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Deflection of Pin - Jointed Truss by Unit Load Method

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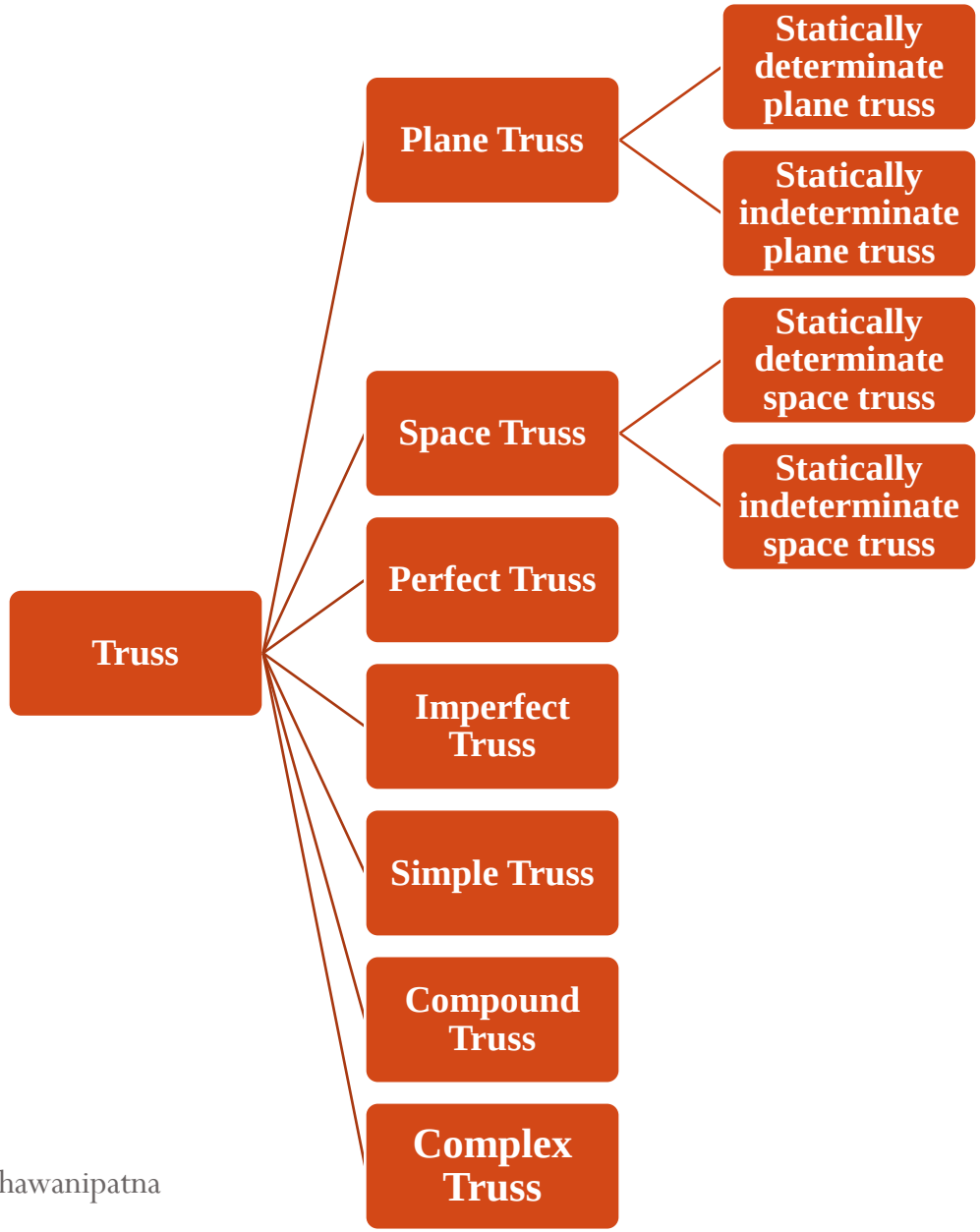
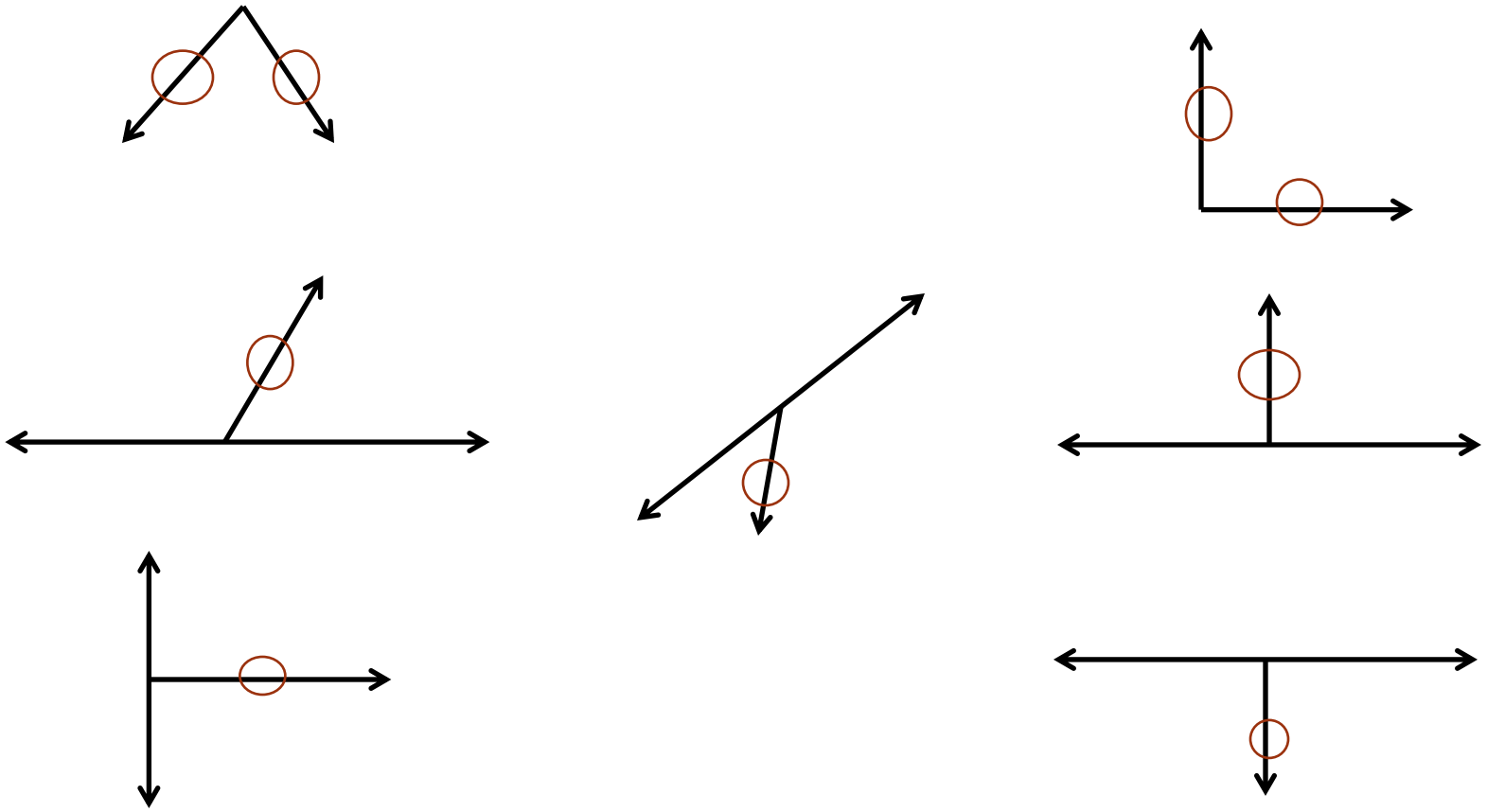
Introduction

