Logic Gates and Minimization of Boolean Functions

Chapter - 3

→ Logic gates are the fundamental building block of digital systems. The name logic gate is derived from the ability of such device to make decisions, it produces one output level when some combinations of input levels are present, and a different output level when other combinations of input levels are present.

→ The interconnection of gates to perform a variety of logical operations is called logic design.

→ Input and outputs of logic gates can occur only in two levels.
These levels are termed as

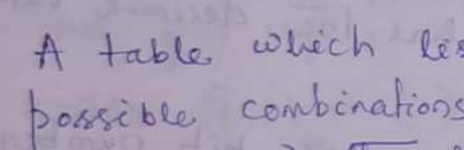
0	1
LOW	HIGH
OFF	ON
FALSE	TRUE

→ A table which lists all possible combinations of input variables and the corresponding output is called a truth table.

Basic Gates

1. AND Gate :- An AND gate has two or more inputs but only one output. The output assumes the logic 1 state only when each one of its input is at logic 1 state.

→ This gate is also called all or nothing gate.

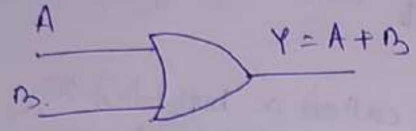


Truth table

I/P		O/P
A	B	Y = AB
0	0	0
0	1	0
1	0	0
1	1	1

2. OR gate :- An OR gate has two or more inputs but only one output. The output assumes the logic 1 state even if one of its input is in logic 1 state.

→ This gate is also called as any or all gate.



i/p		o/p
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

3. NOT Gate

→ It has only one i/p and one output.

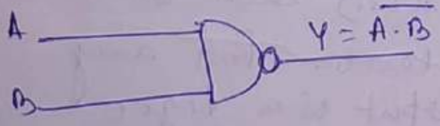
→ Output is always the complement of its input. So, A NOT gate is also called as inverter.

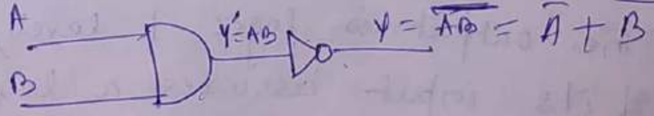
i/p	o/p
A	Y
0	1
1	0

Other logic gates

1. NAND gate :- NAND means NOT AND i.e AND output is NOTed. So, a NAND gate is a combination of an AND gate and a NOT gate.

→ The output is logic 0 level, only when each of the input assumes a logic 1 level state. For any other combination of inputs, the output is a logic 1 level.





i/p		o/p
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

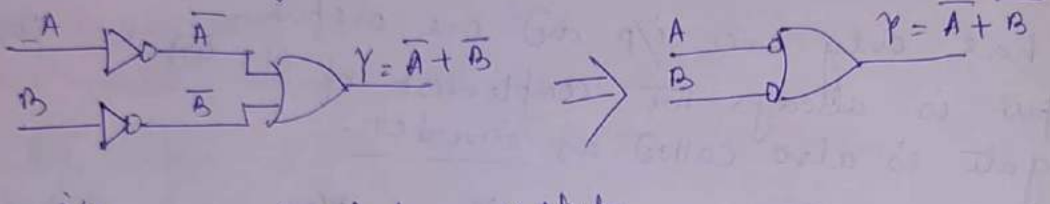
Bubbled OR gate :-

$$Y = \overline{A \cdot B} = \overline{A} + \overline{B}$$

→ NAND function can also be realized by first inverting the inputs and then ORing the inverted inputs.

→ Thus a NAND gate is a combination of two NOT gates and an OR gate.

→ The OR gate with inverted inputs is called a bubbled OR gate / negative OR gate. NAND gate is equivalent to a bubbled OR gate.

