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Arches and Types

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Introduction

- In case of beams supporting uniformly distributed load, the maximum bending moment increases with the square of the span and hence they become uneconomical for long span structures. In such situations arches could be advantageously employed, as they would develop horizontal reactions, which in turn reduce the design bending moment.

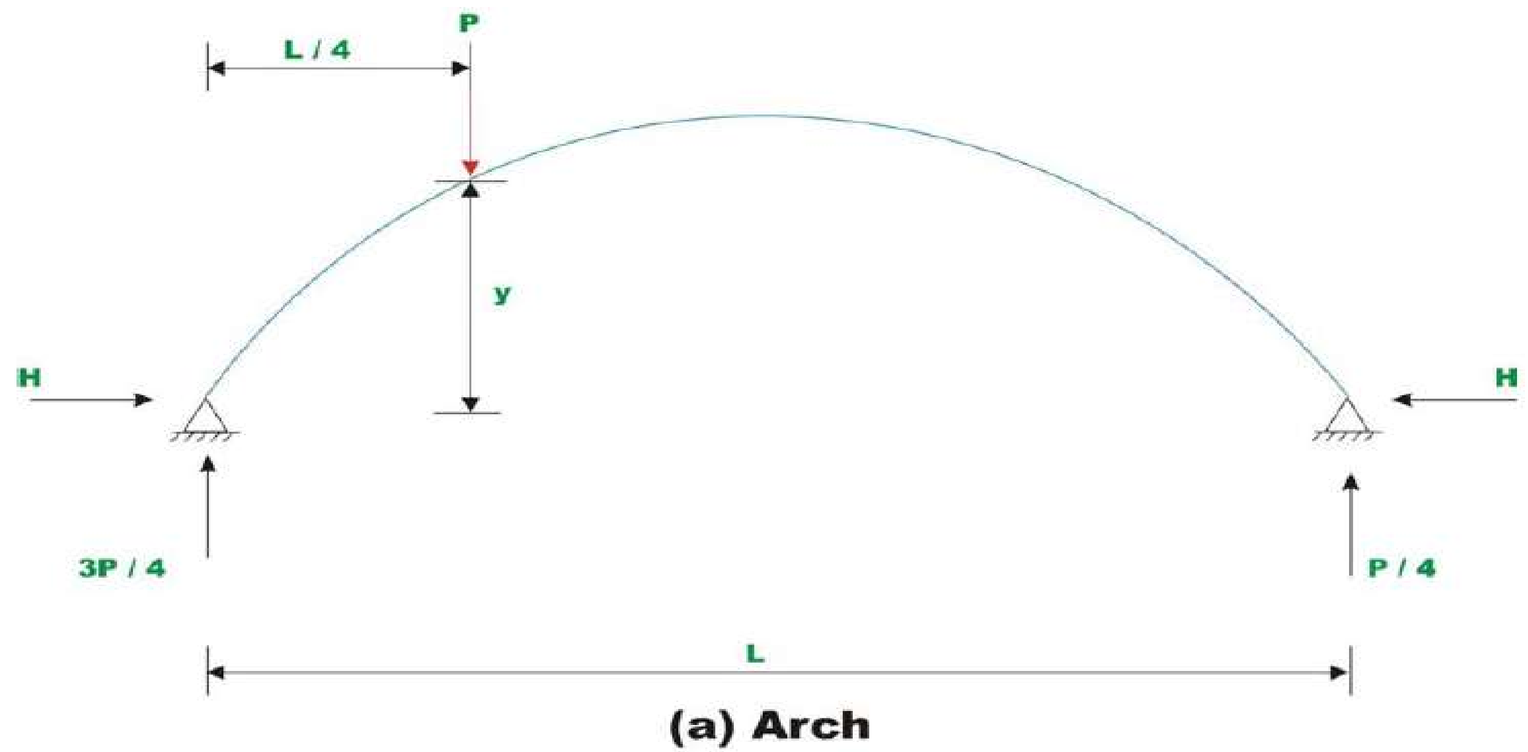


Figure 1: Beam and Arch comparison

- For example, in the case of a simply supported beam shown in Figure 1, the bending moment below the load is $3PL/16$.

- Now consider a two hinged symmetrical arch of the same span and subjected to similar loading as that of simply supported beam. The vertical reaction could be calculated by equations of statics. The horizontal reaction is determined by the method of least work. Now the bending moment below the load is $(3PL/16 - Hy)$.

- It is clear that the bending moment below the load is reduced in the case of an arch as compared to a simply supported beam.

- It is observed that, the cable takes the shape of the loading and this shape is termed as funicular shape.

- If an arch will constructed in an inverted funicular shape then it would be subjected to only compression for those loadings for which its shape is inverted funicular.

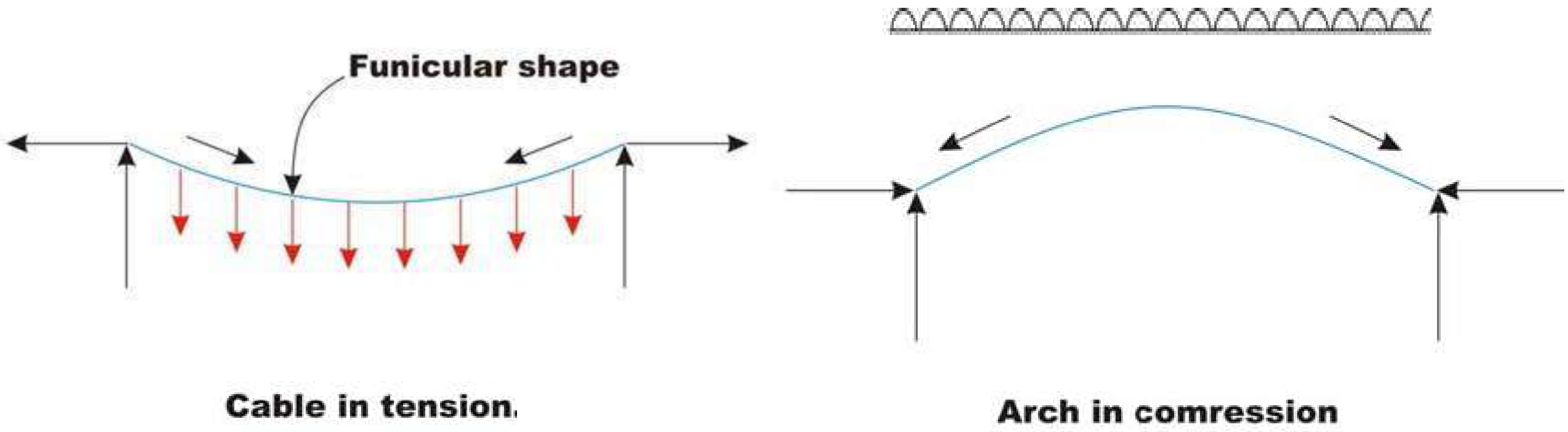


Figure 2: Cable and Arch structure

- Since in practice, the actual shape of the arch differs from the inverted funicular shape or the loading differs from the one for which the arch is an inverted funicular.
- Arches are also subjected to bending moment in addition to compression.
- As arches are subjected to compression, it must be designed to resist buckling.

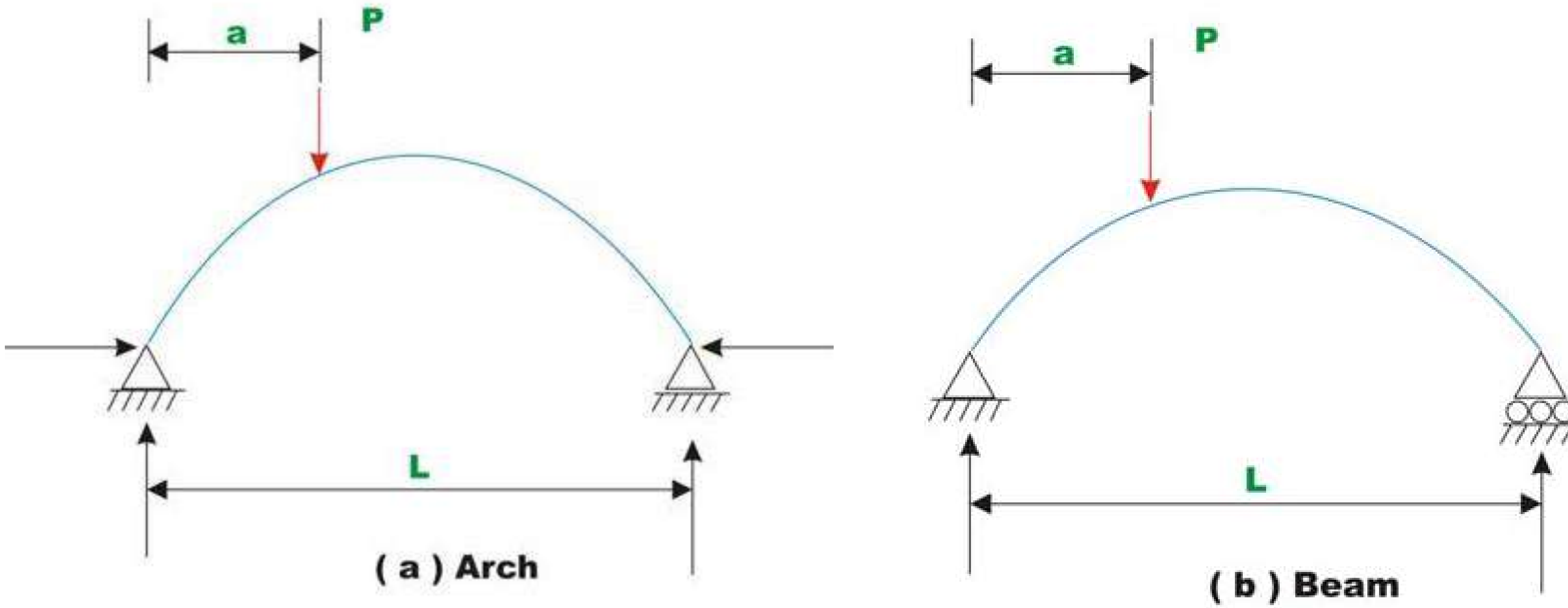


Figure 3: (a) Arch **(b)** Beam

- A structure is classified as an arch not based on its shape but the way it supports the lateral load. Arches support load primarily in compression.
- For example in Figure 3(b), no horizontal reaction is developed. Consequently bending moment is not reduced. It is important to appreciate the point that the definition of an arch is a structural one, not geometrical.

- For large spans like bridges, arches are provided in stead of beams. Arches are curved beams that transfer loads to their planes.
- Arches transfer loads to abutments at springing points. Hinges may be provided at springing points.
- The top most point is called crown.
- The height of the crown above support level is called rise.

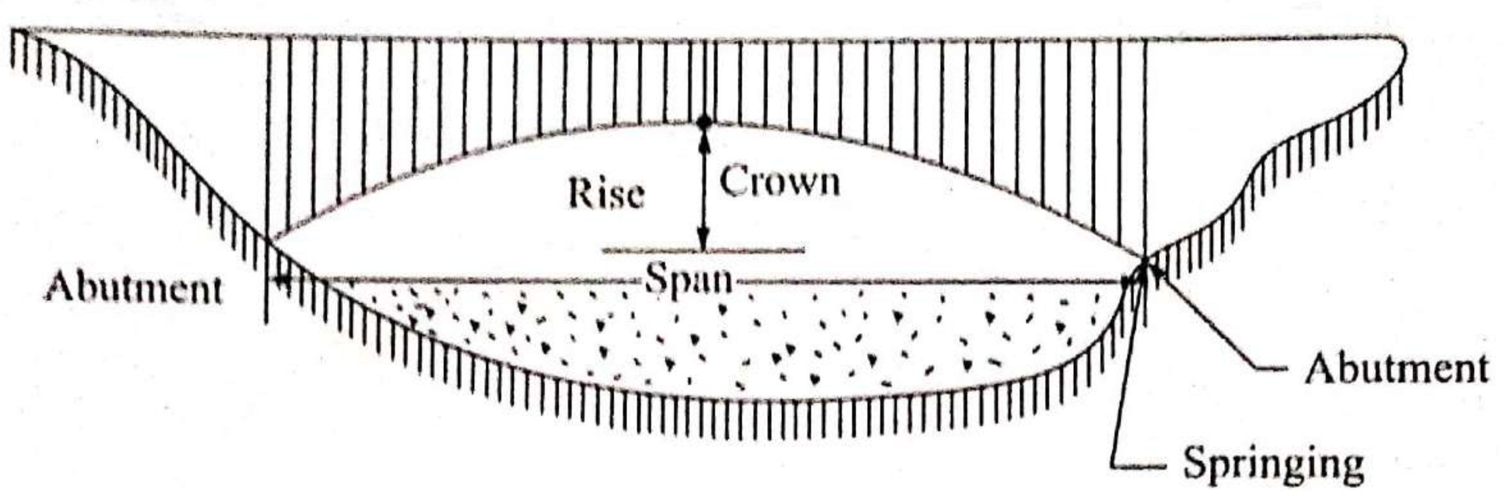


Figure 4: Typical arch bridges

- Due to curved nature of arches, they give rise to horizontal forces.
- Abutments are designed for horizontal forces also.
- Any section in the arch is subjected to normal thrust, radial shear and bending moment.
- Loads in the arches partly transferred by axial compression and partly by flexure.