- A truss consists of the number of members connected at the joint called as pin-jointed truss.
- It is assumed that pin-jointed truss carry axial force only.
- A truss which satisfies the equation ($m = 2j - 3$) is known as perfect truss.
- A perfect truss is one which has just sufficient members to keep the truss in equilibrium under any external force acting at its joints.
- Truss is a structural member that is assembled in such a way that forces are applied only on the ends.

Classification of Truss



Zero Force Members used for analysis of the Truss



- The zero force members may be directly used, during the analysis of the truss.

Deflection of pin-jointed plane truss by unit load method

- Unit load method can be used for finding deflection of a single joint at a time. Deflection of the truss can be find out by

$$\Delta = \int p'e\, dv$$

Where, Δ = Displacement at the point in the direction of unit load applied

P' = Stress due to unit load

e = Strain due to applied load

- In the case of a pin-jointed plane frames, there is only one type of stress, i.e. direct stress. This stress may be different in different members but is constant at all points in a member.
- Hence,

$$\int P'edv = \sum P'e\, AL$$

Where, $\sum$ is to cover all the members

A = Cross sectional area of members

L = Length of the member

P' = Stress due to unit load = $P' = \frac{k}{A}$

Where, k = Force in the member due to unit load

e = Strain due to given load

$$e = \frac{P}{A} \times \frac{1}{E} = \frac{P}{AE}$$

Where, P = force in the member due to given loading

- Displacement at a point in the direction of unit load is

$$\Delta = \sum \frac{k}{A} \times \frac{P}{AE} \times AL$$

$$\Delta = \sum \frac{PkL}{AE}$$

- The method needs analysis of the truss twice, once with the given loading to get 'P' terms, and second time with unit load to get 'k' terms.

Sign Convention

- If the force in the member of the truss is tensile in nature, the arrow mark should be away from the joint.

- If the force in the member of the truss is compressive in nature, the arrow mark should be towards the joint.

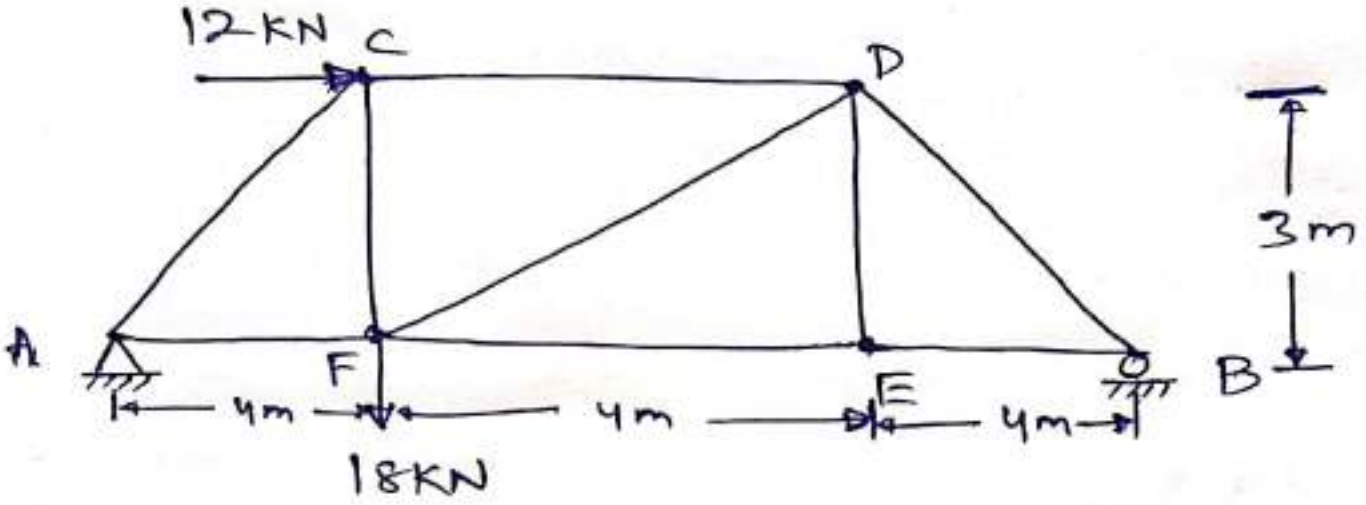
- Use tensile force in the member of the truss positive (+) and compressive force negative (-)
- In the analysis of each joint, use write arrow mark positive and left arrow mark negative. Also, upward arrow mark positive and downward arrow mark negative.
- Clockwise moment positive and anticlockwise moment negative.

