Design for Shear

This section covers the following topics.

- General Comments
- Limit State of Collapse for Shear

General Comments

Calculation of Shear Demand

The objective of design is to provide ultimate resistance for shear (V_{uR}) greater than the shear demand under ultimate loads (V_u). For simply supported prestressed beams, the maximum shear near the support is given by the beam theory. For continuous prestressed beams, a rigorous analysis can be done by the moment distribution method. Else, the shear coefficients in **Table 13** of **IS:456 - 2000** can be used under conditions of uniform cross-section of the beams, uniform loads and similar lengths of span.

Design of Stirrups

The design is done for the critical section. The critical section is defined in **Clause 22.6.2** of **IS:456 - 2000**. In general cases, the face of the support is considered as the critical section.

When the reaction at the support introduces compression at the end of the beam, the critical section can be selected at a distance effective depth from the face of the support.

The effective depth is selected as the greater of d_p or d_s.

d_p = depth of CGS from the extreme compression fiber

d_s = depth of centroid of non-prestressed steel.

Since the CGS is at a higher location near the support, the effective depth will be equal to d_s.

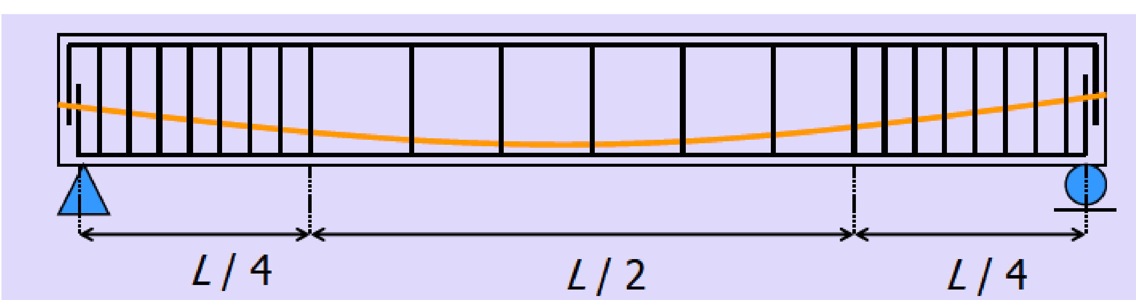
CGC = Centroid of concrete = Centroid of the gross section.

CGS = Centroid of prestressing steel = Centroid of the tendons.

To vary the spacing of stirrups along the span, other sections may be selected for design. Usually the following scheme is selected for beams under uniform load.

1) Close spacing for quarter of the span adjacent to the supports.
2) Wide spacing for half of the span at the middle.

For large beams, more variation of spacing may be selected. The following sketch shows the typical variation of spacing of stirrups. The span is represented by L.



Typical variation of spacing of stirrups

Limit State of Collapse for Shear

The shear is studied based on the capacity of a section which is the limit state of collapse. The capacity (or ultimate resistance) of a section (V_{uR}) consists of a concrete contribution (V_c) and the stirrup contribution (V_S).

$$V_{uR} = V_C + V_S$$

V_c includes V_{cz} (contribution from uncracked concrete), V_a (aggregate interlock) and V_d (dowel action). The value of V_c depends on whether the section is cracked due to flexure. **Section 22.4** of **IS:1343 - 1980** gives two expressions of V_c, one for cracked section and the other for uncracked section. Usually, the expression for the uncracked section will govern near the support. The expression for the cracked section will govern near the mid span. Of course, both the expressions need to be evaluated at a particular section. The lower value obtained from the two expressions is selected.

For uncracked sections,

$$V_c = V_{co}$$

$$V_c = 0.67bD\sqrt{f_t^2 + 0.8f_{cp}f_t}$$

V_{co} is the shear causing web shear cracking at CGC. In the above expression,

b = breadth of the section

= b_w, breadth of the web for flanged sections

D = total depth of the section (h)

f_t = tensile strength of concrete = $0.24\sqrt{f_{ck}}$

f_{cp} = compressive stress in concrete at CGC due to the prestress = P_e/A.

