Digital Systems

Electronics circuits and systems are of two kinds
1. Analog
2. Digital

→ Analog circuits are those in which voltages and currents vary continuously through the given range.

→ A digital circuit is one in which the voltage levels assume a finite number of distinct values. All modern digital circuits have only two discrete voltage levels.

→ Digital circuits are often called <u>switching circuits</u>, because the voltage levels in a digital circuit are assumed to be switched from one value to another instantaneously, that is, the transition time is assumed to be zero.

→ Digital circuits are also called <u>logic circuits</u>, because each type of digital circuit obeys a certain set of logic rules. The manner in which a logic circuit responds to an input is referred to as the circuit's logic.

→ <u>Logic design</u> involves determining how to interconnect basic logic building blocks to perform a specific function.

→ In digital system such as computers, combinations of the two states are used to represent numbers, symbols, alphabetic characters and other type of information.

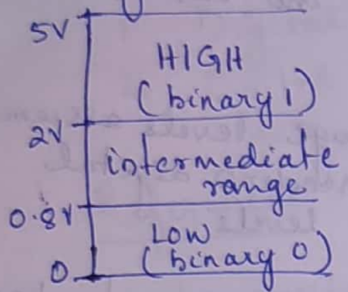
→ The two states are two binary digits 1 and 0 which are represented by two voltage levels.

- Positive logic system
  - 1 is represented by HIGH voltage level
  - 0 is represented by LOW voltage level
- Negative logic system
  - 0 is represented by HIGH voltage level
  - 1 is represented by LOW voltage level

→ The voltages used to represent a 1 and a 0 are called logic levels.

→ Normally, the binary 0 and 1 are represented by the logic voltage levels 0 V and +5 V.

→ In practical digital circuit, because of circuit variations, the 0 and 1 would be represented by voltage ranges instead of particular voltage levels.



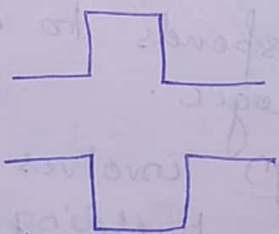
(Logic level ranges of voltage for a logic circuit)

→ Usually, any voltage between 0V and 0.8 V represents the logic 0 and any voltage between 2V and 5V represents logic 1.

→ The range between 0.8 V and 2V is called the intermediate range.

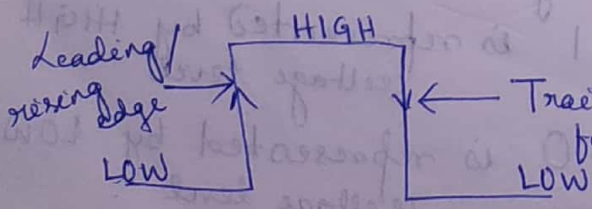
→ In digital circuits and systems, the voltage levels are normally changing back and forth between the HIGH and LOW stages/level.

→ Pulse < Positive pulse / Negative pulse

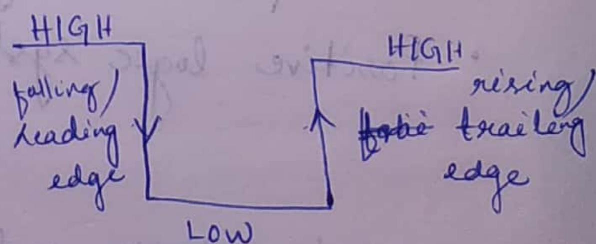


→ A single positive-going pulse is generated when the voltage goes from its normally low level to its HIGH level and then back to its LOW level.

→ The negative-going pulse is generated when the voltage goes from its normally HIGH level to its LOW level and back to its HIGH level.



(Positive pulse)



(Negative pulse)

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→ Most waveforms encountered in digital systems are composed of series of pulses, called pulse trains.