Procedure for calculating truss deflection

- First, calculate the real forces in the member of the truss either by method of joints or by method of sections due to externally applied forces. (P)
- Then, calculate forces in the member of the truss consider the virtual load system such that only a unit load is considered at the joint in the horizontal or in the vertical direction where the deflection is required. (k)
- The length of the truss member is calculated from the geometry. (L)
- The area of the truss member is known in the problem. (A)
- Modulus of Elasticity is known in the problem, (E)
- Displacement at the joint is calculated in the direction of unit load is

$$\Delta = \sum \frac{PkL}{AE}$$

Q.1 Using unit load method or any other method, find the vertical deflection of joint 'E' of a pin-jointed truss loaded & supported as shown in Figure. Take AE = Constant in all member.



The deflection of joint 'E' can be find out using the deflection formula by unit load method

$$\Delta = \sum \frac{PKL}{AE}$$

P = forces in the members due to given loadings

K = Forces in the member due to unit load

L = Length of the member

A = Cross sectional area of the member

E = Modulus of elasticity

Steps to find out the displace in truss

1) Find reaction due to given loading
2) Find forces in the member due to given loading
3) Find reaction due to unit Load applied at Point 'E'
4) Find forces in the member due to unit load at Point 'E'.
5) Find length of the truss members
6) Find all data in a Tabular form
7) Using deflection equation find deflection at Point 'E'.

Support reaction due to given loading

Taking moment about the Point 'A'

$\Sigma M_A = 0$ (Taking clockwise moment +ve)

$12 \times 3 + 18 \times 4 - V_B \times 12 = 0$

$\boxed{V_B = 9 \text{ kN}} \uparrow$

Summation of vertical force is zero

$\Sigma F_y = 0$ (Taking upward direction (+ve)
Downward direction (-ve))

$V_A - 18 + 9 = 0$

$\boxed{V_A = 9 \text{ kN}} \uparrow$

Summation of horizontal force is zero

$\Sigma F_x = 0$ [Right direction (→) Positive
left direction (←) negative]

$H_A + 12 = 0$

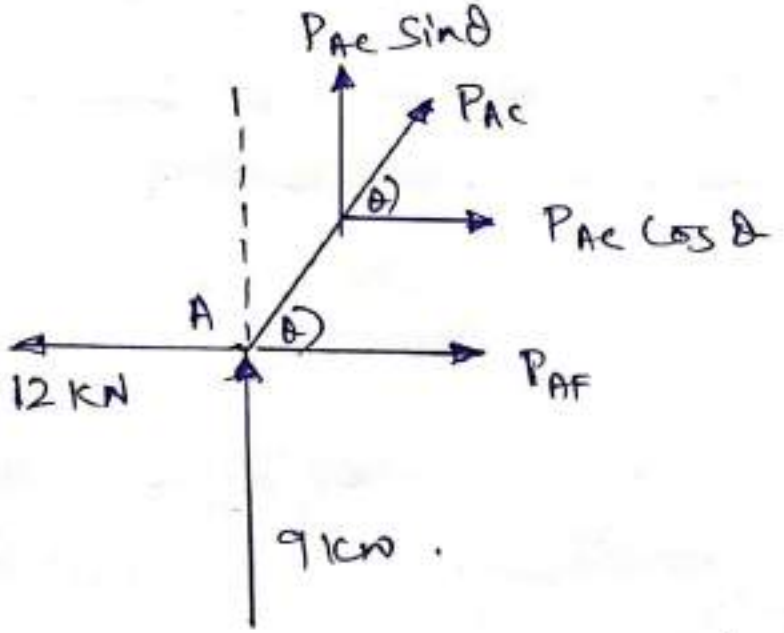
$H_A = -12 \text{ kN}$

$\boxed{H_A = 12 \text{ kN}} (\leftarrow)$

Forces in the member due to given loading

Take each joint and draw free body diagram then find out the forces in the members.

Joint. A



Free Body Diagram of joint 'A'

* First assume the forces in the member tensile. Apply $\Sigma F_x = 0$ & $\Sigma F_y = 0$

If the value comes (-ve), then change the direction of arrow mark and treated as Compressive force.

At each joint for tensile force, give the direction of arrow mark away from the joint. For Compressive force the arrow mark should be towards the joint.

