Measurement of Resistance & Inductance & Capacitance

A bridge circuit in it's simplest form consists of a network of four resistance arms forming a closed circuit with a D.C. source of current applied to two opposite junctions and current detector connected to the other two junctions.

Bridge circuits are extensively used for measuring component values such as R, L & C.

Advantages:-

(i) High measurement accuracy (as the measurement is done by Comparing the unknown value with the standard value.

(ii) The accuracy is indepedent of null detectors characteristics.

(iii) The interchange of the source and detector does not affect the balance condition

(iv) The bridge circuit can be used in control circuits.

Types of bridge:-

① D.C. bridges

These bridges are used to measure the resistances.

Exp:- (1) Wheatstone bridge
(2) Kelvin's bridge

② A.C. bridge:-

These bridges are used to measure the impedances consisting of inductances & capacitance.

Exp:- (1) Maxwell bridge
(2) Hay bridge
(3) Schering bridge
(4) Anderson bridge

Measurement of Resistance

① Classification of resistances

① low resistances:-

All resistances of the order of 1Ω. Such resistances, within armatures and series winding of large machines, cable lengths, contacts.

② Medium resistance:-

These resistances from about 1Ω upwards to $100\,k\Omega$.

③ High resistances:-

Resistances of the order of $100\,k\Omega$.

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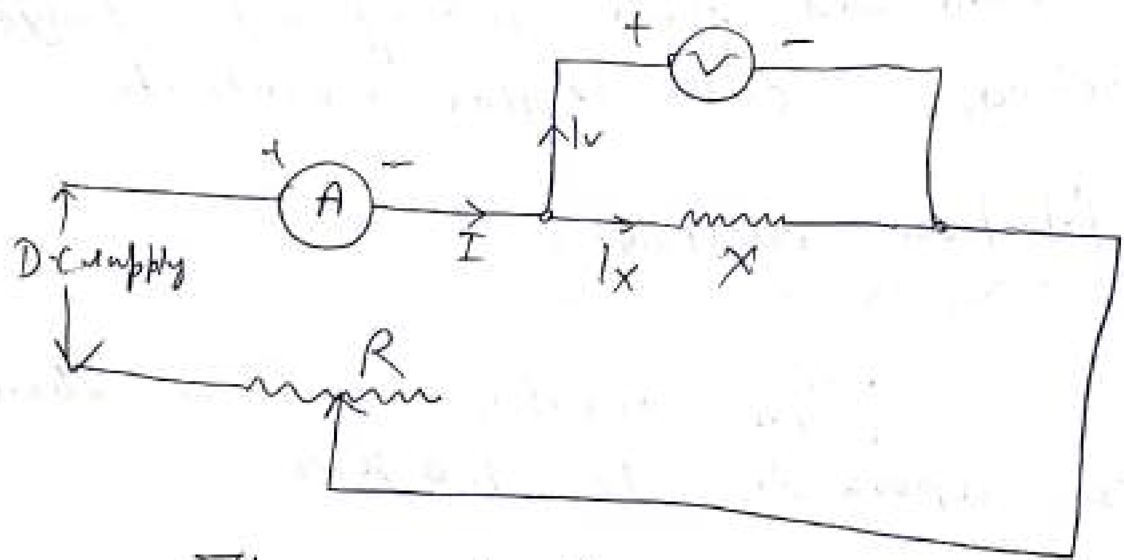
Measurement of low resistances

1. Ammeter - voltmeter method.
2. Potentiometer method.
3. Kelvin double bridge method.

Ammeter - voltmeter method:-



This method, which is simplest of all, is in very common use for the measurement of low resistances.

In this method current through the resistor (X) under test and the potential drop across it are simultaneously measured.

The readings are obtained by ammeter and voltmeter respectively.

From this figure it will observed that there are two ways in which the ammeter & voltmeter may be connected for measurement.

Case-I :- When voltmeter is connected directly across the resistor, the ammeter measures current flowing through the unknown resistance X and the voltmeter.

Current through ammeter = Current through unknown resistance(X) + Current through voltmeter

$$I = I_x + I_v$$

$$I_x = I - I_v$$

The value of unknown resistance

$$X_{true} = \frac{V}{I_x} = \frac{V}{I - I_v} = \frac{V}{I - \frac{V}{R_v}} = \frac{V}{I\left(1 - \frac{V}{IR_v}\right)}$$

$X_{true} = \cancel{\ }$

V = Voltmeter reading

R_V = The resistance of the voltmeter

I = The current indicated by the ammeter.

Case-2

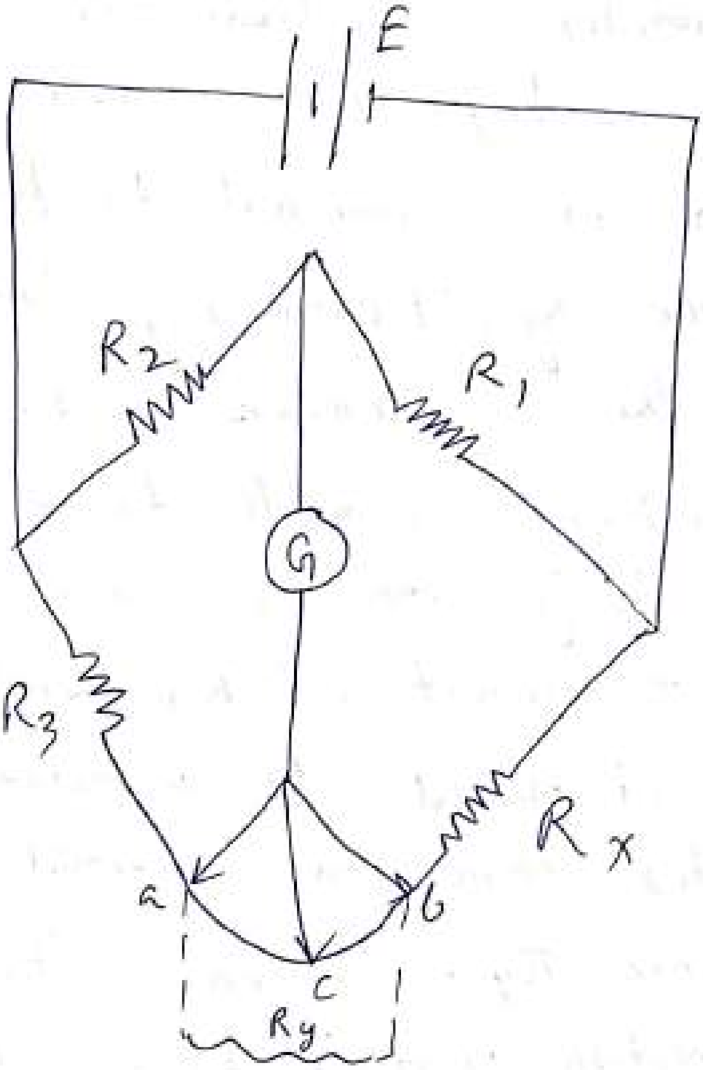
When the ammeter is connected so that it indicates only the current flowing through the unknown resistance the voltmeter measures voltage drop across the ammeter and unknown resistance X. In this case

$$V = IR_A + IX$$
$$= I(R_A + X)$$

$$X_{true} = \frac{V}{I} - R_A.$$

R_A = The resistance of the ammeter.

Kelvin double bridge method



This bridge is a modified version of Wheatstone bridge and provides greatly increased accuracy in the measurement of low value resistance, generally below 1Ω

R_y represents the resistance of connecting lead from R_3 to R_x.

Two galvanometer connections are available, to point a or to point b. When the galvanometer is connected to point a, the resistance R_y (of connecting lead) is added to the unknown resistor R_x. When connection is made to b, R_y is added to bridge arm R_3 and the resulting measurement of R_x will be lower than it should be, actual value of R_3 is higher than its normal value, by resistance R_y. When the galvanometer connection is made to a point c in between the two points a and b, in such a way that the ratio of the resistances from b to c and from a to c equals the ratio of resistors R_1 & R_2,

$$\boxed{\frac{R_{bc}}{R_{ac}} = \frac{R_1}{R_2}} \quad \dots \quad (1)$$

The balance eq^n for the bridge yields,

$$R_x + R_{bc} = \frac{R_1}{R_2}\left(R_3 + R_{ac}\right) \quad \dots \quad (2)$$

Also $R_{ac} + R_{bc} = R_y$, & $\frac{R_{bc}}{R_{ac}} = \frac{R_1}{R_2}$

$$\Rightarrow \frac{R_{bc}}{R_{ac}} + 1 = \frac{R_1}{R_2} + 1$$

$$\text{or } \frac{R_{bc} + R_{ac}}{R_{ac}} = \frac{R_1 + R_2}{R_2}$$

$$\Rightarrow \frac{R_y}{R_{ac}} = \frac{R_1 + R_2}{R_2}$$

$$\text{or } R_{ac} = \frac{R_2 R_y}{R_1 + R_2}$$

$$\Rightarrow R_{bc} = R_y - R_{ac}$$

$$= R_y - \frac{R_2 R_y}{R_1 + R_2}$$

$$\Rightarrow R_{bc} = \frac{R_1 R_y + R_2 R_y - R_2 R_y}{R_1 + R_2} = \frac{R_1 R_y}{R_1 + R_2}$$

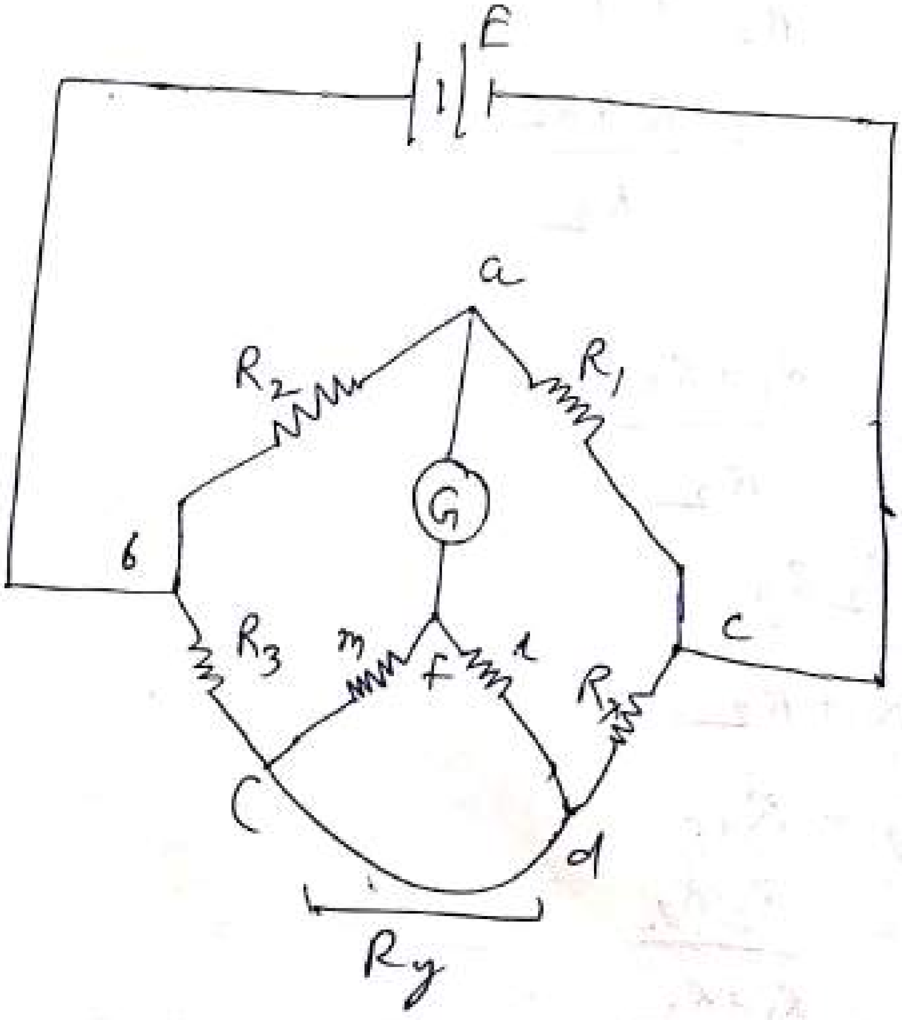
Putting R_{ac} & R_{bc} in eqn ②,

$$\Rightarrow R_x + \frac{R_1 R_y}{R_1+R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2 R_y}{R_1+R_2}\right)$$

$$\text{or } R_x + \frac{R_1 R_y}{R_1+R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_y}{(R_1+R_2)}$$

$$\boxed{\Rightarrow R_x = \frac{R_1 R_3}{R_2}}$$

Kelvin's double bridge:-



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This bridge is so called double bridge since the circuit contains a second set of ratio arms (l and m) as shown in figure. This second set of ratio arms connects the galvanometer to a point f at the appropriate potential between c and d and it eliminates the effect of the yoke resistance R_y. The ratio of the resistances of arms l & m is the same as the ratio of R_1 & R_2.

