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# Civil Engineering

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# Cables and Suspension Bridge

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# Introduction

- Cables are used as temporarily guys during the erection and as permanent guys for supporting masts and towers.
- Cables are used in the suspension bridges. A suspension bridge consists of two cables with the number of suspenders (hangers) which support the roadway.
- Figure 1 shows a typical suspension bridges in which the cable is supported over towers.

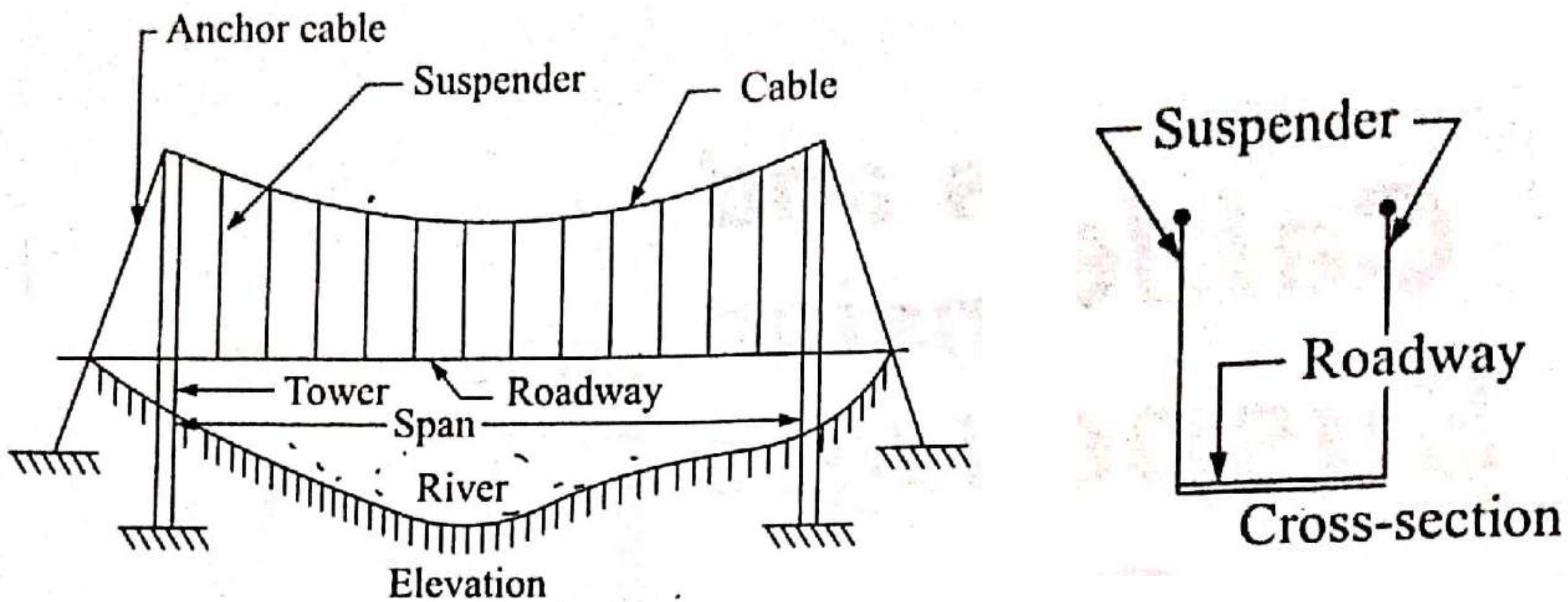
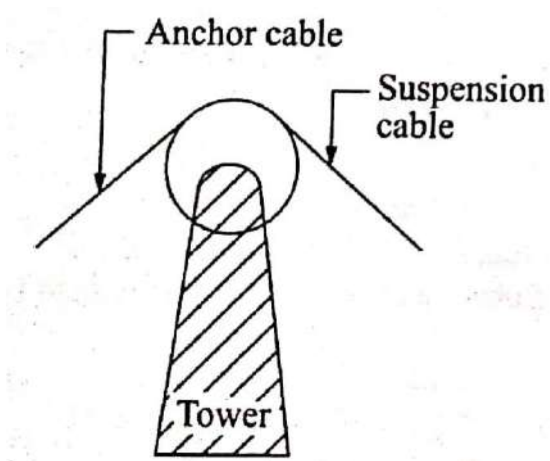
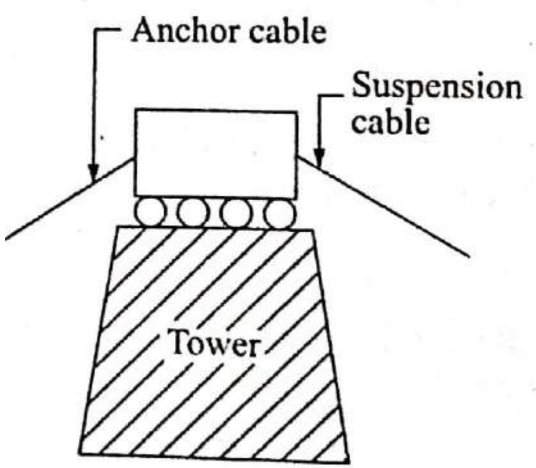



**Figure 1**: A typical suspension bridge

- To reduce the bending moment in the towers anchor cables are provided.
- The central sag or dip of the cable varies from $\left(\frac{1}{10}\right)$ th to $\left(\frac{1}{15}\right)$ th of span.
- The cables will be having either guided pulley support or roller pulley support as shown in Figure 2.




**a**) Guided pulley support




**b**) Roller pulley support

**Figure 2**: Support system

- In case of pedestrian suspension bridges, suspenders support the roadway directly.

- For heavy traffic, large spans stiffening girders are provided to support the roadway as shown in Figure 3.
- Laksman Jhula (Rishikesh) and Howrah bridge (Kalkata) are popular example of suspension bridges.
- Since, the number of suspenders are very large, the load on the cable may be taken as uniformly distributed load.