First find the angle 'θ' from the geometry of truss.

$$\tan\theta = \frac{3}{4}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) \Rightarrow \theta = 36.86^\circ$$

Find the forces in the member, first take

$$\Sigma F_y = 0$$

$$9 + P_{AC} \sin\theta = 0$$

$$9 + P_{AC} \sin(36.86) = 0$$

$$P_{AC} = -15 \text{ KN}$$

$\boxed{P_{AC} = 15 \text{ KN}}$ Compressive force (C) (due to -ve answer)

$$\Sigma F_x = 0$$

$$P_{AF} + P_{AC} \cos\theta - 12 = 0$$

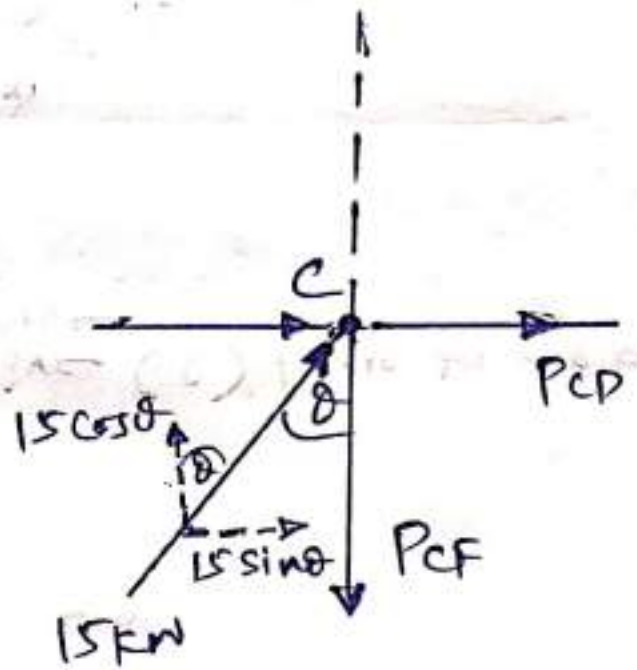
$$P_{AF} + (-15 \times \cos 36.86) - 12 = 0$$

$\boxed{P_{AF} = 24 \text{ KN}}$ Tensile force (T) (due to +ve answer)

Joint 'C'

$\tan\theta = \frac{4}{3}$

$\theta = 53.13^\circ$



$\Sigma F_y = 0$

$15 \cos(53.13) - P_{CF} = 0$

$\boxed{P_{CF} = 9\ \text{KN}}$ (T)

$\Sigma F_x = 0$

$P_{CD} + 15 \sin 53.13 + 12 = 0$

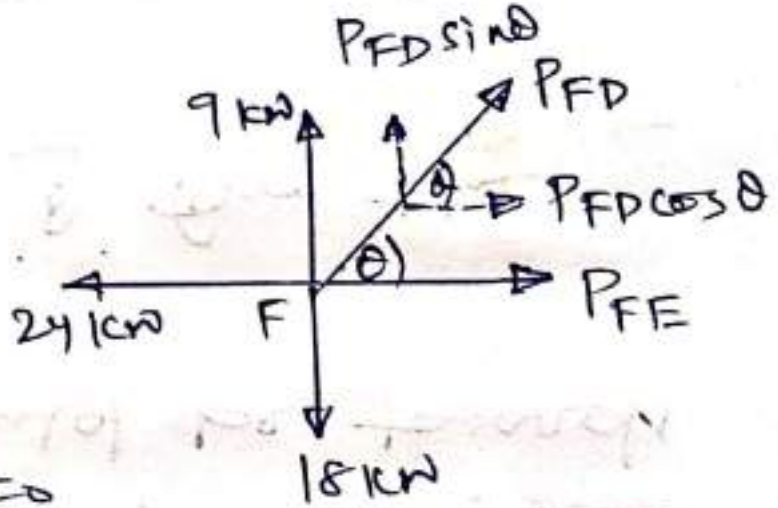
$P_{CD} = -24\ \text{KN}$

$\boxed{P_{CD} = 24\ \text{KN}}$ (C)

Joint 'F'

$\tan\theta = \frac{3}{4}$

$\theta = 36.86^\circ$



$\Sigma F_y = 0$

$9 - 18 + P_{FD}\sin\theta = 0$

$9 - 18 + P_{FD}\sin(36.86^\circ) = 0$

$\boxed{P_{FD} = 15\ \text{kN}}$ (T)

$\Sigma F_x = 0$

$P_{FE} + P_{FD}\cos\theta - 24 = 0$

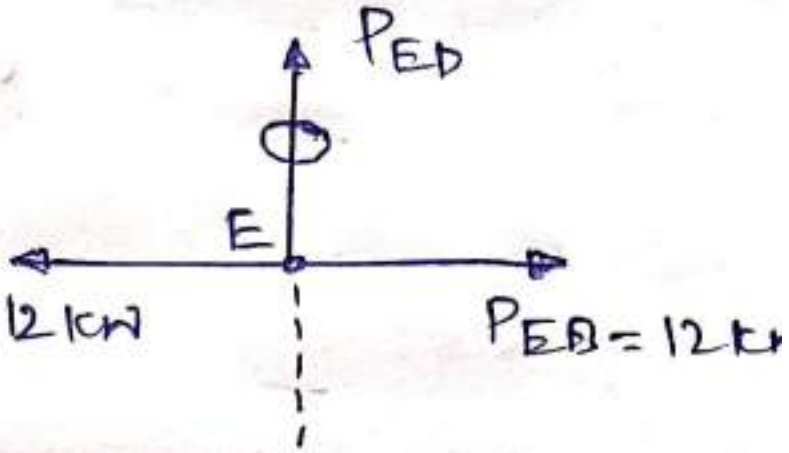
$P_{FE} + 15\cos(36.86) - 24 = 0$

$\boxed{P_{FE} = 12\ \text{kN}}$ (T)