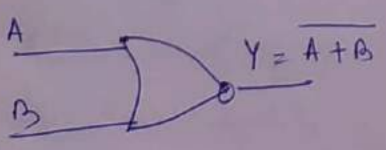


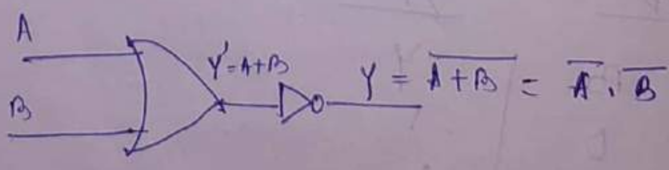
i/p		inverted		Output
A	B	$\overline{A}$	$\overline{B}$	Y
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

2. NOR Gate :-

→ NOR means NOT OR i.e the OR output is NOTed. So, a NOR gate is a combination of an OR gate and a NOT gate. ~~It is represented as~~

→ The output is logic 1 level, only when each one of its input assumes a logic 0 level. For any other combinations of inputs, the output is a logic 0 level.





T.T (Bubbled AND gate)

i/p		inverted		o/p
A	B	$\overline{A}$	$\overline{B}$	Y
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

T.T (NOR gate)

i/p		o/p
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

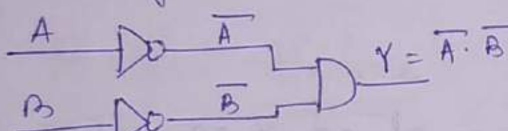
Bubbled ~~ABR~~ AND gate

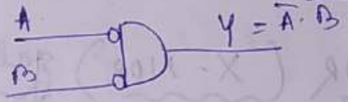
$$Y = \overline{A} \cdot \overline{B} = \overline{A + B}$$

→ A NOR gate is equivalent to an AND gate with inverted inputs and the corresponding output is $Y = \overline{A}\,\overline{B}$.

→ NOR function can also be realized by first inverting the inputs and then ANDing those inverted inputs.

Thus a NOR gate is a combination of two NOT gates and one AND gate.





→ The AND gate with inverted inputs is called bubbled AND gate.

3. Buffer :-

→ The binary value of the output is equal to the binary value of the input.

→ It is equivalent to two NOT gates/inverters connected in cascade.

i/p	o/p
A	Y
0	0
1	1

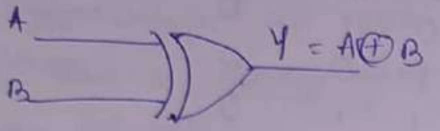
★ NAND and NOR gates are universal gates.

4. Exclusive - OR (X-OR) Gate : (special gate)

→ The X-OR gate is a two input, one output logic circuit.

→ An X-OR gate produces an output 1 only when the inputs are not equal, it is called an anti-coincidence gate or inequality detector.

→ If the input variables are represented by A and B, and the output variable by Y, the expression for the output of this gate is given by $Y = A \oplus B = A\bar{B} + \bar{A}B$

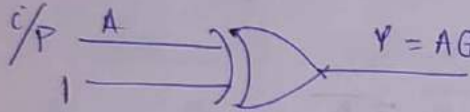


T.T

i/p A	i/p B	o/p Y
0	0	0
0	1	1
1	0	1
1	1	0

→ Three or more variable X-OR gates does not exist.

* An X-OR gate can be used as an inverter by connecting one of the input terminals to logic 1 and feeding the input sequence to be inverted to other terminal.



$Y = A \oplus 1 = \bar{A}1 + \bar{1}\cdot A = \bar{A}$

5. Exclusive - NOR (X-NOR) gate

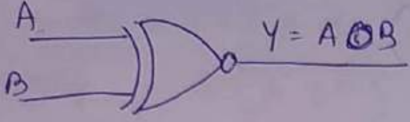
→ An X-NOR gate is a combination of an X-OR gate and a NOT gate.

→ The X-NOR gate is a two input, one output logic circuit, whose output assumes a 1 state only when both the inputs assume a 0 state or when both the inputs assume a 1 state.

→ It is also called a coincidence gate or equality detector.

→ If the input variables are represented by A and B and the output variable by Y, the expression for the output of this gate is given by

$$Y = A \odot B = \overline{A \oplus B} = \overline{\bar{A}B + A\bar{B}}$$
$$= \left(\overline{\bar{A}B}\right)\left(\overline{A\bar{B}}\right)$$
$$= (A + \bar{B})(\bar{A} + B)$$
$$= A\cdot\bar{A} + AB + \bar{A}\bar{B} + B\bar{B}$$
$$= AB + \bar{A}\bar{B}$$



$Y = A \odot B = \overline{A \oplus B}$

i/p		o/p
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

→ Three or more variable X-NOR gates does n't exist.