$\therefore E_{ab} = E_{bcf}$ ... (1)

$\Rightarrow E_{ab} = \dfrac{R_2}{R_1+R_2} \times E$ ... (2)

$\Rightarrow E = I\left[R_3 + R_x + \dfrac{(l+m)R_y}{(l+m)+R_y}\right]$ ... (3)

Putting the value of E in eq (2)

We get $\Rightarrow E_{ab} = \dfrac{R_2}{R_1+R_2} \times I\left[R_3 + R_x + \dfrac{(l+m)R_y}{(l+m)R_y}\right]$

$\Rightarrow E_{bcf} = I\left[R_3 + \dfrac{m}{l+m}\left\{\dfrac{(l+m)R_y}{(l+m)+R_y}\right\}\right]$

$\Rightarrow E_{ab} = E_{bef}$

$$\therefore \frac{1R_2}{R_1+R_2}\left[R_3+R_x+\frac{(l+m)R_y}{(l+m)+R_y}\right] = 1\left[R_3+\frac{m}{l+m}\left\{\frac{(l+m)R_y}{(l+m)+R_y}\right\}\right]$$

$$\text{or } R_3+R_x+\frac{(l+m)R_y}{(l+m)+R_y} = \frac{R_1+R_2}{R_2}\left[R_3+\frac{mR_y}{(l+m)+R_y}\right]$$

$$\text{or } R_3+R_x+\frac{(l+m)R_y}{(l+m)+R_y} = \left(\frac{R_1}{R_2}+1\right)\left(R_3+\frac{mR_y}{l+m+R_y}\right)$$

$$\text{or } R_x+\frac{(l+m)R_y}{l+m+R_y}+R_3 = \frac{R_1R_3}{R_2}+R_3+\frac{mR_1R_y}{R_2(l+m+R_y)}+\frac{mR_y}{l+m+R_y}.$$

$$\Rightarrow R_x = \frac{R_1R_3}{R_2}+\frac{mR_1R_y}{R_2(l+m+R_y)}-\frac{lR_y}{l+m+R_y}$$

$$\Rightarrow R_x = \frac{R_1R_3}{R_2}+\frac{mR_y}{l+m+R_y}\left[\left(\frac{R_1}{R_2}-\frac{l}{m}\right)\right]$$

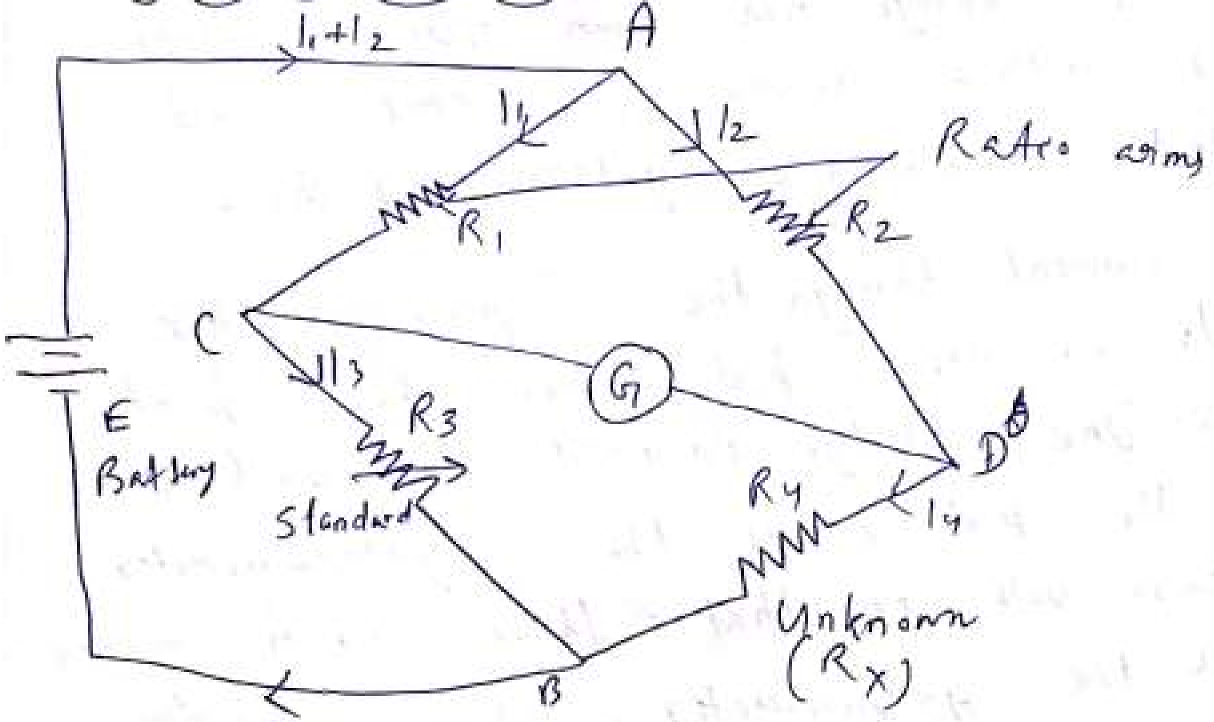
$$\Rightarrow \frac{R_1}{R_2} = \frac{l}{m}$$

$$\boxed{\Rightarrow R_x = \frac{R_1R_3}{R_2}}$$

Measurement of Medium Resistance:-

1. Ammeter-voltmeter method
2. Substitution method
3. Wheatstone bridge
4. Carey-Foster slide-wire bridge method.

Wheatstone bridge:-



The simplest form of bridge is for the purpose of measuring resistance and is called the Wheatstone bridge.

The Wheatstone bridge is widely used for precision measurement of resistance from approximately 1Ω to the low megaohm range.

This bridge has four resistive arms, together with a source of emf and null detector, usually a galvanometer (G).
The current through the galvanometer depends on the p.d. between the points C & D. The bridge is said to be balanced when the p.d. across the galvanometer is zero volt so that there is no current through the galvanometer. Hence bridge is balanced when p.d. between C & D is equal

$$\Rightarrow I_1 R_1 = I_2 R_2$$

when the current through galvanometer is zero, the following conditions should be satisfied

$$I_1 = I_3 = \frac{E}{R_1 + R_3}$$

$$\Rightarrow I_2 = I_4 = \frac{E}{R_2 + R_4}$$

Putting the values of I_1 & I_2 in eq (1), we get,

$$\frac{E}{R_1 + R_3} \times R_1 = \frac{E}{R_2 + R_4} \times R_2$$

$$\Rightarrow \frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4}$$

$$\Rightarrow R_1 (R_2 + R_4) = R_2 (R_1 + R_3)$$

$$\Rightarrow R_1 R_2 + R_1 R_4 = R_1 R_2 + R_2 R_3$$

$$\Rightarrow \boxed{R_1 R_4 = R_2 R_3}$$

This eq^n is the well known expression for balance the wheatstone bridge.

If R_4 is unknown $= R_x$

$$\boxed{R_x = \frac{R_2}{R_1} \cdot R_3}$$

The resistors R_2 & R_1 are called ratio arms while the resistor R_3 is called the standard arm of the bridge.

Applications

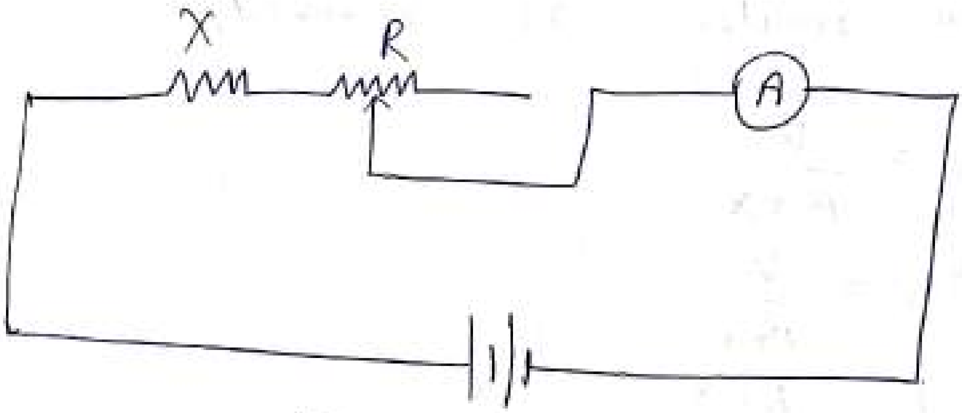
(1) Measure the D.C. resistance of various types of wire either for the purpose of quality control of the wire itself, like measure of the resistance of motor windings, transformers, solenoids & relay coils. (2)

Limitations

(1) While measuring high resistance, the resistance of the bridge becomes so large that the galvanometer becomes insensitive to imbalance. Consequently, a power supply has to replace the battery and D.C. VTVM replaces the galvanometer.

Measurement of Medium resistances

② Substitution method:-



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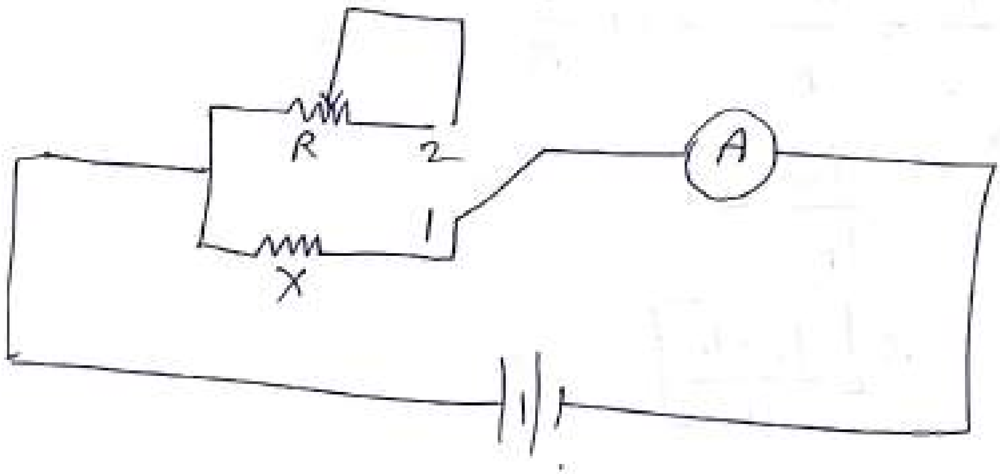


ⓑ

Let R be a variable resistance which can be changed in small steps, say of 0.1Ω.

(i) First resistance X is put in the circuit and the value of current noted.

Then X is removed and it is substituted by a known variable resistance R, which is varied so that the value of the current is same in both the cases. This value of R is equal the unknown resistance.

