Air Refrigeration cycles

- → Refrigerant → air
- → Refrigeration capacity of air is less (because it cannot change it's phase).
- → Now obsolete because of their low cop.

Units of Refrigeration

→ Expressed as 'Tonnes of Refrigeration'.

1 Tonne of Refrigeration (1 TR)

→ Defined as the amount of Refrigeration effect produced by uniform melting of 1 tonne (1000 kg) of ice from and at 0°c in 24 hrs.

$$1\ TR = 1000 \times 335\ kJ \text{ in } 24\ hrs$$

$$= \frac{1000\times 335}{24\times 60} = 232.6\ kJ/min \quad or \quad \frac{232.6}{60} = 3.87\ kJ/s$$

But for numerical purposes (in actual Practice)

$$\underline{1\ TR = 210\ kJ/min \quad or \quad 3.5\ kJ/s}$$

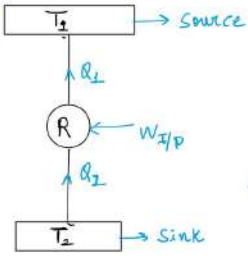
COP of a Refrigerator

$$\text{Theoritical COP} = \frac{\text{Amount of heat extracted in the Refrigerator}}{\text{Work Input (work done on the Refrigerant)}}$$

$$= \frac{Q \longrightarrow kJ/kg}{W_{I/p} \rightarrow kJ/kg} \quad or \quad \frac{\dot{m}Q}{\dot{m}W_{I/p}} = \frac{\text{Ref. capacity} \rightarrow kW}{\text{Power} \rightarrow kW}$$

$$\text{Relative COP} = \frac{\text{Actual COP}}{\text{Theoritical COP}}$$

Q)



<u>given</u> :- $W_{I/p} = 80\ kJ/kg$

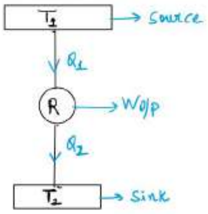
Refrigeration effect (Q_2) = 160 kJ/kg.

<u>To Find</u> :- COP of Refrigerator

<u>solⁿ</u>

$$(COP)_R = \frac{\text{Refrigeration effect}}{\text{Work Input}} = \frac{Q_2}{W_{I/p}} = \frac{160}{80} = 2$$

Difference b/w Heat Engine, Refrigerator & Heat pump.



Heat Engine

$$\eta_E = \frac{\text{Work o/p}}{\text{Heat supplied}} = \frac{W_{o/p}}{Q_1}$$

$$Q_1 = W_{o/p} + Q_2$$

$$\Rightarrow W_{o/p} = Q_1 - Q_2$$

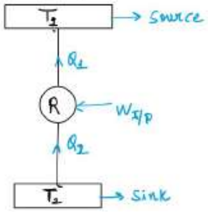
$$\Rightarrow \eta_E = \frac{Q_1 - Q_2}{Q_1}$$

$$\Rightarrow \boxed{\eta_E = 1 - \frac{Q_2}{Q_1}}$$

For a reversible heat engine

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\Rightarrow \boxed{(\eta_E)_{Rev} = 1 - \frac{T_2}{T_1}}$$



Refrigerator

$$(COP)_R = \frac{\text{Heat absorbed from Lower temp Reservoir(sink)}}{\text{Work Input}} = \frac{Q_2}{W_{I/p}}$$

$$Q_1 = Q_2 + W_{I/p}$$

$$\Rightarrow W_{I/p} = Q_1 - Q_2$$

$$\Rightarrow \boxed{(COP)_R = \frac{Q_2}{Q_1 - Q_2}}$$

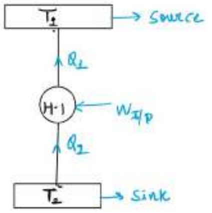
$$= \frac{Q_2/Q_1}{1 - Q_2/Q_1}$$

For a Reversible cycle

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\Rightarrow (COP)_{R,rev} = \frac{T_2/T_1}{1 - T_2/T_1}$$

$$\Rightarrow \boxed{(COP)_{R,rev} = \frac{T_2}{T_1 - T_2}}$$



Heat pump

$$(COP)_{H.P} = \frac{\text{Heat transferred to source}}{\text{Work Input}} = \frac{Q_1}{W_{I/p}}$$

$$Q_1 = Q_2 + W_{I/p}$$

$$\Rightarrow W_{I/p} = Q_1 - Q_2$$

$$\Rightarrow \boxed{(COP)_R = \frac{Q_1}{Q_1 - Q_2}}$$

$$= \frac{1}{1 - Q_2/Q_1}$$

For a Reversible cycle

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

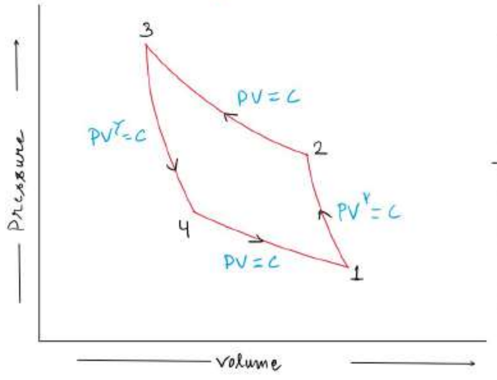
$$\Rightarrow (COP)_{H.P,rev} = \frac{1}{1 - T_2/T_1}$$

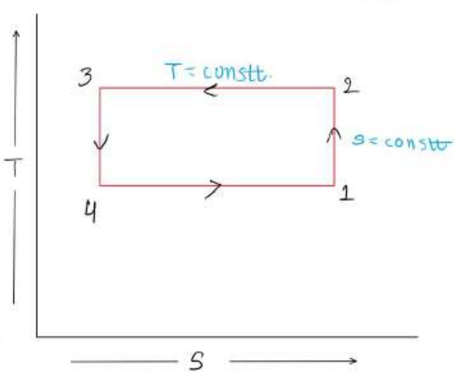
$$\Rightarrow \boxed{(COP)_{H.P,rev} = \frac{T_1}{T_1 - T_2}}$$

Air Refrigerator working on Reversed carnot cycle

Reversed carnot cycle

→ maximum Possible COP.
→ Impossible cycle but Just used as a standard For comparison.