Joint 'E'

$\Sigma F_y = 0$

$$\boxed{P_{ED} = 0}$$

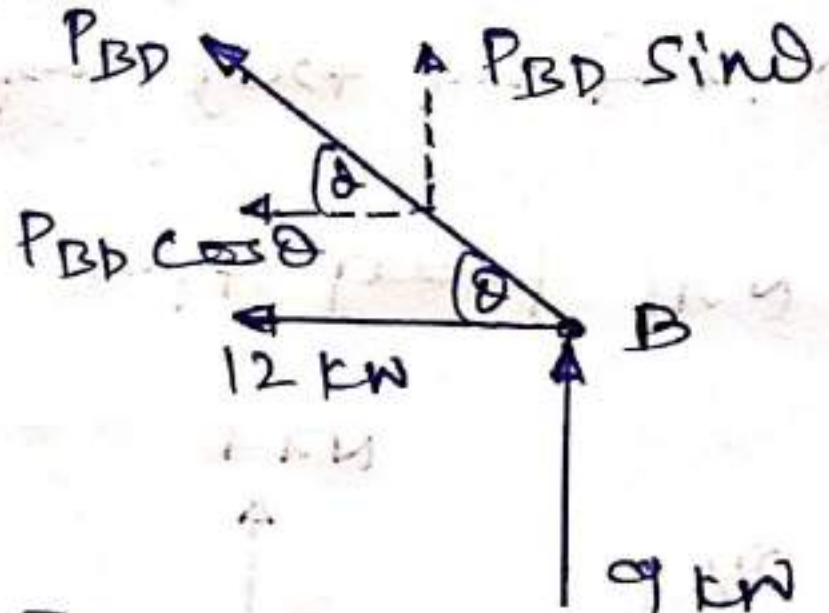


$\Sigma F_x = 0$

$-12 + P_{EB} = 0$

$$\boxed{P_{EB} = 12 \text{ kN}} \quad (T)$$

Joint 'B'