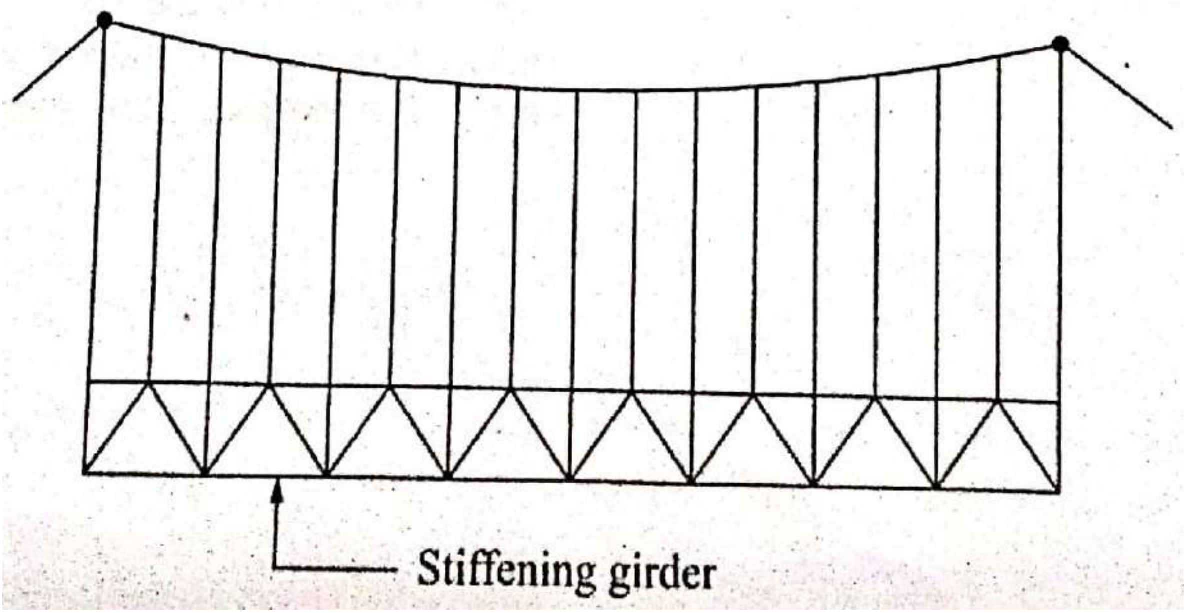



**Figure 3**: Equilibrium of Cable

# Equilibrium of Cable

- A cable is a flexible structure which can not resist Bending Moment.
- In deflected shape of cable, the bending moment at any point of cable is zero which is achieved by developing horizontal thrust at the support.

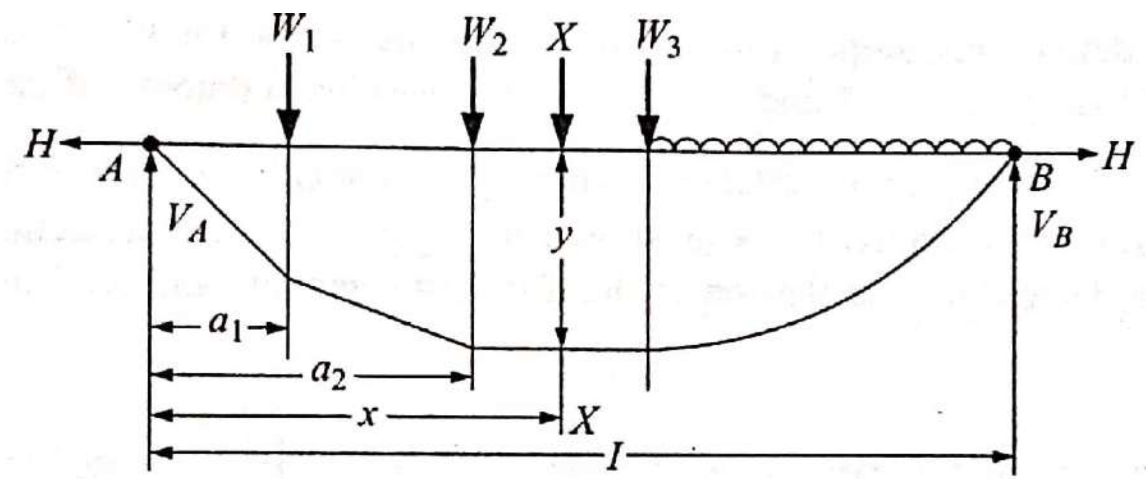



**Figure 4**: Equilibrium of Cable

- Consider the cable shown in Figure 4, which is subjected to various loads.
- Let the horizontal force developed at support is *H*
- Let the vertical reactions at support A and B is $V_A$ and $V_B$ respectively.

At section X-X, let the deflection be '$y$'

- Moment at section x-x = $M_x = V_A x - W_1 (x-a_1) - W_2 (x-a_2) - H y$
- Since the cable is flexible, $M_x = 0$
- Therefore, $H y = V_A x - W_1 (x-a_1) - W_2 (x-a_2)$
- $H y$ = Beam Moment
- The loaded cable can be analyzed by using above equation at any segment of cable.

# Cable Subjected to Concentrated Loads

- Consider the cable of length $L$ spanning over a horizontal gap $l$ subjected to the concentrated loads as shown in Figure 5.
- Let $V_A$ and $V_B$ be the vertical reactions and $H$ be the horizontal reactions at supports.

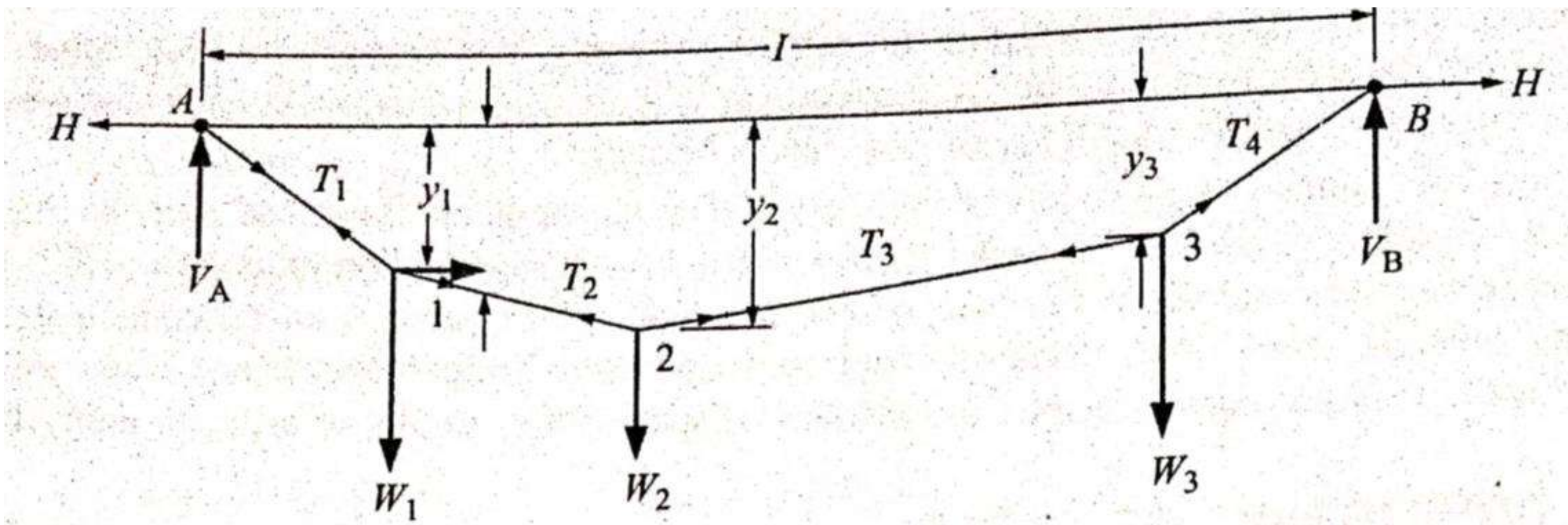



**Figure 5**: Cable subjected to concentrated loads

- The equilibrium condition is $H\,y = M_{\text{beam}}$

$$\text{or} \quad \boldsymbol{y} = \frac{\boldsymbol{M}_{\textbf{beam}}}{\boldsymbol{H}}$$