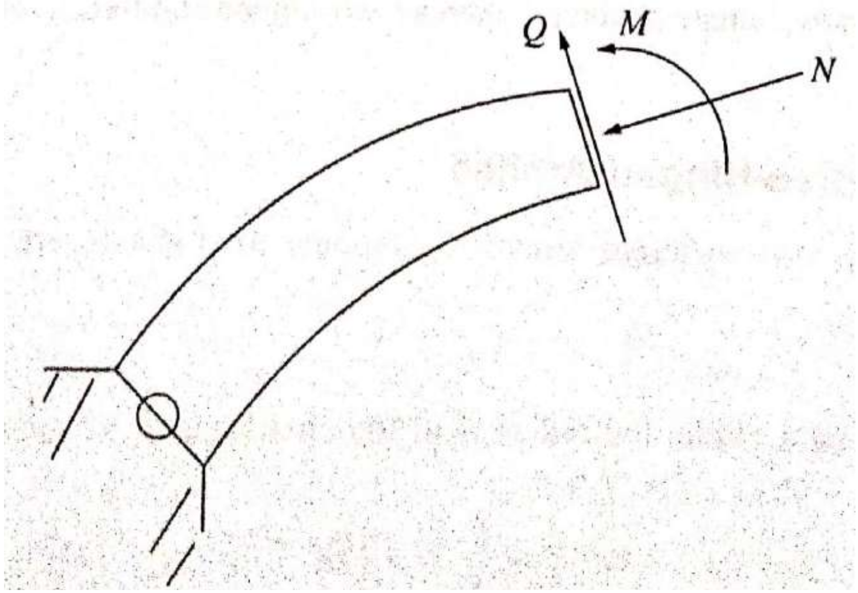


Figure 5: Arch with section showing forces

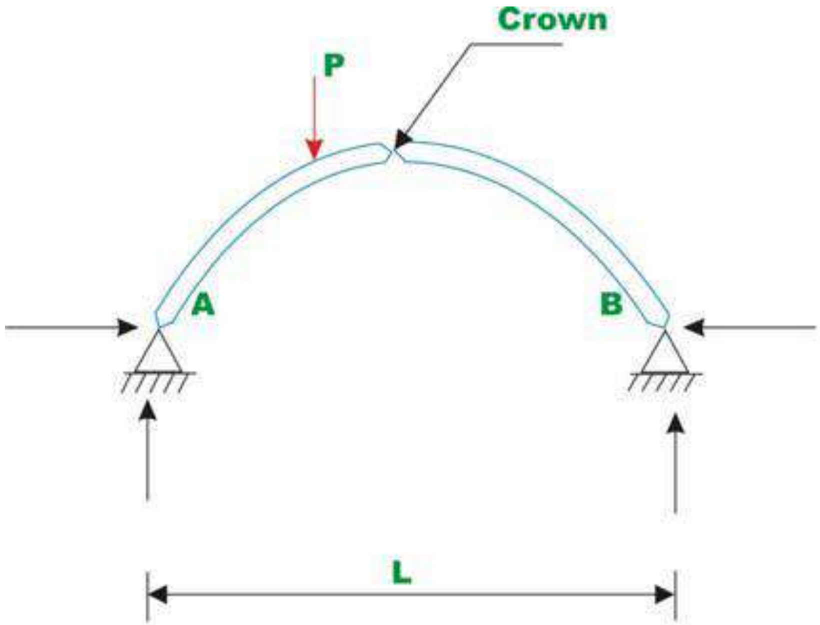
Types of Arches

There are mainly three types of arches that are commonly used in practice:

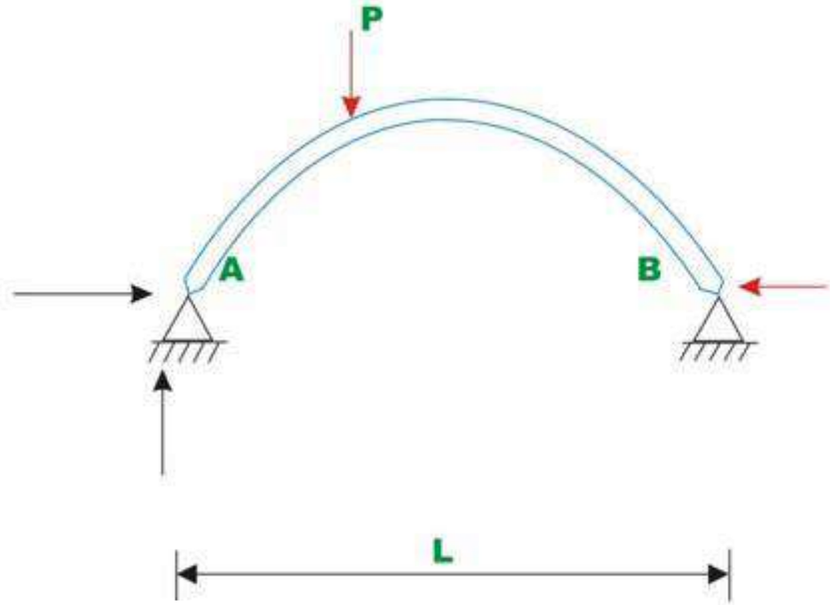
- Three hinged arch,
- Two-hinged arch
- Fixed-fixed arch or Hingeless arch

- Three-hinged arch is statically determinate structure and its reactions / internal forces are evaluated by static equations of equilibrium.

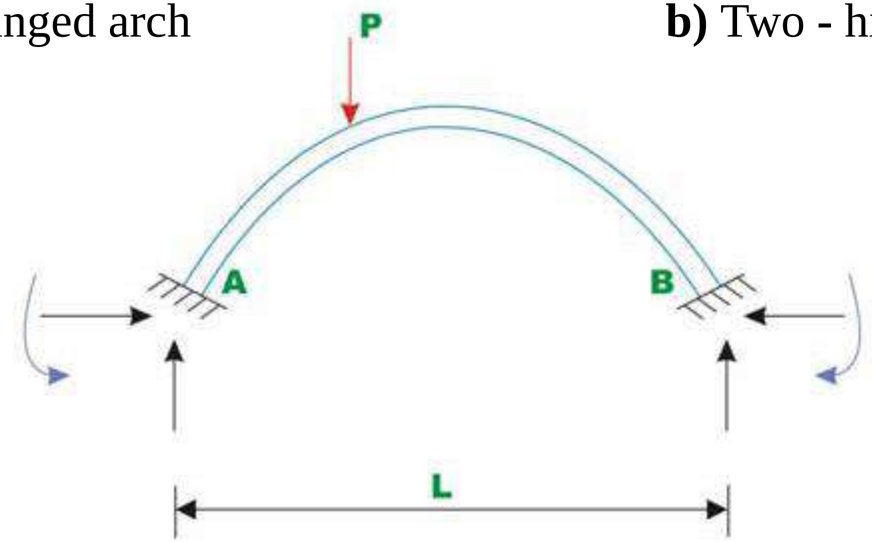
- Two-hinged arch and fixed-fixed arch are statically indeterminate structures. The indeterminate reactions are determined by the method of least work or by the flexibility matrix method.



a) Three – hinged arch



b) Two - hinged arch



c) Fixed – Fixed arch

Figure 6: Types of arches

Thanks

Three-Hinged Arches

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Introduction

- Three hinged arches is statically determinate structure and its reaction and internal forces are evaluated by static equations of equilibrium.
- Arches transfer loads to abutments at springing points.
- The horizontal distance between one support to another support is called span.
- The Top most point of the arch is called crown.
- The height of the crown above the support level is called rise.

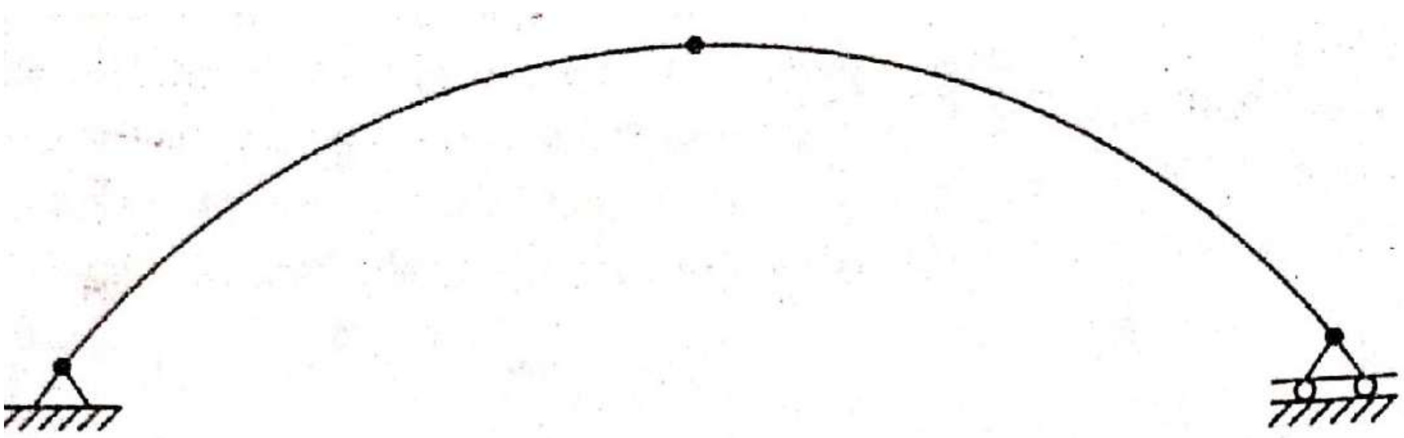

Figure 1:Three-hinged arch

Types of three-hinged arches

- There are two types of three-hinged arches according to the shape of the structure.
- Circular three-hinged arch
- Parabolic three-hinged arch

Circular three-hinged arch

- From the property of a circle, the radius 'R' of the circular arch of span 'L' and rise 'h'

$$\frac{L}{2} \times \frac{L}{2} = h\,(2R - h)$$

$$R = \frac{L^2}{8h} + \frac{h}{2}$$

Circular three-hinged arch

- Taking '*A*' as origin at support, the coordinates of any point '*D*' on the arch is:

$$x = \left[\frac{L}{2} - R\sin\theta\right]$$

$$y = R\cos\theta - (R - h)$$

$$= h - R(1 - \cos\theta)$$

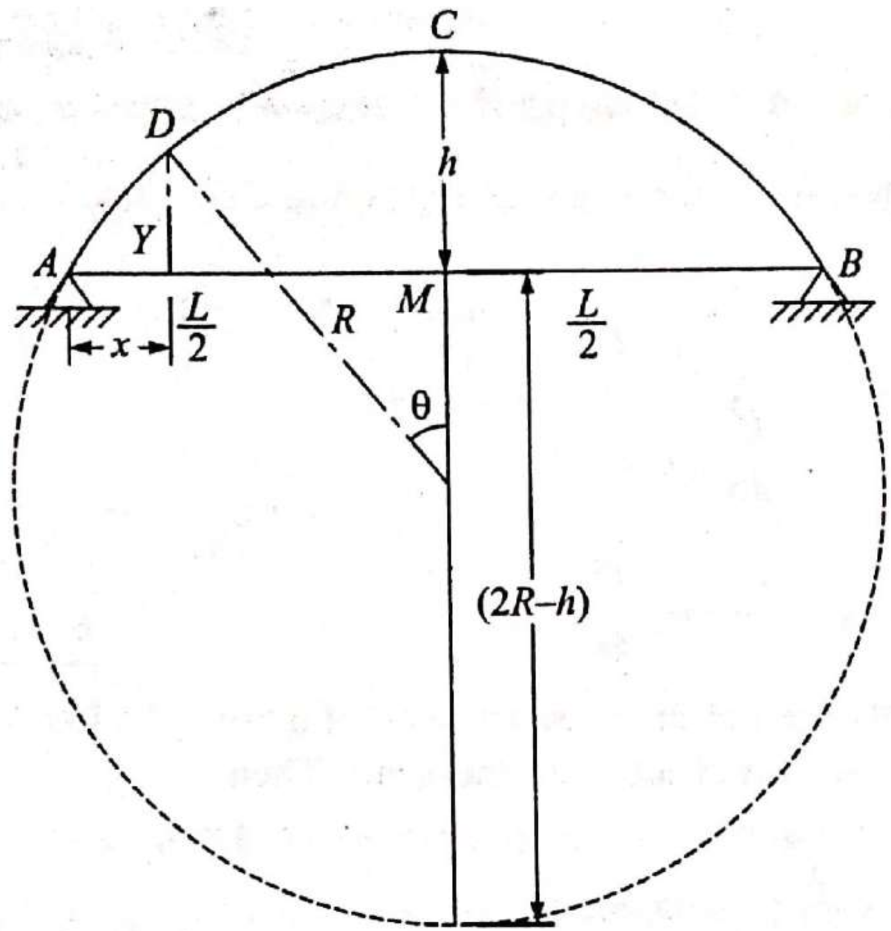

Figure 2: Coordinates of '*D*'