The value of f_{cp} is taken as positive (numeric value). Note that, a reduced effective prestress needs to be considered in the transmission length region of a pre-tensioned beam.

$$V_{c0} = \frac{Ib}{Q}\sqrt{f_t^2 + f_{cp}f_t}$$

$$\rightarrow 0.67bD\sqrt{f_t^2 + 0.8f_{cp}f_t}$$

To be conservative, only 80% of the prestressing force is considered in the term $0.8f_{cp}$. For a flanged section, when the CGC is in the flange, the intersection of web and flange is considered to be the critical location. The expression of V_{c0} is modified by substituting $0.8f_{cp}$ with 0.8 × (the stress in concrete at the level of the intersection of web and flange).

In presence of inclined tendons or vertical prestress, the vertical component of the prestressing force (V_p) can be added to V_{c0}.

$$V_c \rightarrow V_{c0} + V_p$$
$$= 0.67bD\sqrt{f_t^2 + 0.8f_{cp}f_t} + V_p$$

For **cracked** sections,

$$V_c = V_{cr}$$
$$V_c = \left(1 - 0.55\frac{f_{pe}}{f_{pk}}\right)\tau_c bd + M_0\frac{V_u}{M_u}$$
$$\geq 0.1bd\sqrt{f_{ck}}$$

V_{cr} is the shear corresponding to flexure shear cracking. The term $(1 – 0.55f_{pe}/f_{pk})\ \tau_c bd$ is the additional shear that changes a flexural crack to a flexure shear crack.

The notations in the previous equation are as follows.

f_{pe} = effective prestress in the tendon after all losses

$\leq 0.6f_{pk}$

f_{pk} = characteristic strength of prestressing steel

τ_c = ultimate shear stress capacity of concrete, obtained from **Table 6** of **IS:1343 - 1980**. It is given for values of A_p / bd, where d is the depth of CGS.

The values are plotted in the next figure.

b = breadth of the section

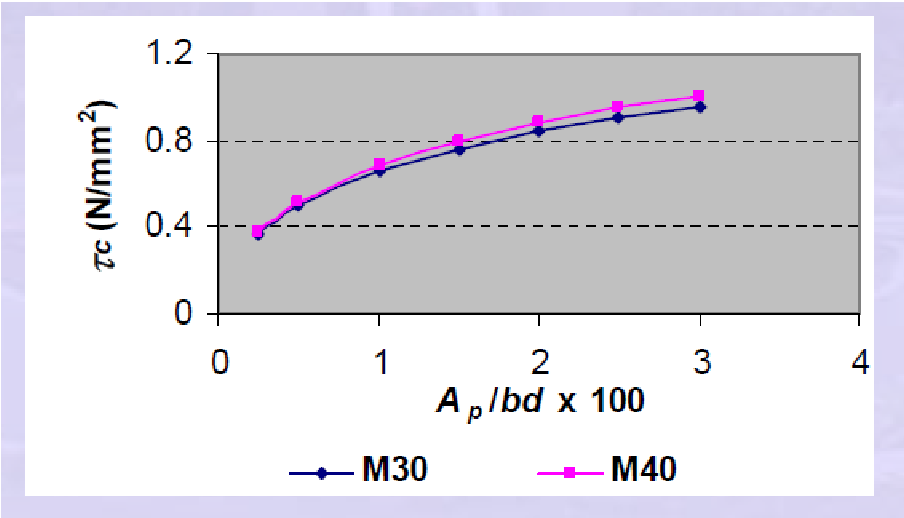
= b_w, breadth of the web for flanged sections

d = distance from the extreme compression fibre to the centroid of the tendons at the section considered

M_0 = moment initiating a flexural crack

M_u = moment due to ultimate loads at the design section

V_u = shear due to ultimate loads at the design section.



Variation of shear strength of concrete

The term $(M_0/M_u)V_u$ is the shear corresponding to the moment M_0, that decompresses (nullifies the effect of prestress) the tension face and initiates a flexural crack. The expression of M_0 is given below.

$$M_0 = 0.8 f_{pt} \frac{I}{y}$$

In the above expression,

f_{pt} = magnitude of the compressive stress in concrete at the level of CGS due to prestress only.

An equal amount of tensile stress is required to decompress the concrete at the level of CGS. The corresponding moment is $f_{pt} I / y$.

In the expression of M_0,

I = gross moment of inertia

y = depth of the CGS from CGC.

