{(\Delta p)^2}{2m} + \frac{1}{2} K (\Delta x)^2$$

$$= \frac{1}{4m}\left[\frac{\hbar^2}{\Delta x^2}\right] +$$

$$E = \frac{\hbar^2}{8m(\Delta x)^2} + \frac{1}{2} m\omega^2 (\Delta x)^2$$

In ground state $\Delta x \simeq x$ & $\Delta p \geq p$

$$E = \frac{\hbar^2}{8mx^2} + \frac{1}{2} m\omega^2 x^2 \quad \text{--- (1)}$$

For minimum value of energy $\left.\frac{\partial E}{\partial x}\right|_{x=0} = 0$

$$\frac{-\hbar^2 \cdot 2}{4\,8 m x^3} + m\omega^2 x = 0$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$x = \left(\frac{\hbar^2}{m\omega^2}\right)^{\frac{1}{4}}$$

$$x = \left(\frac{\hbar}{2m\omega}\right)^{1/2}$$

$$\Rightarrow \quad \frac{-\hbar^2}{4 m x^3} + m\omega^2 x = 0 \qquad x^2 = \frac{1}{2}\frac{\hbar}{m\omega}$$

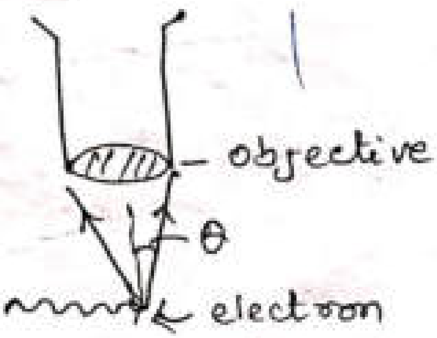
Putting value of x^2 in eq^n (1)

$$E = \frac{\hbar^2}{8m \times \frac{1}{2}\frac{\hbar}{m\omega}} + \frac{1}{2} m\omega^2 \frac{1}{2}\frac{\hbar}{m\omega}$$

$$= \frac{1}{4}\hbar\omega + \frac{1}{4}\hbar\omega = \frac{1}{2}\hbar\omega$$

Illustration of Uncertainity Principle.

(a) Heisenberg's γ-ray microscope:



Imagine a hypothetical gamma ray microscope with high resolving power. If the electrons is to be detected by the microscope, the incident gamma ray photon should be scattered by the electron and enter into objective of microscope with in the cone of illumination. The resolving power

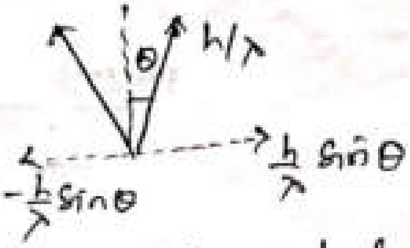
$$R.P = \frac{1.22\lambda}{2\sin\theta}$$

Let the electron be initially at rest, and the γ-ray photon be incident on it along X-direction

The minimum uncertainity in the position of the electron is decided by the R.P

$$\Delta x = R.P = \frac{1.22\lambda}{2\sin\theta}$$

Since the scattered gamma ray photon can enter the objective of the microscope anywhere the cone of illumination with linear momentum h/λ, the x-component of its linear momentum can have value $-\frac{h}{\lambda}\sin\theta$ and $\frac{h}{\lambda}\sin\theta$

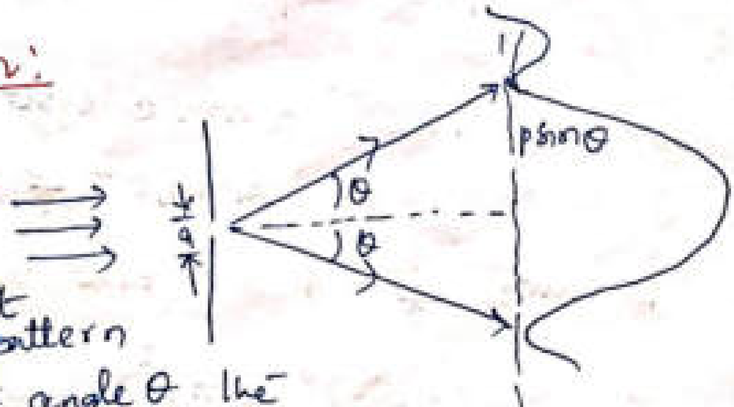


so $\Delta p_x = \frac{2h}{\lambda}\sin\theta$

$$\Delta x \cdot \Delta p_x = \frac{1.22\lambda}{2\sin\theta} \times \frac{2h\sin\theta}{\lambda} = 1.22h$$

or $\Delta x \cdot \Delta p_x \approx h$

(b) Electron diffraction:



$\Delta y = a$

If the position of first minima of diffraction pattern on a screen occurs at angle θ, the diffraction condition gives

$$a\sin\theta = \lambda$$

$$\Delta p_y = 2p\sin\theta$$

$$\Delta y \cdot \Delta p_y = a(2p\sin\theta) = 2p(a\sin\theta) = 2p\cdot\lambda$$

$$= 2p\cdot\frac{h}{p} = 2h$$

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2. why Compton effect is not observable with visible light ?