$$\tan\theta = \frac{3}{4}$$

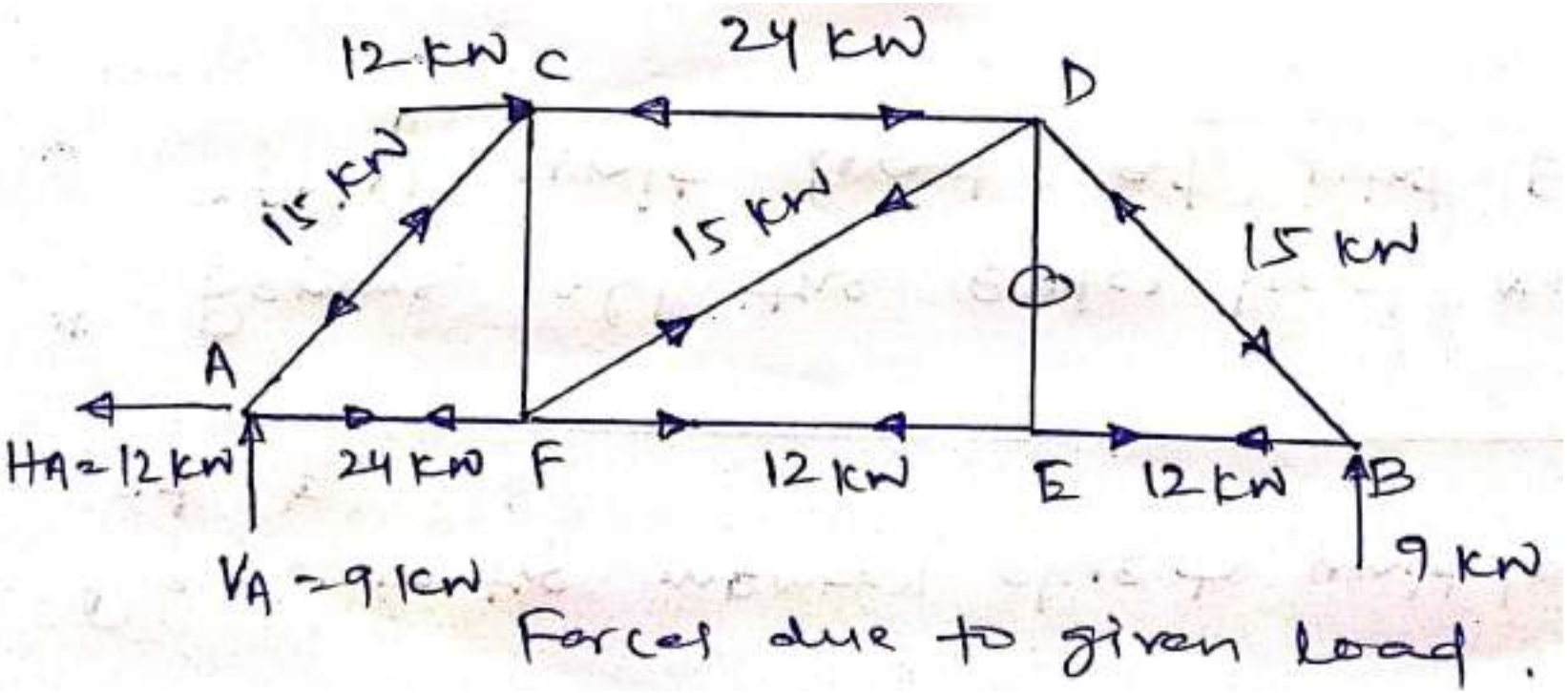
$$\theta = 36.86^\circ$$

$$\Sigma F_y = 0$$

$$P_{BD}\sin(36.86^\circ) + 9 = 0$$

$$P_{BD} = -15 \text{ KN}$$

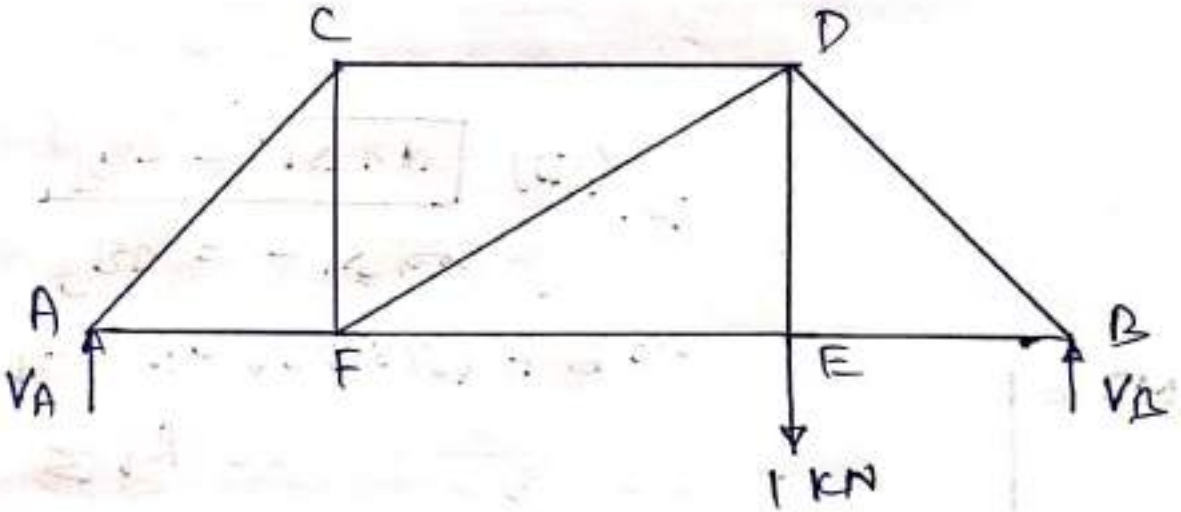
$$\boxed{P_{BD} = 15 \text{ KN}} \quad (C)$$



Forces due to given load.

Forces in the members due to unit load at 'E'

* Remove all the external load and apply unit load at joint 'E'



Truss due to unit load 1KN at 'E'

Find the reaction first by taking moment about 'A' = 0

$\Sigma M_A = 0$

$1 \times 8 - V_B \times 12 = 0$

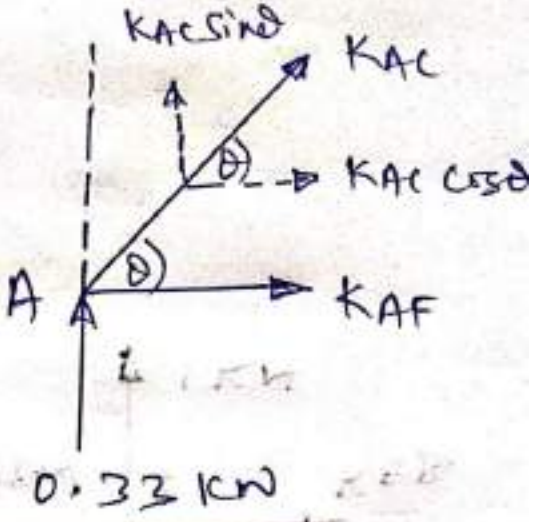
$$\boxed{V_B = 0.67 \text{ kN}} (\uparrow)$$

$\Sigma f_y = 0$

$V_A - 1 + 0.67 = 0$

$$\boxed{V_A = 0.33 \text{ kN}} (\uparrow)$$

Joint 'A'



$$\tan\theta = \frac{3}{4}$$

$$\theta = 36.86^\circ$$

$$\Sigma F_y = 0$$

$$0.33 + K_{AC}\sin\theta = 0$$

$$0.33 + K_{AC}\sin(36.86) = 0$$

$$K_{AC} = -0.55 \text{ KN}$$

$$\boxed{K_{AC} = 0.55 \text{ KN}} \quad (C)$$

$$\Sigma F_x = 0$$

$$K_{AF} + K_{AC}\cos\theta = 0$$

$$K_{AF} + (-0.55 \times \cos 36.86^\circ) = 0$$

$$K_{AF} = 0.44 \text{ KN}$$

$$\boxed{K_{AF} = 0.44 \text{ KN}} \quad (T)$$

Joint 'C'