The factor 0.8 implies that M_0 is estimated to be 80% of the moment that decompresses the concrete at the level of CGS. Since the concrete is cracked and the inclination of tendon is small away from the supports, any vertical component of the prestressing force is not added to V_{cr}.

Maximum Permissible Shear Stress

To check the crushing of concrete in shear compression failure, the shear stress is limited to a maximum value ($\tau_{c,max}$). The value of $\tau_{c,max}$ depends on the grade of concrete and is given in **Table 7** of **IS:1343 - 1980**.

$$\frac{V_u}{bd_t} \le \tau_{c,max}$$

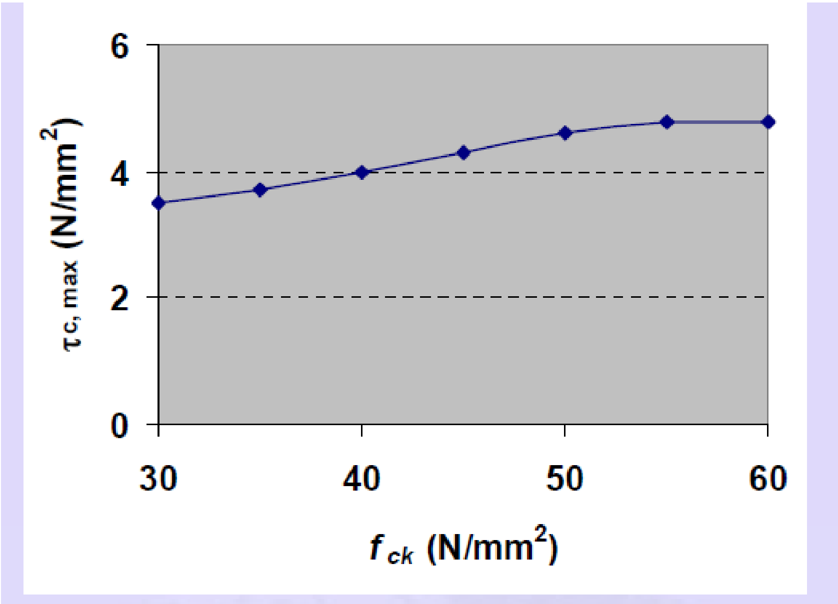
In the previous expression,

d_t = greater of d_p or d_s

d_p = depth of CGS from the extreme compression fiber

d_s = depth of centroid of regular steel

V_u = shear force at a section due to ultimate loads.



Variation of maximum permissible shear stress in concrete

Design of Transverse Reinforcement

When the shear demand (V_u) exceeds the shear capacity of concrete (V_c), transverse reinforcements in the form of stirrups are required. The stirrups resist the propagation of diagonal cracks, thus checking diagonal tension failure and shear tension failure.

The stirrups resist a failure due to shear by several ways. The functions of stirrups are listed below.

1) Stirrups resist part of the applied shear.
2) They restrict the growth of diagonal cracks.
3) The stirrups counteract widening of the diagonal cracks, thus maintaining aggregate interlock to a certain extent.
4) The splitting of concrete cover is restrained by the stirrups, by reducing dowel forces in the longitudinal bars.

After cracking, the beam is viewed as a plane truss. The top chord and the diagonals are made of concrete struts. The bottom chord and the verticals are made of steel reinforcement ties. Based on this truss analogy, for the ultimate limit state, the total area of the legs of the stirrups (A_{sv}) is given as follows.

$$\frac{A_{sv}}{s_v} = \frac{V_u - V_c}{0.87 f_y d_t}$$

The notations in the above equation are explained.