Process 1-2 → Isentropic compression of air
Process 2-3 → Isothermal compression of air $[Q_R = T_3(s_2 - s_3) = T_2(s_2 - s_3)]$
Process 3-4 → Isentropic expansion of air
Process 4-1 → Isothermal expansion of air $[Q_A = T_4(s_1 - s_4) = T_4(s_2 - s_3) = T_1(s_2 - s_3)]$

$$COP_R = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{Q_A}{Q_R - Q_A} = \frac{T_1(s_2 - s_3)}{T_2(s_2 - s_3) - T_1(s_2 - s_3)} = \frac{T_1\cancel{(s_2 - s_3)}}{(T_2 - T_1)\cancel{(s_2 - s_3)}}$$

$$\Rightarrow \boxed{(COP)_R = \frac{T_1}{T_2 - T_1}}$$

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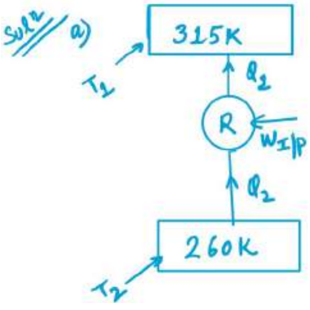
ex For a machine working on Carnot cycle operating between 315K & 250K Determine the COP when it is operated as
a) Refrigerator
b) Heat pump
c) Heat Engine.

Solⁿ a)



For Reversed carnot cycle

$$(COP)_R = \frac{Q_2}{W_{I/P}}$$

By Energy balance

$$Q_2 + W_{I/P} = Q_1$$

$$\Rightarrow W_{I/P} = Q_1 - Q_2$$

$$\Rightarrow (COP)_R = \frac{Q_2}{Q_1 - Q_2}$$

$$\Rightarrow (cop)_R = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

$$(cop)_R = \frac{1}{\frac{T_1}{T_2} - 1}$$

$$= \frac{T_2}{T_1 - T_2} = \frac{250}{315 - 250} = \frac{250}{65}$$

$$\Rightarrow (cop)_R = 3.846$$

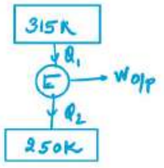
b) Heat pump

$$(COP)_{H.P} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2} = \frac{315}{315 - 250} = \frac{315}{65} = 4.846$$

alternately

$$(COP)_{HP} = 1 + (COP)_R = 1 + 3.846 = 4.846$$

c) Heat Engine



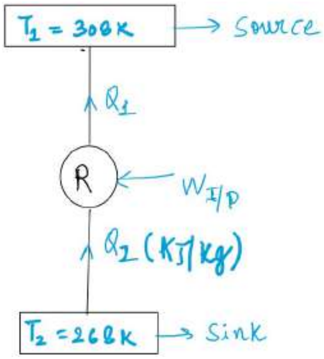
$$\eta_E = \frac{W_{O/P}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\Rightarrow \eta_E = 1 - \frac{T_2}{T_1} = 1 - \frac{250}{315} = \frac{315 - 250}{315}$$

$$\Rightarrow \boxed{\eta_E = 0.206}$$

Q) A cold storage is to be maintained at -5°C while the surroundings are at 35°C. The heat Leakage From the surroundings into the cold storage is 29KW. The actual COP of refrigeration plant is 1/3rd of an ideal plant working between same temperature limits. Find the power required to drive the plant

Soln



given Heat Leakage From surroundings = 29kW

⇒ To maintain constt temperature 29 KW of heat needs to be removed From cold storage maintained at 268K.

$\Rightarrow \dot{m} Q_2 = 29\text{ KW}$ $\quad [\text{Kg/s} \times \text{KJ/kg} = \text{KJ/s} = \text{KW}]$

& $(COP)_{actual} = \frac{1}{3} (COP)_{ideal}$

$$(COP)_{ideal} = \frac{T_2}{T_1 - T_2} = \frac{268}{308-268} = 6.7$$

$$\Rightarrow (COP)_{actual} = \frac{1}{3} \times 6.7 = 2.233$$

$$\& \quad COP = \frac{\text{Refrigeration effect (R.E)}}{W_{I/P}} = \frac{\dot{m} \times R.E}{\dot{m} \times W_{I/P}} = \frac{\text{Refrigeration capacity (R.C)}}{\text{Power Input (P)}}$$

$$(COP)_{actual} = \frac{R.C}{P} = \frac{29}{P} = 2.233$$

$$\Rightarrow P = 29/2.233 = 12.987\text{ KW}$$

$$\Rightarrow \boxed{P_{I/P} = 12.987\text{ KW}}$$

Q) given capacity of a Refrigerator = 200 TR

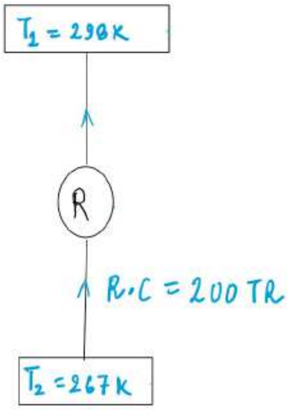
Lower & upper temperature limits = -6°C & 25°C

Latent heat of Ice is 335 kJ/kg.

To Find

a) mass of Ice produced/day from water at 25°C.

b) Power required to drive the unit.



a) $R.C = 200\text{ TR}$

$= (200 \times 3.5)\text{ KJ/s}$

$= 700\text{ KJ/s} = 700\text{ KW}.$

Heat removed From 1 kg water to Form ice.

= Sensible heat (From 25°C – 0°C) + Latent heat (at 0°C)

$= mC\Delta T + m.L$

$= 1 \times 4.186 \times 25 + 1 \times 335 = 439.7$ KJ/kg

$\Rightarrow$ mass of ice produced/s $= \frac{700}{439.7}\left(\frac{KJ/s}{KJ/kg}\right) = 1.591$ Kg/s.

mass of ice produced per day $= 1.591 \times 60 \times 60 \times 24 = 137548$ Kg $= 137.55$ tonnes.

b) $(COP)_{Rev.\,carnot} = \frac{T_2}{T_1 - T_2} = \frac{267}{298-267} = 8.613$

$COP = \frac{R.C}{P_{I/p}} = \frac{700\text{ KW}}{P} = 8.613$

$\Rightarrow$ $\boxed{\text{Power Input} = 81.27\text{ KW}}$

Q) A cold storage plant is required to store 20 tonnes of Fish. The fish is supplied at a temperature of 30°C. The specific heat of a Fish above freezing point is 2.93 KJ/Kg-K. The specific heat of Fish below Freezing Point is 1.26 KJ/Kg-K. The Fish is stored in cold storage which is maintained at −8°C. The Freezing point of Fish is −4°C. The Latent heat of Fish is 235 KJ/Kg. If the plant requires 75 KW to drive it. Find:

a) The capacity of the plant.

b) Time taken to achieve cooling.