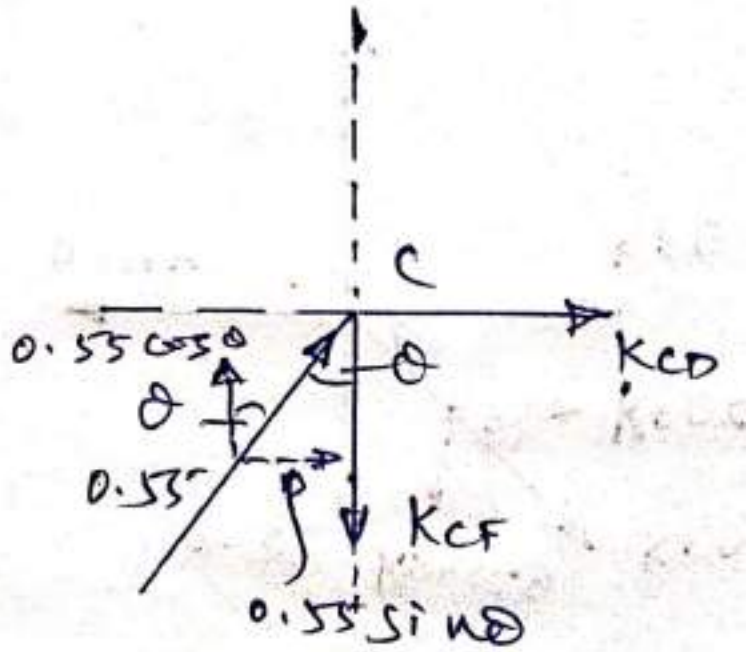
$\tan\theta = \frac{4}{3}$

$\theta = 53.13^\circ$

$\Sigma F_y = 0$

$0.55 \cos(53.13) - K_{CF} = 0$

$\boxed{K_{CF} = 0.33 \text{ kN}}$ (T)



$\Sigma F_x = 0$

$0.55 \sin\theta + K_{CD} = 0$

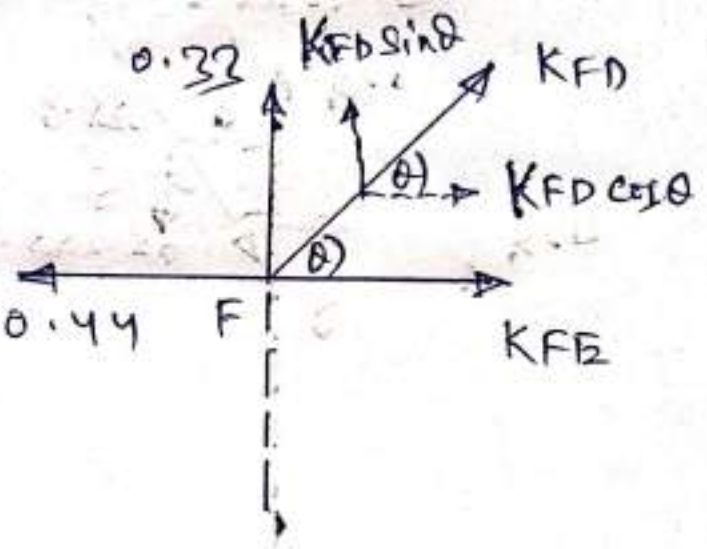
$K_{CD} = -0.44 \text{ kN}$

$\boxed{K_{CD} = 0.44 \text{ kN}}$ (C)

Joint 'F'

$\tan\theta = \frac{3}{4}$

$\theta = 36.86^\circ$



$\Sigma F_y = 0$

$0.33 + K_{FD}\sin(36.8^\circ) = 0$

$K_{FD} = -0.55$ KN

$\boxed{K_{FD} = 0.55 \text{ KN}}$ (C)

$\Sigma F_x = 0$

$K_{FE} + K_{FD}\cos\theta = 0$

$K_{FE} + [-0.55 \times \cos(36.8^\circ)] = 0$

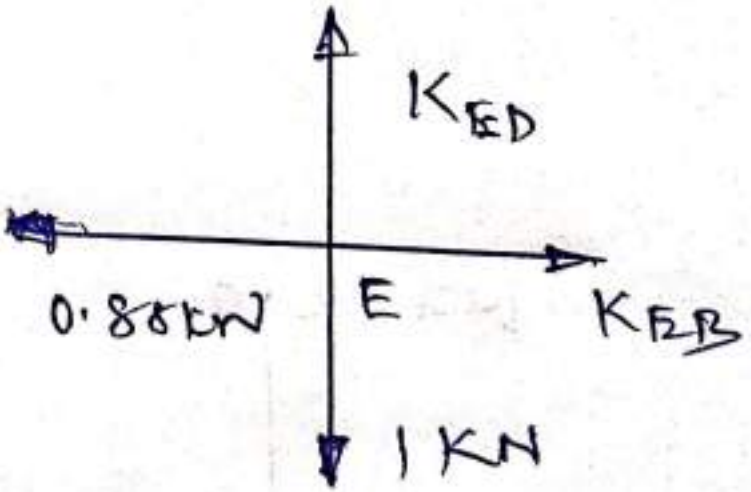
$\boxed{K_{FE} = 0.88 \text{ KN}}$ (T)

Joint 'E'

$\Sigma F_y = 0$

$K_{ED} - 1 = 0$

$\boxed{K_{ED} = 1\ KN}\ (T)$



$\Sigma F_x = 0$

$K_{EB} - 0.88 = 0$

$\boxed{K_{EB} = 0.88\ KN}\ (T)$

Joint 'B'