Parabolic three-hinged arch

- In the case of a parabolic arch, taking the springing point as the origin its equation = $y = \frac{4hx}{L^2}(L\text{-}x)$

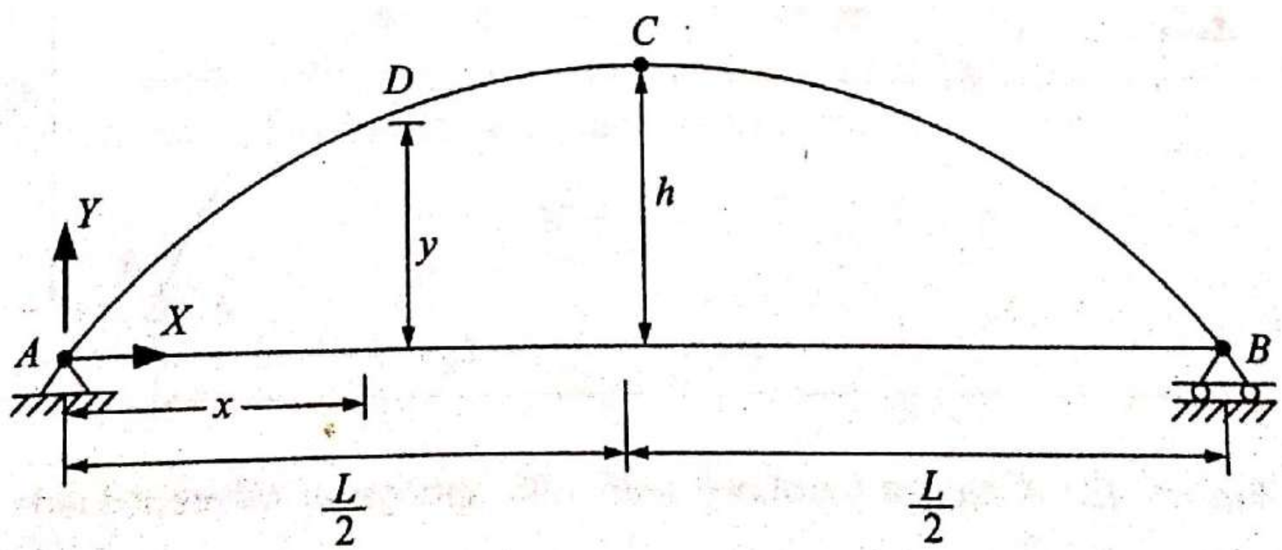

Figure 3: Parabolic arch with origin at '*A*'

- If the crown is taken as the origin, the equation of the parabolic curve $= \frac{x^2}{y} = a$, where a = Constant

If the springing point are at the same level:

$$x = \frac{L}{2}, y = h,$$

$$\frac{L^2}{4h} = a$$

Hence, the equation is $$\frac{x^2}{y} = \frac{L^2}{4h}$$

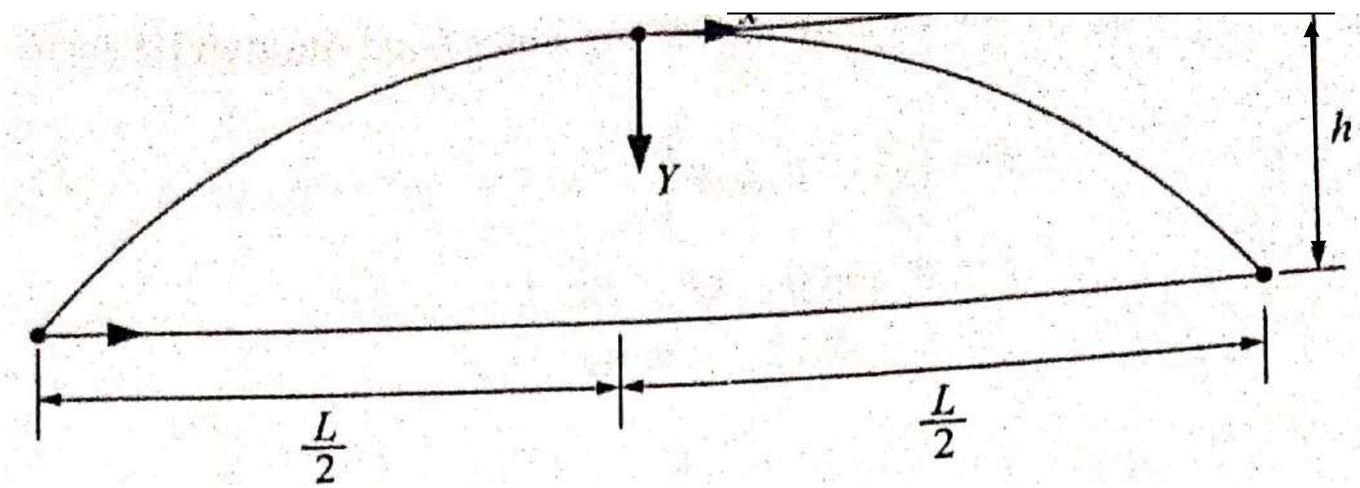

Figure 4: Parabolic arch with origin at crown

If the springing points are not at the same level:

- Let h_1 and h_2 is the depth of the abutments from the crown and let 'L' is the span.

$$\frac{x^2}{y} = \text{constant}$$

$$\frac{x}{\sqrt{y}} = \text{constant}$$

Applying this equation to points A and B, we get

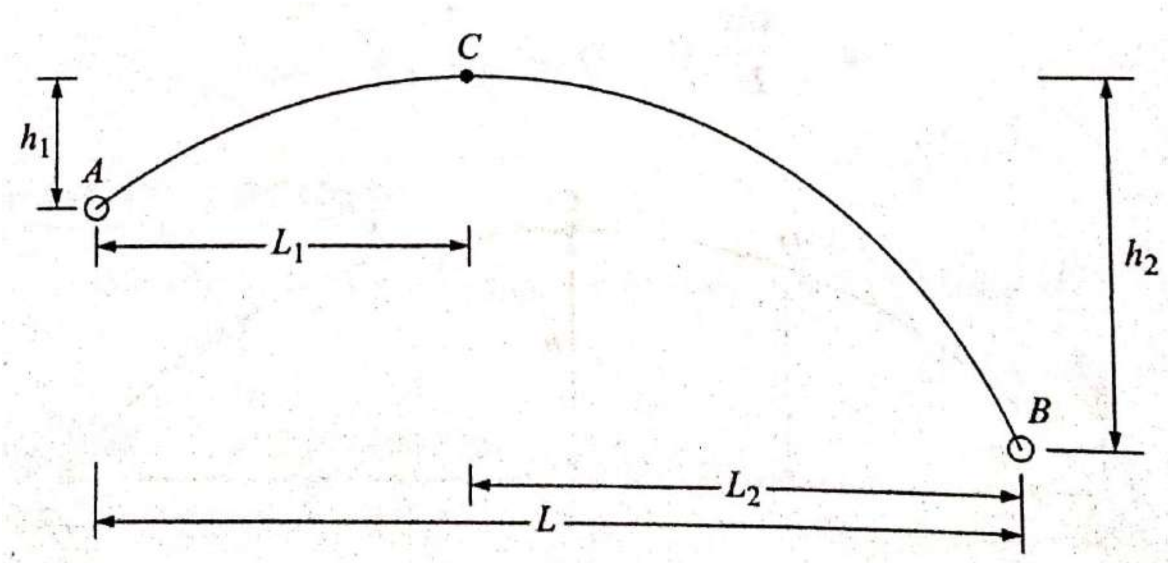

Figure 5: Parabolic arch with springing at different level

$$\text{Constant} = \frac{L_1}{\sqrt{h_1}} = \frac{L_2}{\sqrt{h_2}} = \frac{L_1 + L_2}{\sqrt{h_1} + \sqrt{h_2}} = \frac{L}{\sqrt{h_1} + \sqrt{h_2}}$$

$$L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}}$$

$$L_2 = \frac{L\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}}$$

Analysis for Static Loads

- Consider a three-hinged arches as shown in Figure 6.
- The ends are hinged there is two reaction components at each end namely vertical and horizontal.
- There are four reaction components i.e. V_A, H_A, V_B and H_B.

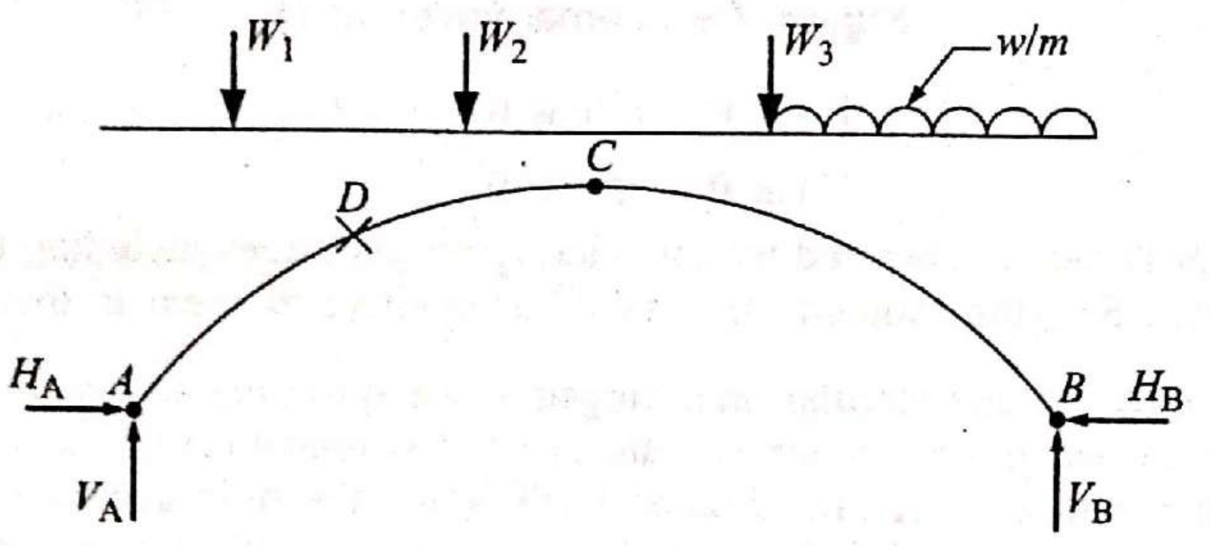

Figure 6: Three-hinged arch with load

- For any plane structure, there are three independent equilibrium equations, i.e.

$$\sum F_{\mathrm{H}} = 0$$

$$\sum F_{\mathrm{V}} = 0$$

$$M_{\mathrm{A}} \text{ or } M_{\mathrm{B}} = 0$$

In this case, the fourth equation is $M_c = 0$, since C is a hinge

- If horizontal load is not acting, $H_{\mathrm{A}} = H_{\mathrm{B}} = H$
- In this case the following equations are used

$$\sum F_{\mathrm{V}} = 0$$

$$M_{\mathrm{A}} \text{ or } M_{\mathrm{B}} = 0$$

$$M_{\mathrm{C}} = 0$$

- Now, consider a section at *D*
- Let *V* = Vertical shear, *Q* = Radial shear, *N* = Normal thrust