Pulse train < Periodic / Non-periodic

→ An important characteristic of a periodic digital waveform is its duty cycle, which is the ratio of the pulse width (t_w) to the period (T).

$$\boxed{\text{Duty cycle} = \left(\frac{t_w}{T}\right) 100\,\%}$$

Number Systems

1. Decimal Number system
2. Binary " "
3. Octal " "
4. Hexadecimal " "

Decimal Number system

→ The Decimal system contains 10 unique symbols 0 – 9. The base / radix is 10. Each symbol in the number is called digit.

→ A number with a decimal point is represented by a series of coefficients.

$$a_5 a_4 a_3 a_2 a_1 a_0 \cdot a_{-1} a_{-2} a_{-3}$$

a_j = coefficient – any value between (0 – 9).
j = gives the place value, the power of 10 by which the coefficient must be multiplied.

→ The value of a digit is determined by its position in the number and has assigned weight.

$$10^3\ 10^2\ 10^1\ 10^0 .\ 10^{-1}\ 10^{-2}\ 10^{-3}$$

→ The leftmost digit in any number representation has the greatest positional weight called most significant digit (MSD). The right most digit has least positional weight, called least significant digit (LSD).

Binary Number Systems

→ The coefficients of the binary number system have only two possible values 0 and 1. The base/radix is 2. The binary digit is called bit. Eg :- $(110)_2$

→ The position of a 1 or 0 in a binary number indicates its weight. The weights in a binary number are based on powers of 2.

Eg $d_3\, d_2\, d_1\, d_0 \,.\, d_{-1}\, d_{-2}\, d_{-3}$

$$d_3 \times 2^3 + d_2 \times 2^2 + d_1 \times 2^1 + d_0 \times 2^0 + d_{-1} \times 2^{-1} + d_{-2} \times 2^{-2} + d_{-3} \times 2^{-3}$$

→ The right most bit is the least significant bit (LSB) and the left most bit is called the most significant bit (MSB).

* Computer capacity is usually given in bytes.

~~A bg~~ 1 byte = 8 bits

1 Nibble = 4 bits

1 KB = 2^{10} bytes = 1024 byte

1 MB = 2^{20} bytes

1 GB = 2^{30} bytes.

Octal Number system

→ The octal number system has a base of 8. It has eight digits (0 – 7). Eg : $(123.7)_8$.

Base $8 = 2^3$ — Every 3 bit group of binary can be represented by an octal digit.

Hexadecimal Number system

→ The base/radix is 16. It has 16 independent symbols : ~~(0 – 15).~~ The 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. Eg : $(F10.2C)_{16}$.

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Base $16 = 2^4$ — Every 4 bit group of binary can be represented by an hexadecimal digit.

Number - Base Conversion

1. Conversion of $(\ )_r$ to decimal number :-

→ The conversion of a number in base r to decimal equivalent is done by multiplying each coefficient to its position weight and the product terms are added.

eg $(x_3 x_2 x_1 x_0 . x_{-1} x_{-2})_r$

$= x_3 \times r^3 + x_2 \times r^2 + x_1 \times r^1 + x_0 \times r^0 + x_{-1} \times r^{-1} + x_{-2} \times r^{-2}$

a. Binary to decimal :-

eg $(1101.11)_2 = (\ )_{10}$

$2^3 \ 2^2 \ 2^1 \ 2^0 \ 2^{-1} \ 2^{-2}$

$= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$

$= 8 + 4 + 1 + 0.5 + 0.25 = 13.75$

b. octal to decimal :-

eg $(234.25)_8 = (\ )_{10}$

$8^2 \ 8^1 \ 8^0 \ 8^{-1} \ 8^{-2}$

$\Rightarrow 2 \times 8^2 + 3 \times 8^1 + 4 \times 8^0 + 2 \times 8^{-1} + 5 \times 8^{-2} =$

c. Hexadecimal to decimal :-

eg $(FAB.2C)_{16} = (\ )_{10}$

$16^2 \ 16^1 \ 16^0 \ 16^{-1} \ 16^{-2}$

$\Rightarrow F \times 16^2 + A \times 16^1 + B \times 16^0 + 2 \times 16^{-1} + C \times 16^{-2} =$

d $(321.22)_5$ to decimal

$\Rightarrow 3 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 + 2 \times 5^{-1} + 2 \times 5^{-2} =$

2. Conversion of decimal number to $(\ )_r$.

→ The decimal integer number is converted to the number in base r by successive division by r, and the decimal fraction is converted to fraction number in base r by successive multiplication by r.