Assume $(COP)_{actual} = 0.3 \times (COP)_{carnot}$.

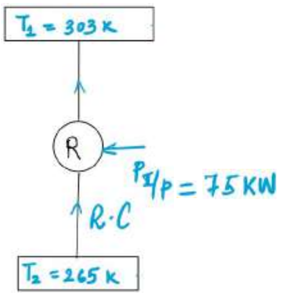
Soln given :- $m = 20 \times 1000 = 20{,}000$ Kg.

Freezing pt. of Fish $= -4°C$

C → Below Freezing Pt. → 1.26 KJ/Kg-K.

C → above Freezing Pt. → 2.93 KJ/Kg-K

a)



carnot $COP = \frac{T_2}{T_1 - T_2} = \frac{265}{303-265} = \frac{265}{38} = 6.97$

Actual $COP = 0.3 \times 6.97 = 2.091$

$COP = \frac{R.C}{P} = 2.091 = \frac{R.C}{75}$ $\Rightarrow$ $\boxed{R.C = 156.8\text{ KW}}$

$R.C = \frac{156.8}{3.5} = 44.8$ TR $\boxed{R.C = 44.8\text{ TR}}$

b) Time taken to achieve cooling

$$\underset{30°C}{\text{Fish at}} \xrightarrow{\text{Sensible heat}} \underset{-4°C}{\text{Fish at}} \xrightarrow{\text{Latent heat}} \underset{-4°C}{\text{Fish at}} \xrightarrow{\text{Sensible heat}} \underset{-8°C}{\text{Fish at}}$$

$[m\,c\,\Delta T]$ $[20000 \times 2.93 \times 34]$ $Q_1 = 1.992 \times 10^6$ KJ

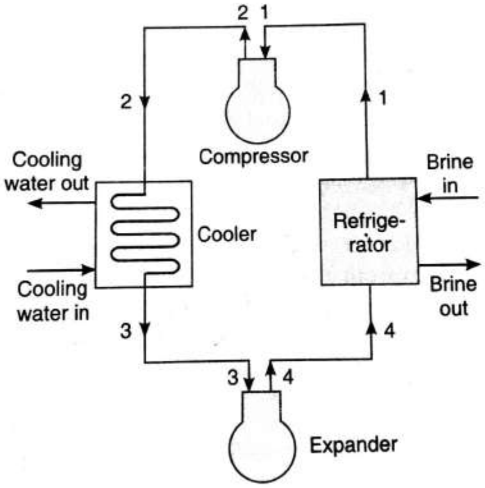
$[m(\Delta h)]$ $[20000 \times 235]$ $Q_2 = 4.7 \times 10^6$ KJ

$[m\,c\,\Delta T]$ $[20000 \times 1.26 \times 4]$ $Q_3 = 0.101 \times 10^6$ KJ

Total heat Removed From Fish $= Q_1 + Q_2 + Q_3 = 6.793 \times 10^6$ KJ.

Time taken To achieve cooling $= \frac{R\cdot E}{R\cdot C} = \frac{6\cdot 793\times 10^{6}}{156\cdot 8} = \frac{[illegible]}{3600} = 12\cdot 03h$

Bell-coleman cycle (Reversed Brayton cycle or Joule cycle)



Process 1-2 → Reversible, adiabatic compression of air in the Refrigerator

Process - 2-3 → Reversible, Isobaric heat rejection in the compressor.

Process → 3-4 → Reversible, adiabatic expansion in a turbine

Process → 4-1 → Reversible, adiabatic heat absorption in Refrigerator

Fig → closed cycle Bell-coleman Refrigerator

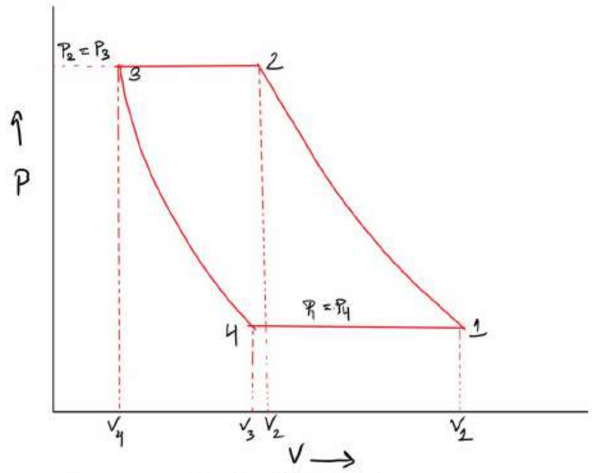


Fig → P-V diagram

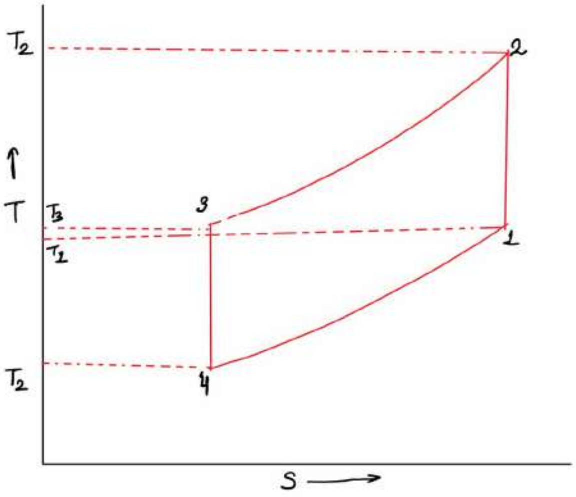


Fig → T-S diagram.

Derivation of expression For COP of a Reversed Brayton cycle.

Coefficient of Performance (COP) = $\dfrac{\text{Heat absorbed in Refrigerator}}{\text{Work Input}} = \dfrac{q_{abs}}{q_{rej} - q_{abs}}$

$$q_{abs} = q_{4-1} = h_1 - h_4 = C_p(T_1 - T_4)$$

& $$q_{rej} = q_{2-3} = h_2 - h_3 = C_p(T_2 - T_3)$$

$$\Rightarrow \quad COP = \frac{C_p(T_1 - T_4)}{C_p(T_2 - T_3) - C_p(T_1 - T_4)} = \frac{(T_1 - T_4)}{(T_2 - T_3) - (T_1 - T_4)}$$

$$= \frac{T_4\left(\frac{T_1}{T_4} - 1\right)}{T_3\left(\frac{T_2}{T_3} - 1\right) - T_4\left(\frac{T_1}{T_4} - 1\right)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma}$$

&

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\gamma-1/\gamma} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma}$$

as $P_2 = P_3$ & $P_4 = P_1$

as 1-2 & 3-4 are Isentropic Processes.