$\tan\theta = \frac{3}{4}$

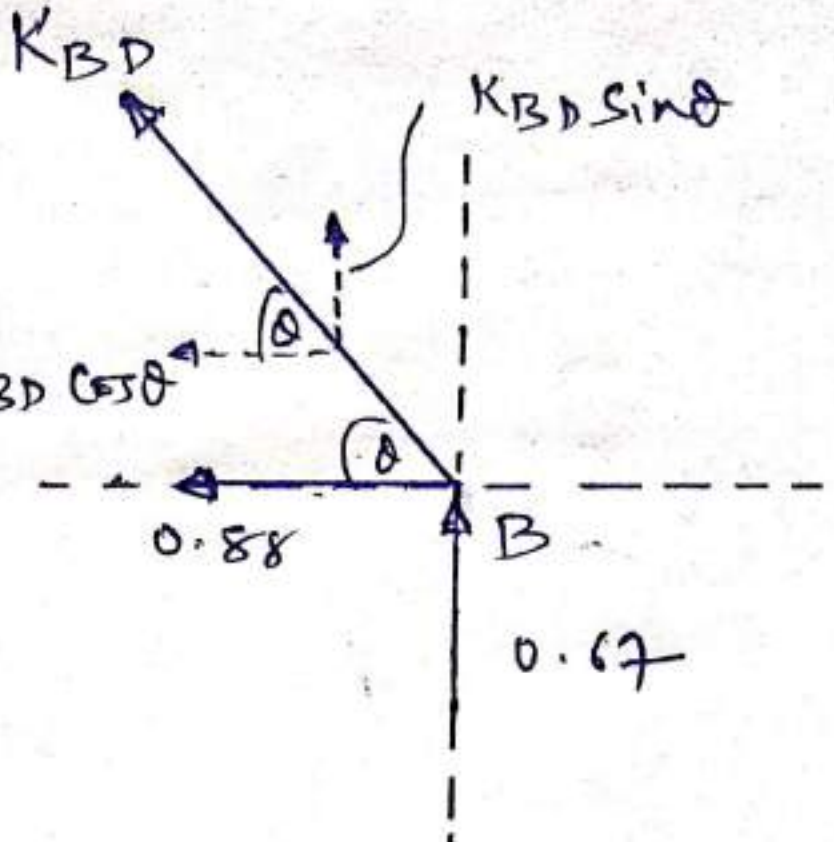
$\theta = 36.86^\circ$

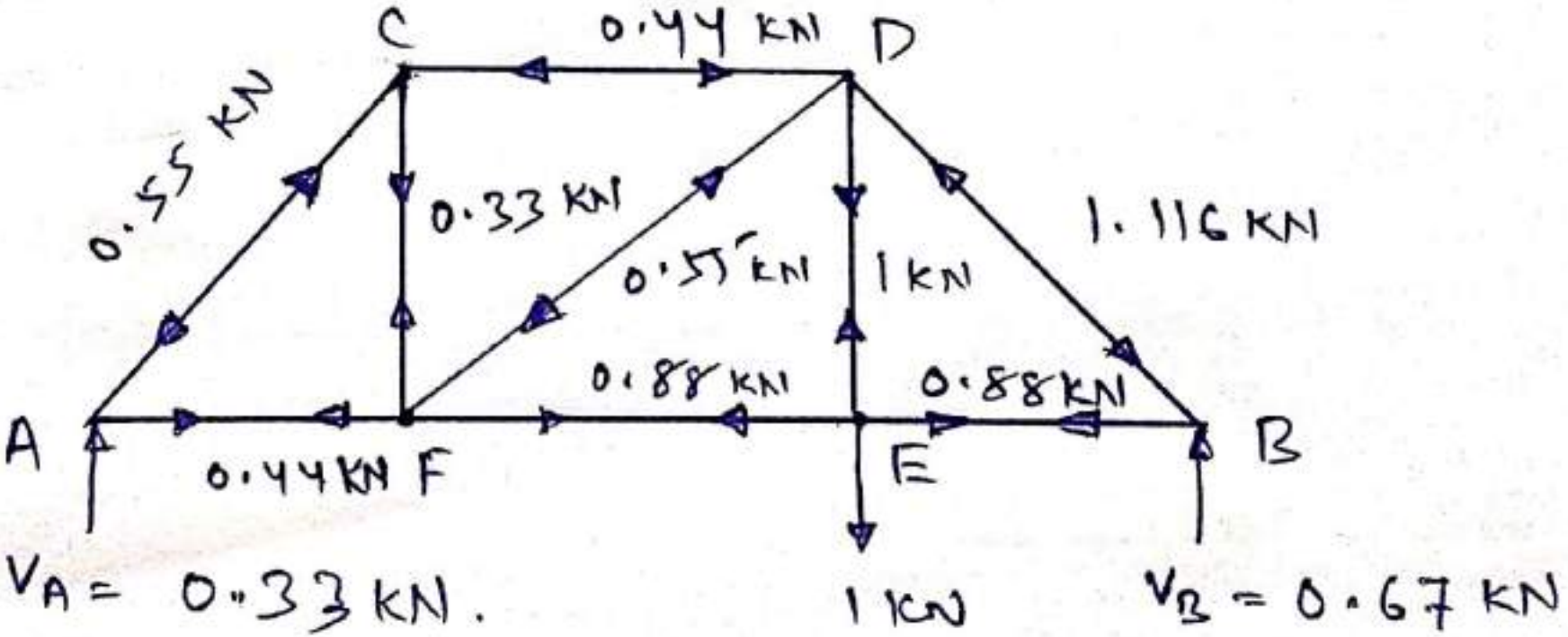
$\Sigma F_y = 0$

$0.67 + K_{BD} \sin(36.8^\circ) = 0$

$K_{BD} = -1.1166\,kN$

$\boxed{K_{BD} = 1.1166\,kN}$ (C)





Force in the Member of a truss due to unit load at 'E'

Calculation Table :

Member	P (KN)	K (KN)	Length (l) (m)	PKL
AF	24	0.44	4	42.24
AC	-15	-0.55	5	41.25
CF	9	0.33	3	8.91
CD	-24	-0.44	4	42.24
FD	15	-0.55	5	41.25
FE	12	0.88	4	42.24
DE	0	1	3	0
BD	-15	-1.111	5	83.745
BE	12	0.88	4	42.24

$\Sigma PKl = 261.615$

Vertical deflection of joint 'E': $\Delta = \sum \frac{PKl}{AE}$

$$\Delta = \frac{261.6 L^3}{AE}$$

Thanks