- Hence, the deflected shape is similar to the beam moment diagram.
- If $M_1$, $M_2$ and $M_3$ are the beam moments at load points 1, 2 and 3 respectively.
- $y_1$, $y_2$ and $y_3$ are the deflections at 1, 2 and 3 respectively
- $y_1$, $y_2$ and $y_3$ can be found using above equation i.e. $y_1 = \frac{M_1}{H}$ , $y_2 = \frac{M_2}{H}$ and $y_3 = \frac{M_3}{H}$
- If the horizontal thrust is known or position of cable at any one point is known, the deflections at all points can be calculated.
- The actual length of the cable is the sum of lengths of each segments.
- After finding the deflections, slope of the various segments can be found.
- Using equilibrium equations of load points 1, 2 and 3, forces in the various segment of cable can be found.

# Cable Subjected to a Uniformly Distributed Load

- Let a cable of length *L* be supported at points *A* and *B* which are at a horizontal distance *l* and are at the same level as shown in Figure 6.
- The cable is subjected to a uniformly distributed load *w*/unit horizontal length.

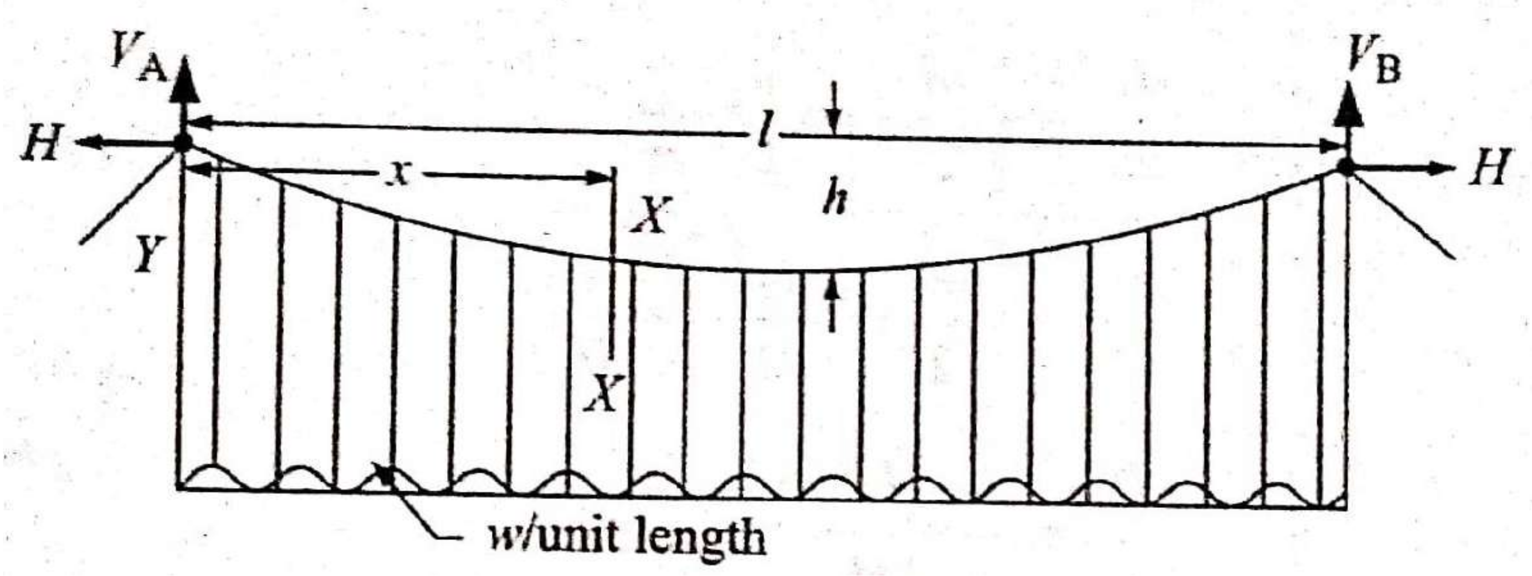



**Figure 6**: A typical cable subjected to *udl*

- The vertical reactions at $A$ and $B$ is $V_A$ and $V_{B.}$

$$V_A = V_B = \frac{wl}{2}$$

- Taking moment at central point and equate to zero (Though the bending moment is zero at all points in the cable)

$$H\,h - \frac{wl}{2} \times \frac{l}{2} + \frac{wl}{2} \times \frac{l}{4} = 0$$

or $$H = \frac{wl^2}{8h}$$

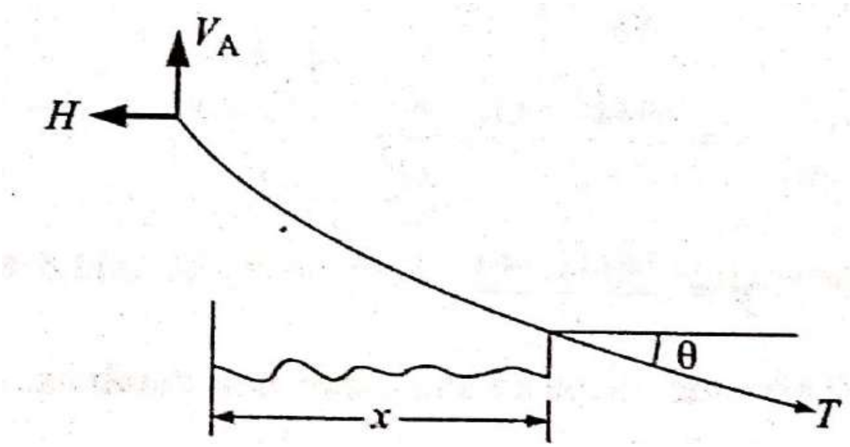



**Figure 7**: Free body diagram of cable

- If $V$ is shear force at any section X-X distance $x$ from $A$ as shown in Figure 7.