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \Rightarrow \quad \boxed{\frac{T_1}{T_4} = \frac{T_2}{T_3}}$$

the above expression thus reduces to.

$$COP = \frac{T_4}{T_3 - T_4} = \frac{1}{\frac{T_3}{T_4} - 1} = \frac{1}{\left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} - 1} = \frac{1}{(r_p)^{\gamma-1/\gamma} - 1}$$

$$\Rightarrow \quad \boxed{COP = \frac{1}{(r_p)^{\gamma-1/\gamma} - 1}} \qquad \text{where } \gamma = \frac{C_p}{C_v} = 1.4$$

Q) **Derive COP if compression & expansion process takes place according to the Law $PV^n = C$.**

Soln

Work done by the compressor during Process 1-2 per kg of air

$$W_c = \frac{n}{n-1}\left[P_2V_2 - P_1V_1\right] = \frac{n}{n-1}\left[RT_2 - RT_1\right]$$

Work done by the turbine during process 3-4 per kg of air

$$W_T = \frac{n}{n-1}\left[P_3V_3 - P_4V_4\right] = \frac{n}{n-1}\left[RT_3 - RT_4\right]$$

Net work supplied = $W_{net} = W_c - W_T$

$$\Rightarrow \quad W_{net} = \frac{nR}{n-1}\left[(T_2 - T_1) - (T_3 - T_4)\right]$$

$$COP = \frac{\text{heat absorbed } (q_{4-1})}{W_{net}} = \frac{C_p(T_1 - T_4)}{\frac{nR}{n-1}\left[(T_2 - T_1) - (T_3 - T_4)\right]}$$

$$R = C_v(\gamma - 1)$$

$$\Rightarrow COP = \frac{C_p(T_1-T_4)}{\frac{n}{n-1}\times C_v(\gamma-1)\left[(T_2-T_1)-(T_3-T_4)\right]}$$

$$= \frac{\gamma(T_1-T_4)}{\frac{n}{n-1}\times(\gamma-1)\times\left[T_2-T_1)-(T_3-T_4)\right]} = \frac{(T_1-T_4)}{\frac{n}{n-1}\times\frac{\gamma-1}{\gamma}\times\left[(T_2-T_1)-(T_3-T_4)\right]}$$

$$\Rightarrow \boxed{COP = \frac{(T_1-T_4)}{\frac{n}{n-1}\times\frac{\gamma-1}{\gamma}\times\left[(T_2-T_1)-(T_3-T_4)\right]}}$$

For isentropic processes → $n=\gamma$

$$\Rightarrow \boxed{COP = \frac{(T_1-T_4)}{(T_2-T_3)-(T_1-T_4)}}$$

Q) For a refrigeration plant working on Bell-coleman cycle.

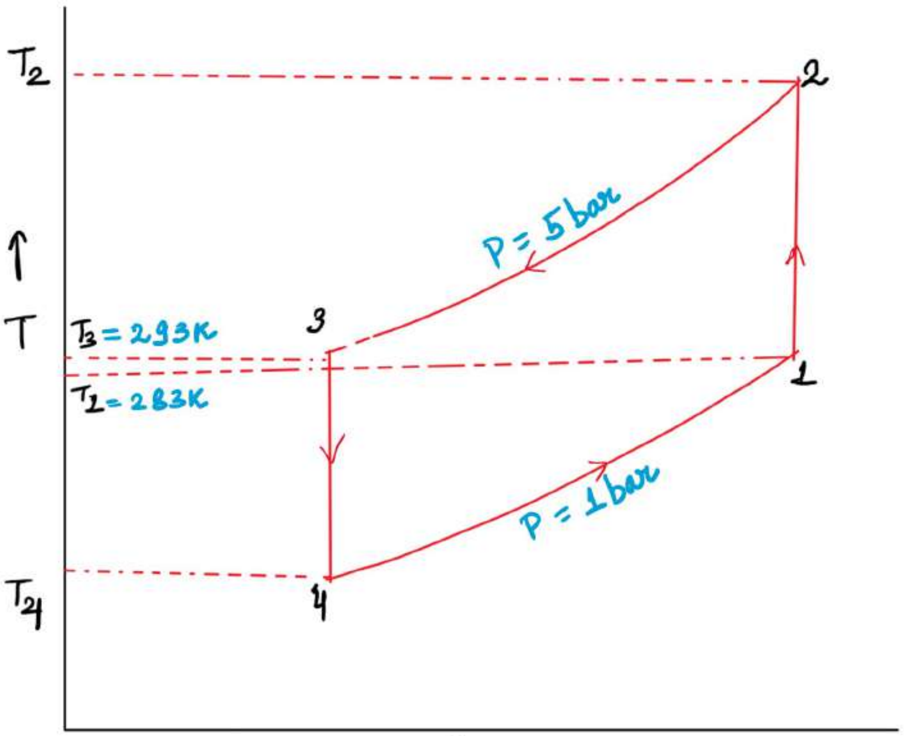


Fig → T-S diagram.

given :- $P_1 = P_4 = 1$ bar
$P_2 = P_3 = 5$ bar.

Initial Temperature $(T_1) = 10°C = 283K$

$T_3 = 20°C = 293K$

$C_p = 1.005\ KJ/kg.K$ & $C_v = 0.718\ K$.