(ii) ⓐ For resistances X and R in series

ⓑ When resistance X is removed.

$$\Rightarrow I_1 = \frac{V}{R+X}$$

$$\Rightarrow I_2 = \frac{V}{R}.$$

$$\Rightarrow \frac{I_2}{I_1} = \frac{R+X}{R} = 1 + \frac{X}{R}$$

$$\Rightarrow \frac{X}{R} = \frac{I_2}{I_1} - 1$$

$$X = R\left[\frac{I_2 - I_1}{I_1}\right].$$

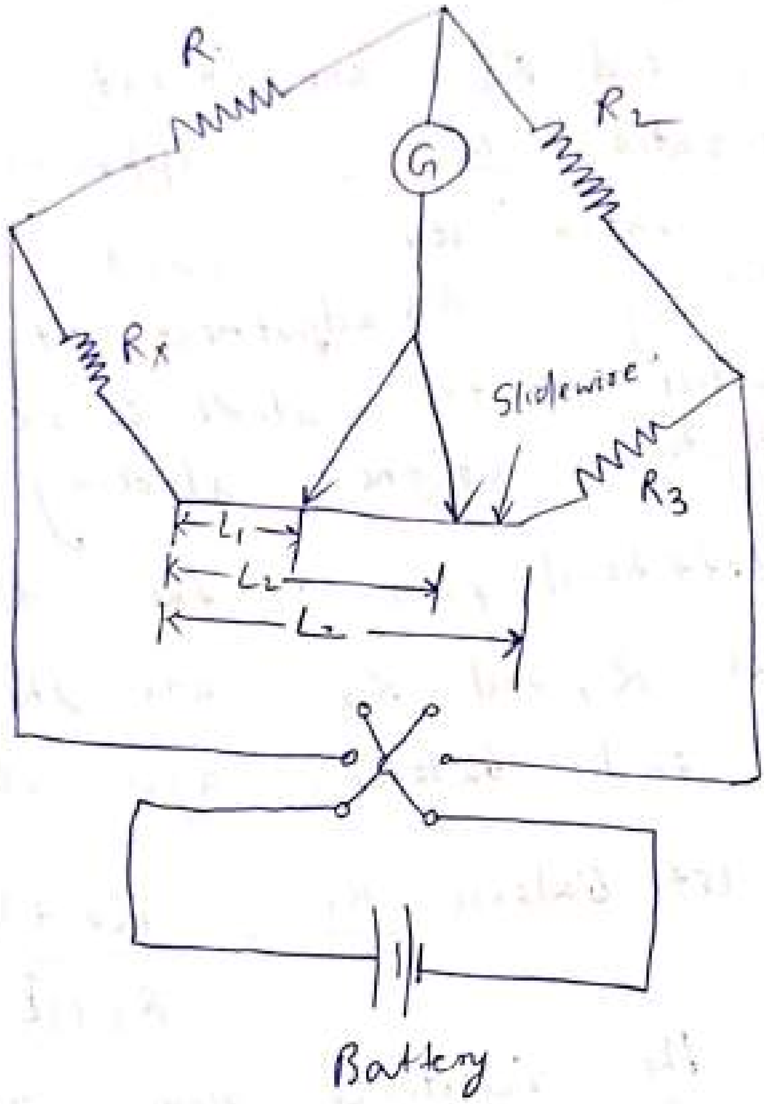
(ii) The two way switch 1st makes contact with 1 and then with 2 and let there readings be I_1 and I_2.

$$\Rightarrow I_1 = \frac{V}{X} \text{ (where X is in the ckt)}$$

$$\Rightarrow I_2 = \frac{V}{R} \text{ (When R is in the ckt.}$$

$$\Rightarrow \frac{I_2}{I_1} = \frac{X}{R} \Rightarrow X = R \times \frac{I_2}{I_1}$$

Carey- Foster slide-wire bridge method:



This bridge circuit is an elaboration of the wheatstone bridge and is specially suited for comparing two nearly equal resistances.

R_1 & R_2 are nominal ratio arms, R_x under test, R_3 are standard resistance.

A slide wire of length L and uniform cross-section is included between R_x and R_3 as shown in figure.

Resistances R_1 and R_2 are first adjusted so that the ratio $\frac{R_1}{R_2}$ is approximately equal to the ratio $\frac{R_x}{R_3}$. Exact balance is obtained by adjustment of the sliding contact on the slide wire. Let l_1 be the distance of the sliding contact from the left hand end of the slide wire. The resistances R_x and R_3 are then interchanged and balance again obtained.

For the 1st balance, $\frac{R_1}{R_2} = \frac{R_x + l_1 r}{R_3 + (L - l_1) r}$ — ①

Where, r is the resistance per unit length of the slide wire.

For the second balance $\frac{R_1}{R_2} = \frac{R_3 + L_2 r}{R_x + (L - l_2) r}$ ②

Comparing both eqn:-

$$\frac{R_x + l_1 r}{R_3 + (L - l_1) r} = \frac{R_3 + l_2 r}{R_x + (L - l_2) r}$$

or $\dfrac{R_x + l_1 r}{R_3 + (L - l_1) r} + 1 = \dfrac{R_3 + l_2 r}{R_x + (L - l_2) r} + 1$

or $\dfrac{R_x + l_1 r + R_3 + L r - l_1 r}{R_3 + (L - l_1) r} = \dfrac{R_3 + l_2 r + R_x + L r - l_2 r}{R_x + (L - l_2) r}$

or $\dfrac{R_x + R_3 + L r}{R_3 + (L - l_1) r} = \dfrac{R_3 + R_x + L r}{R_x + (L - l_2) r}$

or $R_3 + (L - l_1) r = R_x + (L - l_2) r$

$\Rightarrow R_3 + L r - l_1 r = R_x + L r - l_2 r$

$$\boxed{\Rightarrow R_3 - R_x = r (l_1 - l_2)}$$

The difference between R_3 and R_x is obtained from the resistance of the slide-wire between the two balance points.

The slide wire is calibrated. r is obtained by shunting R_3 by a known high resistance, of the slide wire between the two balance points.

The slidewire is calibrated, r is obtained by shunting R_3 by a known high resistance, thus reducing the effective value to R_3'. Now balance points l_1' and l_2' are then obtained by repeating the whole procedure previously described.

$$R_3' - R_X = r(l_1' - l_2')$$

$$\Rightarrow \boxed{\frac{R_3 - R_X}{R_3' - R_X} = \frac{l_1 - l_2}{l_2' - l_1'}}$$

$$\boxed{\Rightarrow R_X = \frac{R_3(l_1' - l_2') - R_3'(l_1 - l_2)}{(l_1' - l_2' - l_1 + l_2)}}$$

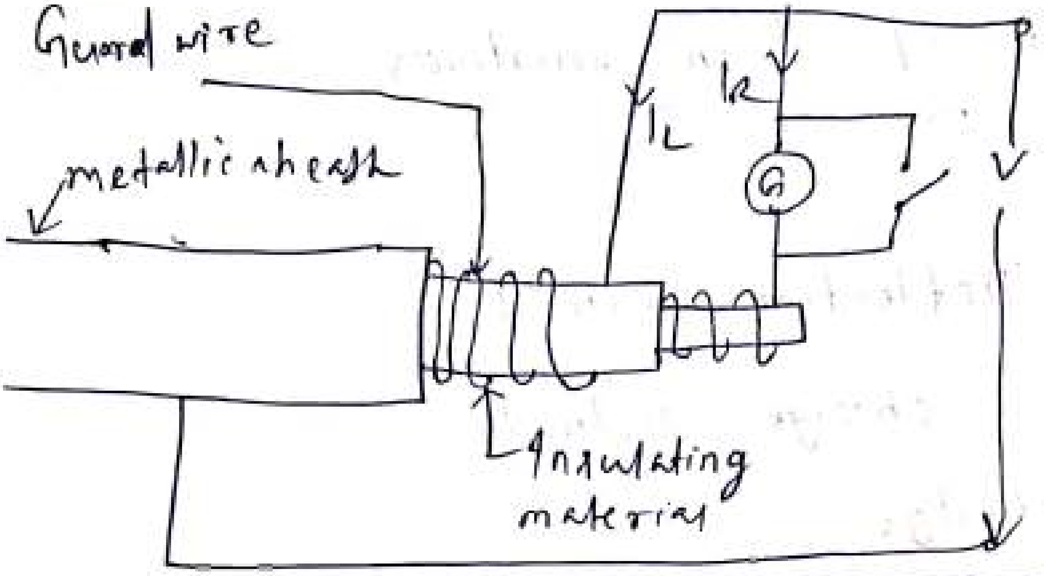
Measurement of high resistances

① Direct Deflection method.
② loss of charge method.
③ Megohm bridge
④ Megger

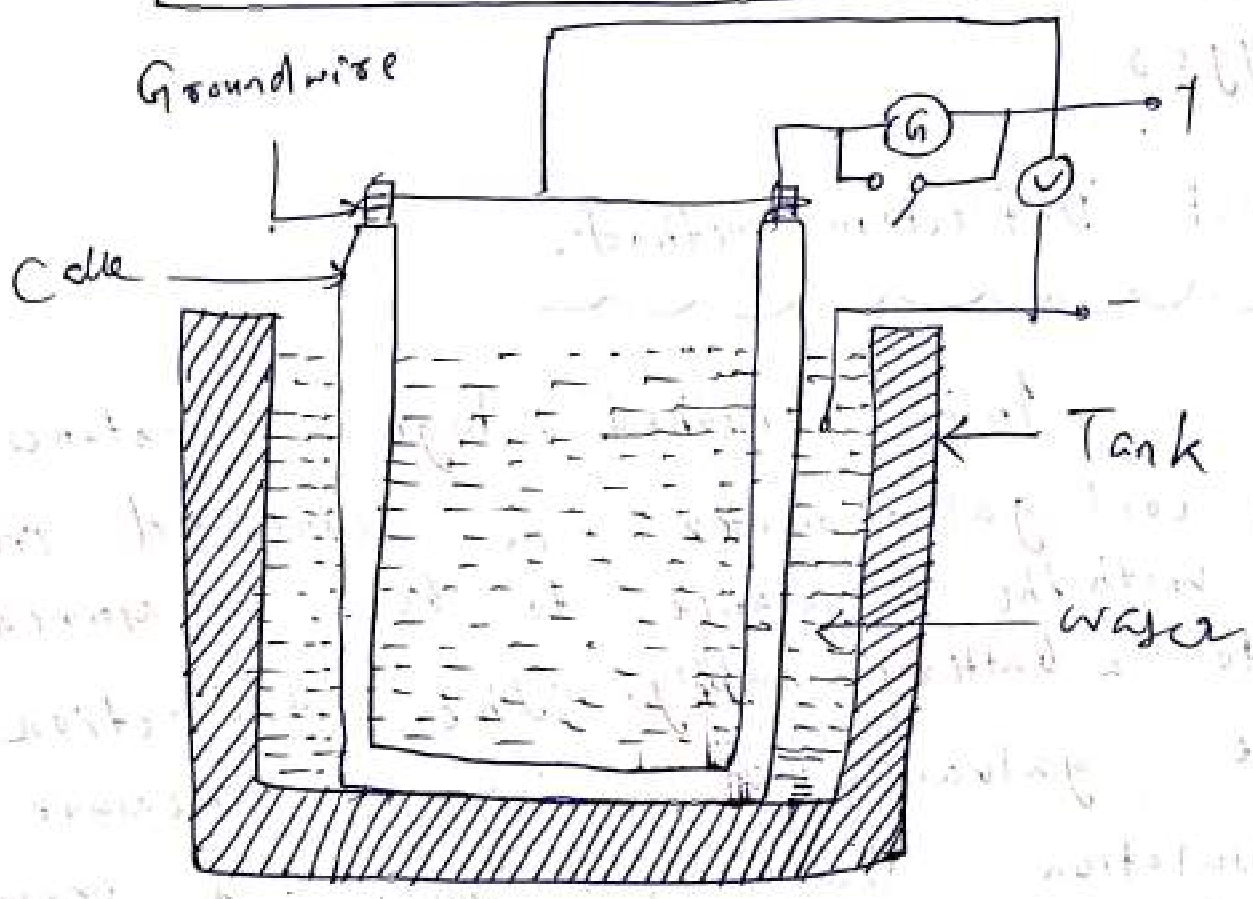
① Direct Deflection method:-

In this method, high resistance moving coil galvanometer is connected in series with the resistance to be measured, and to a battery supply. The deflection of the galvanometer gives a measure of the insulation resistance. This is very simple but rough method of measuring insulation resistance.

The galvanometer G measures the current I_R between conductor core and metal sheath; Leakage current I_L over the surface of the insulating material are carried by the guard wire wound on the insulation & doesn't flow through the galvanometer



cable having sheath



Cable having no conducting sheath.