Then, $T = \sqrt{V^2 + H^2}$

$$V_{\max} = \frac{wl}{2} \text{ at support}$$

Therefore, $T_{\max} = \sqrt{\left(\frac{wl}{2}\right)^2 + \left(\frac{wl^2}{8h}\right)^2} = \frac{wl}{2}\sqrt{\left(1 + \frac{l^2}{16h^2}\right)}$

$$V_{\min} = 0 \text{ at centre}$$

$$T_{\min} = \sqrt{0 + H^2} = H$$

- At any point, since cable can not resist shear

$$V = T \sin\theta$$

- Now to find the shape of the cable, consider the portion on left side of section X-X. Let θ be the slope. Then,

$$\sum H = 0,\ \ T\cos\theta = H$$

$$\sum V = 0,\ \ T\sin\theta = V_A - w\,x$$

or $$T \sin \theta = \frac{wl}{2} - w\,x$$

Therefore, $$\tan \theta = \left[\frac{wl}{2} - wx\right] \times \frac{1}{H}$$

i.e. $$\frac{dy}{dx} = \left[\frac{wl}{2} - wx\right] \times \frac{1}{H}$$

Therefore, $$y = \left[\frac{wl}{2}x - \frac{wx^2}{2}\right] \times \frac{1}{H}$$

or $$y = \frac{wx(l-x)}{2H}$$

Substituting the value of H = $\frac{wl^2}{8h}$, we get

$$y = \frac{wx(l-x)}{2} \times \frac{8h}{wl^2} = \frac{4hx(l-x)}{l^2}$$

- Which is a parabola. Thus the shape of the cable is a parabola.
- To find the length of the cable in any curve ($L$)

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 1 + \frac{1}{2}\left[\frac{4h(l-2x)}{l^2}\right]^2$$

Therefore, Length of the cable = $L = \int_0^l ds = l + \frac{8h^2}{3l}$

# Forces on Anchor Cables and Towers

- The forces on anchor cable and towers depends upon the type of support given to cables.

  There are two types of support:
- Guided Pulley Support
- Roller Support

## Guided Pulley Support:

- Let the inclination of main cable to horizontal be **'θ'.**
- Inclination of anchor cable to horizontal be **'α'**
- Assuming the pulley as friction less.
- Tension in anchor cable = tension in main cable.
- Let the tension be *T*

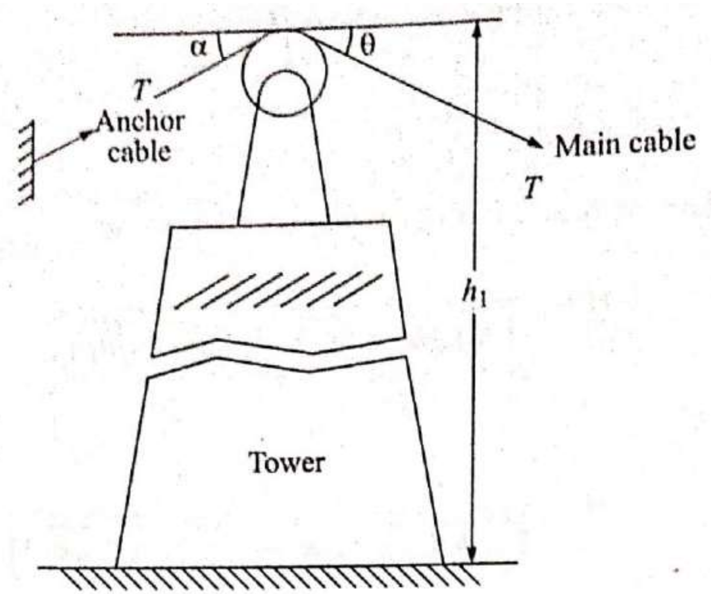



**Figure 8**: Guided Pulley Support

Vertical load transmitted to tower = $T \sin\theta + T \sin\alpha$

Vertical load transmitted to tower = $T(\sin\theta + \sin\alpha)$

Horizontal load transmitted to tower = $T \cos\theta - T \cos\alpha$

Horizontal load transmitted to tower = $T(\cos\theta - \cos\alpha)$

Bending moment on the tower = Horizontal force on tower × Height of tower

**Bending moment on the tower = $T(\cos\theta - \cos\alpha) \times h_1$**

## Roller Support:

- In this case, the suspensible cable and the anchor cables are connected to a saddle resting on a tower.

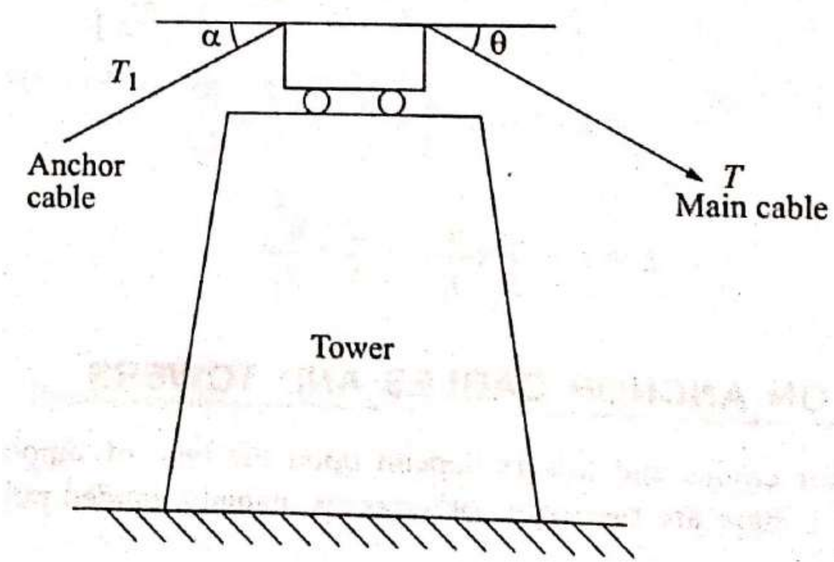



**Figure 9**: Roller Support

- In this arrangement, the two cables need not have the same tension.
- Let $T$ be the tension in main cable and $T_1$ in the anchor cable.
- Assume saddle have frictionless rollers

$$T_1 \cos \alpha = T \cos \theta$$

$$T_1 = T \left(\frac{\cos \theta}{\cos \alpha}\right)$$

- Since, saddle is having frictionless rollers, there is no horizontal force and hence, no bending moment on tower,

Vertical force on the tower = $T_1 \sin \alpha + T \sin \theta$

# Thanks

# Cables and Suspension Bridge (Solved Problems)

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Q1. A bridge cable is suspended from towers 80 m apart and carries a load of 30 kN/m on the entire span. If the maximum sag is 8 m, calculate the maximum tension in the cable. If the cable is supported by saddles which are stayed by wires inclined at 30° to the horizontal, determine the forces acting on the towers. If the same inclination of back stay passes over pulley, determine the forces on the towers.