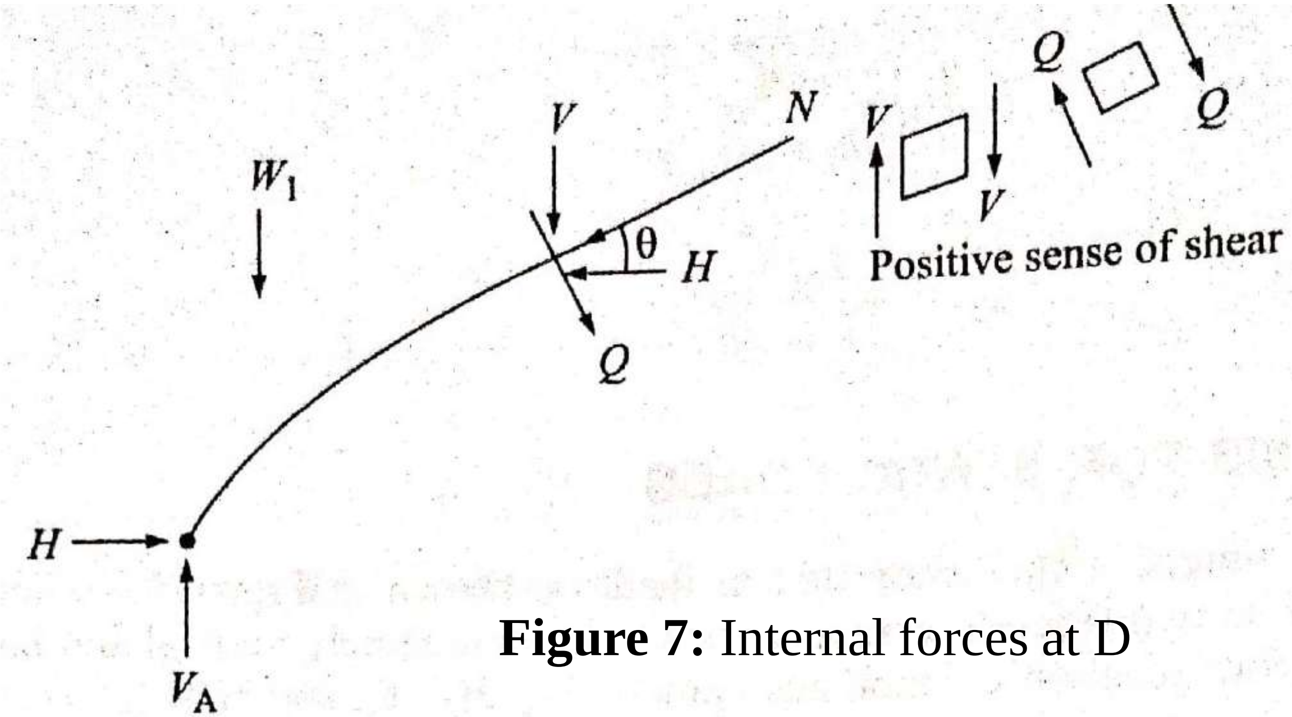

Figure 7: Internal forces at D

- In the above Figure 7, All these forces are indicate positive sign, The normal thrust and radial shear is calculated by

$$N = V \sin\theta + H \cos\theta$$

$$Q = V \cos\theta - H \sin\theta$$

Thanks

Three-Hinged Arches Solved Problems

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Q1. A three-hinged circular arch hinged at the springing and crown points has a span of 40 m and a central rise of 8 m. It carries a uniformly distributed load 20 kN/m over the left-half of the span together with a concentrated load of 100 kN at the right quarter span point. Find the reactions at the supports, normal thrust and shear at a section 10 m from left support.

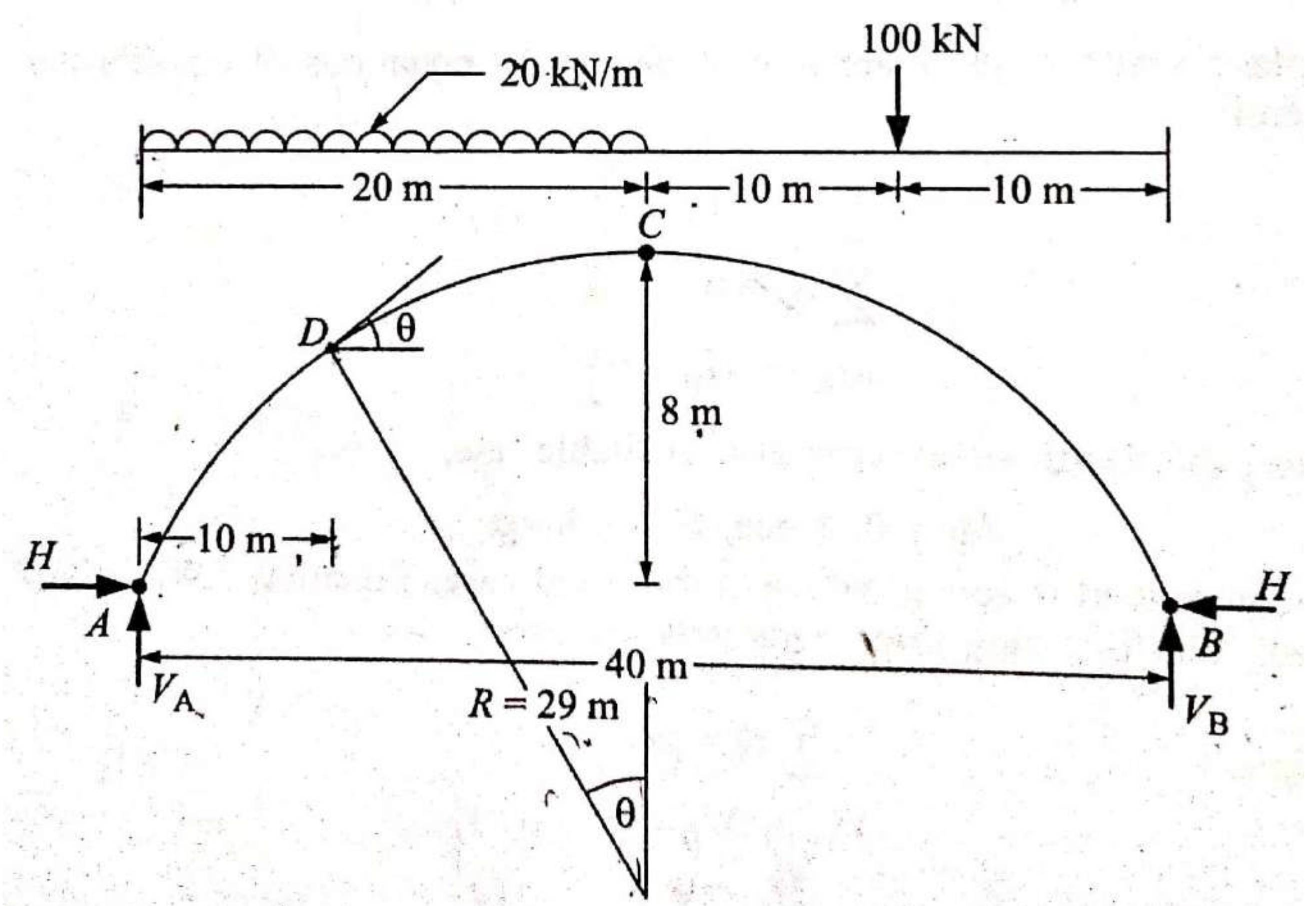


Figure 1

- First find the reactions by using equilibrium equations.
- By taking moment about B and equate to zero i.e. $\sum M_B = 0$

$$V_A \times 40 - 20 \times 20 \times 30 - 100 \times 10 = 0$$

$$V_A = 325 \text{ kN}$$

- Summation of vertical force is zero i.e. $\sum F_V = 0$

$$V_A + V_B = 20 \times 20 + 100$$

$$V_B = 500 - 325 = 175 \text{ kN}$$

- Since, C is hinged, at hinge $M_C = 0$ (Take moment for right portion or left portion of hinge and equate the moment to zero, and find H)

$$V_B \times 20 - 100 \times 10 - H \times 8 = 0$$

$$175 \times 20 - 100 \times 10 - H \times 8 = 0$$

$$H = 312.5 \text{ kN}$$

- Let D be the point 10 m from the left support where the normal thrust and radial shear are to be found. Now from the property of circles.

$$h\,(2R - h) = \frac{L}{2} \times \frac{L}{2}$$

$$8\,(2R - 8) = \frac{40}{2} \times \frac{40}{2} = 400$$

$$R = 29 \text{ m}$$

$$\text{Slope at } D = \theta = \sin^{-1}\left(\frac{10}{R}\right) = \sin^{-1}\left(\frac{10}{29}\right)$$

$$\theta = 20.171^\circ$$

- Vertical shear at D, $V = V_A - 20 \times 10$

  $V = 325 - 200 = 125$ kN

- Normal thrust at D, $N = V \sin\theta + H \cos\theta$

  $N = 125 \sin 29.171^\circ + 312.5 \cos 20.171^\circ = 336.437$ kN

- Radial shear at D, $Q = V \cos\theta - H \sin\theta$

  $Q = 125 \cos 20.171^\circ - 312.5 \sin 20.171^\circ = 9.575$ kN

Q2. A circular arch to span 20 m with a central rise 5 m is hinged at the crown and springing. It carries a point load of 100 kN at 6 m from the left support. Calculate 1) The reactions at the supports, 2) The reactions at crown, 3) Moment at 5 m from the left support.

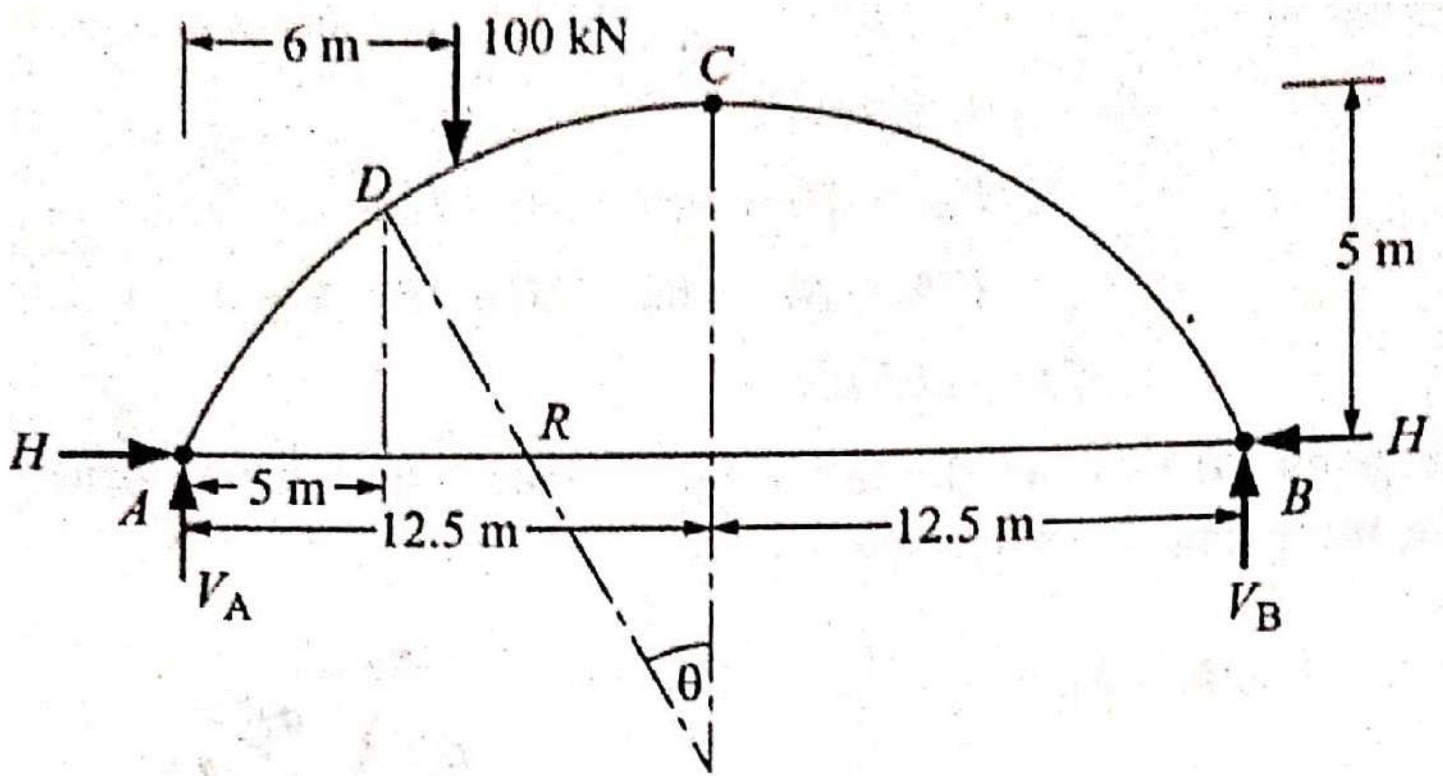


Figure 1

- First find the reactions using equilibrium equations
- Taking the moment about *B*, we get