To Find :- a) COP

b) Refrigeration effect (R.E)

Sol^n

a) $$COP = \frac{\text{heat absorbed}}{W_{I/P}} = \frac{h_1 - h_4}{q_{rej} - q_{abs}} = \frac{C_p(T_1 - T_4)}{C_p(T_2 - T_3) - C_p(T_1 - T_4)} = \frac{T_1 - T_4}{(T_2 - T_3) - (T_1 - T_4)}$$

For process 1-2

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} \Rightarrow T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} = 283 \times (5)^{0.4/1.4}$$

$$\Rightarrow \boxed{T_2 = 448.272K}$$

For Process 3-4

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\gamma-1/\gamma} \Rightarrow \frac{293}{T_4} = (5)^{0.4/1.4} \Rightarrow \boxed{T_4 = 185K}$$

$$COP = \frac{T_1 - T_4}{(T_2 - T_3) - (T_1 - T_4)} = \frac{283 - 185K}{(448.272 - 293) - (283 - 185)}$$

$$COP = \frac{98}{155.272 - 100} = 1.77$$

alternately

$$COP = \frac{1}{(r_p)^{\gamma-1/\gamma} - 1} = \frac{1}{(5)^{0.4/1.4} - 1} = \frac{1}{1.584 - 1} = 1.712$$

b) Net Refrigeration effect

$$R.E = h_1 - h_4 = C_p(T_1 - T_4) = 1.005 \times (283 - 185)$$

$$\boxed{R.E = 98.5\ KJ/kg}$$

eg In the above Problem determine COP if expansion & compression follows the law $PV^{1.3} = C$.

$$\boxed{COP = \frac{(T_1 - T_4)}{\frac{n}{n-1} \times \frac{\gamma-1}{\gamma} \times \left[(T_2 - T_3) - (T_1 - T_4)\right]}} = \frac{283 - 185}{\frac{1.3}{0.3} \times \frac{0.4}{1.4} \times \left[(448.272 - 293) - (283 - 185)\right]}$$

$$\Rightarrow \boxed{COP = 1.382}$$

Q) A refrigerating machine of 6 tonnes capacity working on Bell-Coleman cycle has an upper limit of pressure 5.2 bar. The pressure & temperature at the start of compression are 1 bar & 16°C respectively. The compressed air is cooled at constt pressure to a temperature of 41°C, enters the expansion cylinder. Assuming both expansion & compression processes to be isentropic with $\gamma = 1.4$. Calculate :

a) COP

b) Quantity of air circulated/min.

c) Piston displacement of compressor & expander.

d) Bore of compressor and expansion cylinders. The unit runs at 240 RPM and is double-acting. stroke Length = 200 mm.

e) Power required to drive the unit.

For air, take $\gamma = 1.4$ & $C_p = 1.003$ KJ/kg-K.

soln

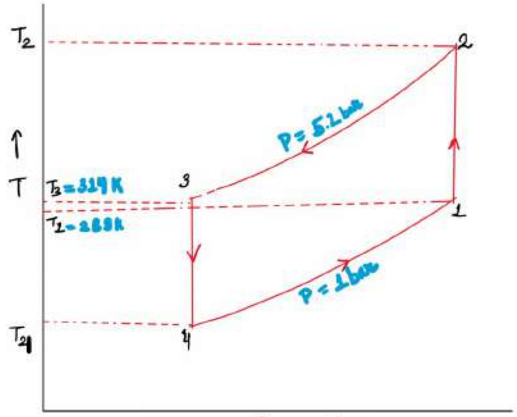


Fig → T-S diagram

given

Refrigeration capacity (R.C) = 6TR
$= 6 \times 3.5$ KJ/s
$= 21$ KW

$P_2 = P_3 = 5.2$ bar

$P_1 = P_4 = 1$ bar

$T_1 = 16°C = 289$ K

$T_3 = 41°C = 314$ K

$\gamma = 1.4$

a)

$$COP = \frac{1}{(r_p)^{\gamma-1/\gamma} - 1} = \frac{1}{(5.2)^{0.4/1.4} - 1} = 1.662$$

b) Quantity of air circulated/min.

$$\underset{(KJ/s)}{\text{Refrigeration capacity}} = \underset{(kg/s)}{\text{mass Flow rate}} \times \underset{(KJ/kg)}{\text{Refrigeration effect}}$$

$$R.C = \dot{m} \times R.E$$

$$(6 \times 3.5) = \dot{m} \times (h_1 - h_4)$$

$$\Rightarrow \dot{m} = \frac{6 \times 3.5}{C_p(\underset{\checkmark}{T_1} - \underset{?}{T_4})}$$

$T_1 = 289$ K

$T_4 = ?$

For process 3-4

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{0.4/1.4} \Rightarrow \frac{314}{T_4} = (5.2)^{0.4/1.4}$$

$$\Rightarrow \boxed{T_4 = 196 \text{ K}}$$

$$\dot{m} = \frac{6 \times 3.5}{C_p(T_1 - T_4)} = \frac{6 \times 3.5}{1.003 \times (289 - 196)} = 0.225 \text{ kg/s}$$

$$\Rightarrow \boxed{\dot{m} = 13.50\ kg/min}$$

c) Piston displacement of compressor & expander/min.

using Ideal gas equation

$$P_1 V_1 = m_a R_a T_1$$

$$V_1 = \frac{m_a R_a T_1}{P_1} = \frac{13.50 \times 287 \times 289}{1 \times 10^5} = 11.19\ m^3/min$$

Piston displacement of compressor $= \dot{m} v_1 = V_1 = 11.19\ m^3/min$

For Process 4-1 (Isobaric process)

$$P = C$$

$$PV = mRT \quad \Rightarrow \quad V \propto T$$

$$\Rightarrow \frac{V_1}{T_1} = \frac{V_4}{T_4} \quad \Rightarrow V_4 = \frac{11.19}{289} \times 196$$

$$\Rightarrow V_4 = 7.59\ m^3/min$$

Piston displacement of Turbine $= \dot{m} v_2 = V_2 = 7.59\ m^3/min$

d) Bore of compressor and expansion cylinders.

$$V_1 = \frac{\pi}{4} \times D^2 \times L \times N \times K$$

K=2] → For double acting compressor.

$$11.19 = \frac{\pi}{4} \times D^2 \times 0.2 \times 240 \times 2$$

$$11.19 = 75.4 D^2$$

$$\Rightarrow D = 0.385\,m \ - \boxed{D = 386\,mm} \rightarrow \text{For compressor cylinder}$$

Similarly for Expansion cylinder

$$V_4 = \frac{\pi}{4} \times D^2 \times L \times N \times K$$

$$7.59 = \frac{\pi}{4} \times D^2 \times 0.2 \times 240 \times 2$$

$$\Rightarrow \boxed{D = 318\ mm}$$

e) Power required to drive the unit.