s_v = spacing of the stirrups

d_t = greater of d_p or d_s

d_p = depth of CGS from the extreme compression fiber

d_s = depth of centroid of non-prestressed steel

f_y = yield stress of the stirrups

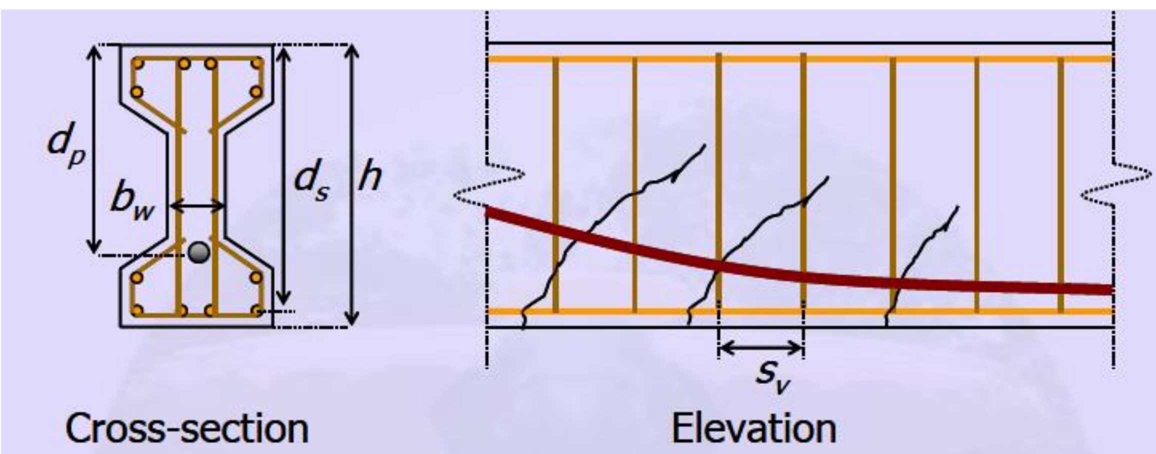
The grade of steel for stirrups should be restricted to Fe 415 or lower.

Detailing Requirements

The detailing requirements for the stirrups in **IS:1343 - 1980** are briefly mentioned.

Maximum Spacing of Stirrups

The spacing of stirrups (s_v) is restricted so that a diagonal crack is intercepted by at least one stirrup. This is explained by the following sketch.



Cross-section and elevation of a beam showing stirrups

As per **Clause 22.4.3.2**, the maximum spacing is $0.75d_t$ or $4b_w$, whichever is smaller. When V_u is larger than $1.8V_c$, the maximum spacing is $0.5d_t$.

The variables are as follows.

- b_w = breadth of web
- d_t = greater of d_p or d_s
- d_p = depth of CGS from the extreme compression fiber
- d_s = depth of centroid of non-prestressed steel
- V_u = shear force at a section due to ultimate loads
- V_c = shear capacity of concrete.

Minimum Amount of Stirrups

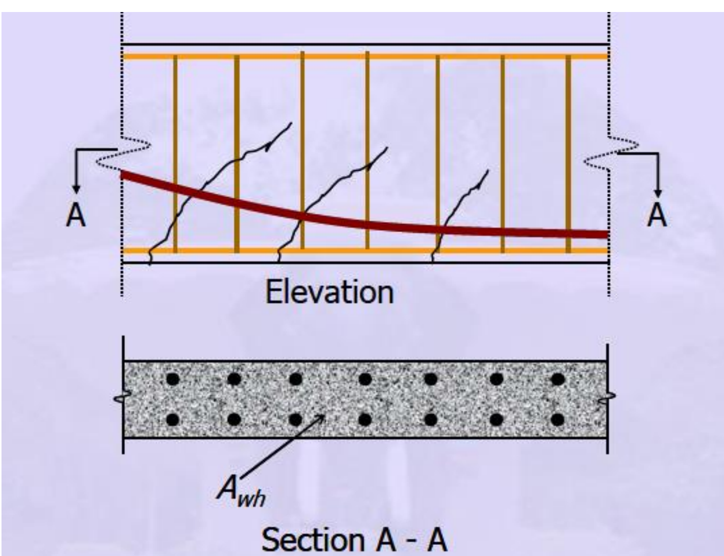
A minimum amount of stirrups is necessary to restrict the growth of diagonal cracks and subsequent shear failure. For $V_u < V_c$, minimum amount of transverse reinforcement is provided based on the following equation.

$$\frac{A_{sv}}{bs_v} = \frac{0.4}{0.87f_y}$$

b = breadth of the section = b_w, breadth of the web for flanged sections.

If $V_u < 0.5V_c$ and the member is of minor importance, stirrups may not be provided.

Another provision for minimum amount of stirrups ($A_{sv,min}$) is given by **Clause 18.6.3.2** for beams with thin webs. The minimum amount of stirrups is given in terms of A_{wh}, the horizontal sectional area of the web in plan. The area is shown in the following sketch.