$$V_A \times 25 = 100 \times (25 - 6)$$

$$V_A = 76 \text{ kN}$$

- Taking summation of vertical force is zero ($\sum V = 0$), we get

$$V_A + V_B = 100$$

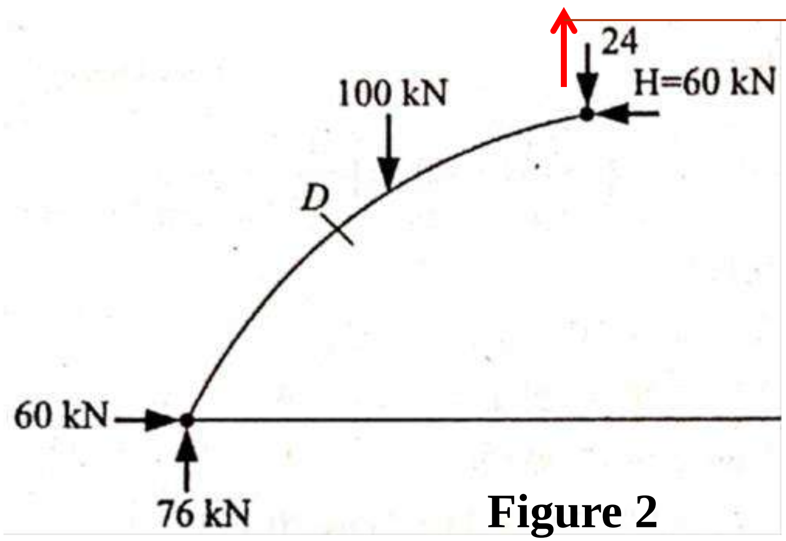
$$V_B = 100 - 76 = 24 \text{ kN}$$

- For horizontal reaction, taking moment about *C*, we get

$$24 \times 12.5 - H \times 5 = 0$$

$$H = 60 \text{ kN}$$

- Considering the equilibrium of the left half of the arch, the reaction at crown can be found from Figure 2.



Correct direction of V_C = 24 kN

Figure 2

- By taking summation of horizontal force zero, H = 60 kN (←)
- By taking summation of vertical force zero,
- $76 - 100 - V_C = 0$
- $V_C = 76 - 100 = -24$ kN = 24 kN (upward direction) (↑)

Moment at 5 m from the left support:

- From the property of circle, we get

$$H\,(2R - h) = \frac{L}{2} \times \frac{L}{2}$$

$$5\,(2R - 5) = \frac{25}{2} \times \frac{25}{2}$$

$$R = 18.125 \text{ m}$$

- From Figure 1, we can find, $R \sin\theta = 12.5 - 5 = 7.5$

$$\sin\theta = \frac{7.5}{18.125} = 0.4138$$

$$\theta = 24.443^\circ$$

$$y_D = h - R\,(1 - \cos\theta) = 5 - 18.125\,(1 - \cos 24.443^\circ)$$

$$y_D = 3.375 \text{ m}$$

Moment at $D = M_D = V_A \times 5 - H \times y_D$

$$M_D = 76 \times 5 - 60 \times 3.375 = 177.5 \text{ kNm}$$

Q3. A three-hinged parabolic arch hinged at the supports and at the crown has a span of 24 m and a central rise of 4m. It carries a concentrated load of 50 kN at 18 m from left support and a uniformly distributed load of 30 kN/m over the left half portion. Determine the moment, thrust and radial shear at a section 6 m from the left support.

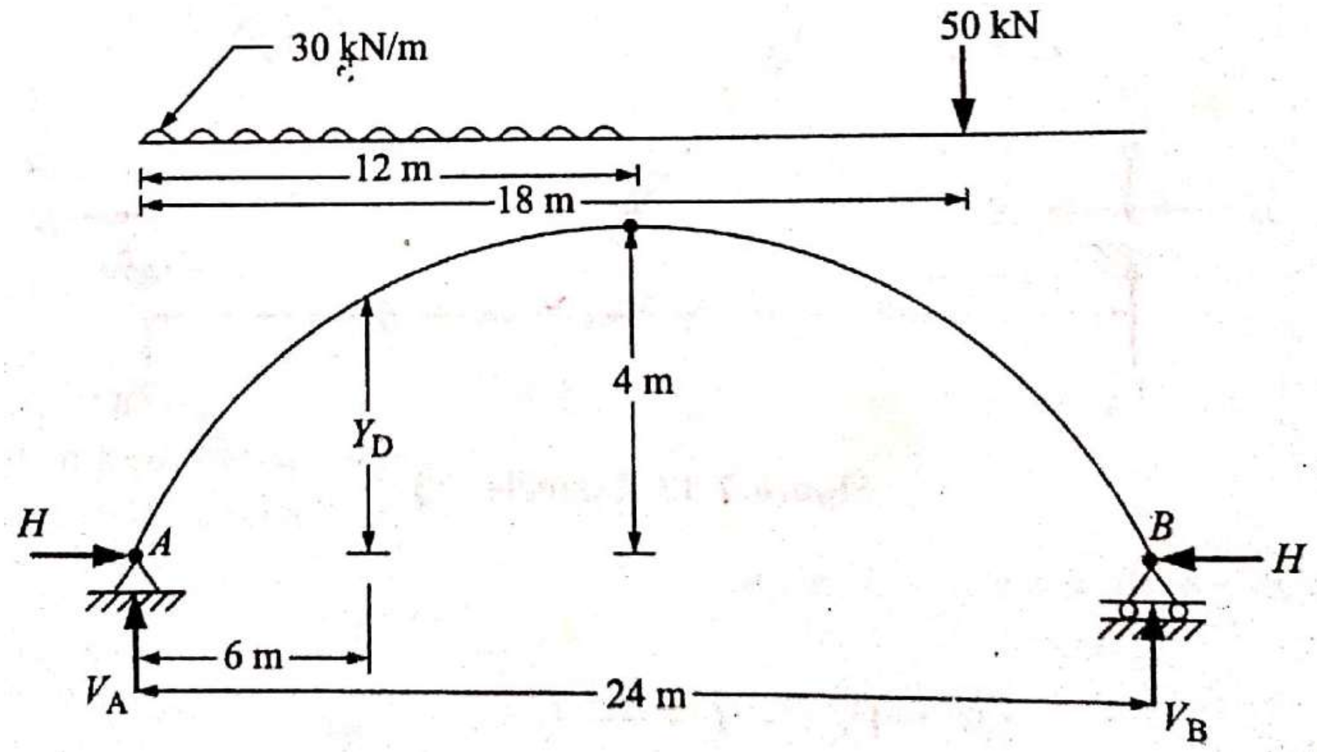


Figure 1

- First find the Reactions, By taking moment about *B*, we get

$$V_A \times 24 - 30 \times 12 \times 18 - 50 \times 6 = 0$$

$$V_A = 282.50 \text{ kN}$$

- Then by taking summation of vertical force is zero, we get

$$V_A + V_B = 30 \times 12 + 50$$

$$V_B = (30 \times 12 + 50) - 282.50 = 127.5 \text{ kN}$$

- By taking moment about crown *C* is zero (Right half)

$$V_B \times 12 - H \times 4 - 50 \times 6 = 0$$

$$127.5 \times 12 - H \times 4 - 50 \times 6 = 0$$

$$H = 307.5 \text{ kN}$$

- Moment at a distance 6 m from the left support,

$$M_D = V_A \times 6 - H \times y_D - 30 \times \frac{6^2}{2}$$

- In the parabolic arch,

$$y = \frac{4hx\,(L - x)}{L^2}$$

- Therefore, at x = 6 m,

$$y_D = \frac{4 \times 4 \times 6(24-6)}{24^2} = 3 \text{ m}$$

$$M_D = 282.5 \times 6 - 307.5 \times 3 - 30 \times \frac{6^2}{2}$$

$$M_D = 232.5 \text{ kNm}$$

- Vertical shear at D,

$$V_D = V_A - 30 \times 6$$

$$V_D = 282.5 - 30 \times 6 = 102.5 \text{ kN}$$

- To find the θ, differentiating y with respect to x

$$\frac{dy}{dx} = \tan\theta = \frac{4h(L-2x)}{L^2}$$

- Therefore, at x = 6 m, $\tan\theta = \frac{4\times4(24-2\times6)}{24\times24}$

$$\theta = 18.435^\circ$$

- Normal thrust = $N = V\sin\theta + H\cos\theta$

$$N = 102.5\sin 18.435^\circ + 307.5\cos 18.435^\circ$$

$$N = 324.133 \text{ kN}$$

- Radial shear = $Q = V\cos\theta - H\sin\theta$

$$Q = 102.5\cos 18.435^\circ - 307.5\sin 18.435^\circ$$

$$Q = 0$$

Q4. show that the parabolic shape is a funicular shape for a three hinged arch subjected to a uniformly distributed load over to its entire span.

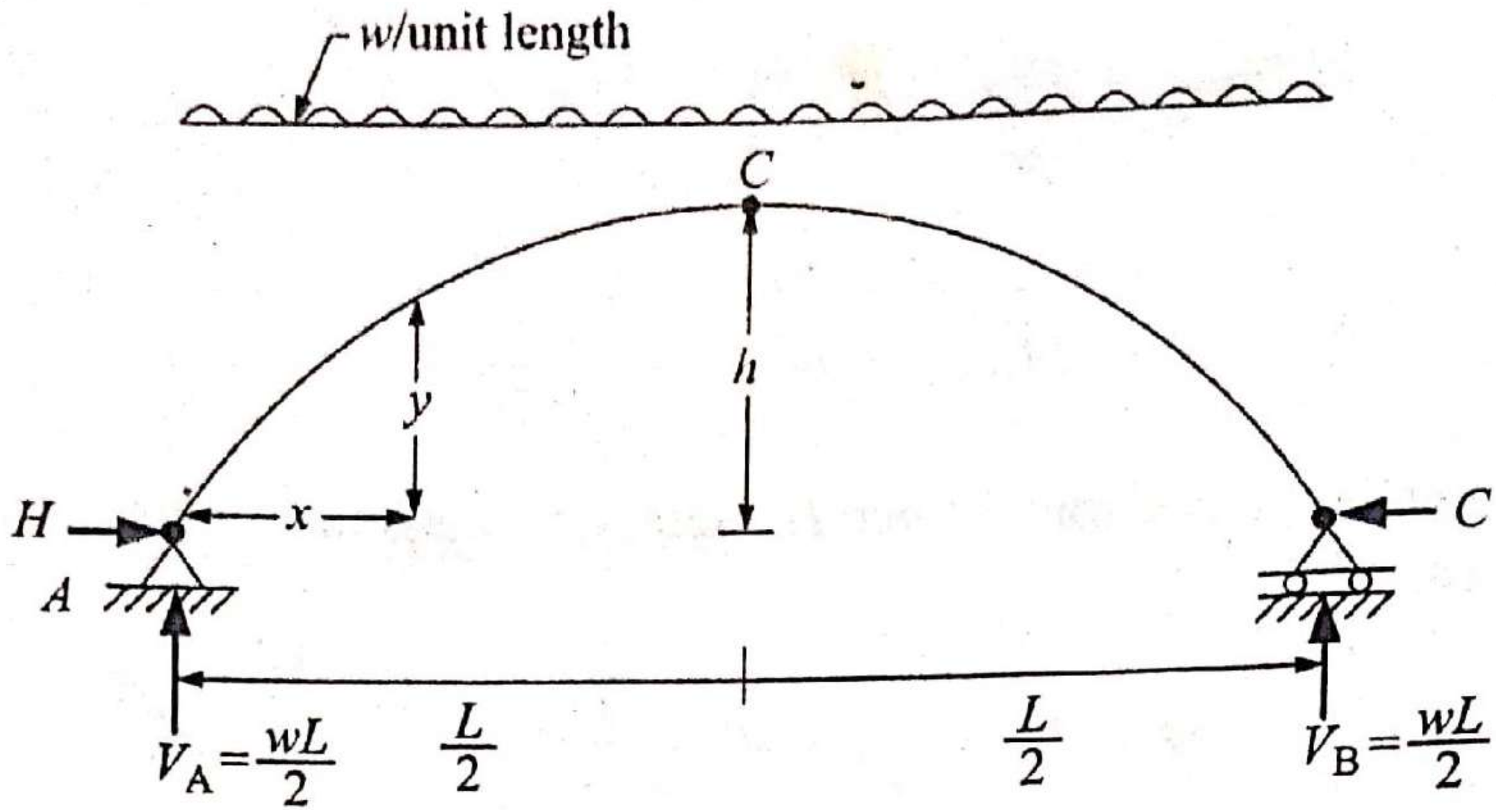


Figure 1

- Let the span of the arch be L and rise be h as shown in Figure.
- Due to symmetric loading, $V_A = V_B = \frac{1}{2} \times$ total load

$$V_A = V_B = \frac{1}{2} \times w \times L = \frac{wL}{2}$$

- Taking moment about C, we get (Left side of the hinge C)

$$M_C = V_A \times \frac{L}{2} - H \times h - w \times \frac{L}{2} \times \frac{L}{4} = 0$$

$$\frac{wL}{2} \times \frac{L}{2} - H \times h - \frac{wL^2}{8} = 0$$

$$H = \frac{wL^2}{8}$$

- Moment at any section distance *x* from *A*,

$$M = V_A \times x - H \times y - \frac{wL2}{2}$$

- But in a parabolic arch,

$$y = \frac{4hx(L-x)}{L^2}$$

$$M = \left(\frac{wL}{2}x\right) - \frac{wL^2}{8} \times \frac{4hx(L-x)}{L^2} - \left(\frac{wL^2}{2}\right)$$

$$M = \left(\frac{wLx}{2}\right) - \frac{w}{2}x(L\text{-}x) - \left(\frac{wL^2}{2}\right) = 0$$

- Thus, for a parabolic arch subjected to a uniformly distributed load over its entire span, the bending moment at any section is zero.
- Hence, the parabolic shape is a funicular shape for a three hinged arch subjected to uniformly distributed load over entire span.