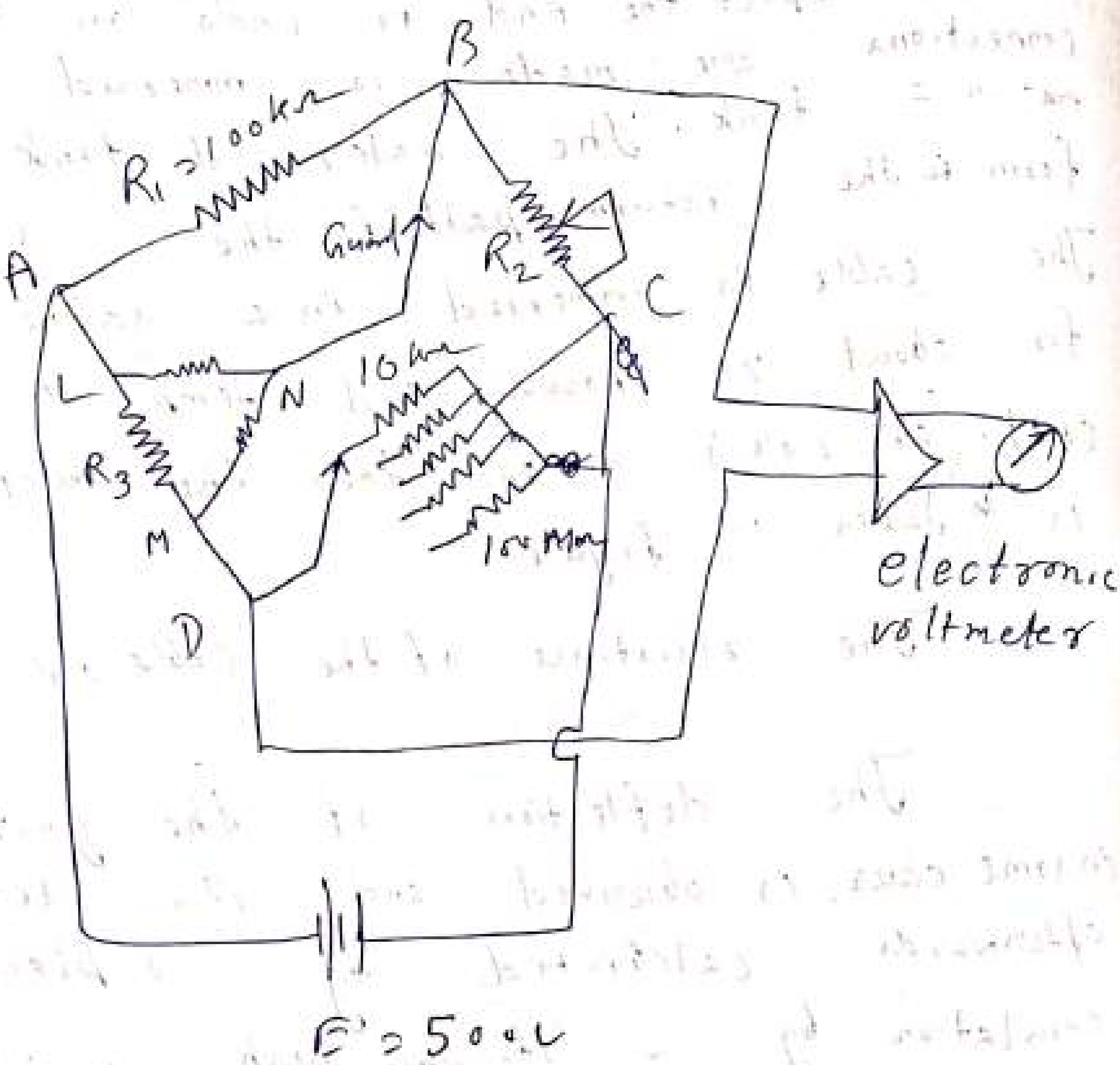
The cables without metal sheath canbe tested in a similar way of the cable expect the end or ends on which connections are made, is immersed in water ~~or~~ in a tank. The water and tank then form ~~a~~ the return path for the current. The cable is immersed in a saline water for about 24 hours and temp. is kept const. (at 20°c), and then the measurement is ~~to~~ taken in figure

The resistance of the cable, $R = \frac{V}{I_R}$

The deflection of the galvenometer, in some cases, is observed and its scale is afterwards calibrated by replacing the insulation by a standard high resistance, the galvenometer shunt being varied, as required to give a deflection of the same order as before.

While conducting tests on cables, the galvenometer should be short-circuited before applying the voltage. The short-circuiting connection is removed only after sufficient time has elapsed so that charging and absorbtion currents cease to flow.

Megohm bridge:-



This figure shows the circuit of a completely self contained Megohm bridge, it includes the following:

- power supplies
- Bridge members
- Amplifiers
- Indicating instrument

Sensitivity for balancing against high resistance is obtained by use of adjustable high voltage supplies of 500V or 1000V and the use of a sensitive null indicating arrangement such as a high gain amplifier with an electronic voltmeter or a CRO.

The dial on R_2 is calibrated 1-10-100-1000 MΩ with main decade 1-10 occupying greater part of the dial space. Since unknown resistance

$R_3 = \frac{R_1 R_4}{R_2}$, the arm R_2 is made tappered, so that the dial calibration is approximately logarithmic in main decade 1-10.

Arm R_4 gives five multipliers 0.1, 1, 10, 100, 1000.

The junction of ratio arms R_1 and R_2 is brought on the main panel and is designated as 'Guard' terminal.

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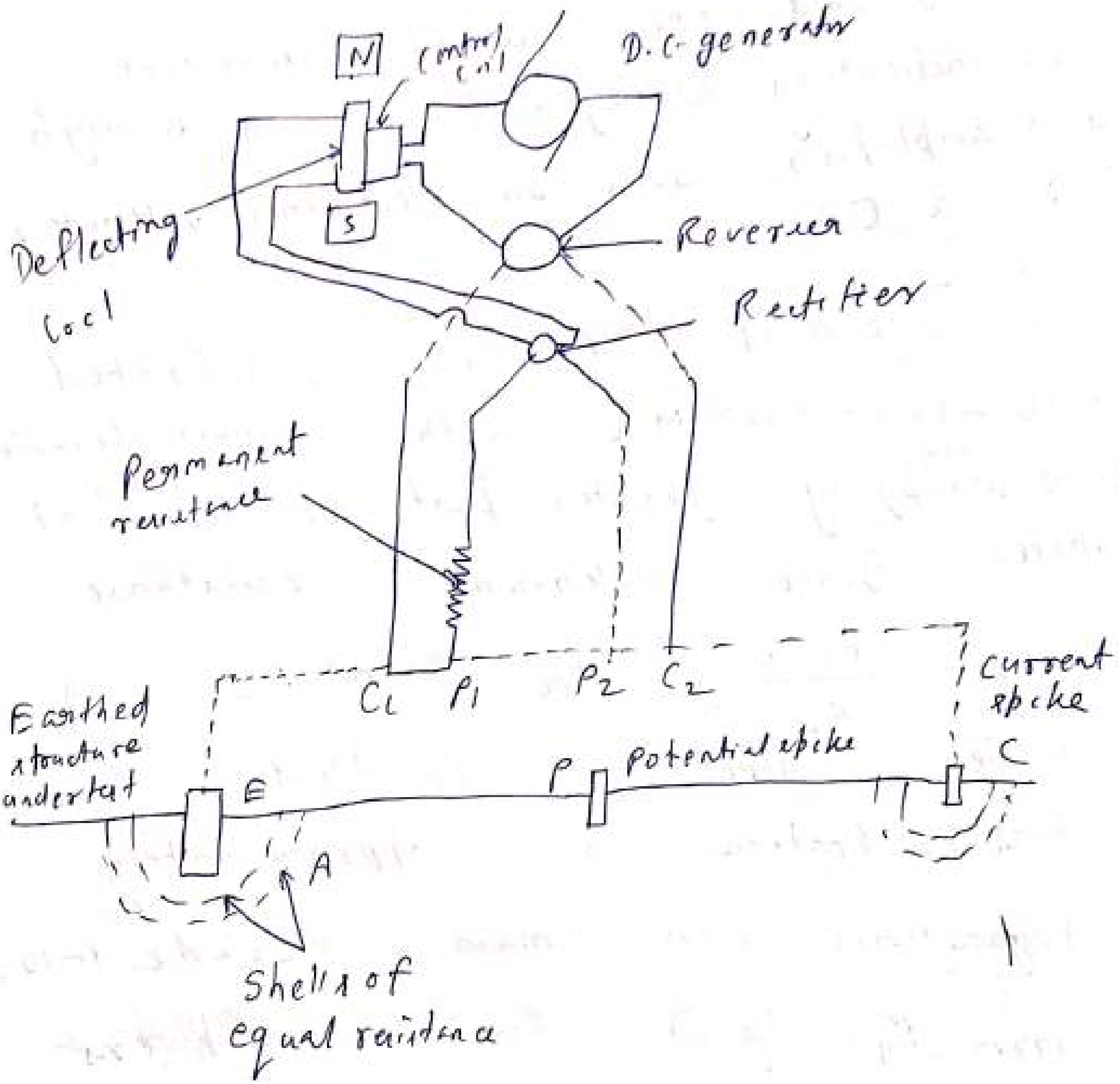
Megger earth tester:-



The earth resistance can be measured with the help of earth tester or a Megger earth tester.

The Megger earth tester is essentially a direct reading ohmmeter and a hand driven generator which supplies the testing current.

Construction :-

The ohmmeter consists of two coils (current coil & potential coil.) mounted at a fixed angle to each other on a common axle. The current coil carries current proportional to the current flowing in the test circuit, while the potential coil carries current proportional to the potential across the resistance under test. Thus the potential coil acts as a voltmeter while current coil acts as an ammeter

Since the deflection of the needle is proportional to the ratio of the current in the two coils, it gives resistance directly.

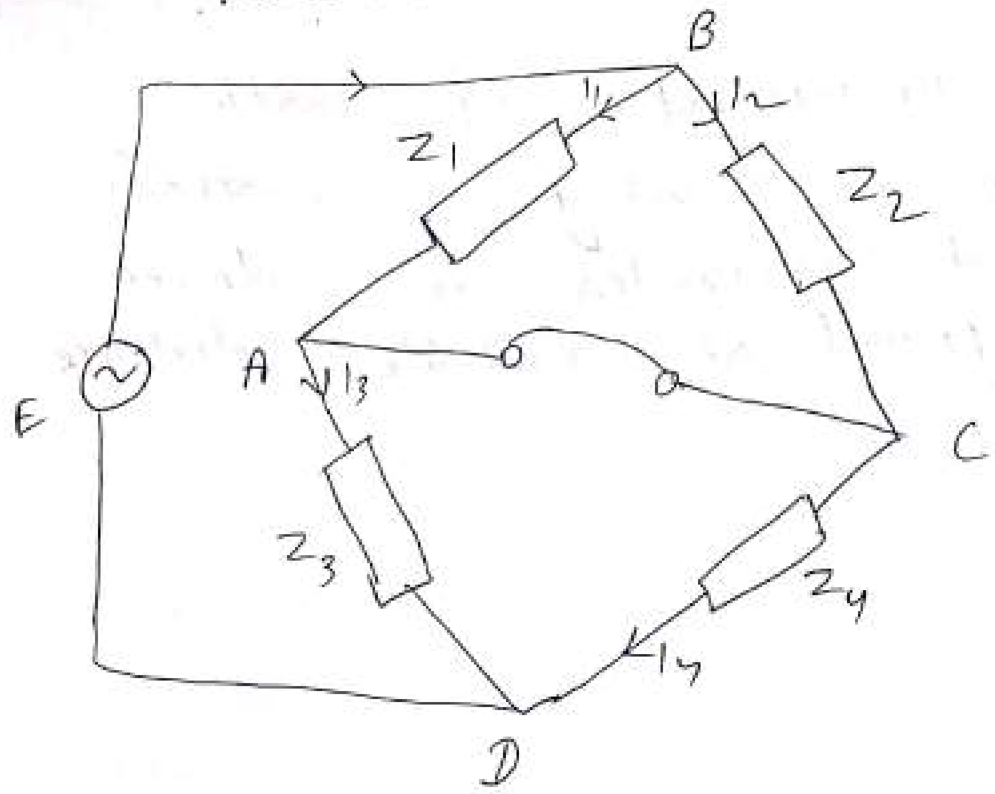
The hand-operated generator produces the direct current, but to eliminate the effect of electrolytic emf. , it is necessary to pass alternating current through the coil, so to change the D.C. into an alternating supply a rotary current reverser is mounted on the same shaft of the generator. The alternating current in the soil will produce an alternating drop in the soil but the potential to be applied across the moving coil must be direct because the ohmmeter in a moving coil instrument works on D.C. alone, so for changing the alternating drop into the direct drop a synchronous rotary rectifier is also attached as shown in figure

Working:-

For measurement of earth resistance two spikes acting as current and potential electrodes are driven into the ground at a suitable distance

(C_2

A.C. bridge



Four bridge arms:-

An A.C. bridge in its basic form consists of the following elements.