$$COP = \frac{\text{heat absorbed}}{\text{Work Input}} = \frac{\dot{m}_a C_p (T_1 - T_4)}{\dot{m}_a W_{I/p}} = \frac{1.003 \times (289 - 196)}{P_{I/p}}$$

$$1.662 = \frac{20.987}{P_{I/p}}$$

$$\Rightarrow \boxed{P_{I/p} = 12.628\ KJ/s}$$

Q) A dense air refrigeration cycle operates between pressures of 4 bar
16 bar. The air temperature after heat rejection to the surroundings
is 37°C and the air temperature at the exit of refrigerator is
The Isentropic efficiences of turbine and compresser are 0.85 & 0.8 re
Determine :-

a) Determine Compresser & Turbine work per TR.
b) COP
c) Power/TR

Sol^n

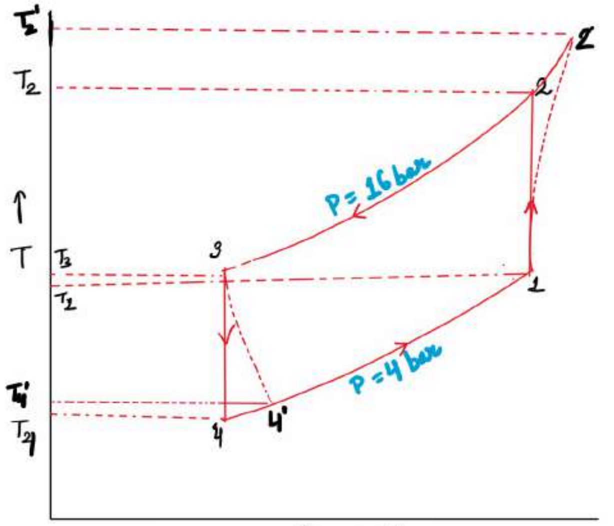


Fig → T-S diagram.

given :-

$T_3 = 37°C = 310K$
$T_1 = 7°C = 280K$
$\eta_T = 0.85$
$\eta_c = 0.8$
$\gamma = 1.4$
$C_p = 1.005\ KJ/Kg\text{-}K$

a) **compressor work per TR**

compressor work $(W_c) = (h_2' - h_1) = \underset{?}{\dot{m}_a}\, C_p(\underset{?}{T_2'} - \underset{✓}{T_1})$ KW.

here, $m_a \rightarrow$ mass of air required to produce 1TR

Finding T_2'

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} = \left(\frac{16}{4}\right)^{1.4-1/1.4} = (4)^{0.4/1.4} = 1.486$$

$\Rightarrow T_2 = 1.486 \times 280$

$\Rightarrow T_2 = 416K$

As $\eta_c = 0.8$

& $\eta_c = \frac{T_2 - T_1}{T_2' - T_1} = \frac{416 - 280}{T_2' - 280} = 0.8$

$\Rightarrow \boxed{T_2' = 450K}$

Finding T_4'

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\gamma-1/\gamma} = (4)^{0.4/1.4} = 1.486$$

$\Rightarrow T_4 = T_3/1.486 = 310/1.486$

$\Rightarrow T_4 = 208.613K$

As $\eta_T = 0.85$

$\Rightarrow \eta_T = \frac{T_3 - T_4'}{T_3 - T_4} = \frac{310 - T_4'}{310 - 208.613} = 0.85$

$\Rightarrow \boxed{T_4' = 223.82K}$

Refrigeration effect (R.E) $= h_1 - h_4'$

$= C_p(T_1 - T_4')$

$= 1.005 \times (280 - 223.82)$

here Refrigeration capacity = 1 TR = 3.5 KW

$R.C = \dot{m}_a \times R.E$

$\Rightarrow \dot{m}_a = \frac{3.5}{56.179} = 0.0623\ kg/s$ or $3.74\ kg/min$

$\Rightarrow$ compressor work $(W_c) = (h_2' - h_1) = \dot{m}_a C_p (T_2' - T_1)$ KW.

$= 0.0623 \times 1.005 \times (450 - 280)$

$\boxed{W_c = 10.644\ KW}$

$\Rightarrow$ Turbine work $(W_T) = (h_3 - h_4') = \dot{m}_a C_p (T_3 - T_4')$ KW

$= 0.0623 \times 1.005 \times (310 - 223.82)$

$\Rightarrow \boxed{W_T = 5.4\ KW}$

Net power $= W_{net} = W_c - W_T$

$= 10.644 - 5.4$

$\Rightarrow 5.244$ KW.

Airplane Refrigeration system

→ Airplanes fly at an altitude of 10000 m where the ambient Temperature is -50°C and pressure is about 0.15 bar.

⇒ The higher we go the temperature as well as pressure decreases.

Requirement of cooling in aeroplanes

→ **Ramming of air**

Due to high Velocity of surrounding air the air entering the system is rammed. During this process the Kinetic energy of entering air is converted into enthalpy, thus enthalpy of air rises which Leads to increase in Temp.

→ **To account for the solar Radiation.**

The solar radiation incident on the airplane in the upper Troposphere is about 1353 W/m².

→ **To Remove the heat dissipated by control devices.**

→ **To Remove the heat dissipated by occupants.**

Merits of Air-refrigeration system

→ No cost of Refrigerant involved as air is available free of cost

→ Air is non-toxic and non-flammable.

→ The air-cycle equipment is extremely reliable, reducing maintenance cost and system down-time.

- → Air is light in weight per tonne of Refrigeration.
- → Minor Leakage of air is not a problem.
- → Chilled air is directly used for cooling thereby eliminating the cost of a separate evaporator.
- → Since the pressure in the system is quite Low, therefore the Piping, ducting are simple to design.

Demerits of Air-Refrigeration systems

- → The COP of air-refrigeration system is very low (0.3-0.5).
- → The rate of air-circulation is very large.
- → Problem of freezing of moistre.

Simple air-cooling system.