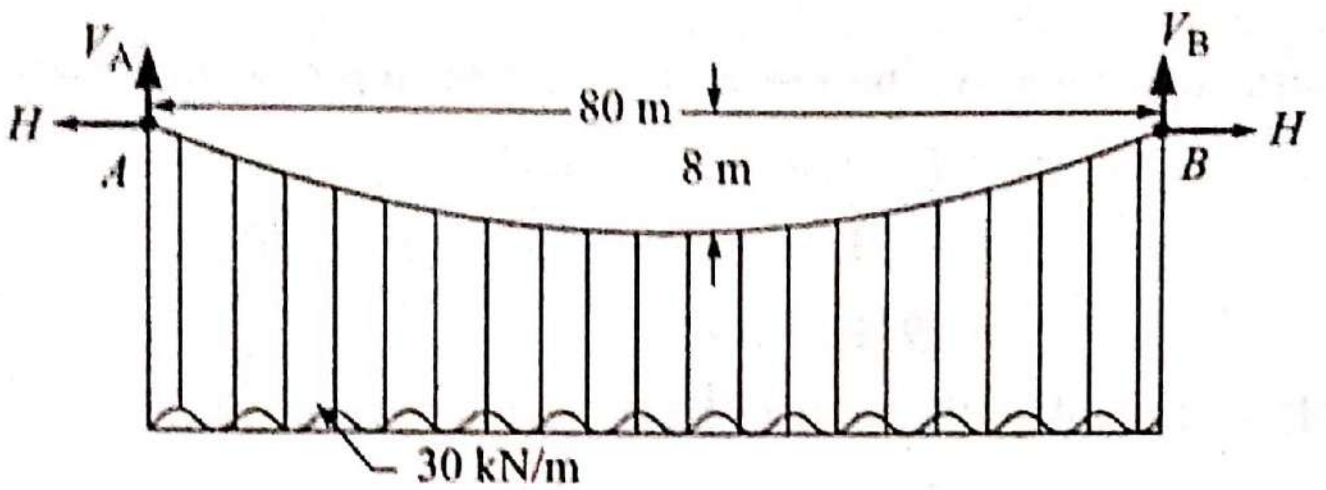


**Figure 1(a):** Example 1

- Reaction at both ends = $V_A = V_B = \frac{wl}{2} = \frac{30\times80}{2}$ = 1200 kN
- For horizontal reaction, taking moment about central point '$C$'

$$H \times 8 - \frac{wl}{2}\times\frac{l}{2} + \frac{wl}{2}\times\frac{l}{4} = 0$$

or $$H = \frac{wl^2}{64} = \frac{30\times80^2}{64} = 3000 \text{ kN}$$

- Maximum tension occurs at support

$$T_{max} = \sqrt{V^2 + H^2} = \sqrt{1200^2 + 3000^2}$$

$$T_{max} = 3231.1 \text{ kN}$$

$$H = T_{max} \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{H}{T}\right) = \cos^{-1}\left(\frac{3000}{3231.1}\right) = 21.80^{\circ}$$

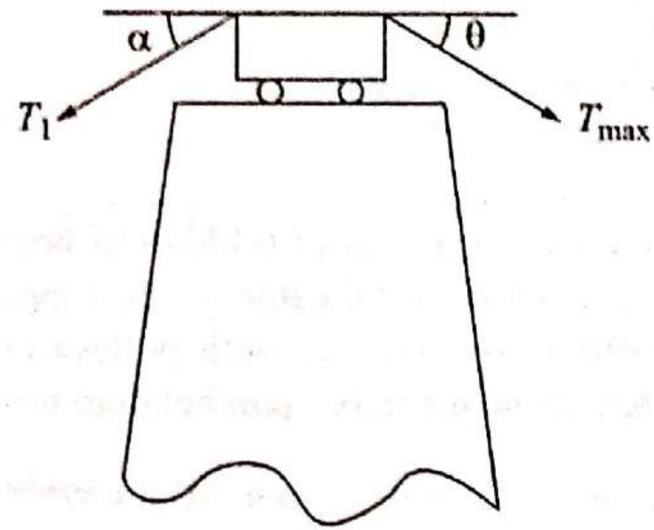


**Figure 1(b):** saddle support

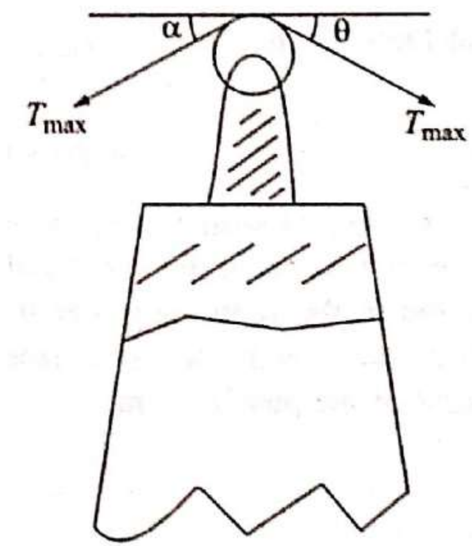


**Figure 1(c) :** Pulley support

**If the cable is supported by saddle** (Figure 1b)

The anchor cable tension $T_1$ can be found by equating horizontal tension

$$T_1 \cos \alpha = T_{max} \cos \theta$$

$$T_1 \times \cos 30^\circ = 3231.80 \times \cos 21.80^\circ$$

$$T_1 = 3464.1 \text{ kN}$$

- There is no horizontal force on the tower.
- The vertical force on the tower = $T_1 \sin \alpha + T_{max} \sin \theta$

Vertical force = 3464.1 sin 30° + 3231.1 sin 21.80° = 2931.98 kN

**If the cable is supported over pulley** (Figure 1c)

- The vertical force on tower = $T_{max} (\sin \alpha + \sin \theta)$

Vertical force = 3231.1 (sin 30°+ sin 21.80°) = 2815.48 kN

- Horizontal force on the tower = Tmax (cos θ – cos α)

Horizontal force = 3231.1 (cos 21.8° - cos 30°) = 201.82 kN

Q2. A cable of span 120 m and dip 10 m carries a load of 6 kN/m of horizontal span. Find the maximum tension in the cable and the inclination of the cable at the support. Find the forces transmitted to the supporting pier if the cable passes over smooth pulleys on top of the pier. The anchor cable is at 30° to the horizontal. Determine the maximum bending moment for the pier if the height of the pier is 15 m.