* An X-NOR gate can be used as an inverter by connecting one of the two input terminals to logic 0.

$Y = A \odot 0 = \bar{A}\,\bar{0} + A \cdot 0 = \bar{A}$

* X-NOR of three variables A, B, C is not equal to the complement of the X-OR of A, B, and C.

$$A \odot B \odot C \neq \overline{A \oplus B \oplus C}$$

Boolean Algebra

→ Boolean algebra is a system of mathematical logic.

→ It is an algebraic system consisting of the set of elements (0, 1), two binary operators called OR and AND and one unary operator called NOT.

→ It is a way to express the logic functions algebraically.

→ In Boolean algebra, the multiplication and addition of the variables and functions are also only logical. They represent logical operations.

→ Logical multiplication is same as the AND operations, and logical addition is same as the OR operation.

Laws of Boolean Algebra

1) Complementation Laws

(a) $\bar{0} = 1$

(b) $\bar{1} = 0$

(c) $A = 0$ then $\bar{A} = 1$

(d) $A = 1$ then $\bar{A} = 0$

d) $\overline{\overline{A}} = A$ (double complementation law)

2> <u>AND Laws</u>

(a) $A \cdot 0 = 0$ (Null law)

(b) $A \cdot 1 = A$ (Identity law)

(c) $A \cdot A = A$

(d) $A \cdot \overline{A} = 0$

3> <u>OR laws</u>

(a) $A + 0 = A$ (Null law)

(b) $A + 1 = 1$ (Identity law)

(c) $A + A = A$

(d) $A + \overline{A} = 1$

4> <u>Commutative Laws</u>

(a) $A + B = B + A$

(b) $A \cdot B = B \cdot A$

5> <u>Associative Laws</u>

(a) $(A+B)+C = A+(B+C)$

(b) $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

6> <u>Distributive Laws</u>

(a) $A(B+C) = AB + AC$

(b) $A + BC = (A+B)(A+C)$

Proof: $(A+B)(A+C)$

$= A \cdot A + AC + AB + BC$

$= A + AC + AB + BC = A(1 + C + B) + BC$

$= A + BC$

7> <u>Redundant literal Rule</u> (RLR)

(a) $A + \overline{A}B = A + B$

Proof:

$A + \overline{A}B$

$= (A + \overline{A})(A+B)$ (using distributive law)

$= 1 \cdot (A+B)$

$= A + B$

(b) $A(\overline{A}+B) = AB$

Proof: $A(\overline{A}+B) = A\cdot\overline{A} + AB = AB$

8) <u>Idempotence Laws</u>

(a) $A\cdot A = A$ (b) $A + A = A$

9) <u>Absorption Laws</u>

(a) $A + A\cdot B = A$

Proof: $A + AB$
$= A(1+B)$
$= A$

(b) $A(A+B) = A$

Proof: $A(A+B)$
$= A\cdot A + AB$
$= A + AB = A(1+B)$
$= A$

10) <u>Consensus theorem</u>

(a) $xy + x'z + yz = xy + x'z$

Proof: $xy + x'z + yz(x+x')$
$= xy + x'z + xyz + x'yz$
$= xy(1+z) + x'z(1+y)$
$= xy + x'z$

(b) $(x+y)(x'+z)(y+z) = (x+y)(x'+z)$

11) <u>Transposition theorem</u>

(a) $AB + \overline{A}C = (A+C)(\overline{A}+B)$

12) <u>De Morgan's theorem</u>

(a) $\overline{A+B} = \overline{A}\cdot\overline{B}$

— This law states that the complement of a sum of variables is equal to the product of their individual complements.

(b) $\overline{AB} = \overline{A}+\overline{B}$

— This law states that the complement of the product of variables is equal to the sum of their individual complements.

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Duality

→ By changing OR $(+)$ ⟶ AND $(\cdot)$, AND $(\cdot)$ ⟶ OR $(+)$

→ By complementing all 0s and 1s.

→ The variables are not complemented in this process.