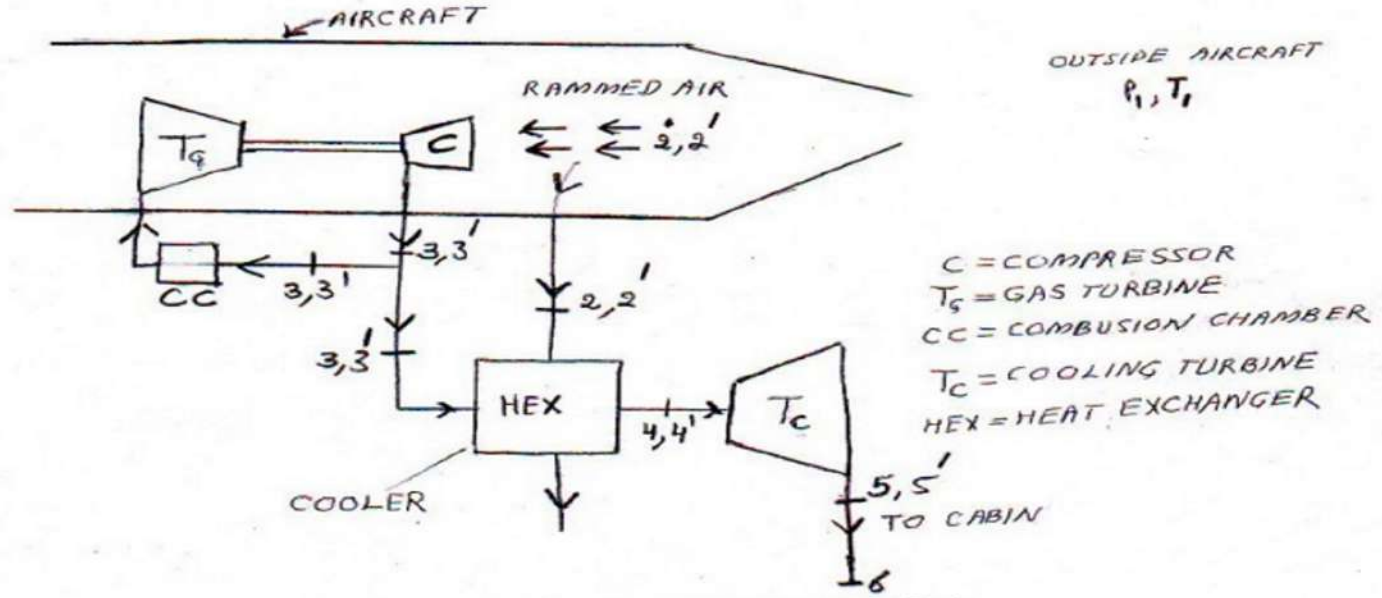


Fig. LINE DIAGRAM OF SIMPLE COOLING AIRCRAFT SYSTEM

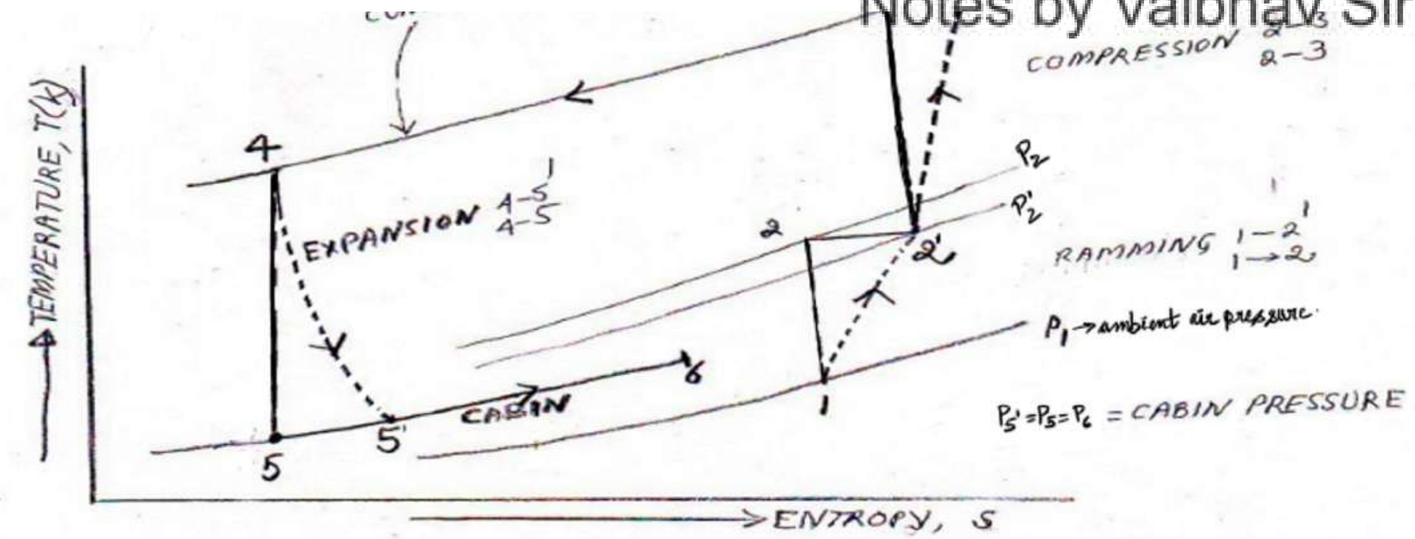


Fig. SIMPLE COOLING AIRCRAFT SYSTEM ON T-S CHART

Description of Various processes

1) Ramming Process

1-2 → Ideal Ramming process (Isentropic)

1-2' → Actual Ramming process

↳ not isentropic due to internal friction due to Irreversibilities.

NOTE

Assume Ramming process to be Isentropic unless mentioned in Problem.

During the ideal or the actual ramming Process, the total ener

or enthalpy remains constt.

$$\Rightarrow \quad h_2 = h_2' \quad \Rightarrow \quad T_2 = T_2'$$

using steady flow energy equation for Ramming Process.

$$h_1 + \frac{C_1^2}{2000} + \cancel{gz_1} + \cancel{\dot{q}}^{0} = h_2 + \frac{C_2^2}{2000} + \cancel{gz_2 + \dot{w}_{cv}}^{0}$$

assuming $\Delta P.E = 0$ & $\dot{q}$ & $\dot{w}_{cv} = 0$ & $C_2 <<< C_1$

$$\Rightarrow \quad h_2 - h_1 = C_1^2$$

$$\overline{2000}$$

$$C_p(T_2-T_1) = \frac{C_1^2}{2000}$$

$$\Rightarrow \quad T_2 - T_1 = \frac{C_1^2}{2000\,C_p}$$

$$T_2 = T_1 + \frac{C_1^2}{2000\,C_p} \quad \Rightarrow \quad \frac{T_2}{T_1} = 1 + \frac{C_1^2}{2000\,C_p T_1}$$

$$\Rightarrow \quad \frac{T_2'}{T_1} = 1 + \frac{C_1^2}{2000\,C_p T_1}$$

as $C_p = \frac{\gamma R}{\gamma - 1}$ $\Rightarrow$ $\frac{T_2}{T_1} = \frac{T_2'}{T_1} = 1 + \frac{C_1^2(\gamma-1)}{2000\,\gamma R T_1}$

here R → KJ/kg-K

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{T_2'}{T_1} = \frac{C_1^2(\gamma-1)}{2\gamma R T_1} \qquad R \rightarrow J/kg\,K$$

$$= \frac{C_1^2(\gamma-1)}{2a^2}$$

$$= \frac{\gamma-1}{2} \times M^2$$

$a = \sqrt{\gamma R T}$
↳ Local sonic velocity
M → Mach no. of Flight
$M = \left(\frac{\text{air craft vel.}}{\text{sound velocity}}\right)^2$

$T_2 = T_2'$ → stagnation temp of ambient air entering the compressor.