As we have discussed compton effect occurs significantly with x-rays which are very short wave length radiations. If we use visible light $\lambda \sim (4000 Å \sim 7000 Å)$ in place of x-rays and calculate compton shift $(\Delta\lambda = \lambda' - \lambda)$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\phi)$$

$$= 0.0242 \times 10^{-10} m\,(1-\cos\phi)$$

For maximum shift $\phi = 180°$

$$(\Delta\lambda)_{max} = 0.0484 \; Å$$

The percentage of Compton shift for $\lambda = 4000 Å$

$$\frac{(\Delta\lambda)_{max}}{\lambda} \times 100 \simeq 0.001\%$$

Similarly the percentage Compton shift for large wavelength of visible light, i.e. for $\lambda = 7000 Å$ would be 0.0007%. These values are not significant. Thus x-rays are appropriate for realizing Compton effect.

3. Calculate the work function, stopping potential and maximum velocity of photo electrons for a light of wavelength 4350 Å when it incidents on a sodium surface. Consider the threshold wavelength of photo-electrons to be 5420 Å.

$$\lambda_0 = 5.42 \times 10^{-7} m \qquad \lambda = 4.35 \times 10^{-7} m$$

$$\phi_0 = \frac{hc}{\lambda_0} = h\nu_0$$

$$\frac{1}{2} m v_{max}^2 = h\nu - h\nu_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$eV = h\nu - h\nu_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$\phi_0 = \frac{hc}{\lambda_0} = \frac{6.62\times10^{-34} \times 3\times10^{8}}{5.42\times10^{-7}} = 3.664\times10^{-19}\,J$$

$$\frac{1}{2}mv_{max}^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$= \frac{2hc}{m}\left(\frac{\lambda_0 - \lambda}{\lambda_0\lambda}\right)$$

$$v_{max}^2 = \frac{2\times6.62\times10^{-34} \times 3\times10^{8}\,(5.42-4.35)\times10^{-7}}{9.1\times10^{-31} \times 5.842 \times 4.35\times10^{-14}}$$

$$= 0.1981\times10^{11}\ m^2/sec^2$$

$$v_{max} = 4.45\times10^{5}\ m/sec$$

$$eV_s = \frac{1}{2}mv_{max}^2$$

$$V = \frac{m v_{max}^2}{2e} = \frac{9.1\times10^{-31}\times(4.45\times10^{5})^2}{2\times1.6\times10^{-19}}$$

$$= 0.56\ \text{volt}$$

4. Calculate the wavelength of incident x-ray photon which produces recoil electron of energy 4.0 KeV in Compton effect. The electron recoils in the direction of incident photon and photon is scattered at an angle of 180°

$\phi = 180°$ and energy of recoiled electron $= 4000\,eV$

Let λ be the wavelength of incident photon & λ' be the scattered photon, then from the conservation of energy

$$\frac{hc}{\lambda} - \frac{hc}{\lambda'} = \text{K.E of recoiled electron}$$

$$= \frac{1}{2}mv^2 = 4\times10^{3}\times1.6\times10^{-19}\,J$$

$$\frac{hc}{\lambda} - \frac{hc}{\lambda'} = 6.4\times10^{-16}\,J \qquad \text{——— (1)}$$

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According to the principle of conservation of linear momentum in the direction of incident photon

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\phi + mv\cos\theta$$

$$= \frac{h}{\lambda'}\cos 180^\circ + mv\cos 0^\circ$$

$$= -\frac{h}{\lambda'} + mv$$

$$\frac{h}{\lambda} + \frac{h}{\lambda'} = mv$$

$$\frac{1}{2}mv^2 = 6.4\times10^{-16}\ J$$

$$\frac{(mv)^2}{m} = 2\times 6.4\times10^{-16}\ J$$

$$(mv)^2 = 2\times 9.1\times10^{-31}\times 6.4\times10^{-16}$$

$$mv = 34.13\times10^{-24}\ kgm\ sec^{-1}$$

$$\frac{h}{\lambda} + \frac{h}{\lambda'} = 34.13\times10^{-24}$$

$$\frac{hc}{\lambda} + \frac{hc}{\lambda'} = 102.4\times10^{-16} \quad \text{———} \quad (2)$$

Adding (1) & (2)

$$\frac{2hc}{\lambda} = (102.4 + 6.4)\times10^{-16} = 108.79\times10^{-16}$$

$$\lambda = \frac{2hc}{108.79\times10^{-16}} = 0.365\times10^{-10}\ m$$

$$\lambda = 0.365\ \mathring{A}$$

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Example. 5 X-rays with $\lambda = 1\,\mathring{A}$ are scattered from a carbon block. The scattered radiation is viewed at 90° to the incident beam.

(i) What is the Compton shift $\Delta\lambda$?

(ii) What K.E is imparted to the recoil electron?