Eg Duals

$\overline{0} = 1 \longrightarrow \overline{1} = 0$

$0 \cdot 1 = 0 \longrightarrow 1 + 0 = 1$

$0 \cdot 0 = 0 \longrightarrow 1 + 1 = 1$

$A \cdot 0 = 0 \longrightarrow A + 1 = 1$

$A \cdot B = B \cdot A \longrightarrow A + B = B + A$

$A(B+C) = AB + AC \longrightarrow A + (B \cdot C) = (A+B)(A+C)$

$A + \overline{B}C = (A+\overline{B})(A+C) \longrightarrow A(\overline{B}+C) = A\overline{B} + AC$

Complement

→ The complement of a function f is $\overline{f}$ and is obtained from an interchange of 0's for 1's and 1's for 0's in the value of f.

→ The complement of a function may be derived algebraically through De Morgan's theorem.

Q1 Reduce the following expression.

$f = \overline{\overline{AB} + \overline{A} + AB}$

$= \overline{\overline{A} + \overline{B} + \overline{A} + AB}$ $\quad (\because \overline{A} + \overline{A} = \overline{A})$

$= \overline{\overline{A} + \overline{B} + AB}$

$= \overline{(A+\overline{A})(\overline{A}+B) + \overline{B}}$ ← Distributive law

$= \overline{\overline{A} + B + \overline{B}}$ $\quad (\because B + \overline{B} = 1)$

$= \overline{\overline{A} + 1}$ $\quad (\because 1 + A = 1)$

$= \overline{1} = 0$

Q2 $(\overline{A + \overline{BC}})(A\overline{B} + \overline{ABC})$

$= (\overline{A} \cdot \overline{\overline{BC}})(A\overline{B} + \overline{ABC})$ (De-Morgan's law)

$= (\overline{A} \cdot BC)(A\overline{B} + \overline{ABC})$

$= \overline{A}BC \cdot A\overline{B} + \overline{A}BC \cdot \overline{ABC}$

$= \overline{A}BC(\overline{A} + \overline{BC})$

$= \overline{A}BC(\overline{A} + \overline{B} + \overline{C})$

$= \overline{A}BC \cdot \overline{A} + \overline{A}BC\overline{B} + \overline{A}BC\overline{C}$ $\left(\because \overline{A} \cdot \overline{A} = \overline{A},\ B \cdot \overline{B} = 0,\ C \cdot \overline{C} = 0\right)$

$= \overline{A}BC$

Q3 Show that $A\overline{B}C + B + B\overline{D} + AB\overline{D} + \overline{A}C = B + C$

Ans $A\overline{B}C + B + B\overline{D} + AB\overline{D} + \overline{A}C$

$= B(1 + \overline{D} + A\overline{D}) + A\overline{B}C + \overline{A}C$

$= B + A\overline{B}C + \overline{A}C$ ← $(\because 1 + \overline{D} + A\overline{D} = 1)$

$= C(\overline{A} + A\overline{B}) + B$

$= C((A + \overline{A})(\overline{A} + \overline{B})) + B$ ← (Distributive law)

$= \overline{A}C + \overline{B}C + B$

$= \overline{A}C + (B + \overline{B})(B + C)$ ← (Distributive law)

$= \overline{A}C + B + C$

$= C(1 + \overline{A}) + B$ ← $(\because 1 + \overline{A} = 1)$

$= B + C$

Q4 Prove that $\overline{AB + AC} + A\overline{B}C = \overline{A} + \overline{B}$

Ans $\overline{AB + AC} + A\overline{B}C$

$= (\overline{AB})(\overline{AC}) + A\overline{B}C$ ← De-Morgan's law

$= (\overline{A} + \overline{B})(\overline{A} + \overline{C}) + A\overline{B}C$

$= \overline{A} \cdot \overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + A\overline{B}C$ $(\because \overline{A} \cdot \overline{A} = \overline{A})$

$= \overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + A\overline{B}C$

$= \overline{A}(1 + \overline{C} + \overline{B}) + \overline{B}\,\overline{C} + A\overline{B}C$

$= \bar{A} + \bar{B}\bar{C} + A\bar{B}C \qquad (\because 1 + \bar{C} + \bar{B} = 1)$