1. Four bridge arms.
2. A source of excitation

The power source supplies an A.C. voltage to the bridge at the desired frequency.

· For power measurements at low frequencies, the power line may serve as the source of excitation.

At higher frequencies, the electronic oscillators are universally used as the source of supply due to the following reasons:-

(i) The output waveform is very close to sine wave.

(ii) The output frequency is very stable.

(iii) The output frequency can be determined with accuracy and is also easily adjustable.

(iv) The output power is adequate to drive the logic circuits.

③ Null detector:-

It must respond to A.C. unbalance currents. The null / balance detectors commonly used for A.C. bridges are:

(i) Headphones:-

These are used & as detectors at the frequencies of 250 Hz to 3 to 4 kHz

(ii) Tunable amplifier circuits

A tuned detector is the most sensitive detector while working with single frequency. Tunable amplifier detectors are used for frequency range of 10 Hz to 100 kHz

(iii) Vibration galvanometers:-

These are useful for low audio frequency range from 5 Hz to 1000 kHz but are commonly used ~~for~~ below 200 Hz.

General eqn for balance a bridge:-

The balance condition is reached when the detector response is zero, or indicates a null.

The condition for bridge balance requires that potential difference from A to C in the figure is zero. This will be the case when the voltage drop from B to A equals the voltage drop from B to C in both magnitude and phase. In complex notation, we can write

$$\bar{E}_{BA} = \bar{E}_{BC} \quad \ldots (1)$$

$$\bar{I}_1 Z_1 = \bar{I}_2 Z_2 \quad \ldots (2)$$

$\therefore$ Also at balance $\Rightarrow$ $\bar{I}_1 = \bar{I}_3 = \dfrac{\bar{E}}{Z_1 + Z_3} \quad \ldots (3)$

$$\Rightarrow \bar{I}_2 = \bar{I}_4 = \frac{\bar{E}}{Z_2 + Z_4} \quad \ldots (4).$$

Putting the eq^n (3) & (4) in equation (2), we get

$$\frac{\bar{E}}{Z_1 + Z_3} \times Z_1 = \frac{E}{Z_2 + Z_4} \times Z_2$$

$$\Rightarrow \frac{Z_2 + Z_4}{Z_2} = \frac{Z_1 + Z_3}{Z_1}$$

$$\Rightarrow \text{or } 1 + \frac{Z_4}{Z_2} = 1 + \frac{Z_3}{Z_1}$$

$$\boxed{\Rightarrow \bar{Z}_1 \bar{Z}_4 = \bar{Z}_2 \bar{Z}_3}$$

$\Rightarrow \bar{Z} = Z\angle\theta$

Z = magnitude

θ

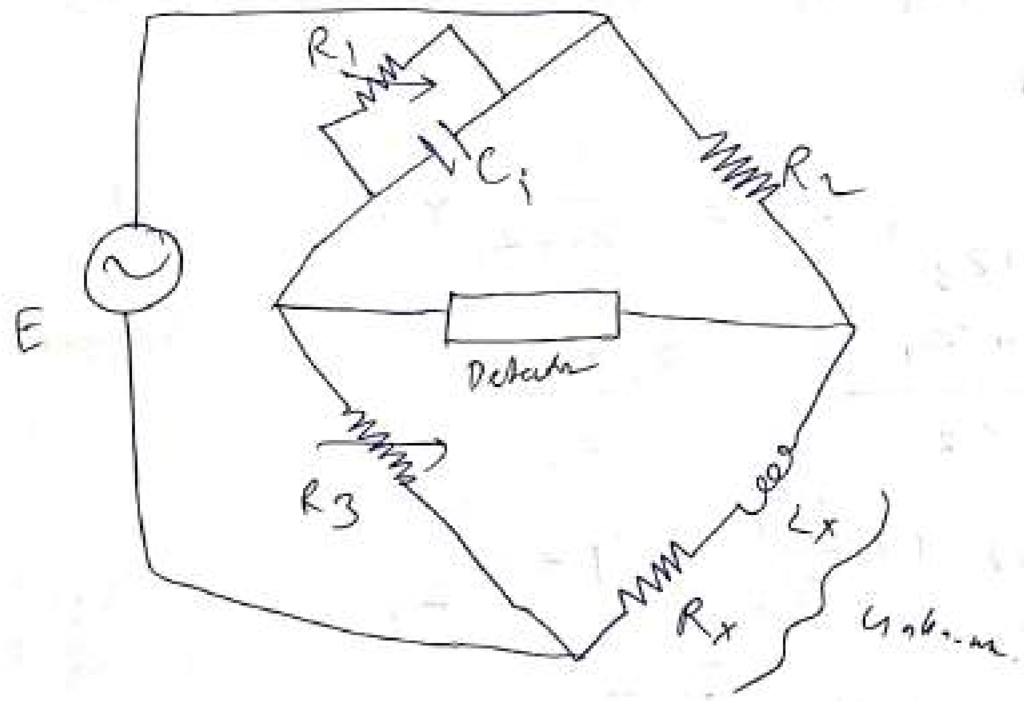
$$\boxed{(Z_1\angle\theta_1)(Z_4\angle\theta_4) = (Z_2\angle\theta_2)(Z_3\angle\theta_3)}$$

$\Rightarrow Z_1 Z_4 \angle(\theta_1+\theta_4) = Z_2 Z_3 \angle(\theta_2+\theta_3)$

$\Rightarrow Z_1 Z_4 = Z_2 Z_3$

$$\boxed{\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3}$$

Maxwell Bridge:-



Maxwell Bridge, measures an unknown inductance in terms of a known capacitor,

- The use of standard arm offers the advantages of compactness and easy shielding.

The capacitor is almost a loss-less component

One arm has a resistance R_1 in parallel with C_1, and hence it is easier to write the balance eq^n using the admittance of arm 1 instead of the impedance.

$$\bar{Z}_1 \bar{Z}_x = \bar{Z}_2 \bar{Z}_3$$

$$\bar{Z}_x = \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_1} = \bar{Z}_2 \bar{Z}_3 \bar{Y}_1 \quad \ldots (1)$$

where $\bar{Z}_1 = R_1$

in parallel with C_1, $\bar{Y}_1 = \frac{1}{Z_1}$

$\Rightarrow \bar{Y}_1 = \frac{1}{R_1} + j\omega C_1$

$\Rightarrow \bar{Z}_2 = R_2$

$\Rightarrow \bar{Z}_3 = R_3$

$\Rightarrow \bar{Z}_x = R_x$ in series with L_x

$(R_x + j\omega L_x)$

Putting this value in eqn -①, we get

$$\Rightarrow R_x + j\omega L_x = R_2 R_3 \left[\frac{1}{R_1} + j\omega C_1\right]$$

$$\boxed{\Rightarrow R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3}$$

Equating real terms & imaginary terms we have

$$\Rightarrow R_x = \frac{R_2 R_3}{R_1}$$

$$\Rightarrow L_x = C_1 R_2 R_3$$

The "quality factor" of the coil =

$$Q = \frac{\omega L_x}{R_x} = \frac{\omega C_1 R_2 R_3}{(R_2 R_3 / R_1)} = \frac{\omega \cdot C \cdot R_2 R_3 R_1}{R_2 R_3}$$

$$\boxed{\Rightarrow Q = \omega C_1 R_1}$$

Maxwell bridge is limited to the measurement of low Q values (1–10).

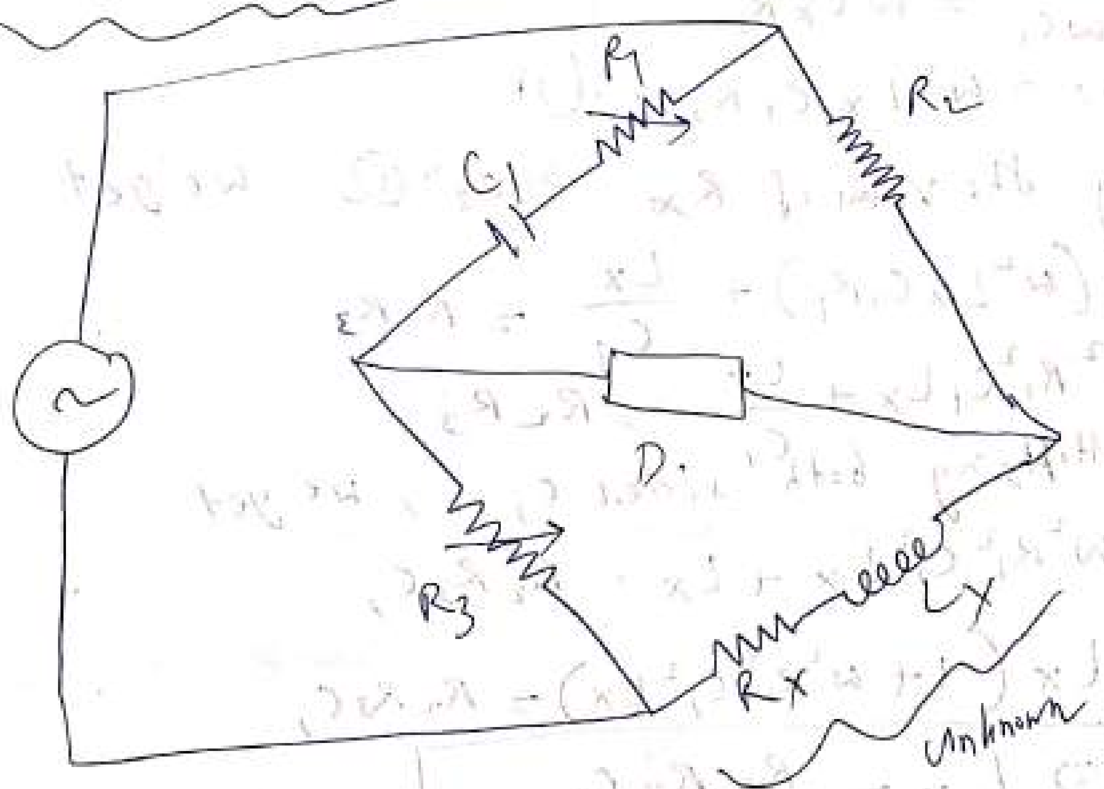
Advantages:-

① The balance eqⁿ is indepedent of losses associated with inductance.

② The measurement is independent of the excitation frequency.

③ A wide range of inductance at power & audio frequencies can be measured.

Hay Bridge:-



· It is used to measure unknown value of inductance.