Q5. A symmetric three-hinged parabolic arch of span 36 m and rise 6 m is subjected to a concentrated load of 120 kN at a point 12 m from left support. Draw the bending moment diagram for the arch.

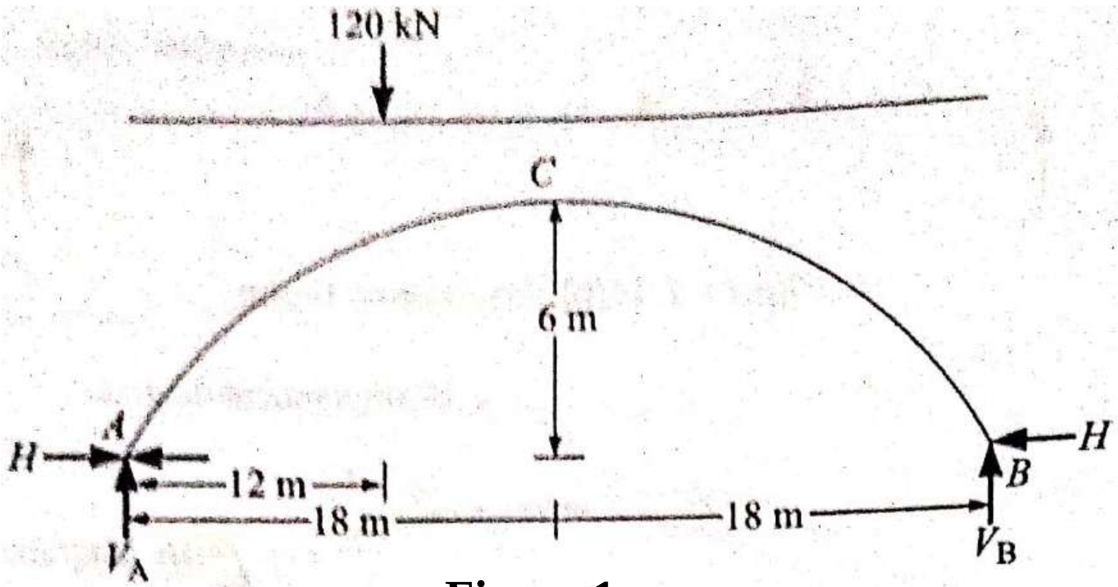


Figure 1

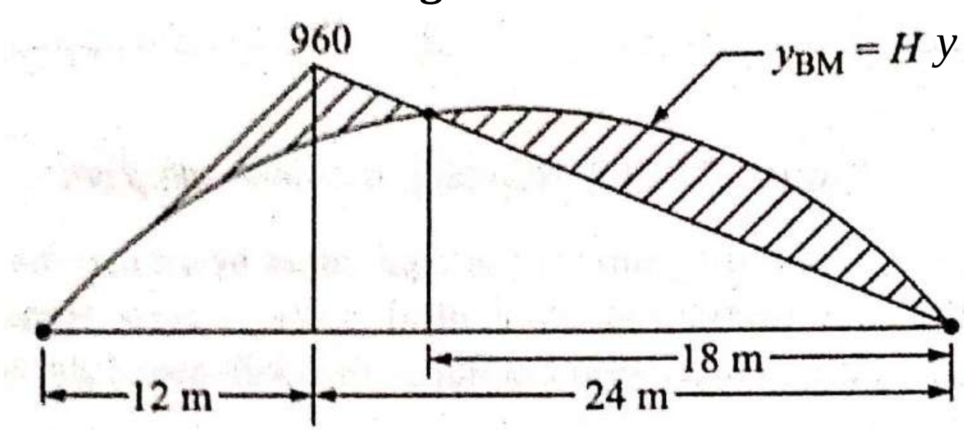


Figure 2: Bending Moment Diagram

- The arch is shown in Figure 1.
- For getting reaction, taking moment about B and equate to zero ($\sum M_B = 0$), gives

$$V_A \times 36 - 120\,(36 - 12) = 0$$

$$V_A = 80 \text{ kN}$$

- By taking summation of vertical force zero ($\sum V = 0$)

$$V_A + V_B = 120$$

$$V_B = 120 - 80 = 40 \text{ kN}$$

- Beam moment diagram is a triangle with maximum ordinate at the load point can be found by
- Beam moment at load point = $\frac{120 \times 12(36-12)}{36}$ = 960 kNm

Note: For simply supported beam of length ***L*** with eccentrically loading (***W***), at a distance ***a*** from support ***A*** and ***b*** from support ***B***, **Maxm. BM at load point = Wab/L** (use this in above calculation)

- The beam moment is drawn first as shown in Figure 2.
- We know at mid span the net bending moment is zero
- The ordinate of the beam moment diagram at mid span can be calculated by two similar triangle = $\frac{960 \times 18}{24}$ = 720 kNm
- Since, $M_C = 0$ in the arch, Hh = 720 kNm

or $$H = \frac{720}{h} = \frac{720}{6} \text{ kN} = 120 \text{ kN}$$

- A parabola is drawn with its central ordinate equal to 720 kNm as shown in Figure 2.

The equation of the parabola is

$$y_{BM} = H \times y = H \times \frac{4hx(L-x)}{L^2}$$

Thanks

ILD for Three-Hinged Arch

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Draw the Influence Line Diagram for the following

1. Horizontal Thrust '*H*'
2. Moment at section '*D*'
3. Normal Thrust at section '*D*'
4. Radial shear at section '*D*'

Consider the three-hinged arch of span *L* and rise *h* as shown in Figure 1. Where '*D*' is the point at distance *z* from the left support.

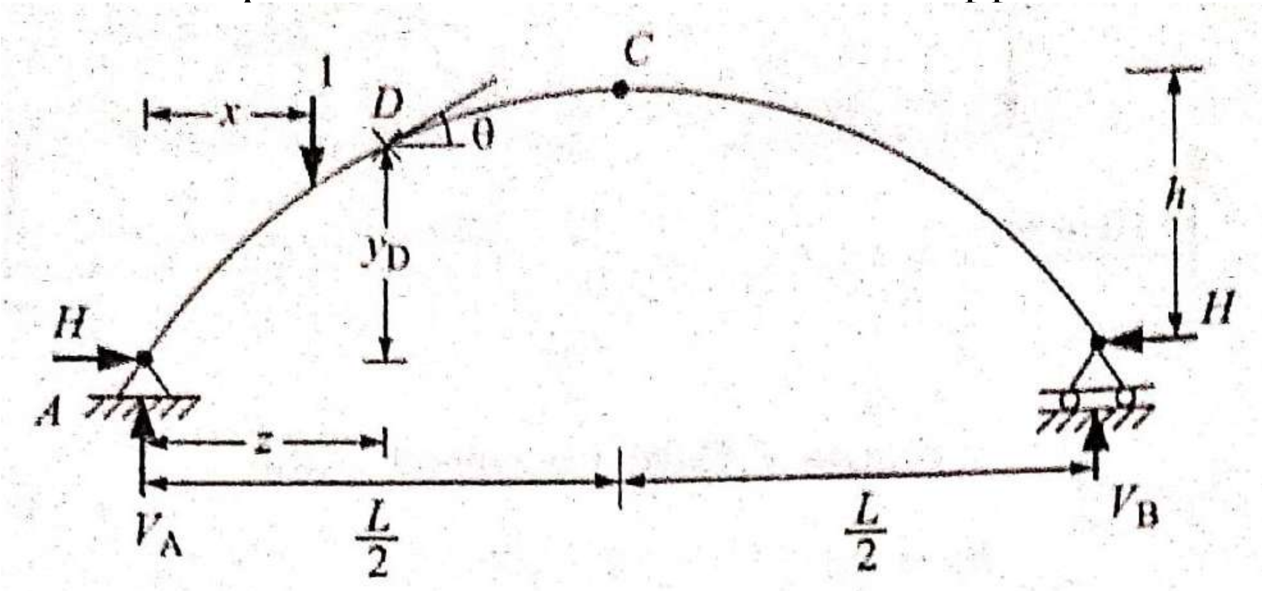

Figure 1: A typical three hinged arch subjected to unit load

Influence Line Diagram for *H*:

- Let x be the distance of the unit load from support A.
- For vertical reactions V_B, Taking the moment about A, we get

$$V_B \times L = 1 \times x$$

$$V_B = \frac{x}{L}$$

- For vertical reactions V_A, Taking the moment about B, we get

$$V_A \times L = 1 \times L\text{-}x$$

$$V_A = \frac{L-x}{L}$$

- When the load is in portion AC taking the moment about hinge C, we get

$$H \times h = V_B \times \frac{L}{2} = \frac{x}{L} \times \frac{L}{2}$$

or $H = \frac{x}{2h}$, Linear variation

- when $x = 0$, $H = 0$
- when $x = \frac{L}{2}$, $H = \frac{L}{4h}$
- When the unit load is in portion *CB*, considering the left half, taking moment about *C*, we get

$$H \times h = V_A \times \frac{L}{2} = \frac{L-x}{L} \times \frac{L}{2} = \frac{L-x}{2}$$

$$H = \frac{L-x}{2h}\text{, Linear variation}$$

- when $x = \frac{L}{2}$, $H = \frac{L}{4h}$
- when $x = L$, $H = 0$

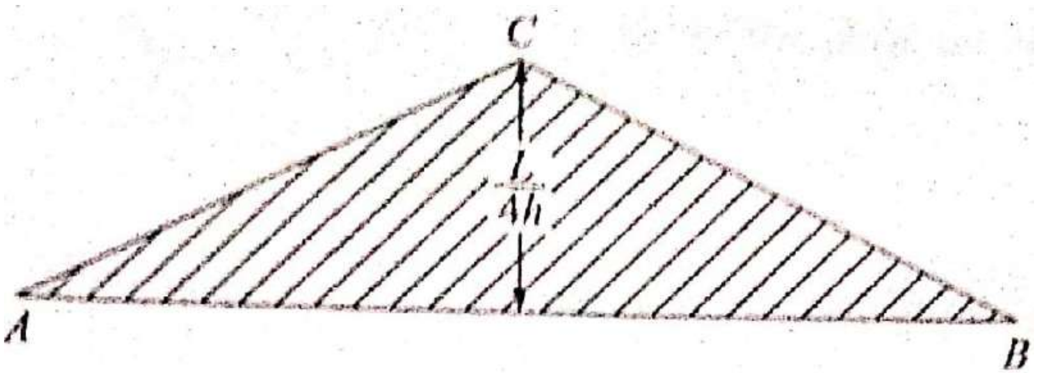

Figure 2: ILD for *H*

- Hence, ILD for H is a triangle with its maximum ordinate equal to $\frac{L}{4h}$ at hinge *C* as shown in Figure 2.