$= \bar{A} + \bar{B}(\bar{C} + AC)$

$= \bar{A} + \bar{B}\left((\bar{C} + C)(\bar{C} + A)\right)$ ← Distributive law

$= \bar{A} + \bar{B}(\bar{C} + A)$

$= \bar{A} + \bar{B}\bar{C} + A\bar{B}$

$= (\bar{A} + A)(\bar{A} + \bar{B}) + \bar{B}\bar{C}$ ← Distributive law

$= (\bar{A} + \bar{B}) + \bar{B}\bar{C}$

$= \bar{A} + \bar{B}(1 + \bar{C}) \qquad (\because 1 + \bar{C} = 1)$

$= \bar{A} + \bar{B}$

Q5 If $\bar{A}B + A\bar{B} = C$

Then prove $\bar{A}C + A\bar{C} = B$

Ans $\bar{A}(\bar{A}B + A\bar{B}) + A(\overline{\bar{A}B + A\bar{B}})$

$= \bar{A}\bar{A}B + \bar{A}A\bar{B} + A\left((\overline{\bar{A}B})(\overline{A\bar{B}})\right)$ ← Demorgan's theorem

$= \bar{A}B + A\left((\bar{\bar{A}} + \bar{B})(\bar{A} + \bar{\bar{B}})\right)$

$= \bar{A}B + A\left((A + \bar{B})(\bar{A} + B)\right)$

$= \bar{A}B + A(A\bar{A} + AB + \bar{A}\bar{B} + B\bar{B}) \qquad (\because A\bar{A} = 0,\ B\bar{B} = 0)$

$= \bar{A}B + A \cdot A \cdot B + \overline{A\bar{A}B} \qquad (\because A \cdot A = A,\ A \cdot \bar{A} = 0)$

$= AB + \bar{A}B$

$= B(A + \bar{A})$

$= B \qquad (\because A + \bar{A} = 1)$

Q6 Apply De Morgan's theorem to

$$\overline{(\overline{A+B})(\overline{C+D})(\overline{E+F})(\overline{G+H})}$$

$$= \overline{(\overline{A+B})(\overline{C+D})} + \overline{(\overline{E+F})(\overline{G+H})}$$

$$= (\overline{A+B})(\overline{C+D}) + (\overline{E+F})(\overline{G+H})$$

$= \cancel{\overline{A + B}}\ \overline{A} \cdot \overline{B}\ \overline{C}\ \overline{D} + \overline{E}\,\overline{F}\,\overline{G}\,\overline{H}$

Q7 Reduce the following Boolean expression to the indicated number of literals.

(a) $\overline{A}\,\overline{C} + ABC + A\overline{C} + A\overline{B}$ to ~~three~~ two literals

(b) $\overline{A}B(\overline{D} + C\overline{D}) + B(A + \overline{A}CD)$ to one literal

(a) $\overline{A}\,\overline{C} + ABC + A\overline{C} + A\overline{B}$

$= \overline{C}(A + \overline{A}) + A(BC + \overline{B})$

$= \overline{C} + A((B + \overline{B})(C + \overline{B}))$

$= \overline{C} + AC + \cancel{B}A\overline{B} = (C + \overline{C})(\overline{C} + A) + A\overline{B}$

$= \cancel{A + A\overline{B} + \overline{C}}\ \overline{C} + A + A\overline{B} = A(1 + \overline{B}) + \overline{C} = A + \overline{C}$

(b) $\overline{A}B(\overline{D} + CD) + B(A + \overline{A}CD)$

$= \overline{A}B((D + \overline{D})(\overline{D} + C)) + B((A + \overline{A})(A + CD))$

$= \overline{A}B\overline{D} + \overline{A}BC + AB + BCD$

$= B(A + \overline{A}\,\overline{D}) + \overline{A}BC + BCD$

$= B((A + \overline{A})(A + \overline{D})) + \overline{A}BC + BCD$

$= AB + B\overline{D} + \overline{A}BC + BCD$

$= B(A + \overline{A}C) + B(\overline{D} + CD)$

$= B((A + \overline{A})(A + C)) + B((D + \overline{D})(C + \overline{D}))$

$= AB + BC + BC + B\overline{D}$

$= AB + BC + B\overline{D}$

$= B(\overline{A}\,\overline{D} + CD + A + \overline{A}CD)$

$= B(CD(1 + \overline{A}) + A + \overline{A}\,\overline{D}) = B(CD + (A + \overline{A})(A + \overline{D}))$

$= B(CD + A + \overline{D})$

Q Find the dual and complement of the following Boolean expression.