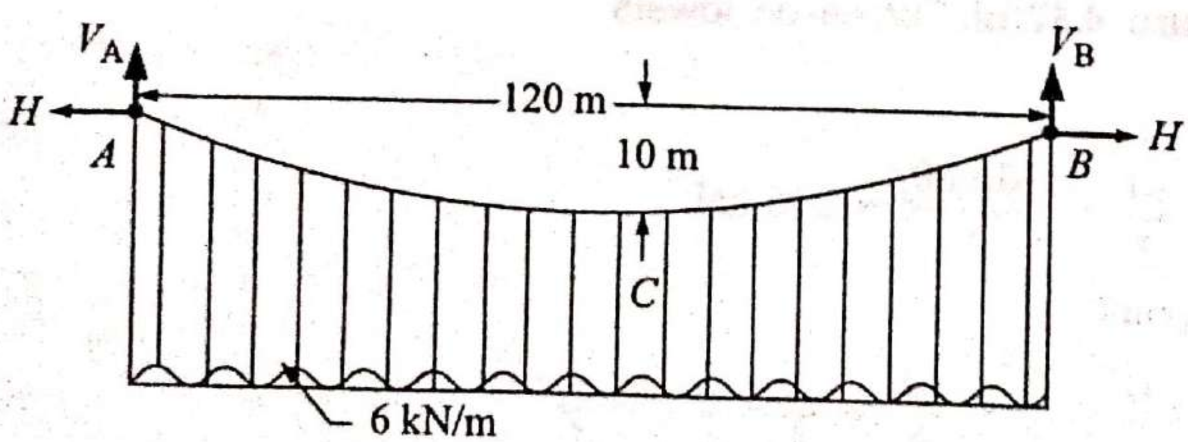


**Figure 2 (a):** Example 2

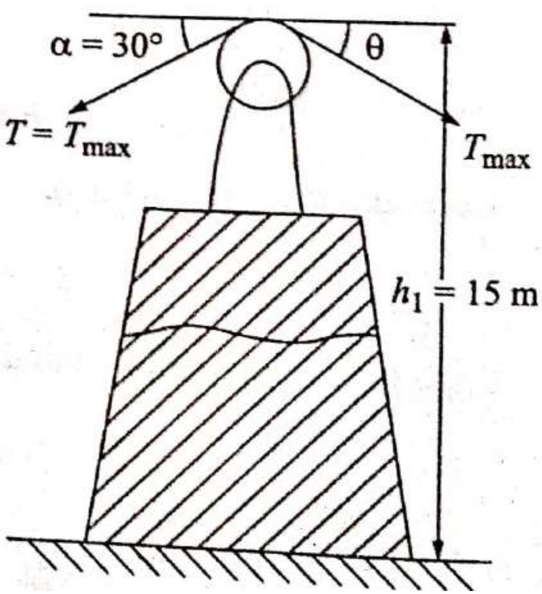


**Figure 2 (b):** Forces on pier

- Due to symmetry, Reaction at A and B is

$$V_A = V_B = \frac{wl}{2} = \frac{6 \times 120}{2} = 360 \text{ kN}$$

- Taking moment about central point C,

$$H \times h - \frac{wl}{2}\times\frac{l}{2} + \frac{wl}{2}\times\frac{l}{4} = 0$$

$$H = \frac{wl^2}{8h} = \frac{6\times120\times120}{8\times10} = 1080 \text{ kN}$$

$$T_{\max} = \sqrt{V^2 + H^2} = \sqrt{360^2 + 1080^2} = 1138.42 \text{ kN}$$

$$\cos\theta = \frac{H}{T_{max}} = \frac{1080}{1138.42}$$

$$\theta = 18.435^\circ$$

- Horizontal force transferred to pier = $T_{\max}$ (cos 18.435° - cos 30°)
- Horizontal force transferred to pier = 1138.42 (cos 18.435° - cos 30°) = 94.099 kN
- Maximum bending moment in the pier = $H\, h_1$ = 94.099 × 15 = 1411.49 kNm
- Vertical force on the pier = $T$ (sin θ + sin α) = 1138.42 (sin 18.435° + sin 30°)

  = 929.21 kN

Q3. A light flexible cable 18 m long is supported at two ends at the same level. The supports are 16 m apart. The cable is subjected to uniformly distributed load of 1 kN/m of horizontal length over its entire span. Determine the reactions developed at the support.