$Z_1 Z_x = Z_2 Z_3$

$\Rightarrow Z_1 = R_1 - \frac{j}{wC_1}$

$Z_2 = R_2$

$\Rightarrow Z_3 = R_3 \quad \Rightarrow Z_x = R_x + jwL_x$

$\left(R_1 - \frac{j}{wC_1}\right)(R_x + jwL_x) = R_2 R_3$

$\Rightarrow R_1 R_x + jwL_x R_1 - \frac{jR_x}{wC_1} + \frac{L_x}{C_1} = R_2 R_3$

Equating Real & Imaginary terms, we get

$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \ldots (2)$

$\Rightarrow \frac{R_x}{wC_1} = wL_x R_1$

$\Rightarrow R_x = w^2 L_x C_1 R_1 \ldots (1)$

Putting the value of R_x in eq^n (2), we get

$\Rightarrow R_1 (w^2 L_x C_1 R_1) + \frac{L_x}{C_1} = R_2 R_3$

$\Rightarrow w^2 R_1^2 C_1 L_x + \frac{L_x}{C_1} = R_2 R_3$

$\Rightarrow$ Multiplying both sides C_1, we get

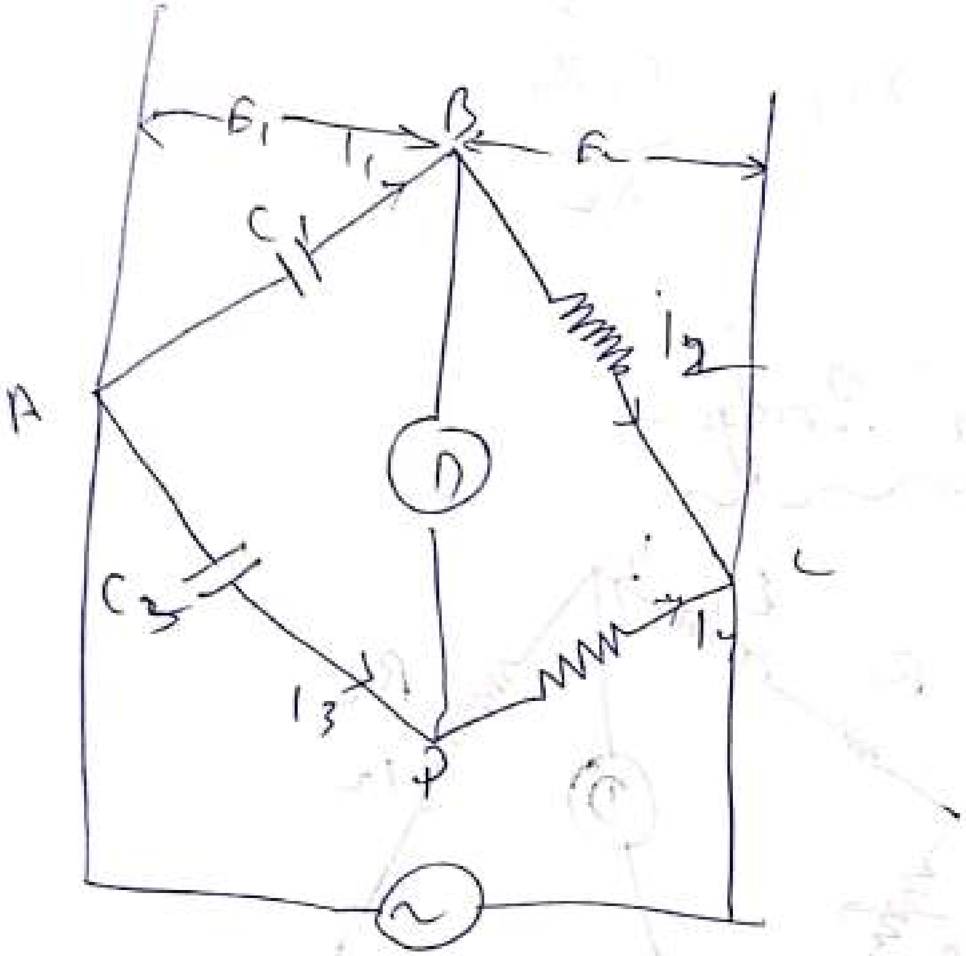
$\Rightarrow w^2 R_1^2 C_1^2 L_x + L_x = R_2 R_3 C_1$

$\Rightarrow L_x (1 + w^2 R_1^2 C_1^2 L_x) = R_2 R_3 C_1$

$$\boxed{\Rightarrow L_x = \frac{R_2 R_3 C_1}{1 + w^2 R_1^2 C_1^2}}$$

$$\Rightarrow R_x = \frac{w^2 C_1^2 R_1 R_2 R_3}{1 + w^2 R_1^2 C_1^2}.$$

De Sauty Bridge:-



This bridge is the simplest method of comparing two capacitance.

C_1 = Capacitor whose capacitance is to be measured

C_3 = Standard capacitor

R_3, R_4 = Non-inductive resistors,

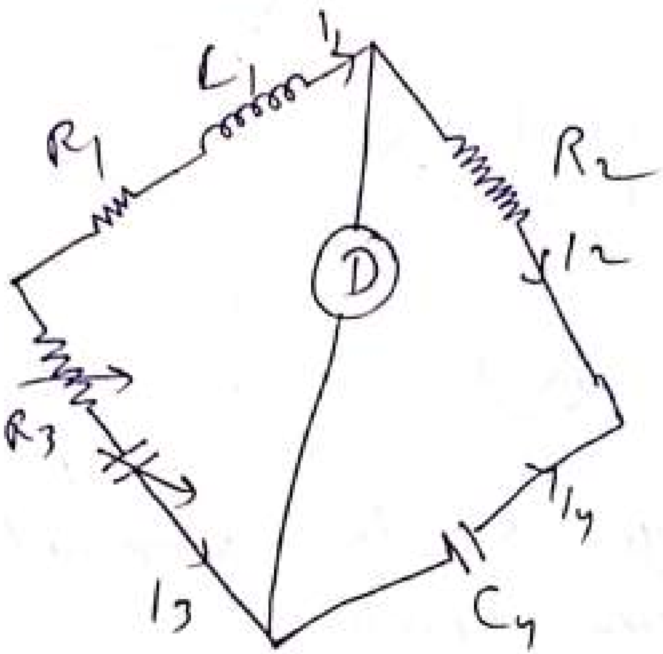
$\Rightarrow I_1 = I_2, \; I_3 = I_4$

$$\Rightarrow \left(\frac{1}{j\omega C_1}\right) R_4 = \left(\frac{1}{j\omega C_3}\right) R_2$$

$$\Rightarrow \frac{R_4}{C_1} = \frac{R_2}{C_3}$$

$$\boxed{\Rightarrow C_1 = \frac{C_3 R_4}{R_2}}$$

Owen Bridge:-



The bridge may be used for measurement of an inductance in terms of capacitance.

L_1 = unknown self inductance of R_1.
R_2 = Fixed non-inductive resistance.
R_3 = Variable non-inductive resistance.

C_3 = Variable standard capacitor

C_4 = Fixed Standard capacitor

When the bridge is balanced

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

$$\Rightarrow (R_1 + j\omega L_1)\left(\frac{1}{j\omega C_4}\right) = R_2 \cdot \left(R_3 + \frac{1}{j\omega C_3}\right)$$

$$\Rightarrow \frac{R_1}{j\omega C_4} + \frac{L_1}{C_4} = R_2 R_3 + \frac{R_2}{j\omega C_3}$$

Separating the real & imaginary terms,

$$\frac{L_1}{C_4} = R_2 R_3$$

$$\Rightarrow L_1 = R_2 R_3 C_4$$

$$\Rightarrow \frac{R_1}{C_4} = \frac{R_2}{C_3}$$

$$\boxed{\Rightarrow R_1 = \frac{R_2 C_4}{C_3}}$$

Advantages:-

1. The bridge can be used over a wide range of measurement of inductance.

2. The bridge balance equations are simple, and donot contain any frequency component.

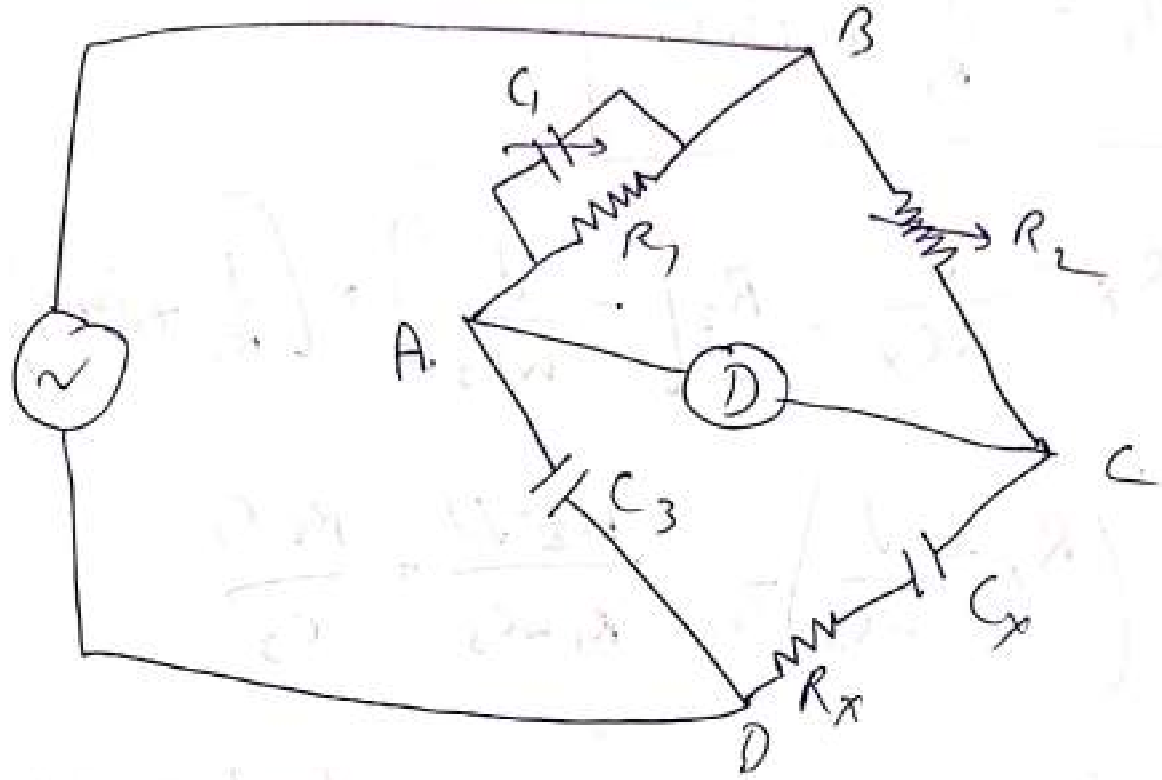
3. Due to R_3 & C_3 the variable elements, are in same arm, convergence to balance condition is much easier

Disadvantages:-

1. While measuring high Q-coil, the value of capacitance C_3 tends to become rather large.

2. The bridge requires a variable capacitor which is an expensive item & also its accuracy is about 1%.

Schering bridge:-



It is widely used for the measurement of unknown capacitors, dielectric loss & power factor.

~~The~~ :

$$\bar{Z}_1 \bar{Z}_x = \bar{Z}_2 \cdot \bar{Z}_3$$

$$\Rightarrow Z_x = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 Y_1$$

$$\Rightarrow Z_x = R_x - \frac{j}{\omega C_x}$$

$$\Rightarrow Z_2 = R_2$$

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$$Z_3 = \frac{-j}{wC_3}$$

$$\boxed{\Rightarrow Y_1 = \frac{1}{R_1} + jwC_1}$$

$$\Rightarrow R_x - \frac{j}{wC_x} = R_2\left[-\frac{j}{wC_3}\right] \times \left(\frac{1}{R_1} + jwC_1\right)$$