Ans $f(x,y,z) = \bar{x}yz + \bar{x}y\bar{z} + x\bar{y}\bar{z} + x\bar{y}z$

Dual of $f(x,y,z)$

$$= (\bar{x}+y+z)(\bar{x}+y+\bar{z})(x+\bar{y}+\bar{z})(x+\bar{y}+z)$$

Complement of $f(x,y,z)$

$$\overline{F}(x,y,z) = \overline{\bar{x}yz + \bar{x}y\bar{z} + x\bar{y}\bar{z} + x\bar{y}z}$$

$$= (\overline{\bar{x}yz})(\overline{\bar{x}y\bar{z}})(\overline{x\bar{y}\bar{z}})(\overline{x\bar{y}z})$$

$$= (\bar{\bar{x}}+\bar{y}+\bar{z})(\bar{\bar{x}}+\bar{y}+\bar{\bar{z}})(\bar{x}+\bar{\bar{y}}+\bar{\bar{z}})(\bar{x}+\bar{\bar{y}}+\bar{z})$$

$$= (x+\bar{y}+\bar{z})(x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+y+\bar{z})$$

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Canonical and Standard forms :-

An Boolean expression can be represented into either of two standard forms 1. Sum - of - products form
2. Product - of - sums form.

Sum - of - Products (SOP)

When two or more products terms are summed by Boolean addition, the resulting expression is a Sum - of - Products (SOP)

eg. $AB + \bar{A}CD$

where A, B, C, D = are the variables.

→ A Boolean function can be expressed algebraically from a give truth table by forming a minterm for each combination of the variables that produces a 1 in the function and then taking the OR of all those term.

→ If in SOP expression, each product term contains all the variables of the function either in complemented or uncomplemented form, then the function is called Standard expression / Canonical expression.

Each product term is called minterm / standard product term.

eg $AB + \bar{A}BC$ → C is missing

Number of variables 3 (A, B and C)

So it is not a standard or canonical SOP form.

$ABC + \bar{A}BC$ → canonical SOP form

→ Therefore, an SOP expression can be implemented by AND-OR logic in which the outputs of a number of AND gates connect to the inputs of an OR gate.

→ An n variable function can have maximum 2^n minterms.

The minterms are denoted as $m_0, m_1, \ldots$

→ So it is also called Sum - of minterms.

Eg

$2^3 = 8$ combinations

A	B	C	Term	Designation	Function (F)
0	0	0	$\overline{A}\,\overline{B}\,\overline{C}$	m_0	0
0	0	1	$\overline{A}\,\overline{B}\,C$	m_1	0
0	1	0	$\overline{A}\,B\,\overline{C}$	m_2	1
0	1	1	$\overline{A}\,B\,C$	m_3	1
1	0	0	$A\,\overline{B}\,\overline{C}$	m_4	0
1	0	1	$A\,\overline{B}\,C$	m_5	1
1	1	0	$A\,B\,\overline{C}$	m_6	0
1	1	1	$A\,B\,C$	m_7	0

(variables (3): A, B, C)

Hence $f = \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C$

$= m_2 + m_3 + m_5 = \sum m(2, 3, 5)$

* A standard product term is equal to 1 for only one combination of variable values.
Let. $\overline{A}B\overline{C}$ is 1 when $\overline{A} = 1, B = 1, \overline{C} = 1$
Hence $A = 0, B = 1, C = 0$.

$\sum m$ represents the sum of all the minterms whose decimal codes are given in the parenthesis.

Converting product terms to standard SOP

→ A non standard SOP expression is converted to standard form by using the Boolean algebra rule $(A + \overline{A}) = 1$.

→ Multiply each non standard product term by a term made up of the sum of a missing variable and its complement. This results in two product terms.