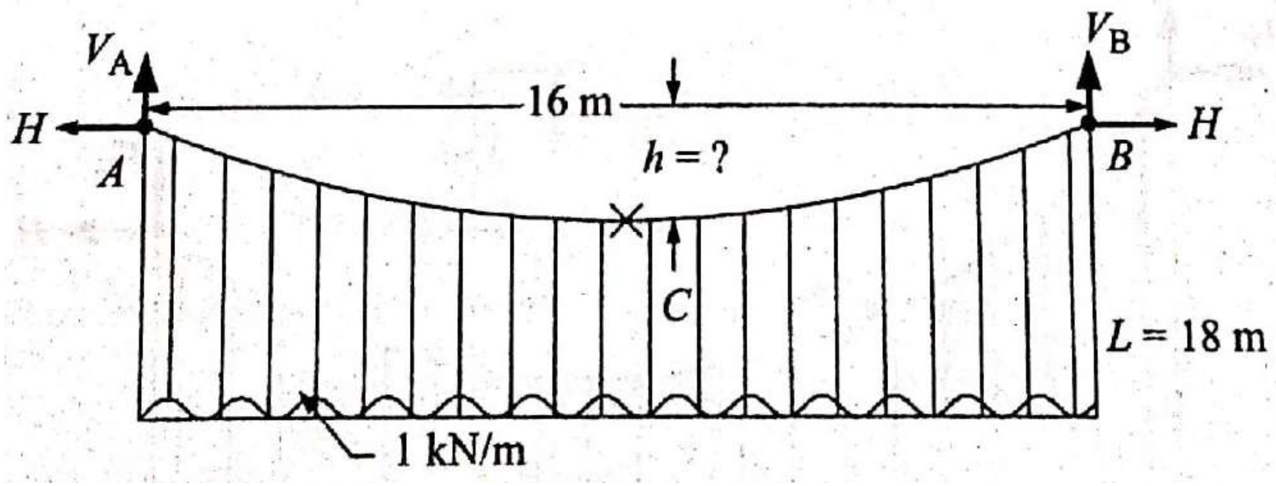


**Figure 3:** Example 3

- The length of the cable = $L = l + \frac{8}{3} \times \frac{h^2}{l}$

  where, $l$ = span, $h$ = central dip

- Applying this we get, $18 = 16 + \frac{8}{3} \times \frac{h^2}{16}$

  or $h = 3.464$ m

- Let $H$ = horizontal force, and $V_A$ = vertical reaction at $A$

$$V_A = \frac{wl}{2} = \frac{1\times16}{2} = 8\text{ kN}$$

$$H \times 3.464 = \frac{wl^2}{8} = \frac{1\times162}{8}$$

$$H = 9.237\text{ kN}$$

$$T_{max} = \sqrt{V^2 + H^2} = = \sqrt{8^2 + 9.237^2} = 12.220\text{ kN}$$

- Inclination 'θ' with horizontal

$$T_{max} \cos\theta = H$$

$$\theta = \cos^{-1}\left(\frac{H}{T_{max}}\right) = \cos^{-1}\left(\frac{9.237}{1.220}\right) = 40.898^\circ$$

# Thanks

# Suspension bridge with three-hinged stiffening Girder

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# Introduction

- Consider the suspension cable stiffened with a three-hinged girder as shown in Figure 1.
- The girder can be a heavy beam or a truss which has three hinges two at the ends and one at the centre.
- The cable and the girder are connected by a number of hangers/suspenders.
- Since the number of suspenders are very large, the load on cable or girder, due to the forces in the suspenders, may be taken as uniformly distributed load.

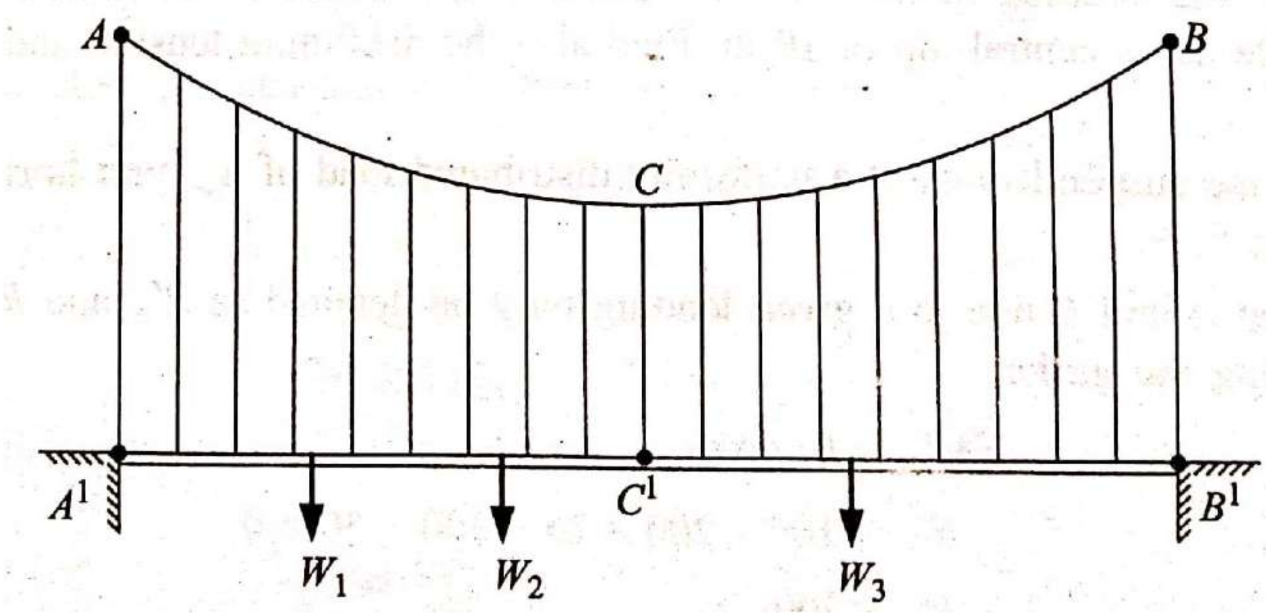


**Figure 1**: Typical suspension bridge with three-hinged stiffening girder

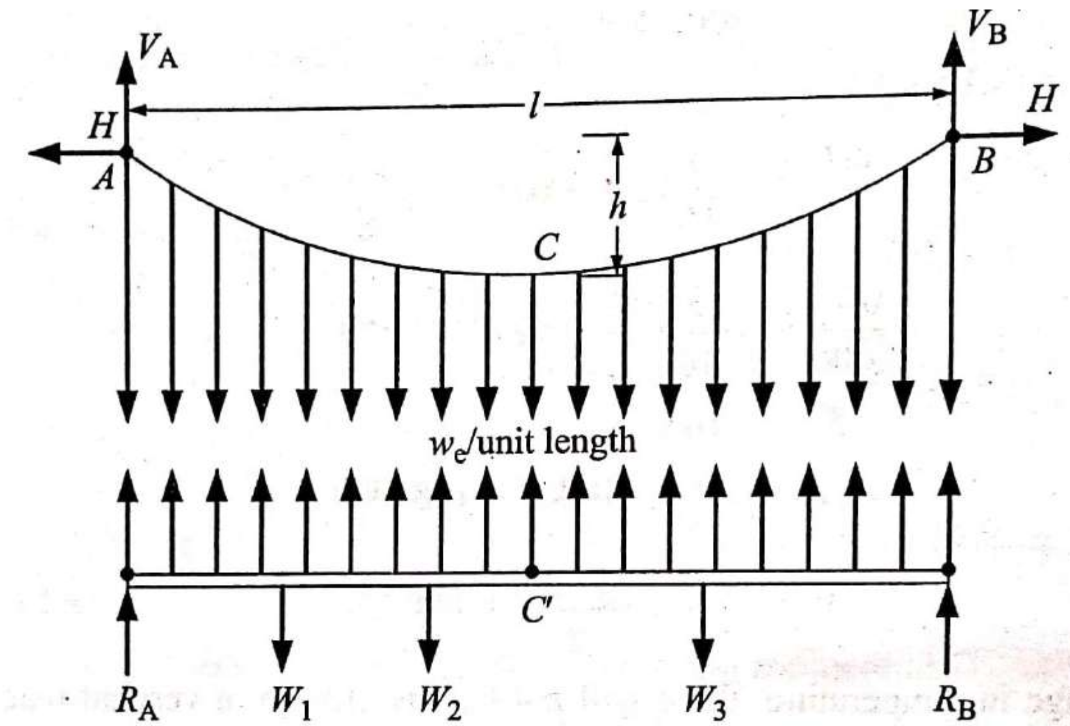


**Figure 2**: Free Body Diagram of cable and girder

- Let the uniformly distributed load = $w_e$ per unit horizontal length
- Let C' is the central hinge of girder
- Let the uniformly distributed load $w_e$ exerted by suspender on the girder
- **The beam may be analyzed for the given load along with $w_e$**

**Due to $w_e$ alone,**

- The bending moment at section x-x = $\frac{w_e x}{8}(l - x)$
- Maximum bending moment at C = $\frac{w_e l^2}{8}$ (Hogging moment)
- The shear force at section x-x = $-w_e\left(\frac{l}{2} - x\right)$
- The cable can be analyzed for the uniformly distributed load $w_e$

Q. A three-hinged stiffening girder of a suspension bridge of span 100 m is subjected to two point loads of 200 kN and 300 kN at the distance of 25 m and 50 m from the left end. Find the shear force and bending moment for the girder at a distance 30 m from the left end. The supporting cable has a central dip of 10 m. Find also the maximum tension and its slope in the cable.