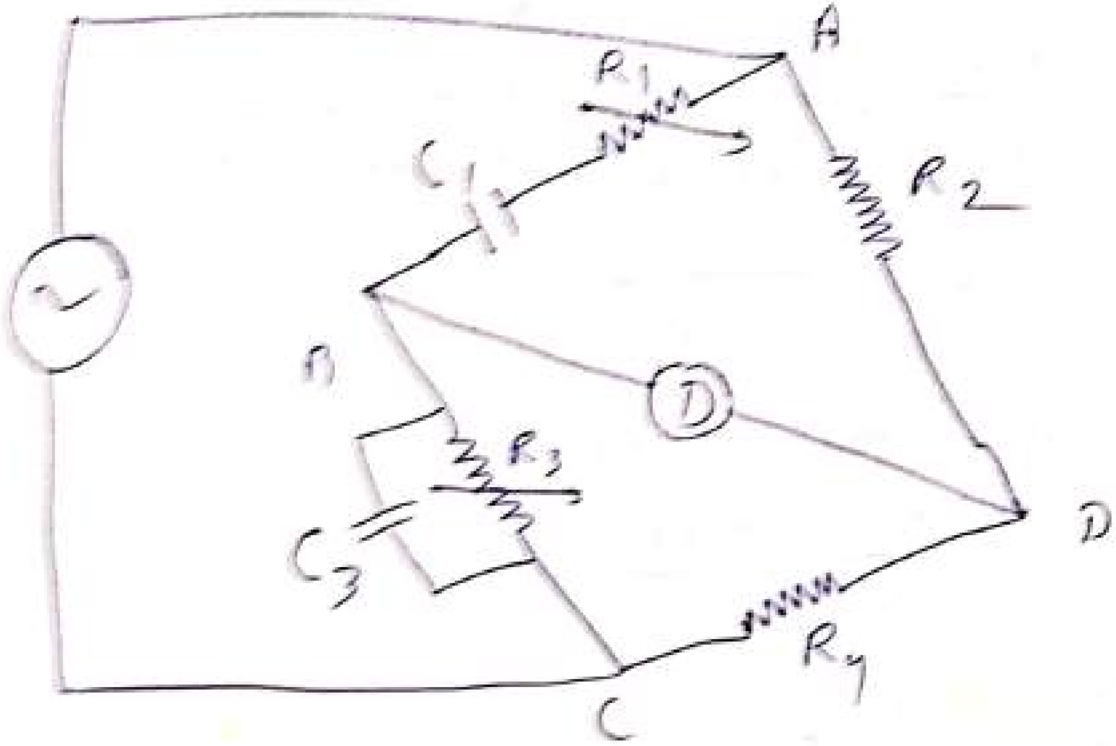
$$\text{or } \left(R_x - \frac{j}{wC_x}\right) = \frac{R_2(-j)}{R_1 wC_3} + \frac{R_2 C_1}{C_3}$$

Equating real & imaginary parts, we get

$$\Rightarrow R_x = \frac{R_2 C_1}{C_3}$$

$$\Rightarrow C_x = \frac{R_1 C_3}{R_2}.$$

Wein's bridge:- (measurement of frequency)



The wein's bridge is known as a "frequency" determining bridge.

$$Z_1 = R_1 - \frac{j}{\omega C_1}$$

$$Y_3 = \frac{1}{R_3} + j\omega C_3$$

$$Z_1 Z_4 = Z_2 Z_3$$

$$\Rightarrow Z_1 Z_4 = Z_2 \times \frac{1}{Y_3}$$

$$\boxed{\Rightarrow Z_2 = Z_1 Z_4 Y_3}$$

$$\Rightarrow R_2 = \left[R_1 - \frac{j}{\omega C_1}\right] R_4 \left[\frac{1}{R_3} + j\omega C_3\right]$$

Expanding this expression, we get

$$R_2 = \frac{R_1 R_4}{R_3} + j\omega R_1 R_4 C_3 - \frac{jR_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1}$$

$$\Rightarrow R_2 = \left[\frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}\right] - j\left[\frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4\right].$$

Equating real terms we get

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}$$

$$\Rightarrow \frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}$$

Equating imaginary parts we have

$$\omega C_3 R_1 R_4 = \frac{R_4}{\omega C_1 R_3}$$

$$\Rightarrow \omega^2 = \frac{1}{C_1 C_3 R_1 R_3}$$

$$\Rightarrow \omega = \frac{1}{\sqrt{C_1 C_3 R_1 R_3}}$$

$\omega = 2\pi f$, therefore

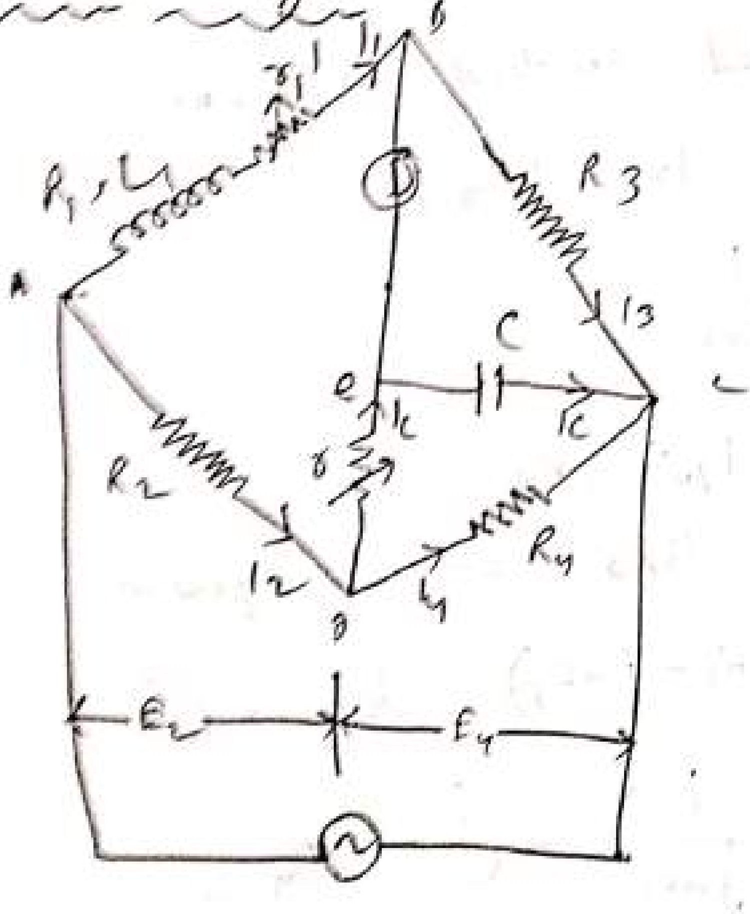
$$2\pi f = \frac{1}{\sqrt{C_1 C_3 R_1 R_3}} \Rightarrow f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}}$$

Application:-

① The bridge is employed for measuring frequency in the audio range. The audio range is normally divided into 20 Hz, 20 kHz, 2 kHz, - 20 kHz.

② The bridge can also be used for measuring capacitances, in this case operational frequency must be known.

Anderson's bridge:-



This bridge is a modification of Maxwell's inductance - capacitance bridge. In this method, the self inductance is measured in terms of capacitor. This method is applicable for precise measurement of self inductance over a wide range of values.

L_1 = Self-inductance to be measured

R_1 = Resistance of self inductor.

r_1 = resistance connected in series with self inductor.

r, R_2, R_3, R_4 = known non-inductive resistance

C = fixed standard capacitor.

At balance

$I_1 = I_3$, $I_2 = I_c + I_4$

$$I_1 R_3 = I_C \times \frac{1}{j\omega C}$$

$$\Rightarrow I_c = I_1 j\omega C R_3.$$

Writing the other balance equation.

$$I_1(r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_c r$$

$$\Rightarrow I_C\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_1)R_4.$$

Substituting the value of I_c in above equations, we have

$$I_1(r_1 + R_1 + j\omega L_1) = I_2R_2 + I_1 j\omega C R_3 r.$$

or $$I_1(r + R_1 + j\omega L_1 - j\omega C R_3 r) = I_2R_2 \quad \cdots (i)$$

$$j\omega C R_3 I_1\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_1 j\omega C R_3) R_4$$

or $$I_1(j\omega C R_3 r + j\omega C R_3 R_4 + R_3) = I_2 R_4 \quad \cdots (ii)$$

From eq^n (i) & (ii), we get

$$\Rightarrow I_1(r_1 + R_1 + j\omega L_1 - j\omega C R_3 r)$$
$$= I_1\left(\frac{R_2R_3}{R_4} + \frac{j\omega C R_2 R_3 r}{R_4} + j\omega C R_3 R_2\right)$$

$\Rightarrow$ Equating real & imaginary part,

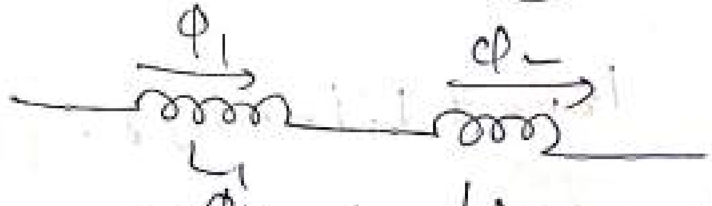
$$R_1 = \frac{R_2R_3}{R_4} - r_1$$

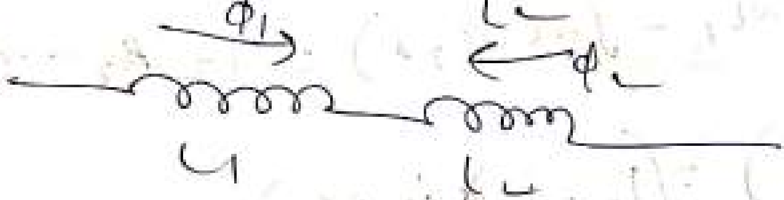
$$L_1 = C\frac{R_3}{R_4}\left[r(R_4 + R_2) + R_2R_4\right]$$

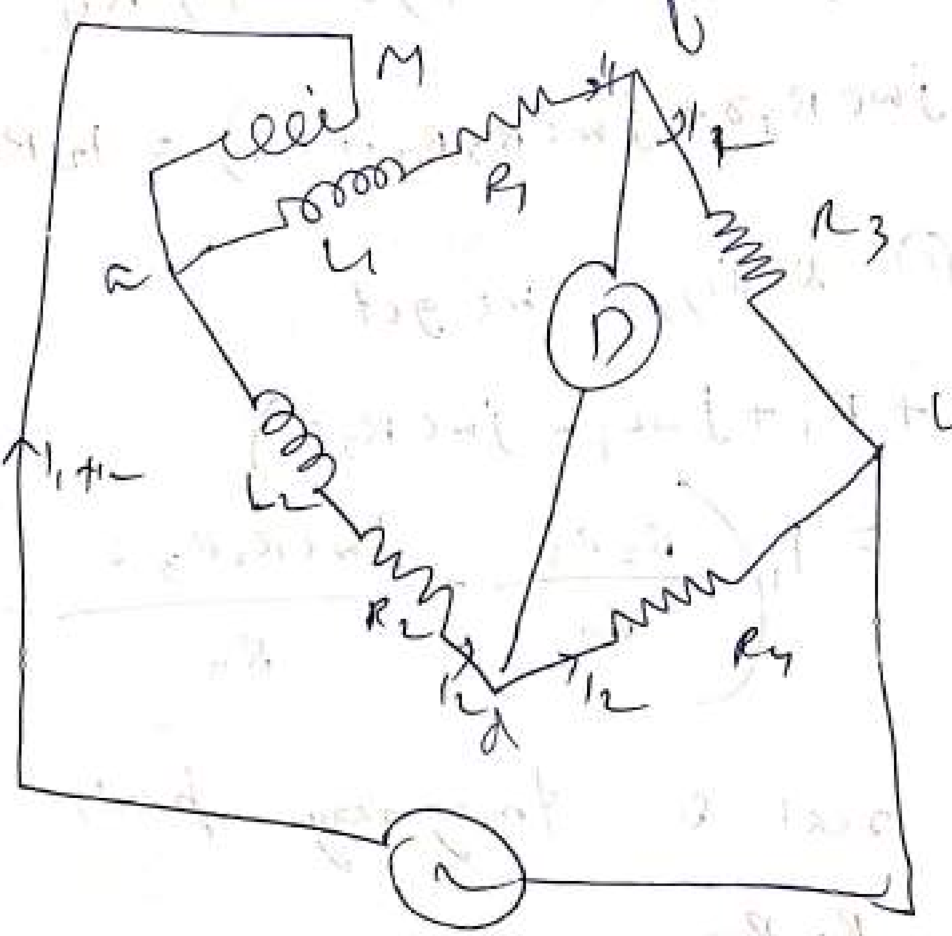
Heaviside

Measurement of Mutual Inductance

Heaviside Mutual Inductance







$$L_{e1} = L_1 + L_2 + 2M$$

$$L_{e2} = L_1 + L_2 - 2M$$

$$M = \frac{1}{4}\left(L_{e1} - L_{e2}\right)$$

M = ~~An~~ unknown mutual Inductance

L_1 = Self-inductance of secondary of mutual Inductance.