Eg Convert the following Boolean expression into standard SOP form

$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$

Ans $A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$

4 variables A, B, C & D

$A\bar{B}C \rightarrow$ D is missing, so $A\bar{B}C(D+\bar{D}) = A\bar{B}CD + A\bar{B}C\bar{D}$

$\bar{A}\bar{B} \rightarrow$ C & D are missing $\bar{A}\bar{B}(C+\bar{C})(D+\bar{D})$

$= (\bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C})(D+\bar{D})$

$= \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}$

$AB\bar{C}D \rightarrow$ standard product term.

Standard / canonical SOP expression is

$f = A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$

Product - of - Sums (POS) form

When two or more sum terms are multiplied, the resulting expression is a product - of - sums (POS).

Eg. $(A+\bar{B})(C+B)$

→ Implementing a POS expression simply requires ANDing the outputs of two or more OR gates.

→ A sum term which contains each of the n variables in either complemented or uncomplemented form is called maxterm / standard sum term. ~~The SO POS expr~~

→ for n variable, there are 2^n maxterm.

→ The POS expression containing the sum terms having all of the variables is called standard POS expression / canonical POS ~~form~~ expression.

Eg $(A+B)(\bar{A}+B+\bar{C})$ → C is missing.

A, B and C are three variables.

So it is not a standard POS form.

$(A+B+C)(\bar{A}+B+\bar{C})$

all variables are present.

So, it is called standard / canonical POS form.

→ In the truth table, form a maxterm for each combination of variables that produces a 0 in the function, and then form the AND of all those maxterms.

→ Maxterms are often represented as M_0, M_1, M_2 - - - where the suffixes denote their decimal code.

Eg $f(A, B, C)$

A	B	C	Term	Designation	F (A, B, C)
0	0	0	$(A+B+C)$	M_0	0
0	0	1	$A+B+\overline{C}$	M_1	1
0	1	0	$A+\overline{B}+C$	M_2	0
0	1	1	$A+\overline{B}+\overline{C}$	M_3	0
1	0	0	$\overline{A}+B+C$	M_4	1
1	0	1	$\overline{A}+B+\overline{C}$	M_5	1
1	1	0	$\overline{A}+\overline{B}+C$	M_6	0
1	1	1	$\overline{A}+\overline{B}+\overline{C}$	M_7	0

$$F(A, B, C) = M_0 \, M_2 \, M_3 \, M_6 \, M_7$$
$$= \prod M(0, 2, 3, 6, 7)$$

where $\prod$ represents the product of all maxterms whose decimal value is given in the parenthesis.

★ A standard sum term is equal to 0 for only one combination of variable value.

Let $A+\overline{B}+\overline{C} = 0$ means $A = 0$, $\overline{B} = 0$ and $\overline{C} = 0$

Hence $A = 0$, $B = 1$, and $C = 1$.

Converting a sum term to standard POS form

→ A nonstandard POS expression is converted to standard form using Boolean algebra rule $A \cdot \overline{A} = 0$.

→ Add to each nonstandard product term a term made up of the product of the missing variable and its complement.

Then apply rule $A + BC = (A+B)(A+C)$.

Eg Convert the following Boolean expression into standard POS form.

$$(A+\overline{B})(\overline{A}+\overline{B}+\overline{C})(\overline{A}+B+C+\overline{D})$$

Ans 4 variables A, B, C and D

$A+\overline{B} \rightarrow$ C and D are missing.

Hence $(A+\overline{B}+C\overline{C}) = (A+\overline{B}+C)(A+\overline{B}+\overline{C})$

$$\downarrow \qquad \qquad \downarrow$$

$$(A+\overline{B}+C+D\overline{D}) \quad (A+\overline{B}+\overline{C}+D\overline{D})$$

$$= (A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D}) \quad = (A+\overline{B}+\overline{C}+D)(A+\overline{B}+\overline{C}+\overline{D})$$

$\overline{A}+\overline{B}+\overline{C} \rightarrow$ D is missing, so $(\overline{A}+\overline{B}+\overline{C}+D\overline{D}) = (\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+\overline{D})$

Hence, the standard/canonical POS expression is

$$f(A,B,C) = (\overline{A}+B+C+\overline{D})(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})$$
$$(A+\overline{B}+\overline{C}+D)(A+\overline{B}+\overline{C}+\overline{D})(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+\overline{D})$$

Conversion between canonical forms :-

Let a ~~product~~ sum of minterms expression is given

$$f = AB + \overline{A}C$$

It is a non standard SOP expression.