Given $\lambda = 1\times10^{-10}$ m

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\phi)$$

$\phi = 90^\circ$

$$\Delta\lambda = \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos 90^\circ)$$

$$= 2.425\times10^{-12}\ m$$

Let λ be the wavelength of incident x-ray photon and λ' be the scattered photon, then according to the law of conservation of energy

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + E_K = \frac{hc}{\lambda+\Delta\lambda} + E_K \quad \left[\because \lambda'-\lambda = \Delta\lambda\right]$$

$$E_K = \frac{hc}{\lambda} - \frac{hc}{\lambda+\Delta\lambda}$$

$$= \frac{hc\,\Delta\lambda}{\lambda(\lambda+\Delta\lambda)}$$

$$= \frac{6.62\times10^{-34}\times3\times10^{8}\times2.425\times10^{-12}}{1\times10^{-10}\times(1+0.02425)\times10^{-10}}$$

$$= 47.02\times10^{-18}\ J$$

$$= 294\ eV$$

Example · 6 A particle of charge q and mass m is accelerated through a potential difference of V. Find its deBroglie's wavelength. Calculate the wavelength (λ) if the particle is an electron and V = 50 volts.

$$\lambda = \frac{h}{mv}$$

$$E_k = \frac{1}{2}mv^2 = qV \quad \text{or} \quad m^2v^2 = 2mqV$$

$$mv = \sqrt{2mqV}$$

$$\text{so} \quad \lambda = \frac{h}{\sqrt{2mqV}}$$

given $q = 1.6 \times 10^{-19} C$ & $V = 50$ volts

$$\lambda = \frac{6.62 \times 10^{-34} J\text{-}s}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 50}}$$

$$= 1.74 \mathring{A}$$

Example · 7

Calculate the wavelength of thermal neutrons at 27°C, given mass of neutron $= 1.67 \times 10^{-27}$ kg, Boltzmann constant $= 1.37 \times 10^{-23} JK^{-1}$

$$T = 27^\circ C + 273 = 300 K, \quad m = 1.67 \times 10^{-27} kg$$

$$\lambda = \frac{h}{mv}$$

$$E_k = \frac{1}{2}mv^2 = \frac{3}{2}KT \qquad (mv)^2 = 3mKT$$

$$mv = \sqrt{3mKT}$$

$$\lambda = 1.452 A^\circ$$

Example 8 : A proton and a deuteron have the same kinetic energy. Which has a longer wavelength ?

m_p = mass of proton $m_d = 2m_p$ & v_p & v_d are the velocities of proton and deuteron

K.E of proton $E_p = \frac{1}{2} m_p v_p^2$

$$E_d = \frac{1}{2} m_d v_d^2 = \frac{1}{2}(2m_p) v_d^2$$

$$E_d = m_p v_d^2$$

But $E_p = E_d$

$$m_p v_d^2 = \frac{1}{2} m_p v_p^2$$

$$v_d = \frac{v_p}{\sqrt{2}}$$

de Broglie wavelength corresponding to moving proton and deuteron are

$$\lambda_p = \frac{h}{m_p v_p} \qquad \lambda_d = \frac{h}{m_d v_d} = \frac{h \times \sqrt{2}}{2 m_p v_p}$$

$$= \frac{h}{\sqrt{2} m_p v_p}$$

$$\frac{\lambda_d}{\lambda_p} = \frac{h}{\sqrt{2} m_p v_p} \times \frac{m_p v_p}{h} = \frac{1}{\sqrt{2}}$$

$$\lambda_p = \sqrt{2}\, \lambda_d$$

ie proton has a longer wavelength.

Example - 9 Show that the de Broglie wavelength of a relativistic particle at rest and mass m and K.E E_K is given by

$$\lambda = \frac{hc}{E_K\left(1+\frac{2mc^2}{E_K}\right)^{1/2}}$$

Show that in the non-relativistic limit $\lambda = \frac{h}{mv}$

For a relativistic particle

$$E_K = E - m_0c^2 = \sqrt{p^2c^2+m^2c^4} - mc^2$$

$$(E_K + mc^2)^2 = p^2c^2 + m^2c^4$$

$$\Rightarrow p^2c^2 = E_K^2 + m^2c^4 + 2E_K mc^2 - m^2c^4$$

$$\Rightarrow p^2c^2 = E_K^2\left(1+\frac{2mc^2}{E_K}\right)$$

$$\Rightarrow p = \frac{E_K}{c}\left(1+\frac{2mc^2}{E_K}\right)^{1/2}$$

$$\Rightarrow \lambda = \frac{h}{p} = \frac{hc}{E_K\left(1+\frac{2mc^2}{E_K}\right)^{1/2}}$$

In the non-relativistic limit $E_K = \frac{1}{2}mv^2$

and $1 \ll \frac{2mc^2}{E_K}$ ~~and $1 \ll \frac{2mc^2}{E_K}$~~

$$\text{so, } \lambda = \frac{hc}{\frac{1}{2}mv^2\left(\frac{2mc^2}{mv^2/2}\right)^{1/2}} = \frac{h}{mv}$$