Influence Line Diagram for Moment at D (M_D):

- Bending moment at any given section in the arch = Beam Moment – Hy
- Where, beam moment means, the moment in an equivalent beam considered only due to vertical forces as shown in Figure 4.
- **Bending Moment at D = M_D = Beam moment at D – Hy_D**
- Hence, ILD for M_D in the arch will be drawn as the difference diagram of **beam moment diagram** and the **Hy_D moment diagram**.
- We know that ILD for beam moment at D is a triangle with maximum ordinate $\frac{z(L-z)}{L}$ at D as shown in Figure 5.
- Since ILD for H is a triangle with maximum ordinate of $\left(\frac{L}{4h}\right)$ at hinge C.
- Hy_D diagram is a triangle with $\left(\frac{L}{4h} \times yD\right)$ as the maximum ordinate at C as shown in Figure 5.

- The Hy_D diagram is to be subtracted from the beam moment diagram. Hence, both the triangle is drawn on the same side. The difference diagram is marked as the ILD for the bending moment at *D* as shown in Figure 5.

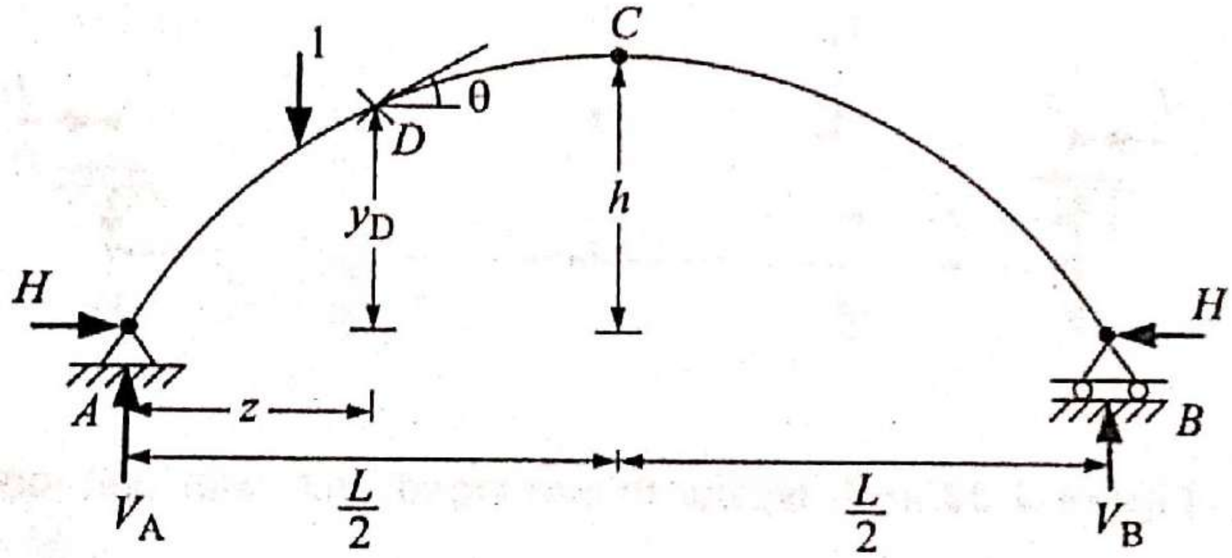

Figure 3: A typical three hinged arch with unit load

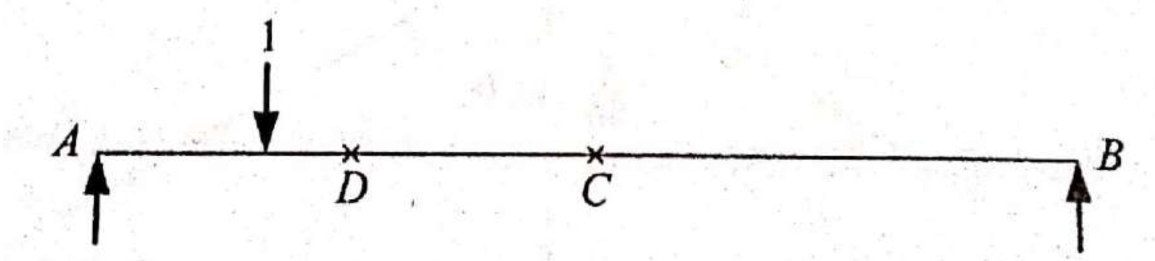

Figure 4: Equivalent Beam

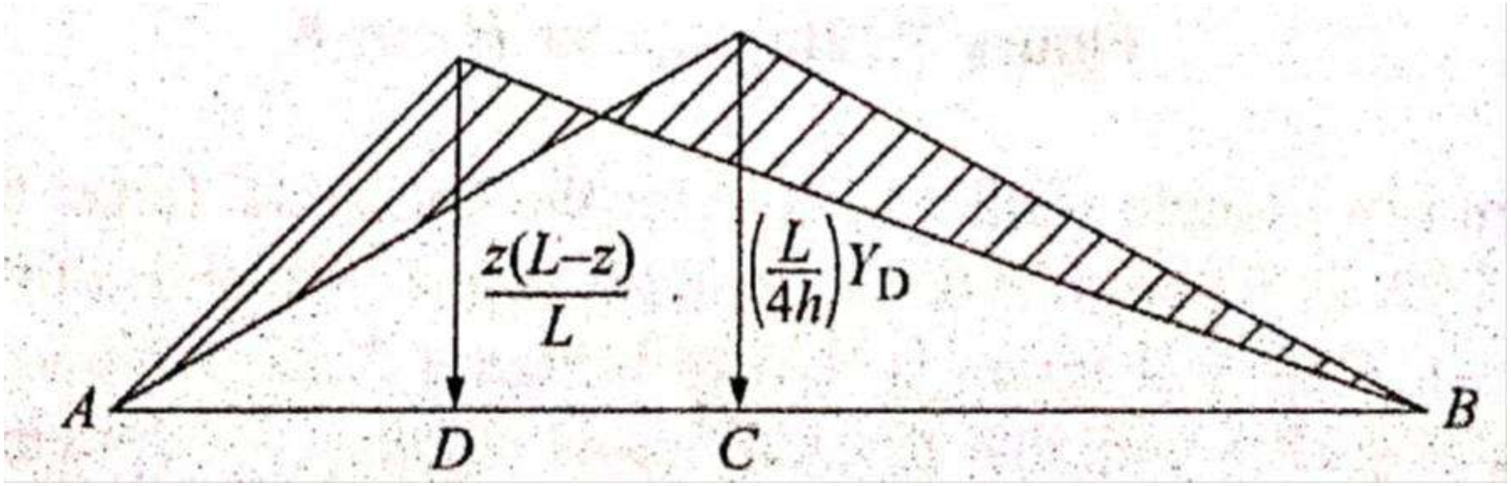

Figure 5: ILD for M_D

- In case of a parabolic arch, we know

$$y_D = \frac{4h(L-z)}{L^2}$$

- Hence, the maximum ordinate for Hy_D term

$$Hy_D = \frac{L}{4h} \times \frac{4h(L-z)}{L^2} = \frac{z(L-z)}{L}$$

- Which is same as that of the beam moment ordinate at *D*.

Influence Line Diagram for normal thrust at *D* (M_D):

- The normal thrust at section $D = N_D = V \sin\theta + H \cos\theta$
- Where, θ is the slope of the arch with the horizontal and V is the vertical shear as shown in Figure 6.

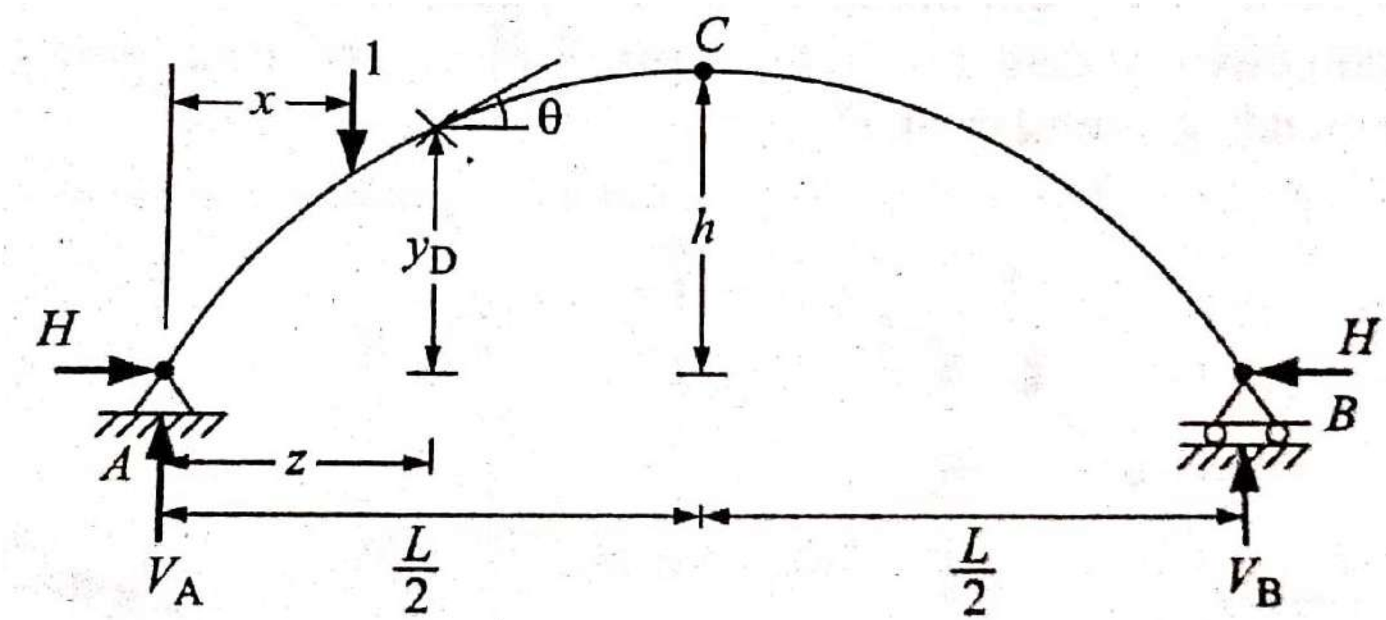

Figure 6: A typical three hinge arch with unit load

- Now, **ILD for *V* sin θ and *H* cos θ** will be drawn so as to get a diagram for the normal thrust.

- $H \cos \theta$ diagram is a triangle similar to ILD for the horizontal thrust but multiplied by $\cos \theta$ as shown in Figure 7.

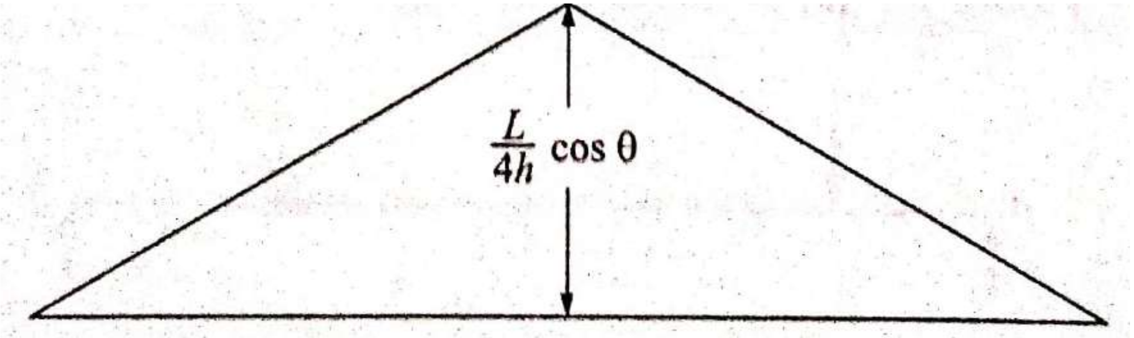

Figure 7: ILD for $H \cos \theta$

- Since, V is the vertical shear at D, which is same as that in an equivalent beam, $V \sin \theta$ diagram is shown in Figure 8.