This method is also known as double - dabble method.

a. Decimal to Binary :- (Successive division by 2 method)

Eg $(105.15)_{10} = (\quad)_2$

2	105	
2	52	→ 1 ← LSB
2	26	→ 0
2	13	→ 0
2	6	→ 1
2	3	→ 0
2	1	→ 1
	0	→ 1 ← MSB

read from bottom to top.

$$
\begin{array}{r}
0.15 \\
\times\ 2 \\
\hline
\boxed{0}.30 \\
\times\ 2 \\
\hline
0.60 \\
\times\ 2 \\
\hline
1.20 \\
\times\ 2 \\
\hline
0.40 \\
\times\ 2 \\
\hline
0.80 \\
\times\ 2 \\
\hline
1.60
\end{array}
$$

read from top to bottom

Ans $(105.15)_{10}$

$= (1101001.001001)_2$

2nd method to convert decimal to binary

Sum-of-weights method :- (normally used for small numbers)

Eg $(75)_{10} = (\quad)_2$

Ans The largest number, which is a power of 2, not exceeding 75 is 64 i.e 2^6

$$64 = 2^6 = (1000000)_2$$

The remainder is

$$75 - 64 = 11$$

The largest number, which is a power of 2, not exceeding 11 is 8 i.e 23

$8 = 2^3 = 1000$

— The remainder is $11 - 8 = 3$

— The largest number, which is a power of 2, not exceeding 3 is $2 = 2^1 = (10)_2$

— The remainder is $3 - 2 = 1$.

$1 = 2^0$

Hence $(75)_{10} = (1000000)_2 + (1000)_2 + (10)_2 + 1$.

$=$

Counting in Binary

Decimal number	Binary number
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000

Decimal number	Binary number
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

* $(2^n)_{10} = (1\ n\ \text{zeros})_2$
* $(2^n - 1)_{10} = (n\ \text{ones})_2$

b. Decimal to octal :-

Eg $(378.93)_{10} = (\quad)_8 = (572.7341)_{10}$

$$\begin{array}{r|l}
8 & 378 \\ \hline
8 & 47 \rightarrow 2 \ \text{LSB} \\ \hline
8 & 5 \rightarrow 7 \\ \hline
 & 0 \rightarrow 5 \ \text{MSB}
\end{array}$$

$0.93 \times 8 = 7.44$

$0.44 \times 8 = 3.52$

$0.52 \times 8 = 4.16$

$0.16 \times 8 = 1.28$

C. Convert $(2598.675)_{10} = (\quad)_{16}$

$$
\begin{array}{r|l l}
16 & 2598 & \\
16 & 162 & \rightarrow 6 \\
16 & 10 & \rightarrow 2 \\
 & 0 & \rightarrow A
\end{array}
$$

$$
\begin{array}{r}
0.675 \\
\times 16 \\
\hline
10.8 \\
\times 16 \\
\hline
12.8 \\
\times 16 \\
\hline
12.8 \\
\times 16 \\
\hline
12.8
\end{array}
$$

Ans $(2598.675)_{10}$

$= (A26.ACCC)_{16}$

D. Octal to Binary

→ Each octal digit is represented by a 3-bit binary number.

Eg

octal digit	Binary digit
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Eg $(7526.766)_8 = (\quad)_2$

Ans $(111\,101\,010\,110 . 111\,110\,110)_2$

E. Binary to octal

→ Start with the right-most group of three bits and moving from right to left, convert each 3-bit group to the equivalent octal digit.

Eg $(11011.1011)_2 = (\quad)_8$

$(011\,011 . 101\,100)_2 = (33.54)_8$

E. Hexadecimal to Binary

→ Replace each hex digit by its 4-bit binary group.