$\rightarrow$ Convert it into canonical form:

$$f = AB(C+\overline{C}) + \overline{A}C(B+\overline{B})$$
$$= ABC + AB\overline{C} + \overline{A}BC + A\overline{B}C$$

$\rightarrow$ Here, 3 variables, $2^3 = 8$ combination.

$$f = ABC + AB\overline{C} + \overline{A}BC + A\overline{B}C$$
$$= \underset{7}{111} \quad \underset{6}{110} \quad \underset{3}{011} \quad \underset{5}{101}$$ (corresponding binary ~~expre~~ values)

$$= \sum m(3,5,6,7)$$

The missing terms are 0, 1, 2, 4.

Hence POS form is $\prod M(0,1,2,4)$

Scanned with CamScanner

→ Same method is applied to convert POS → SOP.

Alternative method for converting SOP → POS & POS → SOP

SOP → POS

→ Take the complement of given SOP expression and expand using De-Morgan's theorem and simplify.

→ Take once again complement of the simplified expression to get POS expression.

Eg $f = A\overline{B} + \overline{A}B$

Step-1 $\overline{F} = \overline{A\overline{B} + \overline{A}B}$

$$= (\overline{A\overline{B}})(\overline{\overline{A}B})$$

$$= (\overline{A} + \overline{\overline{B}})(\overline{\overline{A}} + \overline{B}) = (\overline{A} + B)(A + \overline{B})$$

$$= \cancel{\overline{A}A}^{\,0} + \overline{A}\,\overline{B} + AB + \cancel{B\overline{B}}^{\,0}$$

$$= AB + \overline{A}\,\overline{B}$$

Step-2 $\overline{\overline{F}} = \overline{AB + \overline{A}\,\overline{B}}$

$$= (\overline{\overline{A}\,\overline{B}})(\overline{AB})$$

$$= (\overline{\overline{A}} + \overline{\overline{B}})(\overline{A} + \overline{B})$$

$$= (A + B)(\overline{A} + \overline{B})$$

Observe from the T.T

i/p A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

minterms are $\overline{A}B + A\overline{B}$

Maxterms are $(A+B)(\overline{A}+\overline{B})$

POS → SOP

→ Expand the POS expression.

→ Simplify using theorem of Boolean algebra.

Eq $f = (A+B)(\bar{A}+\bar{B})$

$= \cancel{A\bar{A}}^{0} + A\bar{B} + \bar{A}B + \cancel{B\bar{B}}^{0} = \bar{A}B + A\bar{B}$

Q Expand $A(A+\bar{B})(A+\bar{B}+C)$ to minterms and maxterms.

Sol This is POS form.

3 variables A, B and C

A → both B and C are absent.

$A + B\bar{B} = (A+B)(A+\bar{B})$

$A+B \rightarrow (A+B+C\bar{C}) = (A+B+C)(A+B+\bar{C})$

$A+\bar{B} \rightarrow (A+\bar{B}+C\bar{C}) = (A+\bar{B}+C)(A+\bar{B}+\bar{C})$

2nd term $(A+\bar{B}) \rightarrow C$ is missing

$(A+\bar{B}+C\bar{C}) = (A+\bar{B}+C)(A+\bar{B}+\bar{C})$

Hence the standard POS expression is

$$F(A,B,C) = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})$$
$$(A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+\bar{B}+C)$$

$$= \underset{\substack{000\\0}}{(A+B+C)}\ \underset{\substack{001\\1}}{(A+B+\bar{C})}\ \underset{\substack{010\\2}}{(A+\bar{B}+C)}\ \underset{\substack{011\\3}}{(A+\bar{B}+\bar{C})}$$

$$= \prod M(0,1,2,3)$$

The missing terms are 4, 5, 6, 7

Hence minterms are $\sum m(4,5,6,7)$

Q Expand $A + B\bar{C} + AB\bar{D} + ABCD$ to minterms and maxterms.

Sol minterms $= \sum m(4,5,8,9,10,11,12,13,14,15)$

maxterms $= \prod M(0,1,2,3,6,7)$