L_2 = known self-inductance

R_1, R_2, R_3, R_4 = non-inductive resistor

At balance voltage drop between b and c must equal the voltage drop between d and c. Also the voltage drop across a-b-c must equal the voltage drop across a-d-c.

$$I_1 R_3 = I_2 R_4$$

$$\Rightarrow (I_1 + I_2)(j\omega M) + I_1 (R_1 + R_3 + j\omega L_1) = I_2 (R_2 + R_4 + j\omega L_2).$$

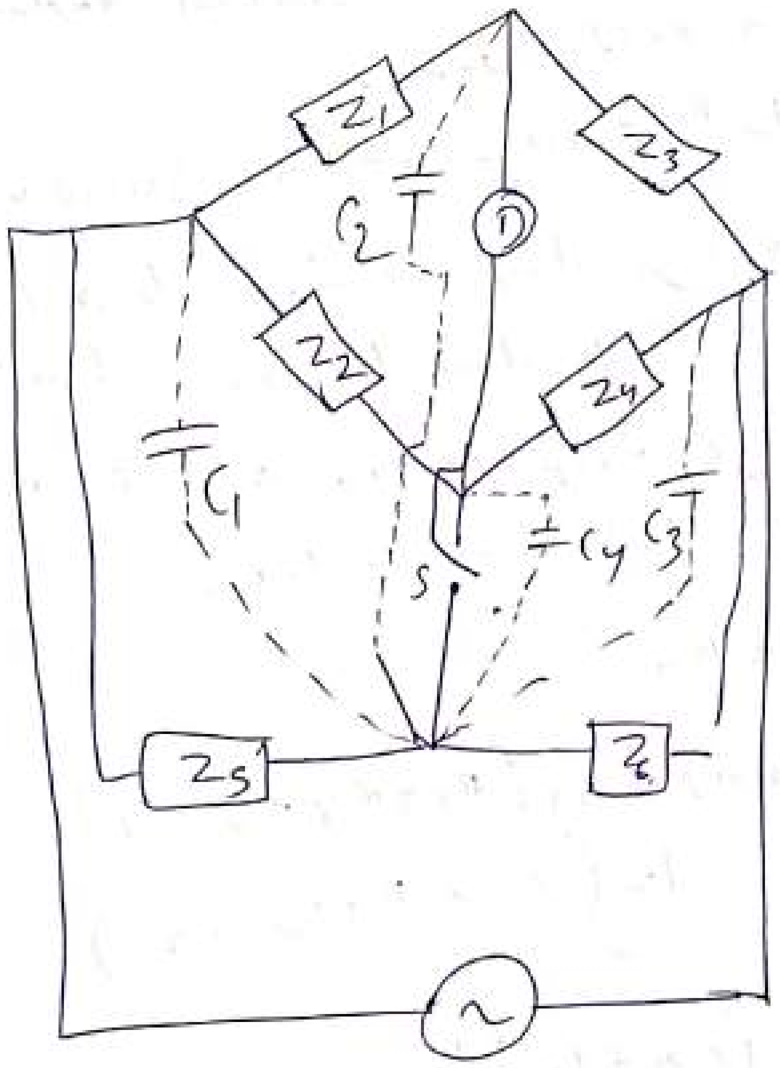
$$\Rightarrow I_2 \left(\frac{R_4}{R_3} + 1\right) j\omega M + I_2 \frac{R_4}{R_3} (R_1 + R_3 + j\omega L_1) = I_2 (R_2 + R_4 + j\omega L_2)$$

$$\Rightarrow j\omega M \left(\frac{R_4}{R_3} + 1\right) + \frac{R_4}{R_3} R_1 + R_4 + j\omega L_1 \frac{R_4}{R_3} = R_2 + R_4 + j\omega L_2$$

$$\Rightarrow R_1 = R_2 R_3 / R_4$$

$$\Rightarrow M = \frac{L_2 - L_1 R_4 / R_3}{\ldots} \Rightarrow \frac{R_3 L_2 - R_4 L_1}{R_3 + \ldots} - (1)$$

Wagner Earthing Device:-



If each component in a bridge has a defining screen connecti to one end, a very high accuracy in measurement is made possible by the addition of a wagner earthing Device. This device removes all the earth capacitance from the bridge network.

This figure shows the connections of the device for use in conjunction with the general form of bridge network, in which Z_1, Z_2, Z_3 & Z_4 are the impedances of the bridge arms, Z_5 & Z_6 are the two variable impedences of the Wagner Earth branch, the centre point of which is earth as shown in figure. These impedences may consists of variable resistances and capacitances similar to those used in the arms of the bridge proper, but not necessarily of known value. The two impedance Z_5 & Z_6 must be capable of forming a balanced bridge with Z_1 and Z_3 or Z_2 & Z_4 and can be a duplicate of either of these pairs of arms, C_1, C_2, C_3, C_4 are the stray earth capacitances appearing at the apexes of the bridge. D is the detector, which in this case is headphones.

If the switch S is on contacts d, balance of the bridge may be obtained by adjustment of the impedances Z_2 & Z_4.

The presence of the earth capacitances will prevent a true balance being obtained but a point of minimum sound can be obtained.

After adjusting the bridge to give minimum sound, the switch S is thrown to contact, e so that the headphones are then connected between b and earth; Z_5 & Z_6 are next adjusted until minimum sound is obtained. The headphones are next reconnected to b, d and Z_2 & Z_4 adjusted to give minimum sound again. The process is repeated until silence is obtained with the switch on d, and silence or the minimum sound is obtained, with the switch on e. All three points b, d & e are at earth potential. Under these conditions no current flows in the earth capacitances C_a & C_4 and since C_1 and C_3 shunt the Wagner arms Z_5 & Z_6, these capacitances are eliminated from the bridge network, Z_1, Z_2, Z_3 & Z_4.

By comparing eq ① and ②; we get

$$L_2 = \frac{M(R_3 + R_4) + R_4 L_1}{R_3}$$

$$\boxed{= M\left(1 + \frac{R_4}{R_3}\right) + \frac{R_4}{R_3} L_1}$$

Factors causing errors:-

(i) Stray-conductance effects, due to imperfect insulation.

(ii) Mutual Inductance effect :- due to magnetic coupling between various components of the bridge.

(iii) Stray-capacitance effects due to electrostatic field between conductors at different potential.

(iv "residues" in components - the existence of small amount of series inductance or shunt capacitance in nominally non-reactive resistors

Measurement of Energy by Energy meter

energy is the total power delivered or consumed over a time interval.

Energy = power x time.

$$W = \int_0^t vi \cdot dt$$

If v is expressed in V, i in A and t in s., the unit of energy is joule or watt over an interval of one second.

Motor meters:-

Motor meters are used for measurement of energy in both d.c. & a.c circuits. For d.c. circuits, the meter may be an ampere hour meter or a watthour meter

In motor meters the moving system revolves continously unlike in the indicating instruments where it deflects through a fraction of a revolution.

The speed of rotation is proportional to power in the case of watthour meters and ampere ~~turns~~ hours in the case of ampere hour meters. Thus the total number of revolutions made by a watthour meter in a given interval of time is proportional to the energy supplied (and in the case of ampere hour meters, to the total quantity of electricity supplied). In this connection a term called meter constant is used. Meter constant is defined as the number of revolutions made per kilowatt-hour (kwh.). The value of meter const. is usually marked on the ~~paper~~ meter.

Braking:-

In a motor meter the speed of the moving system is controlled by a braking torque. The braking system consists of a permanent magnet (braking magnet) so placed that it induces eddy currents in some part of the moving system.

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These eddy currents produce a braking torque which is proportional to the speed of moving system. The part in which eddy currents are produced is usually an aluminium disc. The disc is mounted on the moving system, and therefore, when the moving system revolves this disc cuts through the field of the permanent magnet

Emf generated in the disc $\Rightarrow e = K_1 \phi n$

ϕ = flux of the permanent magnet

n = speed of rotation

K_1 = a const.

$\Rightarrow$ Let r = resistance of the eddy current path.

$\Rightarrow$ So the eddy current produced is

$\Rightarrow i = e/r$

$= K_1 \phi n / r$

The braking torque is produced by the interaction of the eddy current and the field of the permanent magnet.

This torque is directly proportional to the product of flux of flux of the magnet, magnitude of eddy current and the effective radius R from the axis of the disc.

Braking torque $T_B = K_2 \phi \, i R = K_1 K_2 \phi^2 n R / r$

$= K_3 \phi^2 n R / r$

where K_2 & K_3 are const.

If the radius R of the disc is const.

$T_B = K_4 \phi^2 n / r$

where $K_4 =$ a const. $= K_3 R$.

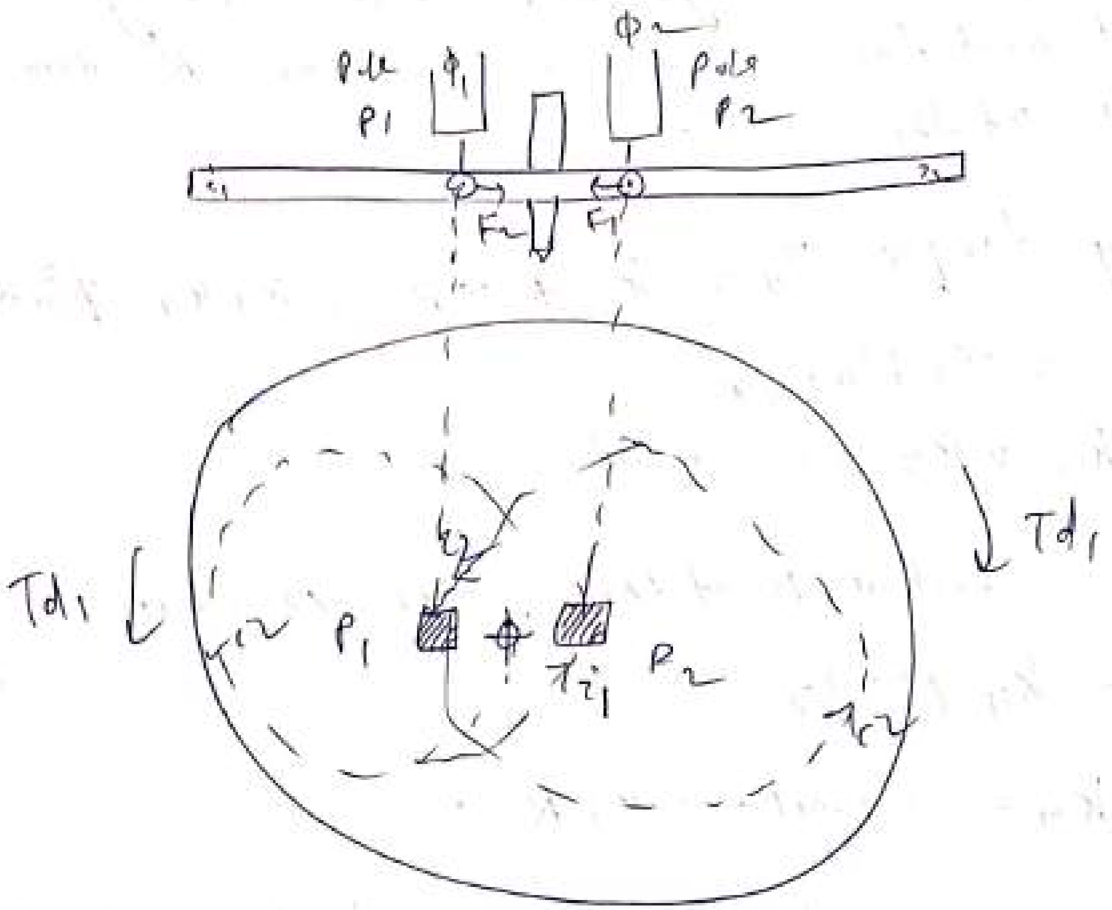
The moving system attains a steady speed when the driving torque is equal to the braking torque.

Breaking torque at steady speed $N \Rightarrow$

$T_B = K_3 \cdot \phi^2 N R / r = K_5 N$

$$\boxed{K_5 = K_3 \phi^2 R / r.}$$

Energy meters for A.C. circuits:



Induction type of energy meters are universally used for measurement of energy in domestic and industrial a.c. circuits. Induction type of meters possess lower friction and higher torque/weight ratio. Also induction type meters are inexpensive and accurate and retain their accuracy over a wide range of loads and temp. conditions