Eg

Hexadecimal digit	Binary digit	Hexadecimal digit	Binary digit
0	0000	9	1001
1	0001	A	1010
2	0010	B	1011
3	0011	C	1100
4	0100	D	1101
5	0101	E	1110
6	0110	F	1111
7	0111		
8	1000		

Eg $(5AB2.10C)_{16} = (\quad)_2$

0101 1010 1011 0010 0001

Ans $(0101101010110010.000100001100)_2$

F. Binary to Hexadecimal :-

→ Make groups of 4 bits, and replace each 4-bit group by a hex digit.

Eg $(11011.11011)_2 = (\quad)_{16} = (1B.D8)_{16}$

0001 → 1, 1011 → B, 1101 → D, 1000 → 8

G. Octal to Hexadecimal :-

Octal → binary → Hexadecimal

Eg $(730.12)_8 = (\quad)_{16}$

$(730.12)_8$

↓

$(111011000.001010)_2$ → 1 D 8 . 2 8

$(1D8.28)_{16}$ (Ans)

H. Hexadecimal to octal

Hexadecimal $\longrightarrow$ binary $\longrightarrow$ octal

eg $(BC2.F3)_{16} = (\quad)_8$

Ans $(BC2.F3)_{16}$

$\downarrow$

$(101111000010.11110011)_2 \longrightarrow (5702.7406)_8$

Questions

1. Find x

(a) $(55)_x = (30)_{10}$ (b) $(257)_x = (140)_{10}$

(c) $(431)_x = (116)_{10}$ (d) $\frac{14}{2} = 5$

(e) $(103)_8 - (45)_8 = (x)_5$ (f) $\frac{41\cancel{3}}{3\cancel{4}} = 13$

(g) $24 + 17 = 40$ (h) $\sqrt{(14)_x} = 3$

(i) $(125)_6 = (\quad)_5$

(j) The solutions to the quadratic equation ~~$x^2-11x+22=$~~ $x^2 - 11x + 22 = 0$ are $x = 3$ and $x = 6$. What is the base of the number ?

Solutions

1(a) $(55)_x = (30)_{10}$

$\downarrow$ convert it into decimal

$5x^1 + 5x^0 = 30 \Rightarrow 5x + 5 = 30$

$\Rightarrow x = \frac{25}{5} = 5 \Rightarrow \boxed{x = 5}$

(b) $(257)_x = (140)_{10}$

$\downarrow$ convert it into decimal

$2x^2 + 5x + 7 = 140$

$2x^2 + 5x - 133 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4\times 2\times(-133)}}{4} =$$

(c) Same procedure

(d) $\frac{14}{2} = 5$

Let the base is x. Then $\frac{(14)_x}{(2)_x} = (5)_x$

$$\frac{x+4}{2} = 5 \Rightarrow x+4 = 10 \Rightarrow \boxed{x=6}$$

(e) $(103)_8 - (45)_8 = (x)_5$

$$\Rightarrow \begin{array}{r} (103)_8 \\ -\ (45)_8 \\ \hline (36)_8 \end{array}$$

$(36)_8 = (\quad)_5$

$(36)_8 = \underbrace{(3\times 8+6)}_{\text{equivalent decimal number}} = (30)_{10}$

Hence $(30)_{10} = (x)_5$

convert it into decimal

$$\begin{array}{r|l} 5 & 30 \\ 5 & 6 - 0 \\ 5 & 1 - 1 \\ & 0 \rightarrow 1 \end{array}$$

$\cancel{x\times 5 = 30}$ $\cancel{\Rightarrow x = 30}$

$(30)_{10} = (110)_5$

$\Rightarrow \boxed{x = 110}$

f $\frac{41\cancel{5}}{3\cancel{4}} = 13$

Let the base is x. Then $\cancel{\frac{(15)_x}{(4)_x} = (13)_x}$

$\cancel{\frac{x+5}{4} = x+3} \Rightarrow \cancel{x+5 = 4x+12}$

$$\Rightarrow \frac{(41)_x}{(3)_x} = (13)_x \Rightarrow \frac{4x+1}{3} = x+3$$

$$\Rightarrow 4x+1 = 3x+9 \Rightarrow \boxed{x=8}$$

g same as (e) and (f).

h) $\sqrt{(14)_x} = 3$

Let the base is x $\sqrt{(14)_x} = (3)_x$

Squaring both sides

$(14)_x = 3 \times x^0 \times 3 \times x^0$

$\Rightarrow x + 4 = 9$ $\boxed{x = 5}$

i) $(125)_6 = (x)_5$

↓

convert it to decimal.