2) compression process

2'–3 → actual compression process.

$$\boxed{W_c = m_a C_p (T_3' - T_2')}$$

3) Cooling Process

3'-4 → actual cooling process in a heat exchanger

$$\boxed{Q_R = m_a C_p (T_3' - T_4)}$$

4) Expansion Process

4-5' → Expansion of cooled air in Turbine.

$$\boxed{W_T = m_a C_p (T_4 - T_{5'})}$$

5) Refrigeration process

5'-6 → Refrigeration effect.

$$R.E = h_6 - h_{5'}$$
$$= m_a C_p (T_6 - T_{5'})$$

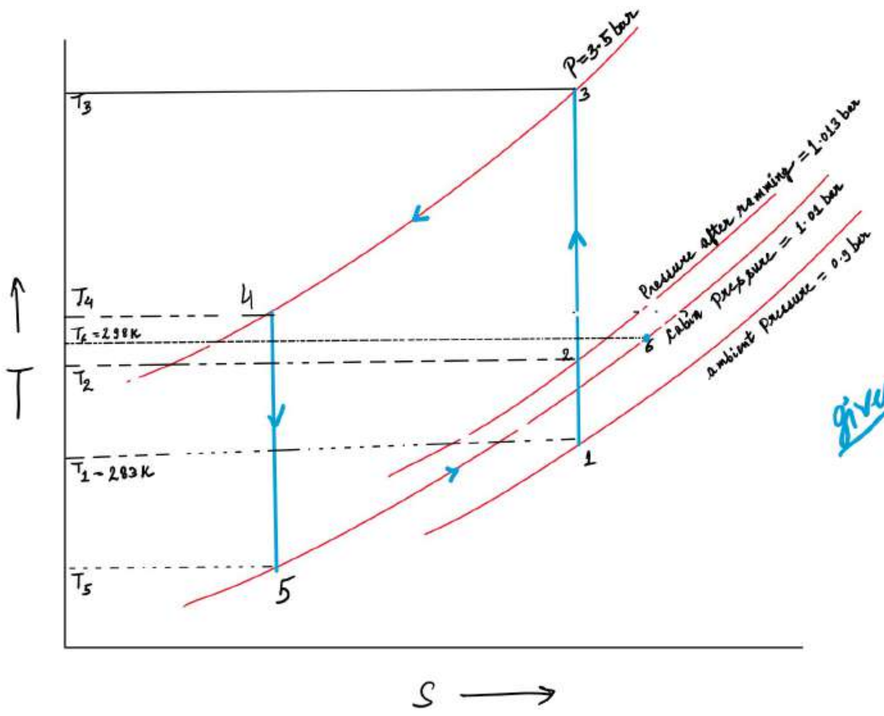
Q) A simple air-cooled system is used for an aeroplane having a load of 10 tonnes. The atmospheric pressure and temperature are 0.9 bar and 10°C respectively. The pressure increases to 1.1013 bar due to ramming. The temperature of the air is reduced by 50°C in the heat exchanger. The pressure in the cabin is 1.01 bar and the temperature of air leaving the cabin is 25°C. Determine :-

a) Power required to take the load of cooling in the cabin.

b) COP of the system.

Assume that all the expansions & compression are Isentropic. The Pressure of compressed air is

Soln



given:- ambient pressure & Temp. i.e P_1

$P_1 = 0.9$ bar , $T_1 = 283K$

Pressure after Ramming $(P_2) = 1.013$ b

cabin pressure & Temp ie P_6 & T_6

$P_6 = 1.01$ bar & $T_6 = 298K$

$T_3 - T_4 = 50°C$

Soln

cooling load OF aeroplane = 10 TR

= 10 X 3.5 KJ/s

= 35 KW

⇒ 35 KW OF heat ~~must be~~ removed to maintain cabin Temp.

⇒ 35 KW is ~~the~~ Refrigeration capacity.

⇒ $\boxed{R.C = 35\,KW}$

$$R.C = \dot{m}_a \times (h_6 - h_5)$$

$$R.C = \underset{?}{\dot{m}_a}\, c_p\, (\underset{✓}{T_6} - \underset{?}{T_5})$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} = \left(\frac{1.013}{0.9}\right)^{0.4/1.4}$$

$$\frac{T_2}{T_1} = 1.0343$$

$$\Rightarrow T_2 = 1.0335 \times 283$$

$$\boxed{T_2 = 292.72\ K}$$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\gamma-1/\gamma} = \left(\frac{3.5}{1.013}\right)^{0.4/1.4}$$

$$\frac{T_3}{T_2} = 1.425$$

$$\Rightarrow \boxed{T_3 = 417.155K}$$

as $T_3 - T_4 = 50$ (given)

$$\Rightarrow \boxed{T_4 = 367.155\ K}$$

$$\frac{T_4}{T_5} = \left(\frac{P_4}{P_5}\right)^{\gamma-1/\gamma} = \left(\frac{3.}{1.0}\right.$$

$$\Rightarrow \frac{T_4}{T_5} = 1.426$$

$$\Rightarrow T_5 = \frac{367.155}{1.426}$$

$$\Rightarrow \boxed{T_5 = 257.472K}$$

$$R.C = \dot{m}_a\, c_p\, (T_6 - T_5)$$

$$35 = \dot{m}_a \times 1.005 \times (298 - 257.472)$$

$$\Rightarrow \dot{m}_a = 0.8593 \text{ kg/s} \quad \text{or} \quad 0.8593 \times 60 = 51.5.5 \text{ kg/min}$$

a) Power required to take the load of cooling in the cabin.

$$W_C = h_3 - h_4$$
$$= C_p(T_3 - T_2)$$
$$= 1.005 \times (417.155 - 292.72)$$
$$\Rightarrow W_C = 124.435 \text{ kJ/kg}$$

$$\text{power} = \dot{m}_a \times W_c$$
$$= 0.8593 \times 124.435$$
$$\Rightarrow \boxed{\text{Power} = 106.92 \text{ KW}}$$

b) COP

$$COP = \frac{R.C}{P} = \frac{10 \times 3.5}{106.92}$$

$$\Rightarrow \boxed{COP = 0.327 \text{ KW}}$$