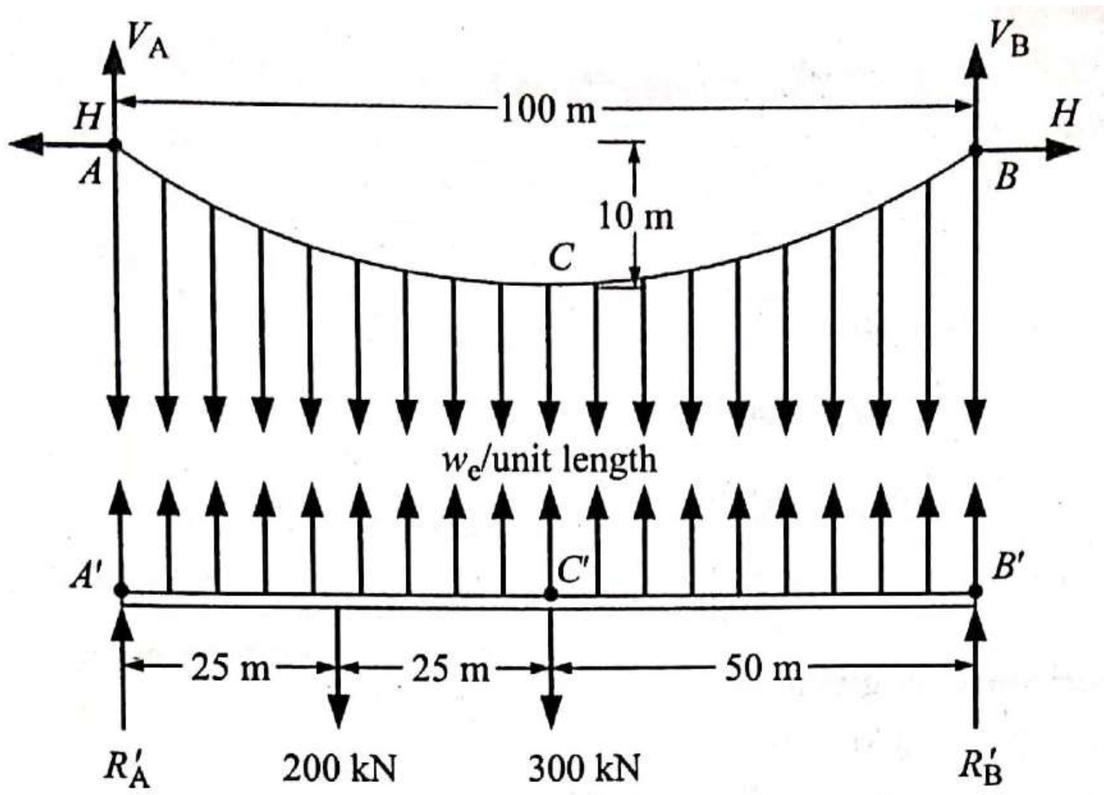


**Figure 3**

- Let the suspenders exert a uniformly distributed load of $w_e$ per unit horizontal length as shown in Figure 3.
- Reactions at $A$ and $B$ due to a given loading only be denoted as $R'_A$ and $R'_B$, respectively.
- To find out the reactions in the girder, take $\sum M'_A = 0$

$$R'_B \times 100 - 200 \times 25 - 300 \times 50 = 0$$

$$R'_B = 200 \text{ kN}$$

- Using $\sum V = 0$, Find $R'_A$

$$R'_A + R'_B = 200 + 300$$

$$R'_A = 300 \text{ kN}$$

- Moment at the central hinge of girder ($C'$) = 0 ($\sum M_{c'} = 0$)
- Bending moment due to given loading + Bending moment due to $w_e$ = 0

$$R'_B \frac{l}{2} - \frac{w_e l^2}{8} = 0$$

$$200 \times \frac{100}{2} = \frac{w_e \times 1002}{8} \text{ (After simplification)}$$

$$w_e = 8 \text{ kN/m}$$

- Shear Force and Bending moment at a distance 30 m from the left end,
- SF = SF due to given loading + SF due to $w_e$

$$SF = R'_A - 200 - w_e\left(\frac{100}{2} - 30\right)$$

$$= 300 - 200 - 8(50-30)$$

$$= -60 \text{ kN} = 60 \text{ kN } (\uparrow)$$

- BM = Moment due to given loading + Moment due to $w_e$

$$BM = 300 \times 30 - 200 \times 5 - \frac{w_e x}{8}(l - x)$$

$$BM = 300 \times 30 - 200 \times 5 - \frac{8 \times 30}{8}(100 - 30) = -400 \text{ kNm}$$

- BM = 400 kN (Hogging)
- For the analysis of cable
- First finding vertical reaction at A and B i.e. $V_A$ and $V_B$, Take $\sum V = 0$

$$V_A = V_B = w_e \times \frac{l}{2} = 8 \times \frac{100}{2} = 400 \text{ kN}$$

- For getting horizontal reaction, taking moment about $C$, we get

$$H \times h = \frac{w_e l}{2} \times \frac{l}{2} - w_e \times \frac{l}{2} \times \frac{l}{4} = \frac{w_e l^2}{8}$$

$$H \times 10 = \frac{w_e l^2}{8} = \frac{8 \times 100^2}{8} = 10000$$

or $\quad H = 1000 \text{ kN}$

- Maximum Tension in the cable = $T_{max} = \sqrt{V^2{}_A + H^2}$

$$T_{max} = \sqrt{400^2 + 1000^2} = 1077.033 \text{ kN}$$

- Its slope to horizontal is $T_{max} \cos\theta = H$

$$\theta = \cos^{-1}\left(\frac{1000}{1077.033}\right) = 21.80^\circ$$

# Thanks