$(125)_6 = 1 \times 6^2 + 2 \times 6^1 + 5 \times 6^0 = 36 + 12 + 5 = (53)_{10}$

$(53)_{10} = (x)_5 = (203)_5$

Then Decimal to a number base 5.

$$\begin{array}{r|l} 5 & 53 \\ 5 & 10 \quad — 3 \\ 5 & 2 \quad — 0 \\ & 0 \rightarrow 2 \end{array}$$

$\boxed{x = 203}$

j) The solutions are $x = 3$ and $x = 6$.

$(x-3)(x-6) = x^2 - 6x - 3x + 18$

Hence, $\Rightarrow x^2 - 9x + 18 = x^2 - 11x + 22$

$\Rightarrow$ comparing both sides

$(9)_{10} = (11)_x$

$\Rightarrow x + 1 = 9$

$\Rightarrow \boxed{x = 8}$

$(18)_{10} = (22)_x$

$2x + 2 = 18$

$2x = 16$

$\boxed{x = 8}$

K) In a number system with radix r, the decimal value of 110 is equal to 4r. The decimal values of 111 and r, respectively are equal to ________.

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Binary Arithmetic

1. Binary Addition :-

The four basic rules for adding binary bits are

$0 + 0 = 0$ — Sum 0 carry 0

$0 + 1 = 1$ — Sum 1 carry 0

$1 + 0 = 1$ — Sum 1 carry 0

$1 + 1 = 10 \rightarrow$ Sum 0 carry 1

Eg 1. $(111.010)_2 + (101.111)_2 = (1101.001)_2$

Ans

$$\begin{array}{r} 1\,1\,1\,.\,0\,1\,0 \\ +\ 1\,0\,1\,.\,1\,1\,1 \\ \hline (1\,1\,0\,1\,.\,0\,0\,1)_2 \end{array}$$

Eg.2 $(1111)_2 + (1111)_2 + (1111)_2 = (\quad)$

$$\begin{array}{r} 1\,1\,1\,1 \\ +\ 1\,1\,1\,1 \\ +\ 1\,1\,1\,1 \\ \hline (1\,0\,1\,1\,0\,1)_2 \end{array}$$

$4 = (100)_2$ carry goes to next higher bit

Ans $(101101)_2$

2. Binary Subtraction :-

The four basic rules for subtracting bits are as follows

$0 - 0 = 0$ Difference 0 borrow 0

$1 - 0 = 1$ Difference 1 borrow 0

$1 - 1 = 0$ Difference 0 borrow 0

$0 - 1 = 1$ Difference 1 borrow 1

Eg $(1010.01)_2 - (111.111)_2 = (\quad)_2$

Ans

$$\begin{array}{r} 1010.010 \\ -\ 111.111 \\ \hline (010.011)_2 \end{array}$$

Eg 2 $(10000)_2 - (1111)_2$

Ans

$$\begin{array}{r} 10000 \\ -\ 1111 \\ \hline (0001)_2 \end{array}$$

3. Binary multiplication :

The four basic rules for multiplying bits are

$0 \times 0 = 0$
$0 \times 1 = 0$
$1 \times 0 = 0$
$1 \times 1 = 1$

Eg $(110)_2 \times (111)_2 = (101010)_2$

$$\begin{array}{r} 110 \\ \times\ 111 \\ \hline 110 \\ 110 \\ 110 \\ \hline (101010)_2 \end{array}$$

4. Binary Division

Same as decimal division.

Eg $(101101)_2 \div (110)_2$