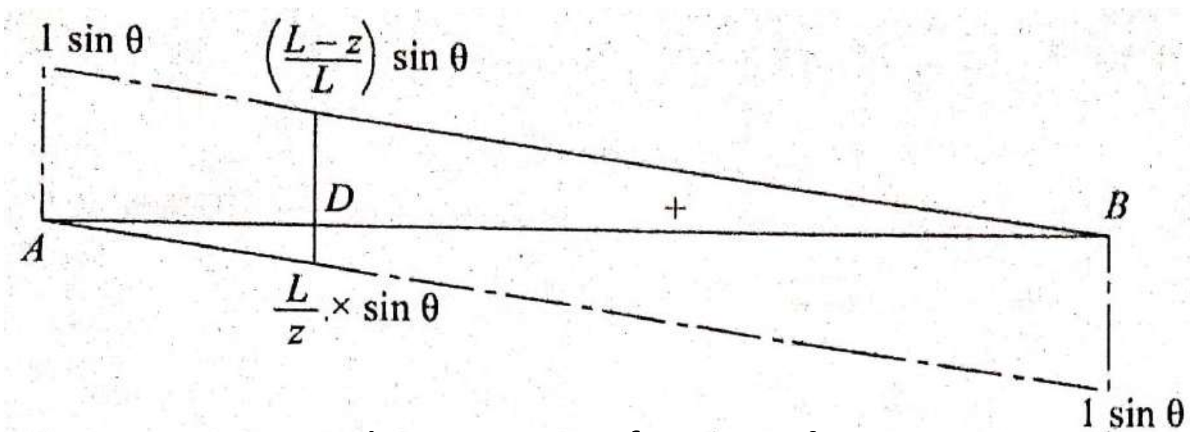

Figure 8: ILD for $V \sin \theta$

- To get ILD for normal thrust at D i.e. N_D, the diagrams are drawn in such a way that the addition is obtained by drawing ILD for $M \cos\theta$ on one side and drawing ILD for $V \sin\theta$ on the opposite side as shown in Figure 9.

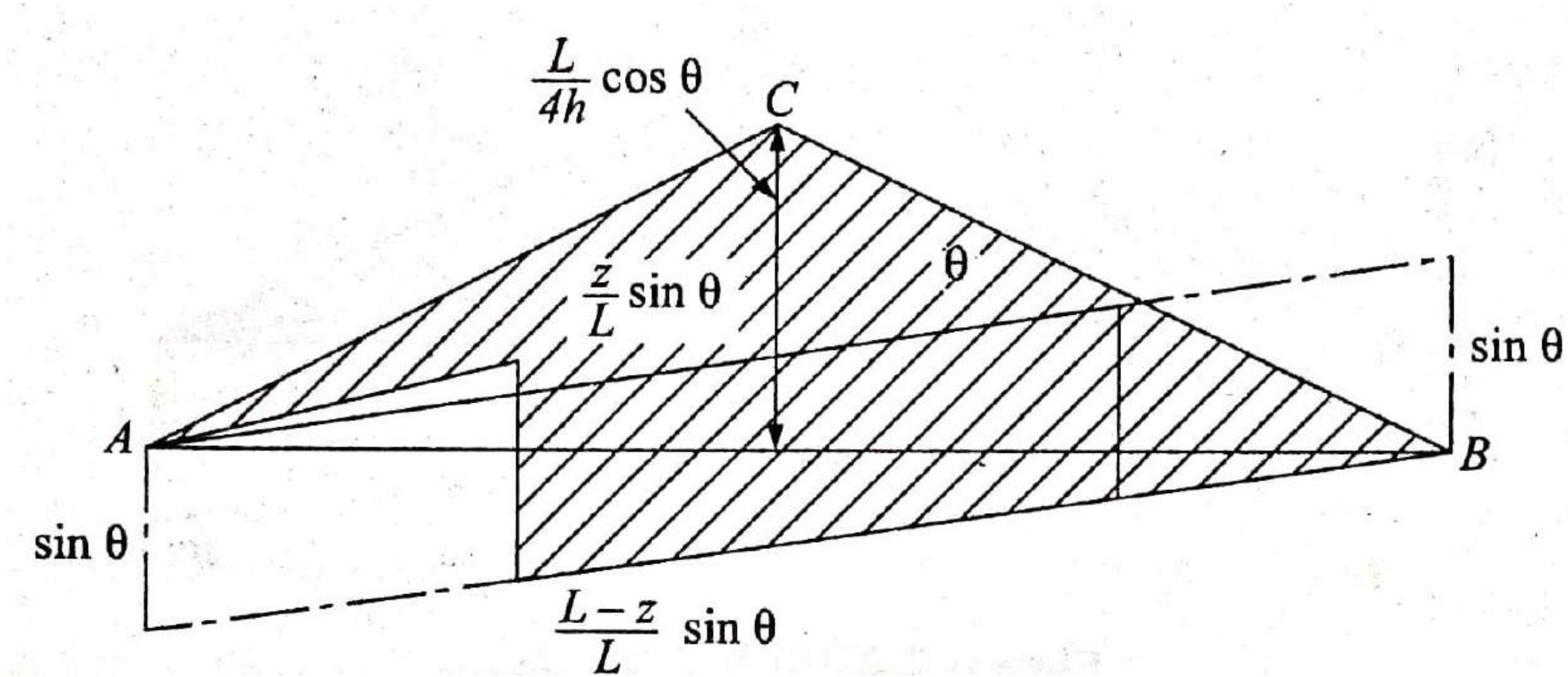

Figure 9: ILD for $N_D = V \sin\theta + H \cos\theta$

Influence Line Diagram for Radial Shear (Q_D):

- Radial shear at $D = Q_D = V \cos\theta - H \sin\theta$
- $H \sin\theta$ diagram is drawn as ILD for H, only multiplied by $\sin\theta$. The ILD diagram is a triangle with maximum ordinate at C as shown in Figure 10.
- $V \cos\theta$ diagram is drawn. This is similar to the ILD for SF in a beam but multiplied by the constant $\cos\theta$ as shown in Figure 11.
- The difference diagram is the ILD for Q_D as shown in Figure 12.

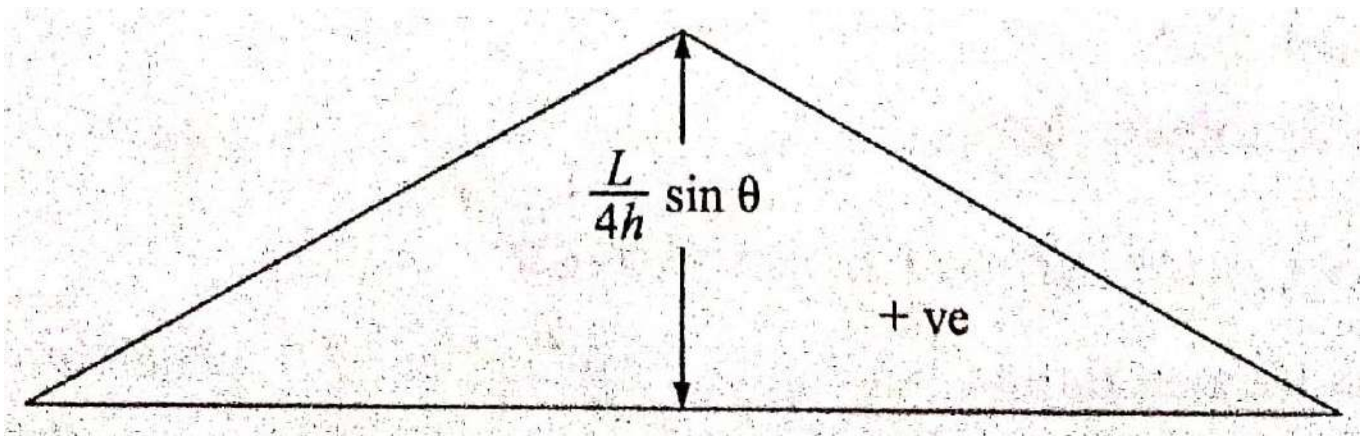

Figure 10: ILD for $H \sin\theta$

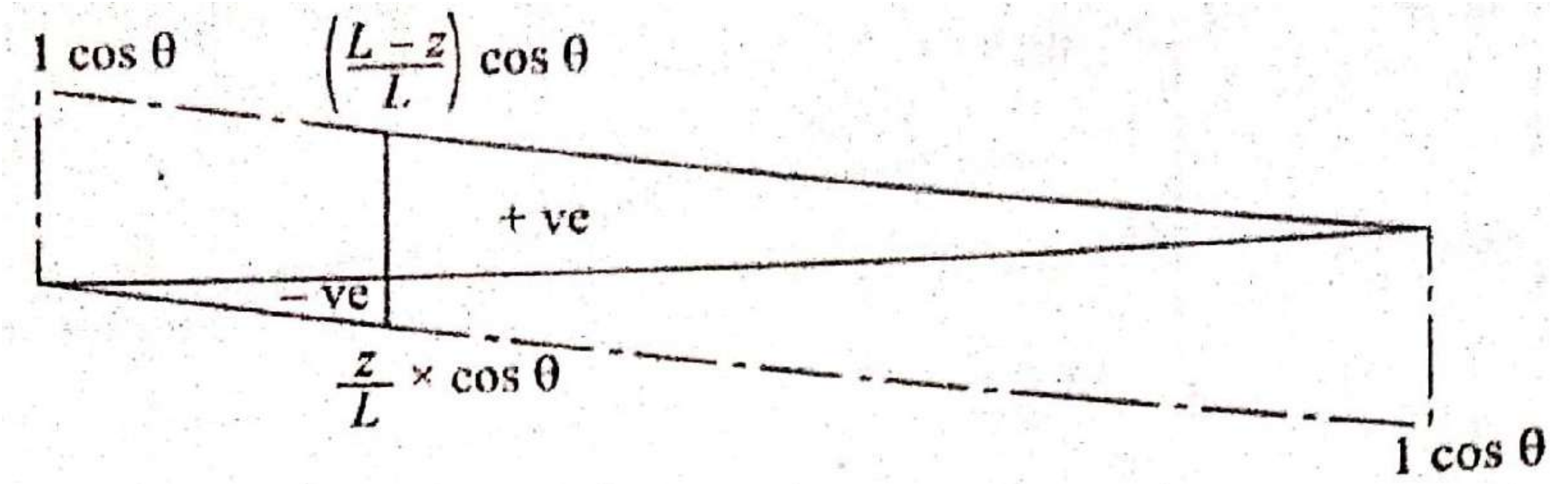

Figure 11: ILD for $V \cos \theta$

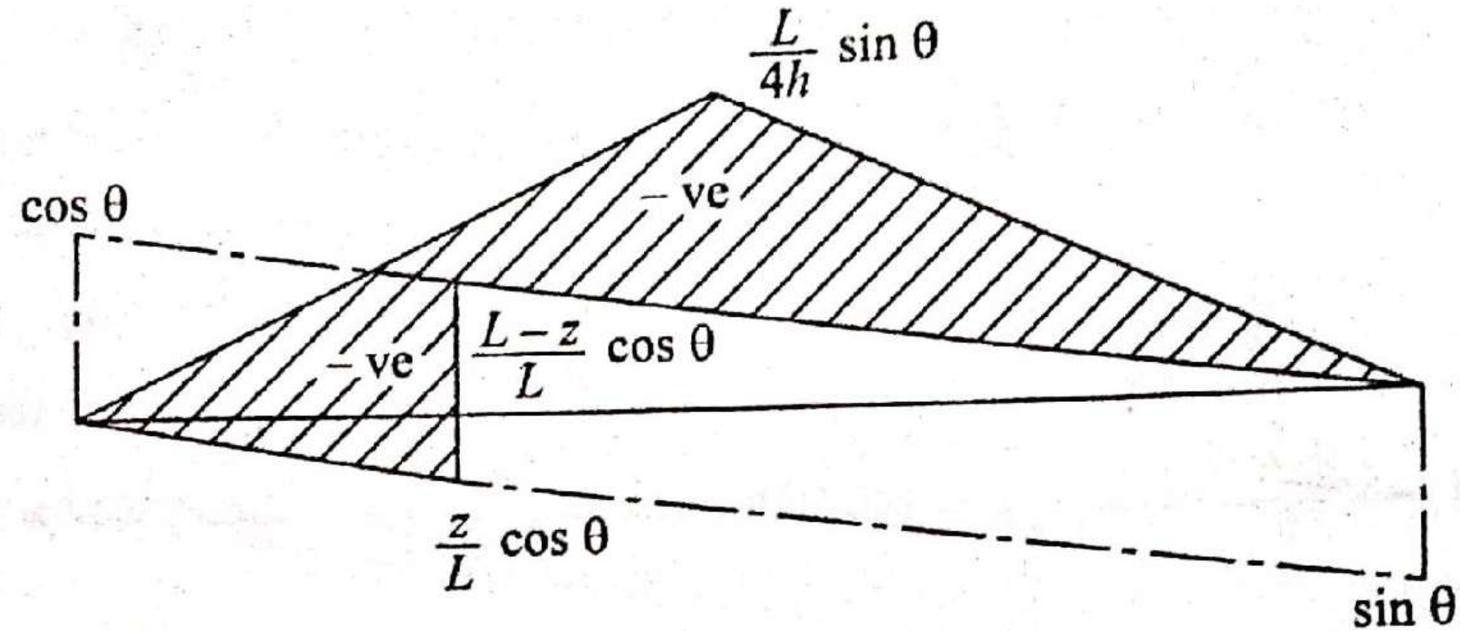

Figure 12: ILD for Q_D

Thanks