Elevation and horizontal section of a beam showing stirrups

In presence of dynamic load,

$$A_{sv,min} = 0.3\%\ A_{wh}$$
$$= 0.2\%\ A_{wh}\text{ , when } h \le 4b_w$$

With high strength bars,

$$A_{sv,min} = 0.2\%\ A_{wh}$$
$$= 0.15\%\ A_{wh}\text{ , when } h \le 4b_w$$

In absence of dynamic load, when $h > 4b_w$

$$A_{sv,min} = 0.1\%\ A_{wh}$$

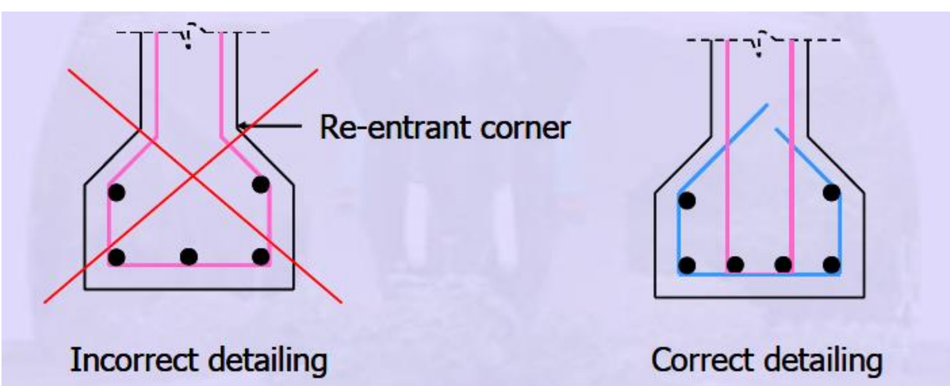
There is no specification for $A_{sv,min}$ when $h \le 4b_w$.

Anchorage of Stirrups

The stirrups should be anchored to develop the yield stress in the vertical legs.

1) The stirrups should be bent close to the compression and tension surfaces, satisfying the minimum cover.
2) Each bend of the stirrups should be around a longitudinal bar. The diameter of the longitudinal bar should not be less than the diameter of stirrups.
3) The ends of the stirrups should be anchored by standard hooks.
4) There should not be any bend in a re-entrant corner. In a re-entrant corner, the stirrup under tension has the possibility to straighten, thus breaking the cover concrete.

The following sketches explain the requirement of avoiding the bend of a stirrup at a re-entrant corner.



Cross-section of the bottom flange of a beam showing stirrups

Minimum Thickness (Breadth) of Web

To check web crushing failure, The **Indian Roads Congress Code IRC:18 - 2000** specifies a minimum thickness of the web for T-sections (**Clause 9.3.1.1**). The minimum thickness is 200 mm plus diameter of the duct hole.

Design Steps

The following quantities are known.

V_u = factored shear at ultimate loads. For gravity loads, this is calculated from V_{DL} and V_{LL}.

V_{DL} = shear due to dead load

V_{LL} = shear due to live load.

After a member is designed for flexure, the self-weight is known. It is included as dead load.

The grade of concrete is known from flexure design. The grade of steel for stirrups is selected before the design for shear. As per **IS:1343 - 1980**, the grade of steel is limited to Fe 415.

The following quantities are unknown.

V_c = shear carried by concrete

A_{sv} = total area of the legs of stirrups within a distance s_v

s_v = spacing of stirrups.

The steps for designing stirrups along the length of a beam are given below.

1) Calculate the shear demand V_u at the critical location.
2) Check $(V_u / bd_t) < \tau_{c,max}$. If it is not satisfied, increase the depth or breadth of the section. Here, b is the breadth of the web (b_w) and d_t is larger of d_p and d_s.
3) Calculate the shear capacity of concrete V_c from the lower of V_{co} and V_{cr}. In presence of inclined tendons or vertical prestress, the vertical component of the prestressing force (V_p) can be added to V_{c0}.
4) Calculate the requirement of shear reinforcement through A_{sv} / s_v. Compare the value with the minimum requirement.
5) Calculate the maximum spacing and round it off to a multiple of 5 mm.
6) Calculate the size and number of legs of the stirrups based on the amount required, type of section and space to accommodate.

Repeat the calculations for other locations of the beam, if the spacing of stirrups needs to be varied.